Theoretical Approaches to One-Shot Games and Their Applications

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Chapter 1

Introduction

Equilibrium, which is the central concept in game theory, is an evolutionary solution which is reached after players have corrected their biased beliefs and adjusted their strategies. Basically, equilibrium models require three components: strategic thinking (formation of beliefs about others' behaviors); optimization (choosing a best response to those beliefs); mutual consistency (adjustment of beliefs and best responses until they are mutually consistent) (Camerer et al., 2004). However, it is often the case that solutions of equilibrium models are not supported by data in lab or the phenomena in the real world. For example, in the Beauty Contest Game, seldom players choose the unique Nash Equilibrium (Camerer, 2003); in the Prisoners' Dilemma, players cooperate about half the time (Sally, 1995); in the Ultimatum Game, low offers are frequently rejected (Camerer & Richard, 1995). Such abnormalities in players' behaviors initiate the exploration of Behavioral Game Theory.

Roughly speaking, development in Behavioral Game Theory has been made in three directions. Firstly, social force and cognitive mechanism are introduced into game-theoretical models. For example, the concepts such as reciprocity (e.g., Fehr & Gachter, 2000; Song, 2008; Neilson, 2009; Regner, 2014), betrayal aversion (Bohnet et al., 2008) and guilty aversion (Beck et al., 2013) are utilized and the incentive mechanism in decision-making is taken into account (Post et al., 2008). Secondly, learning models in games are proposed (Schlag, 1998; Camerer & Ho, 1998, 1999; Camerer et al., 2002). Thirdly, as the gaming situation becomes more complex than individual decision making problems and deviations from the sharp predictions are more likely to happen in reality, the Behavioral Game Theory also utilizes a non-equilibrium approach to model players' behaviors. Several non-equilibrium approaches have been proposed in literature. For example, Nagel (1995), Stahl and Wilson (1995) propose the level-k (LK) model, in which players are divided into different levels corresponding to different depths of reasoning. Following the level-k model, Camerer, Ho and Chong (2004) propose the Cognitive Hierarchy (CH) model to handle the degeneracy of beliefs in LK models, and Goeree and Holt (2004) propose the Noisy Introspection model to consider the reasoning process reversely, i.e. from a sufficiently large order of belief. In summary, these approaches share the same idea of weakening the mutual consistency assumption, in other words, players are assumed to play according to some beliefs at first and learn to play the equilibrium strategies.

The other key common argument of the above three non-equilibrium model is that players are assumed to think 'k' step. However, this raise the question that which step an involved player should stop at. It is rather untraceable and arbitrary. A natural thinking is that a player doesn't take the iterative thinking at all, or once a player started iterative thinking, he/she will continue this process until equilibrium is reached. So we argue the 'k' step thinking way is relatively far from real players' decision making process in a game.

In this research, we propose a new theoretical model – the **One-Shot Game Model** to character real player's decision making process as well as handle deviations in several typical games. Simply speaking, in our model, the decision process of a player consists of two steps. Firstly, a player formulates his/her belief about his/her opponents' actions. Secondly, based on the formulated belief, a player decides his/her optimal choice. It is a non-equilibrium approach because it is not necessary that each player should correctly predict his/her opponents' actions. We share the same idea with the existing non-equilibrium approaches that the mutual consistency assumption is relaxed, but we distinguish our approach in mainly the following two aspects.

Firstly, we enrich the methods of belief formulation. Speaking in detail, we propose several alternatives possibly reflecting real players' thinking process when formulating his/her belief. For example, a player may deduce that his/her opponent should adopt an action which generates a higher average payoff with a higher probability; also, we introduce other alternatives for belief formulation and show how they work in specific games.

Secondly, in solving each player's decision problem, we distinguish us in by seeking a way to capture the mental procedure of a player's decision making process. There are two steps: In the first step, for his/her each possible action, say a, the player evaluates every action chosen by the other players, say b, with considering the likelihood degree of b and the satisfaction level generated by a and b; amongst all possible actions chosen by the other players, the player chooses one which is called as the focus point of the action a (an imagined scenario corresponding to a). In the second step, the player evaluates each action based on its focus point (imagined scenario) and determines the optimal one which can generate the highest satisfaction level when its focus point (imagined scenario) occurs. The reason why the player thinks in this way is that one and only one action (scenario) will be chosen by his/her opponent even if he/she knows the probability distribution of all possible actions. This is the One-Shot Decision Theory based thinking.

