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PAPER

Absorption-based quantum communication with NV centres

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Keywords: quantum communication, NV center, absorption based

Abstract

We propose a scheme for performing an entanglement-swapping operation within a quantum communications hub (a Bell like measurement) using an NV-centre's $|\pm 1\rangle \leftrightarrow |A_2\rangle$ optical transition. This is based on the heralded absorption of a photon resonant with that transition. The quantum efficiency of a single photon absorption is low but can be improved by placing the NV centre inside a micro cavity to boost the interaction time and further by recycling the leaked photon back into the cavity after flipping its phase and/or polarization. Throughout this process, the NV is repeatedly monitored via a QND measurement that heralds whether or not the photon absorption has succeeded. Upon success we know a destructive Bell measurement has occurred between that photon and NV centre. Given low losses and high per-pass absorption probability, this scheme should allow the total success probability to approach unity. With long electron spin coherence times possible at low temperatures, this component could be useful within a memory-based quantum repeater or relay.

1. Introduction

Quantum communication [1–6] is a resource that will be required in the development of tomorrow's quantum internet [7] whether it be to share quantum enabled information over short, medium or long ranges. It will enable a multitude of tasks ranging from quantum key distribution (QKD) [4], device independent QKD [8] to distributed quantum computation and sensing [9, 12, 13]. Especially over long ranges (hundreds or thousands of kilometres) one will require shared entanglement to enable this, which will most likely require the use of quantum repeaters [10, 14–16]. These take the form of a chain of nodes able to perform two basic functions: the first being to store already established entangled links between nodes and the second to merge two links into a new, longer one. Many repeater schemes using various approaches to provide this functionality and various physical systems to implement the nodes of such a repeater have been put forward over the years [2, 3, 11, 15–29], including negatively charged nitrogen vacancy centre in diamond (NV-) [30, 31].

The NV-centre is made up of a vacant carbon position in a diamond lattice adjacent to a substitutional nitrogen atom [32–35] has many desirable properties [30, 36–42], including long coherence times [40] and good controllability of both electronic and nuclear spins at different temperatures [37, 40, 42–53]. The experimentally accessible states are the ground state manifold (GSM) and the excited state manifold (ESM), both of which are spin triplets but the former is composed of orbital angular momentum singlets (A_2 symmetry) while the states of latter form orbital doublets (E) under the action of the defect's symmetry group C_{3v} [35]. The two sets of states are separated by an optical transition with zero-phonon-line wavelength of 637 nm [41] while a set of optically inaccessible intermediate meta spin-singlet states can be reached from the ESM via non-radiative decay processes [35].

NV-centres have already been used as emitter of two different kinds of entangled photon qubits: polarization [54] and time-bin [55]. The former makes use of the special Bell-state like form of the four $m_s = \pm 1$ zero field ESM states. In particular the $|A_2\rangle$ state does not couple to the intermediate singlet states. Since in a dipole interaction the z -component of total the angular momentum must change by ± 1 , spontaneous emission out of

$|A_2\rangle$ creates a photon which is polarization-entangled with the two degenerate $m_S = \pm 1$ states of the GSM [41]. It has been recently demonstrated experimentally, that the time reversal also holds: absorption of a photon acts like a projection onto a joint photon-vacancy spin Bell-state [56]. In principle this could allow for the teleportation of quantum information from an incoming photon to any qubit the absorbing NV- is entangled with.

Our proposal is to use this feature of the NV centre as a core element of a quantum communications network. A node in the network can perform an entanglement swap operation upon heralded absorption of a photon carrying an entangling link to a remote NV. To resolve the ESM sub levels, and also to use the much longer spin coherence times, this scheme would operate at low temperature (4–8 K). The heralding (QND type) measurement is implemented by detecting the phonon-side band photons of a classical laser pulse tuned to the transition between the $m_S = 0$ state of GSM and ESM after a microwave π -pulse resonant with the $|A_2\rangle$ to $|E_{x|y}\rangle$ ($m_S = 0$) transition. To enhance interaction, the NV- would be placed inside a micro cavity but even in this case the single-pass absorption quantum efficiency would be limited to 25%. We can however increase this further by adding a fibre-optic loop to re-cycle leaked photons back into the cavity after potentially flipping its phase and/or polarization.

This paper is structured as follows: in section 2 we describe the main idea of the entanglement swapping scheme, detailing two possible approaches to increase quantum efficiency beyond 25%. We then proceed in section 3 to present a model of the scheme as a discrete-time quantum Markov process including estimations for the various parameters appearing. We show the results of numerical simulations using this model for different parameter regimes, allowing us to make quantitative statements about the expected performance of our scheme taking into account losses and errors. In section 4 we briefly discuss how this heralded Bell measurements can be used in quantum repeaters and relays. We summarize our findings in section 5.

2. Teleportation scheme

Here we will describe in detail the three main ingredients of our scheme, the entangling absorption operation, the QND measurement and photon recycling.