The One-Shot Decision Theory is initially proposed by Guo (2011) which argues that a person makes a decision under uncertainty based on his/her imagined scenario which is most consistent with his/her personality. Different from the existing Expected Utility Theory, the One-Shot Decision Theory argues that a decision maker focuses on a specific state in the future rather than takes all possible states into consideration (Guo, 2014). In order to understand the core argument, let us examine several well-known examples. The Expected Utility Theory tells us that the reason why a person buys a lottery is that his/her utility function is convex while the reason why a person buys insurance is that his/her utility is concave. The One-Shot Decision Theory manifests that the lottery buyer is a decision maker who takes into account the scenario which has a low probability to happen but can bring about a large benefit and the insurance buyer is a decision maker who takes care of the scenario which can cause a large loss even with a low probability. Clearly, the One-Shot Decision Theory based explanation is intuitively acceptable. The One-Shot Decision Theory has been utilized for analyzing a duopoly market of a new product with a short life cycle (Guo, 2010a, Guo et al., 2010), private real estate investment problems (Guo, 2010b), newsvendor problems for innovative products (Guo & Ma, 2014) and multistage one-shot decision making problems (Guo & Li, 2014).

In **Chapter 2**, we make a detailed description of the One-Shot Game Model which divides a player's decision process into the following two steps. In the first step, an involved player formulates his/her belief: different kinds of belief formulation criteria are introduced, based on which players may formulate different beliefs. In the second step, based on the formulated belief, a player evaluates his/her every possible actions: instead of considering all possible actions possibly chosen by his/her opponent, only one of them is focused (called the focus point); a player chooses an action which yields the highest satisfaction associating with its focus point. We introduce two kinds of players, namely the active player and the passive player, an active player focuses on a state yielding high satisfaction with a high probability and a passive player focuses on a state yielding low satisfaction with a high probability. Interestingly, in some simple games, players' choices are rather robust to the formulated beliefs: players' choices are only whether he/she is an active or a passive decision maker.

In **Chapter 3**, we utilize the proposed One-Shot Game Model to examine a typical static game with complete information - the Capacity Allocation Game (Guo & Wang, 2017). In a supply chain, the capacity shortage refers to the situation that retailers' demand is higher than the supplier's inventory, as modifications are infeasible in a short term, the supplier has to divide the limited inventory to each retailer. A classical method is to divide the limited capacity to each retailer proportional to his/her order quantity. In this game, the unique Nash equilibrium is that each retailer submits an infinite order quantity, which is far from experimental findings as well as people's intuition. What is worse, the equilibrium stays the same even if we change the shape of the utility function and parameters of the model. Instead, we utilize the proposed One-Shot Game Model to analyze such a game and analytical results are consistent with the experimental findings.

In **Chapter 4**, we further investigate a static game with incomplete information - First Price Sealed Bid auctions by utilizing the One-Shot Game Model (Wang & Guo, 2015). In the auction model, each bidder is thought to face a decision problem under uncertainty which origins from other bidders' bidding prices. The action set is his/her bidding price and the uncertain factor is his/her opponents' bidding prices. Taking into account the one-time feature of auctions, it is intuitively acceptable that a bidder only imagines one bidding price presented by his/her opponents amongst all possible prices for determining his/her auction price rather than take any kind of average of all possible prices. Analytical results obtained within our framework well explain the deviations observed in first price sealed bid auctions: bidders tend to bid randomly upon drawing a low value and tend to overbid when their valuation is relatively high.

In **Chapter 5,** we make a conclusion of the work mentioned above as well as consider the future path of the proposed framework. Two kinds of games are still unsolvable within our framework: One is the Prisoner's Dilemma, in which dominated strategies are observed about half the time; the other is dynamic games, solutions of which are obtained by backward induction but sometimes are inconsistent to the experimental findings. We potentially improve our approach to handle games in more general horizon and we leave it for the future work.