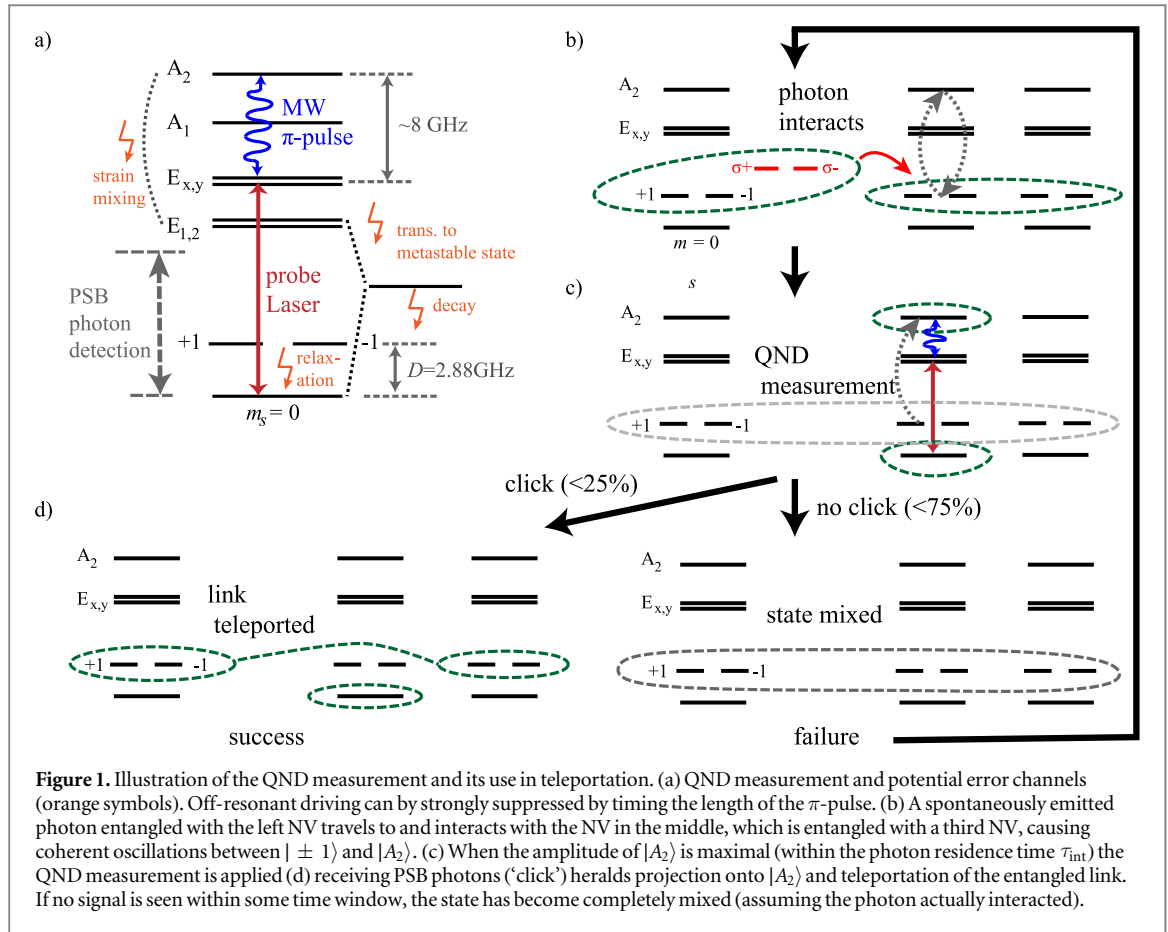
Entangling absorption. At zero magnetic field and zero strain the ESM A_2 state has the form $|A_2\rangle = (|E_+, -1\rangle + |E_-, +1\rangle)/\sqrt{2}$ where E_{\pm} and ± 1 denote the (collective) orbital angular momentum and spin of the NV⁻ vacancy respectively. This is a ψ_+ Bell state between the orbital and spin degree of freedoms. From this $|A_2\rangle$ state there are dipole-allowed optical transitions to the GSM $m_S = \pm 1$ states which due to angular momentum conservation are dependant on the photon polarization.

This property of the A_2 state was exploited in [54] to generate polarization-entangled photons by spontaneous emission out of the $|A_2\rangle$ state. However, this ‘entangling emission’, can also be time-reversed into an ‘entangling absorption’ of an incoming photon. As was demonstrated in a recent experiment absorption into $|A_2\rangle$ is equivalent to a Bell-measurement on the joint photon-vacancy electronic spin system of the form [56]

$$M_{\psi_+} = \frac{1}{2} \left(| +1, \sigma_- \rangle + | -1, \sigma_+ \rangle \right) \left(\langle +1, \sigma_- | + \langle -1, \sigma_+ | \right), \quad (1)$$

where the first quantum number refers to the vacancy spin and the second to the circular photon polarization states σ_{\pm} . This implies, that if the absorbing NV centre starts out maximally entangled with some other qubit, whatever information the photon was carrying will be teleported to that qubit. This is basic process which enables our proposed quantum repeaters and relays.