Chapter 2

One-Shot Game Model

2.1 Belief Formulation

In this section, we describe the belief formulation process of a player. For simplicity, we confine us to a game with two players and investigate how player 1 formulates his/her belief about player 2's actions. Extension to multi-player case is trivial and we omit it to save space.

In a game with complete information, player 1 knows both player 2's actions and player 2's payoffs generated by both players' decisions. Let us make the following notations:

A : Player 1's action set, with $a_i (i = 1,..., n)$ a specific element in it;

B : Player 2's action set, with $b_j (j = 1,..., m)$ a specific element in it;

$v_k(a_i, b_j)$: Player k's $(k = 1,2)$ payoff.

Based on the above definition, we give the following three methods for **Belief Formulation**.

1. Belief Formulation based on the minimum payoff of an action

In this method, player 1 deduces that the larger the minimum payoff an action can generate, the more possible for it to be chosen by player 2. The belief can be written as follows:

$$p_1 : B \to [0,1] \qquad p_1(b_j) = \frac{\exp \min_{a_i \in A} \{ u_2(a_i, b_j) \}}{\sum_j \exp \min_{a_i \in A} \{ u_2(a_i, b_j) \}} \tag{2.1}$$

For example, let us consider the following 2×2 game, the row player is player 1 and the column player is player 2. The payoff matrix is given in Table 1.

Table 1

	b_1	b_2
a_1	(0,5)	(5,4)
a_2	(2,-1)	(3,1)

For player 2, by choosing b_1, his/her minimum payoff is -1; by choosing b_2, his/her minimum payoff is 1. Utilizing (2.1), we can obtain player 1's belief about player 2's actions is as follows:

$$p_1(b_1) = \frac{\exp(-1)}{\exp(-1) + \exp(1)} = 0.12 \tag{2.2}$$

$$p_1(b_2) = \frac{\exp(1)}{\exp(-1) + \exp(1)} = 0.88 \tag{2.3}$$

Comment

We utilize '$\exp$' here to transform the payoff into a strictly positive number. In the above example, as the minimum payoff is a negative number and a positive number respectively, it is difficult to directly handle it without transformation.

2. Belief Formulation based on the maximum payoff of an action

In this method, player 1 deduces that the larger the maximum payoff an action can generate, the more possible for it to be chosen by player 2. The belief can be written as follows:

$$p_1 : B \to [0,1] \qquad p_1(b_j) = \frac{\exp \max_{a_i \in A}\{u_2(a_i, b_j)\}}{\sum_j \exp \max_{a_i \in A}\{u_2(a_i, b_j)\}} \tag{2.4}$$

Let us also consider the 2×2 game given in Table 1. For player 2, by choosing b_1, his/her maximum payoff is 5; by choosing b_2, his/her maximum payoff is 4. Utilizing (2.4), we can obtain player 1's belief about player 2's actions is as follows:

$$p_1(b_1) = \frac{\exp(5)}{\exp(5) + \exp(4)} = 0.73 \tag{2.5}$$

$$p_1(b_1) = \frac{\exp(4)}{\exp(5) + \exp(4)} = 0.27 \tag{2.6}$$

3. Belief Formulation based on the average payoff of an action

In this method, player 1 deduces that the larger the average payoff an action can generate, the more possible for it to be chosen by player 2. The belief can be written as follows:

$$p_1 : B \to [0,1] \qquad p_1(b_j) = \frac{\exp\{\frac{1}{m}\sum_{a_i \in A} u_2(a_i, b_j)\}}{\sum_j \exp\{\frac{1}{m}\sum_{a_i \in A} u_2(a_i, b_j)\}} \tag{2.7}$$

Let us also consider the 2×2 game given in Table 1. For player 2, by choosing b_1, his/her average payoff is 2; by choosing b_2, his/her average payoff is 2.5. Utilizing (2.7), we can obtain player 1's belief about player 2's actions is as follows:

$$p_1(b_1) = \frac{\exp(2)}{\exp(2) + \exp(2.5)} = 0.38 \tag{2.8}$$

$$p_1(b_2) = \frac{\exp(2.5)}{\exp(2) + \exp(2.5)} = 0.62 \tag{2.9}$$

So far, we have discussed several belief formulation methods, it should be mentioned that the above methods can be directly extended to the multi-player case, however, it is difficult to extend the above methods to the continuous case, i.e., player's action set is continuous, for the continuous case, we will discuss it in the following two chapters combining specific games. In the following, we will introduce the One-Shot Decision Theory to character player's decision making process.