QND measurement. It is important to realize, however, that the teleportation does not happen until NV centre is detected in the $|A_2\rangle$, i.e. until the state is projected onto $|A_2\rangle$. Without this, the interaction with the photon will simply cause coherent flip-flops between the NV’s initial state (locally a completely mixed state of $m_S = \pm 1$) and $|A_2\rangle$ at a frequency determined by the interaction strength. We propose to implement this projective QND measurement via a microwave π -pulse resonant with the transition from $|A_2\rangle$ to the ESM $m_S = 0$ states $|E_{x|y}\rangle$ and simultaneous excitation with a probe laser beam tuned to the transition between the $m_S = 0$ states in the GSM and ESM (≈ 637 nm). Detection of phonon-side-band (PSB) photons would then herald, that the NV indeed was in $|A_2\rangle$, while seeing no signal can with high confidence be interpreted to mean the system is still in its initial state. A schematic of the QND measurement and how it is employed as part of the teleportation scheme is depicted in figure 1. In addition to the ideal process, figure 1(a) also shows possible error channels which limit the fidelity of the QND measurement, causing either false positive (off-resonant driving, relaxation) or indeterminate errors (strain mixing plus transition to dark state). The branching ratio between the number of photons emitted into the ZPL and into the PSB is of the order 1:19 and is largely temperature independent. This means the PSB photon detection is viable with preliminary experiments showing approximately 1000 photons per second detected for a driving laser power is 10 nW. This is far above the background rate even at room temperature. However due to the longer spin coherence times and lower

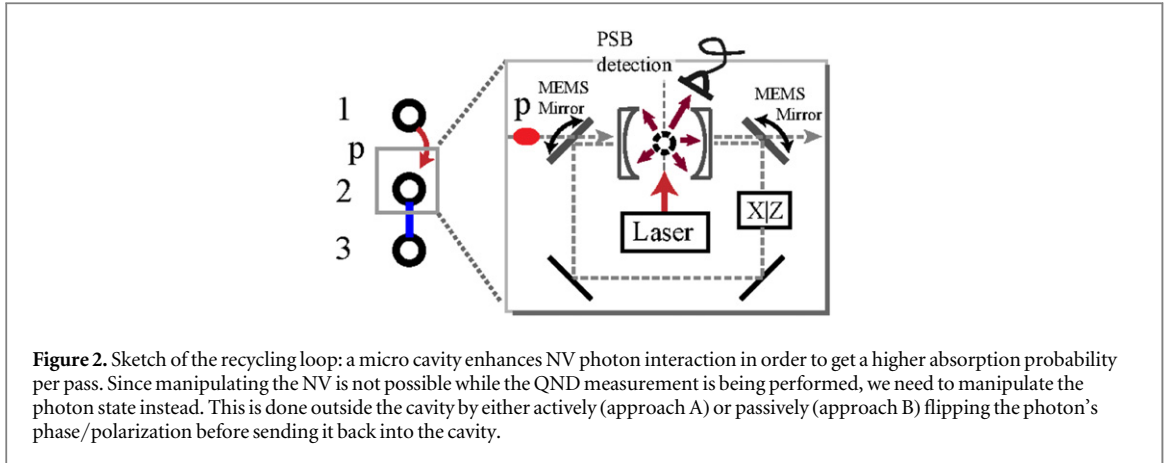


transition rates of the $|A_2\rangle$ state to the metastable states at low temperatures (4–8 K), we envisage operating our node this at these lower temperatures, which also increases the signal-to-noise ratio slightly. We should point out, that since the measurement is projective, the wave function changes not only in the desired case of seeing a heralding signal. Rather, also in the case where no signal is received a change has occurred, given the photon actually interacted, which can be described by the operator $1 - M_{\psi_{\pm}}$ with $M_{\psi_{\pm}}$ the entangling measurement operator from above. The application of the QND measurement π pulse would be timed to coincide with the time of maximum probability to find the NV^- in $|A_2\rangle$. However for a single free photon passage past the NV centre, the probability of absorption will be quite low. Therefore, we envision to place the NV^- inside a micro cavity which enhances both interaction strength and interaction time (residence time) between photon and NV^- . The cavity would have to be carefully designed to fulfil this purpose while at the same time allowing the photon to enter without being reflected.

A Mismatch problem and the recycling loop. Even with perfect absorption the incoming photon can only be absorbed with at most 25% probability of success, if no further measures are taken. This is easily understood by looking at the full initial state $|\psi_0\rangle = \frac{1}{2}(|+1, \sigma_{-}\rangle + |-1, \sigma_{+}\rangle)_{1p}(|+1, +1\rangle + |-1, -1\rangle)_{23}$, where '2' labels the NV the photon p interacts with while 1 and 3 are remote qubits, which in the following we will always assume to be other NV centres. Rewriting $|\psi_0\rangle$ in the Bell-basis between qubits 1 and 3 we have

$$|\psi_0\rangle = \frac{1}{2} \left(|\phi_{+}\rangle_{13} |\psi_{+}\rangle_{2p} + |\phi_{-}\rangle_{13} |\psi_{-}\rangle_{2p} + |\psi_{+}\rangle_{13} |\phi_{+}\rangle_{2p} + |\psi_{-}\rangle_{13} |\phi_{-}\rangle_{2p} \right). \quad (2)$$

where the $|\phi_{\pm}\rangle$ and $|\psi_{\pm}\rangle$ denote the typical even and odd parity Bell-states respectively. Only the $|\psi_{+}\rangle_{2p}$ -term allows a dipole transition to the $|A_2\rangle$ state of the 2nd NV centre, the dipole operator matrix elements $\langle x | \vec{E} \cdot \vec{r} | A_2 \rangle$ for the three other states ($x = \psi_{+}, \phi_{\pm}$) are zero. Their symmetry matches the other three $m_S = \pm 1$ states of the ESM, which are however detuned by at least about 3 GHz results in a relative transition probability ratio between p_{abs, A_1} and p_{abs, A_2} of at most 10^{-4} but likely to be less. If we want a high overall success probability we therefore need to somehow turn the other 2p Bell states into $|\psi_{-}\rangle_{2p}$ by applying some operation to either the spin or the photon. As illustrated in figure 2, our idea to raise the quantum efficiency closer to 1 is to let the photon interact with the NV centre inside a cavity with residence time τ_{int} determined by the cavity Q-factor, and then, via the QND measurement, determine whether the NV has transitioned into $|A_2\rangle$. If the heralding signal is seen the link is teleported with very high probability reduced only by the probability for a false positive,



which we estimate to be low (see appendix). If no heralding signal is seen within time τ_{int} we can assume, also with high probability, that no absorption occurred and the photon left the cavity after time τ_{int} . Outside the cavity, the photon is sent into a fibre-optic loop which serves route it back into the cavity. Furthermore the loop will contain an integrated (active or passive) switch allowing us to flip the photons's polarization, phase or both. if this 'recycling' process takes time τ_{out} , the total (average) time per cycle is $\tau = \tau_{\text{int}} + \tau_{\text{out}}$. We would repeat this procedure a predetermined number of times N , before finally abandoning the teleportation attempt. In the ideal case of perfect absorption per pass we need to re-cycle the photon only three times, corresponding to $N = 4$, to get maximal probability of success. However, in practice absorption cannot be perfect even with a cavity, and we need to recycle multiple times per polarization/phase setting.

2.1. Two approaches

Within the framework described above there are, at least, two ways of solving our mismatch problem: the technically simpler one, which we will call approach A, is to flip only either the phase or polarization of the photon on each cycle. After an even number $N = 2k$ of cycles without absorption we measure both the photon and the vacancy spin in the XX basis, in case we were applying phase flips or in the ZZ basis in case we were applying polarization flips. Measurement of the vacancy electronic spin is made more intricate by the fact that we are working in a degenerate subspace, but it can nonetheless be done with high fidelity as was demonstrated in [56]. Assuming we flip the phase, even parity outcomes ('++' or '--') correspond to a the Bell state $|\psi_+\rangle_{13}$ and odd parity ('+-' or '-+') to $|\psi_-\rangle_{13}$. As we will see later, while technically simpler to implement, this scheme requires high per-pass absorption p_{abs} to achieve good fidelities for the teleported link as well as highly efficient single photon detectors.

In another approach, dubbed B, both phase and polarization are flipped periodically after N_z and N_x rounds respectively. Choosing, e.g., $N = 2N_x = 4N_z$ gives each of the four possible Bell-states a (roughly) equal chance of being heralded by the QND measurement, making a final XX or ZZ measurement un-necessary. The advantages of approach B are that link fidelities are almost independent of p_{abs} and decrease with the number of cycles L only due to dephasing of the electronic spin, dark counts (false positive signals) and transitions to ESM states other than A_2 . Unlike approach A, it can thus be used in the low per-pass absorption probability regime but has the downsides of greater technical complexity as well as a lower total success probability p_{success} as well as, due to the i.g. larger values of N necessary, even more stringent requirements on the loss probability in the recycling loop.

3. Modelling

In this section we present a discrete time Quantum Markov model that we used to obtain quantitative results about the performance of our recycling-loop teleportation scheme when applied to a situation as shown in figure 2 taking into account real world imperfections. We restricted the description to the 32 dimensional effective Hilbert space spanned by $\{|\phi_{\pm}\rangle, |\psi_{\pm}\rangle\}_{13} \otimes \{|\phi_{\pm}\rangle, |\psi_{\pm}\rangle; |A_2\rangle, |A_1\rangle, |\pm 1\rangle\}_{2|p}$. Starting in the state $\rho_0 = |\psi_0\rangle\langle\psi_0|$ with ψ_0 as defined in the previous section we loop through the following steps N times:

- (1) Absorption: with probability $p_{\text{abs}} \langle\psi_+|\rho|\psi_+\rangle_{2p}$ the photon will be absorbed and the NV transitions to the A_2 state.

- (2) QND measurement: the NV is measured and with probability $p_{\text{click}} = p_{\text{QND}} \langle A_2 | \rho | A_2 \rangle_2 + p_{\text{Dark}}$ absorption is heralded, and with prob. $1 - p_{\text{click}}$ it is not. In either case the state ρ is updated accordingly.
- (3) Photon loss: with probability p_{loss} , the photon is lost during the recycling process.
- (4) Dephasing: a dephasing operation with $\eta_2 = \exp(-\tau^2/T_2^2)$ is applied to all the NV centres' electron spins (1–3) in the $(|+1\rangle \pm |-1\rangle)/\sqrt{2}$ basis where τ is the time per cycle.
- (5) Flipping: every N_z th (N_x th) loop a phase (polarization) flip is applied to the photon.