2.2 One-Shot Decision Theory

Following the above setting, let us consider player 1's decision process. For preparation, we make the following normalizations.

Firstly, we normalize the probability distribution function $p_1(b_j)$ as the following relative likelihood function $p_1(b_j)$:

$$\pi_1(b_j) = \frac{p_1(b_j)}{\max_{b_j} p_1(b_j)} \tag{2.10}$$

Secondly, we normalize the payoff function $v_1(a_i, b_j)$ as the following satisfaction function $u_1(a_i, b_j)$:

$$u_1(a_i, b_j) = \frac{v_1(a_i, b_j) - \min_{a_i, b_j} v_1(a_i, b_j)}{\max_{a_i, b_j} v_1(a_i, b_j) - \min_{a_i, b_j} v_1(a_i, b_j)} \tag{2.11}$$

The decision process of player 1 consists of the following two steps:

Step 1. Determine the focus point

In this step, for each action $a_i \in A$, player 1 focuses on one b_j, the reason is that although player 2 has many alternatives, for player 1, only one of them will be realized. Instead of taking an average, player 1 focuses on only one b_j and we call such a b_j the focus point of a_i. Player 1 of different personalities may have different focus points, generally, we suggest the following two criteria for choosing the focus point.

Criterion 1: $b_j^1(a_i) = \arg\max_{b_j} \min(u_1(a_i, b_j), \pi_1(b_j))$ (2.12)

Player 1 using this criterion focuses on the situation with a higher relatively likelihood level and a higher satisfaction level, and we refer this kind of player 1 as an active decision maker.

Criterion 2: $b_j^2(a_i) = \arg\min_{b_j} \max(u_1(a_i, b_j), 1 - \pi_1(b_j))$ (2.13)

Player 1 using this criterion focuses on the situation with a higher relatively likelihood level and a lower satisfaction level, and we refer this kind of player 1 as a passive decision maker.

Step 2. Obtain the optimal action

After determining the focus point, player 1 evaluates his/her each action under its corresponding focus point. The optimal actions under the proposed 2 criteria are defined as follows respectively:

Optimal action under criterion 1:

$$a^{1,*} = \arg\max_{a_i \in A} u_1(a_i, b_j^1(a_i)) \quad (2.14)$$

Optimal action under criterion 2:

$$a^{2,*} = \arg\max_{a_i \in A} u_1(a_i, b_j^2(a_i)) \quad (2.15)$$

Example.

We utilize the example given in Table 1 to illustrate player 1's decision process. Assume player 1 formulates his/her belief based on the maximum payoff of player 2, that is, for player 1, the probability distribution of player 2 choosing b_1 and b_2 is given in Table 2.

Table 2

	b_1	b_2
$p_1(b_j)$	0.73	0.27

Based on the probability distribution and (2.10), we can obtain the relative likelihood function in Table 3.

Table 3

	b_1	b_2
$\pi_1(b_j)$	1	0.37

Based on Table 1 and (2.11), player 1's satisfaction is given in Table 4.

Table 4

	b_1	b_2
a_1	0	1
a_2	0.4	0.6

Assume player 1 is an active decision maker, now we can obtain player 1's focus point of a_1 in Table 5.

Table 5

	b_1	b_2
$\pi_1(b_j)$	1	0.37
$u_1(a_1,b_j)$	0	1
$\min(u_1(a_1,b_j),\pi_1(b_j))$	0	0.37

Based on Table 5, we know the focus point of a_1 is b_2 and by choosing a_1, the satisfaction of player 1 is $u_1(a_1,b_2)=1$.

Similarly, we can obtain player 1's focus point of a_2 in Table 6.

Table 6

	b_1	b_2
$\pi_1(b_j)$	1	0.37
$u_1(a_2,b_j)$	0.4	0.6
$\min(u_1(a_2,b_j),\pi_1(b_j))$	0.4	0.37

Based on Table 6, we know the focus point of a_2 is b_1 and by choosing a_2, the satisfaction of player 1 is $u_1(a_2,b_1)=0.4$.