Since imperfections in the phase and polarization flip operations are not likely to play a major role, we did not include them in this model. Furthermore, as is readily apparent, the success probability depends very strongly on the photon actually arriving at and entering into the our cavity and recycling loop structure. In fact, the chance that the photon is lost are quite high for fibre bound communications over many kilometres. However, since here we are interested only in the performance of our scheme as a component of a network we do not take this initial loss probability into account.

3.1. Parameter estimation

We have five main parameters relevant to the performance during each cycle (p_{abs} , p_{QND} , p_{Dark} , p_{loss} and η_2) and, depending on the approach (A or B), one or three more: total number of rounds N (A and B), polarization-flip period N_x and phase-flip period N_z (only B).

For both approaches we have a fraction of false-negatives approximately $1 - p_{\text{QND}}$ of cases in which we discard a correctly teleported link (see appendix B), which is then erased when we try again. But since the QND measurement has high fidelity this is not likely to be a limiting factor.

This brings us to the question: what are realistic values for the parameters in the model? As we already stated, the QND measurement is likely to be high and the value of $p_{\text{QND}} = 99\%$ we used in our simulations is likely to be realistic. For p_{Dark} we use an equally conservative 0.01%. While a photon loss of 1 dB per element is usually regarded as good, here we cannot tolerate more than ≈ 1 dB for the total structure consisting of cavity, phase/polarization modulator and connecting optical fibre. The fibre loop itself does not contribute much, since even for a delay of $\tau_{\text{out}} = 10$ ns, we have a loop length of only 2 m and thus a fibre loss of only $\approx .03$ dB (at a wavelength of 700 nm standard telecommunication fibre has an attenuation of about 15 dB km^{-1}). As to the residence time τ_{int} , $\tau_{\text{int}} = 2$ ns is more than enough to allow the switching to occur (this could be done on a sub-nanosecond timescale). We investigated values from 0 to 0.5 dB but all explicit references to performance estimates are for the challenging but potentially achievable value of $p_{\text{loss}} = 0.3$ db. Furthermore, assuming a dephasing time of $T_2 = 100 \mu\text{s}$ and a very conservative total time per round of $\tau = 200$ ns we find that the dephasing error is about $1 - \eta_2 = 4 \times 10^{-6}$ and thus quite small.

This leaves open the probability of absorption (per pass) p_{abs} and the number of rounds N which in the next section we will treat as the variable of our analysis.