Based on the above analysis, we know for an active player 1, choosing a_1 yields a satisfaction of 1 while choosing a_2 yields a satisfaction of 0.4, so for an active player 1, a_1 is his/her optimal choice. Readers can also verify that a passive player 1's optimal choice is a_2.

2.3 Belief Independent Decision

In this section, we further consider several simple games within our framework, however, in those games, the involved players' choices are only determined by whether the player is an active decision maker or a passive decision maker, which is to say, players' actions are independent of their initial beliefs.

1 The Stag Hunt Game

In game theory, the stag hunt game describes a situation as follows:

Two hunters are going out on a hunt. They must decide whether to hunt a stag or a hare without knowing the choice of the other. The cooperation is necessary in succeeding in hunting a stag while each hunter can succeed in hunting a less valuable hare individually. The payoff of such a 2×2 game is given in Table 7

Table 7

	S	H
S	(3,3)	(0,2)
H	(2,0)	(2,2)

We will show that the following result holds within our framework.

Theorem 1 Whatever initial belief an active player holds, he/she will choose S; in contrast, whatever initial belief a passive player holds, he/she will choose H.

Proof. Assume the belief of an active player is $p^{act}(x)$ $x \in \{S, H\}$, after the normalization procedure, the relatively likelihood function is then given as $\pi^{act}(x)$, for an active player, his/her payoff matrix is given in Table 8.

Table 8

	S	H
S	3	0
H	2	2

So the satisfaction matrix is then given in Table 9.

Table 9

	S	H
S	1	0
H	0.67	0.67

Let us then examine the active focus point of S.

It is easily to verify the following equalities:

$$\min(u(S,H),\pi^{act}(H)) \le u(S,H) = 0 \tag{2.16}$$

$$\min(u(S,S),\pi^{act}(S)) = \pi^{act}(S) > 0 \tag{2.17}$$

So the active focus point of S is given as follows:

$$x^{act}(S) = \arg\max_{x\in\{S,H\}} \min(u(S,x),\pi^{act}(x)) = S \tag{2.18}$$

So for an active player, by choosing S, his/her (imagined) satisfaction is:

$$u(S,x^{act}(S)) = u(S,S) = 1 \tag{2.19}$$

However, by choosing H, his/her satisfaction is 0.67.

Based on the above analysis, for an active player, he/she chooses S independent of the initial beliefs.

Assume the belief of a passive player is $p^{pas}(x)$, $x \in \{S,H\}$, after the normalization procedure, the relatively likelihood function is then given as $\pi^{pas}(x)$, let us examine the passive focus point of S.

It is easily to verify the following equalities:

$$\max(u(S,S),1-\pi^{pas}(S)) = u(S,S) = 1 \quad (2.20)$$

$$\max(u(S,H),1-\pi^{pas}(H)) = 1-\pi^{pas}(H) < 1 \quad (2.21)$$

So the passive focus point of S is given as follows:

$$x^{pas}(S) = \arg\min_{x\in\{S,H\}} \max(u(S,x),1-\pi^{pas}(x)) = H \quad (2.22)$$

So for a passive player, by choosing S, his/her (imagined) satisfaction is:

$$u(S,x^{pas}(S)) = u(S,H) = 0 \quad (2.23)$$

However, by choosing H, his/her satisfaction is 0.67.

Based on the above analysis, for a passive player, he/she chooses H independent of the initial beliefs.

The proof is completed. □

Analysis

From the above result, we can see in the stag hunt game, two active players can achieve the payoff dominant equilibrium while two passive players can achieve the risk dominant equilibrium. However, an active player and a passive player can't reach any equilibrium and in this situation, an active player suffers more.

2 The Chicken Game

In game theory, the chicken game describes a situation as follows:

Two drivers are heading for a single line bridge. Each driver can choose to swerve or to go straight. The best situation for a driver is that he/she goes straight while his/her opponent swerves and the worst situation is that both drivers go straight, which leads to a conflict. The payoff of such a 2×2 game is given in Table 10.

Table 10

	S	G
S	(2,2)	(1,4)
G	(4,1)	(0,0)

We will show that the following result holds within our framework.