3.2. Analysis

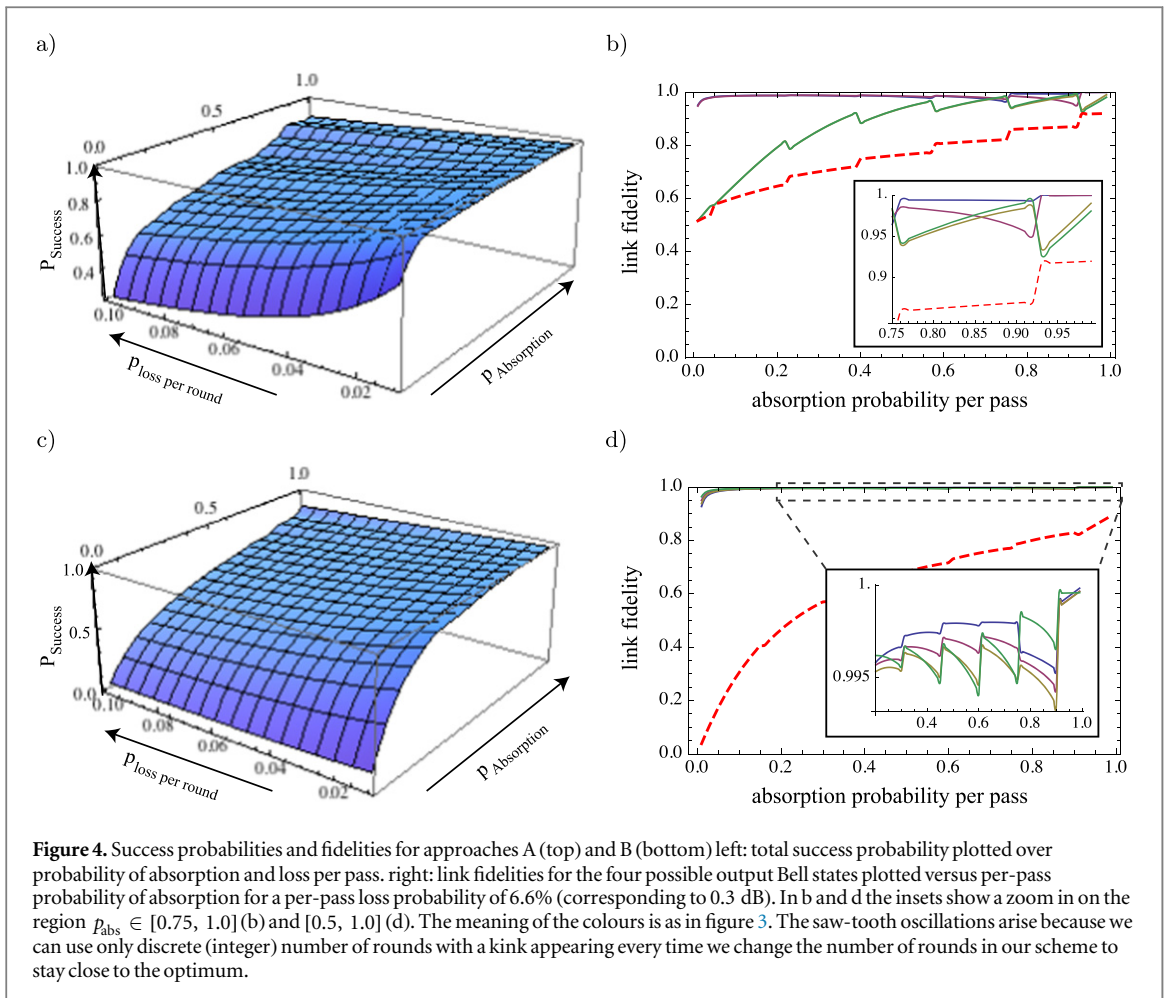
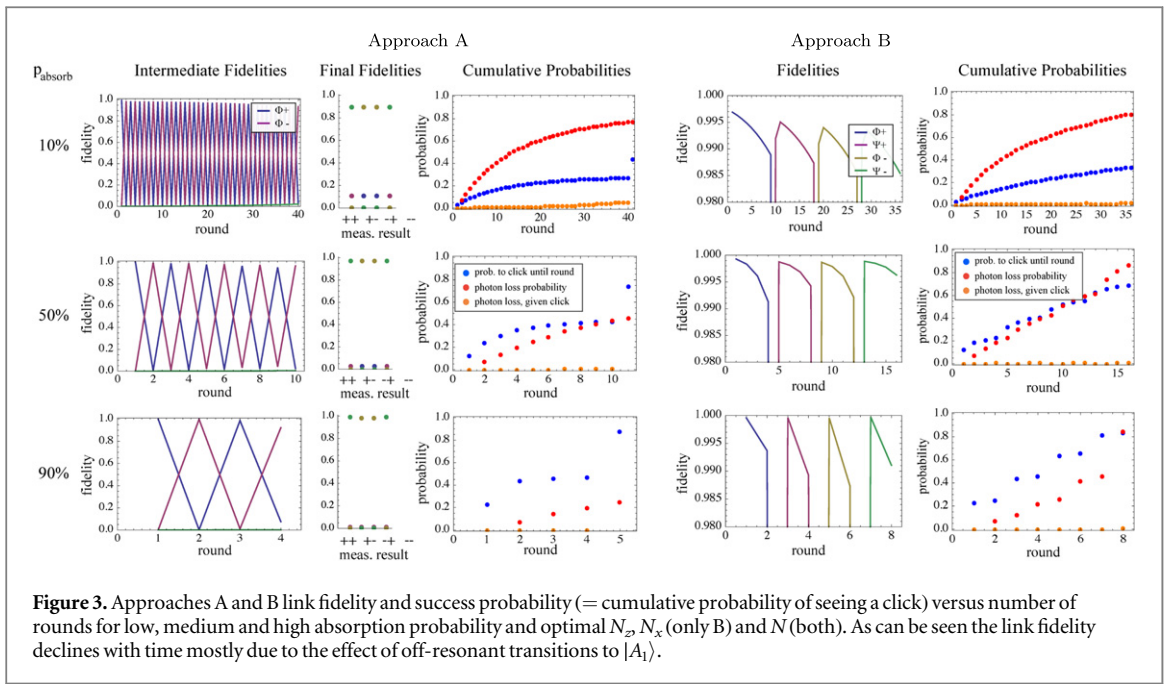
With parameters as determined in the previous section, we tested multiple combinations of $p_{\text{absorption}}$ and N as well as N_x and N_z (only approach B). In the latter case the best fidelities are naturally obtained when attempts are equally distributed over all four possible Bell states between NV centres 1 and 3.

The resulting success probabilities (cumulative probability to see a heralding signal) and average link fidelities for low, medium and high absorption probability and optimal N (and $N = 2N_x = 4N_z$ in case of approach B) are shown in figure 3 as a function of recycling round.

We find, perhaps unsurprisingly, that the probability of absorption is indeed of critical importance. However the simulations also show that there are clearly diminishing returns: the two figures of merit total success probability and average link fidelity increase much more from the low to medium p_{abs} -regime than from the medium to high regime. Consequently it seems advisable to try to increase p_{abs} to, if possible, at least around 50%, but it might not be worthwhile pushing far beyond this. At this point it is unclear, how far p_{abs} can be improved by use of a cavity, but should it prove difficult to reach or go much beyond 50% technical efforts should rather be focused on minimizing the per cycle losses p_{loss} .

We also investigated the susceptibility to photon loss by performing a scan of the success probability and link fidelity for $p_{\text{absorption}}$ between 1 and 99% and per-cycle loss p_{loss} between 1 and 10%. The results, shown in figure 4, show that while at least in approach B link fidelity does not strongly depend on p_{loss} , the overall success probability quickly deteriorates with increasing p_{loss} confirming that it is indeed paramount to limit photon loss as far as possible.

Figure 4 also reveals a crossover between the two approaches A and B. While for low per-pass absorption p_{abs} , B yields superior fidelities, this advantage diminishes as p_{abs} increases and almost disappears for $p_{\text{abs}} > 90$



%, while success probability as defined here is always higher for scheme A. Thus, given the final measurement can be implemented with high reliability, approach A has a performance advantage in the high p_{abs} regime, yielding an about 10% higher p_{success} at comparable link fidelity (for more detailed numbers we refer to the appendix).

4. Relays and repeaters

Our heralded entanglement swapping operation is an extremely useful tool for the creation of long range entangled links. As was depicted in figure 1, this tool can be used in a relay fashion to entangle a photon emitted from a remote NV centre (say at Charlie location) with an already entangled link between two other remote NV centres (at Bob and Alice respective locations). Upon a successful entanglement swapping operation, Alice and Charlie become entangled. David can then send a photon from his location to Charlie's location and the entanglement operation performed again. If it is successful, then Alice and David are entangled. In a relay fashion, longer range entanglement can be created. The probability of success for the creating this longer range links however will decrease exponentially with the numbers of nodes. However using an entangled polarization source of photons, two separate entangled links can be merged together (as is normally done in repeater networks). This in principle allows one to avoid this exponential issue. Further the entanglement swapping operations can be used to enable entanglement purification and so one has all the necessary elements for a quantum network.

5. Conclusion

We are proposing a new approach to performing the entanglement swapping operation in a repeater chain or network made up of nodes of NV^- centres in diamond which is absorption- rather than emission based and which in principle allows (near) deterministic operations. Whereas in the typical emission case, entanglement swapping is in principle bounded by a 50% maximum probability of success, our simulations of our absorption based scheme show that with this approach up to 90% probability of success seem possible, assuming challenging but potentially achievable technical improvements are made, foremost among them the ability to fabricate an optical loop structure capable of flipping polarization or phase on demand and showing low optical coupling losses. But even if a total probability of success of only somewhat above 50% is achieved, this would already be an important step, because if the probability of success is less than 50%, it means operations fail on average while more than 50% means they succeed on average. This has an enormous effect on how the resources scale with the total length of the repeater chain and consequently any scheme capable of surpassing this crucial threshold is a step closer to the goal of scalable quantum communication.

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Appendix A. Parameter estimation continued

The starting state (2) contains four products of pairwise Bell-states btw. 1 and 3 as well as 2 and p respectively. Only one, $|\psi_+\rangle_{2p}$, permits direct transition to $|A_2\rangle$, while the symmetry of the other three matches the other $m_S = \pm 1$ states in the ESM. Since these are split in energy from $|A_2\rangle$ and thus detuned from the incoming photon, these transitions are suppressed by a Lorentzian factor depending on detuning $\Delta\nu$ and the incoming photon's spectral width $\delta\nu$. The energetically closest state is $|A_1\rangle$ which is split by about 3 GHz. Assuming the incoming photon is spontaneously emitted by a remote NV, its lifetime is given by the $|A_2\rangle$ state's lifetime of approximately 10 ns. From this the spectral width is $\delta\nu = 1/(10\pi \text{ ns}) \approx 30 \text{ MHz}$. Thus the transition to the nearest detuned state is already suppressed by a factor $1/(1 + (\Delta\nu/\delta\nu)^2) \approx 10^{-4}$. Transitions to $|E_{1|2}\rangle$ are roughly one order of magnitude smaller still which is why we chose not to include them in our model.

A similar argument can be made to assess the reliability of our QND measurement scheme. Here, there are actually two questions: given the NV centre is in A_2 , how certain are we to detect this, and if the system is not in A_2 , but rather still in $m_S = \pm 1$ of the GSM, how likely are we to see a false positive signal? In our model, the former is included in the form of p_{QND} while the latter is part of what we dubbed dark-count probability p_{dark} . As

Table 1. Teleportation scheme errors: the false negative error causes the protocol to discard a teleported link, but even in the interesting regime $p_{\text{abs}} \gtrsim 0.5$ it is never greater than q_{QND}

p_{abs}	$N_{\text{opt.}}$	$P_{\text{false neg.}}^{\text{max}}/q_{\text{QND}}$	$P_{\text{false pos.}}^{\text{max}}/p_{\text{dark}}$
1%	40	0.282	27.5
10%	20	0.836	7.43
25%	20	0.969	2.98
50%	16	0.990	0.999
90%	4	0.9988	0.111

to the first case, the only way not to see a signal is that the system undergoes a transitions to either a dark state or another $m_S = \pm 1$ state. The first cannot occur for a perfect NV centre but the possibility grows quadratically with applied strain and electric as well as magnetic fields. All three influences can be reduced to the point where they are negligible: electric fields can be applied to cancel any remaining strain, external magnetic fields can be shielded and flip-flop processes with other spins in the sample, most notably nitrogen P1 centres, are suppressed by the energy splitting in the ESM. Transitions to another state could also be caused the driving field itself, in particular from the starting state in the $m_S = \pm 1$ subspace to $m_S = 0$. However, this can be avoided by choosing the Rabi power such that the Fourier transform of the driving field envelope is zero at this transition frequency (2.88 GHz). We therefore choose a conservative $p_{\text{QND}} = 0.99$ and $p_{\text{dark}} = 2 \times 10^{-4}$.

The dephasing times of the NV electronic spin at low temperature have been found to be as long as O(1ms) under the right conditions. In our model we assume a commonly seen value of $100\mu\text{s}$ and a total cycle time of $\tau = \tau_{\text{int}} + \tau_{\text{out}} = 200$ ns. The resulting dephasing error per NV involved is therefore $\gamma_2 = 1 - \exp[-(0.005)^2] \approx 4 \times 10^{-6}$. Note that we assume the remote qubits 1 and 3 to be NV centres too, therefore they can be expected to experience the same kind of dephasing. So the worst case dephasing error is then roughly $3\gamma_2$ or 1.2×10^{-5} per cycle.

The absorption probability is the hardest to model since it is very contingent on the sample and technical details of the implementation. Therefore we chose to use it as one of the ‘variable’ parameters, along with the number of rounds N (both approaches A and B) as well as the phase- and polarization flip periods N_x and N_z (only approach B).

Appendix B. Teleportation errors

Here we look at the ways the teleportation scheme can fail within our model and estimate the likelihood for the two types of error: false negatives and false positives.

B.1. False negative

In this error scenario we do not see a click heralding teleportation, even though the photon got absorbed (and was subsequently spontaneously re-emitted and then lost) and the entanglement it was carrying was in fact teleported to the remote NV-centre 3 (labels as in figure 2). This type of error will cause the whole scheme to fail, since in the absence of a heralding signal node NV2 will demand a new photon to be send from NV1, destroying the entangling link between 1 and 3. Using our discrete model but neglecting dark counts we find an upper bound for the probability of this event

$$\begin{aligned}
 P_{\text{false neg.}}^{\text{max}} &= p_{\text{abs}} q_{\text{QND}} \sum_{l=0}^{N-1} q_{\text{abs}}^l q_{\text{dark}}^l \\
 &= p_{\text{abs}} q_{\text{QND}} \frac{1 - (q_{\text{abs}} q_{\text{dark}})^N}{1 - q_{\text{abs}} q_{\text{dark}}}
 \end{aligned} \tag{B1}$$

with $q_{\text{QND}} \equiv 1 - p_{\text{QND}}$, $q_{\text{abs}} \equiv 1 - p_{\text{abs}}$ and $q_{\text{dark}} \equiv 1 - p_{\text{dark}}$. The real value depend on the state ρ_0 but taking this into account can only reduce p_{abs} and thus lead to a lower total error. Using the result from the previous section, appendix A, we can set $q_{\text{dark}} \approx 1$ for the range of N that are of interest. Thus (B1) simplifies to $P_{\text{false neg.}}^{\text{max}} \approx q_{\text{QND}} (1 - q_{\text{abs}}^N) \approx q_{\text{QND}}$. The latter approximation holds for the interesting regime $p_{\text{abs}} \gtrsim 0.5$. The results for the exact values of $P_{\text{false neg.}}$ for the different p_{abs} regimes investigated in the main text can be found in third column of table 1.

Table 2. Single relay performance (success probability and average link fidelities) of the two approaches for different absorption probabilities and choices of total number of rounds N . Photon-loss per cycle was assumed to be 0.3 dB or 6.6%.

$p_{\text{absorp.}}$	N	Approach A						Approach B					
		p_{success}	$F_{\phi_{\pm}}$	$F_{\psi_{\pm}}$	N_z	N_x	N	p_{success}	$F_{\phi_{+}}$	$F_{\phi_{-}}$	$F_{\psi_{+}}$	$F_{\psi_{-}}$	
10%	40	43.3%	0.97	0.89	9	18	36	33.0%	0.993	0.991	0.991	0.990	
30%	20	61.6%	0.975	0.97	6	12	24	56.9%	0.995	0.996	0.995	0.996	
50%	10	73.8%	0.98	0.97	4	8	16	68.3%	0.996	0.996	0.996	0.997	
70%	6	81.7%	0.975	0.97	3	6	12	76.0%	0.996	0.996	0.997	0.998	
90%	4	86.9%	0.985	0.99	2	4	8	82.8%	0.997	0.994	0.993	0.995	

B.2. False positive

A false positive error occurs when we see a click that was not actually caused by a correctly detected absorption event but a dark count. The latter is a detection event independent of anything else. The exact value again depends on the state ρ but we can find an upper bound in a similar manner as for false negatives:

$$\begin{aligned}
 P_{\text{false pos.}}^{\max} &= p_{\text{dark}} q_{\text{abs}} \sum_{l=0}^{N-1} q_{\text{abs}}^l q_{\text{dark}}^l \\
 &= p_{\text{dark}} q_{\text{abs}} \frac{1 - (q_{\text{abs}} q_{\text{dark}})^N}{1 - q_{\text{abs}} q_{\text{dark}}}.
 \end{aligned} \tag{B2}$$

Making the approximation $q_{\text{dark}} \approx 1$ and $q_{\text{abs}}^N \ll 1$, this simplifies to $P_{\text{false neg.}}^{\max} \approx q_{\text{abs}} p_{\text{dark}} / p_{\text{abs}}$. Thus in the interesting absorption regime $p_{\text{abs}} \gtrsim 0.5$ the overall influence of dark counts is in fact suppressed by the factor $q_{\text{abs}} / p_{\text{abs}} < 1$. The exact values are given in the fourth column of table 1.

Appendix C. Detailed results

Running simulations varying the number of rounds N and the absorption probability per pas p_{abs} we found the success probabilities and fidelities given in table 2.

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