Theorem 2 Whatever initial belief an active player holds, he/she will choose G; in contrast, whatever initial belief a passive player holds, he/she will choose S.

Proof. Assume the belief of an active player is $p^{act}(x)$, $x \in \{S, G\}$, after the normalization procedure, the relatively likelihood function is then given as $\pi^{act}(x)$, for an active player, his/her payoff matrix is given in Table 11.

Table 11

	S	G
S	2	1
G	4	0

So the satisfaction matrix is then given in Table 12:

Table 12

	S	G
S	0.5	0.25
G	1	0

Let us then examine the active focus point of G.

It is easily to verify the following equalities:

$$\min(u(G,G), \pi^{act}(G)) \le u(G,G) = 0 \tag{2.24}$$

$$\min(u(G,S), \pi^{act}(S)) = \pi^{act}(S) > 0 \tag{2.25}$$

So the active focus point of G is given as follows:

$$x^{act}(G) = \arg\max_{x \in \{S,G\}} \min(u(G,x), \pi^{act}(x)) = S \qquad (2.26)$$

So for an active player, by choosing G, his/her (imagined) satisfaction is:

$$u(G, x^{act}(G)) = u(G,S) = 1 \qquad (2.27)$$

However, by choosing S, his/her satisfaction is 0.25 or 0.5, both of which are below 1.

Based on the above analysis, for an active player, he/she chooses G independent of the initial beliefs.

Assume the belief of a passive player is $p^{pas}(x)$, $x \in \{S,G\}$, after the normalization procedure, the relatively likelihood function is then given as $\pi^{pas}(x)$, let us examine the passive focus point of G.

It is easily to verify the following equalities:

$$\max(u(G,S), 1-\pi^{pas}(S)) = u(G,S) = 1 \qquad (2.28)$$

$$\max(u(G,G), 1-\pi^{pas}(G)) = 1-\pi^{pas}(G) < 1 \qquad (2.29)$$

So the passive focus point of G is given as follows:

$$x^{pas}(G) = \arg\min_{x \in \{S,H\}} \max(u(G,x), 1-\pi^{pas}(x)) = G \qquad (2.30)$$

So for a passive player, by choosing G, his/her (imagined) satisfaction is:

$$u(G, x^{pas}(G)) = u(G,G) = 0 \qquad (2.31)$$

However, by choosing S, his/her satisfaction is 0.25 or 0.5, both of which are above 0.

Based on the above analysis, for a passive player, he/she chooses G independent of the initial beliefs.

The proof is completed. □

Analysis

From the above result, we can see in the chicken game, neither two active players nor two passive players can achieve any equilibrium. Among them, two active players yield the worst conflict result. In contrast, an active player and a passive player can an equilibrium which makes the active player better off.

3 The Battle of Sex

In game theory, the battle of sex game describes a situation as follows:

A couple are going out for a date this evening. However, they forget what event they have decided: an opera or a football match. The wife prefers the opera while the husband prefers the football match. Both of them prefer to go to the same place. The payoff of such a 2×2 game is given in Table 13.

Table 13

	O	F
O	(2,3)	(0,0)
F	(0,0)	(3,2)

We will show that the following result holds within our framework.

Theorem 3 Whatever initial belief an active husband (or wife) holds, he/she will choose F (or O); whatever initial belief a passive husband (or wife) holds, his/her focus point by choosing F (or O) will be O (or F).

Proof. Assume the belief of an active husband is $p^{act}(x)$, $x \in \{O, F\}$, after the normalization procedure, the relatively likelihood function is then given as $\pi^{act}(x)$, for an active husband, his payoff matrix is given in Table 14:

Table 14

	O	F
O	2	0
F	0	3

So the satisfaction matrix is then given in Table 15.

Table 15

	O	F
O	0.67	0
F	0	1

Let us then examine the active focus point of F.

It is easily to verify the following equalities:

$$\min(u(F,O),\pi^{act}(O)) \le u(F,O) = 0 \qquad (2.32)$$

$$\min(u(F,F),\pi^{act}(F)) = \pi^{act}(F) > 0 \qquad (2.33)$$

So the active focus point of F is given as follows:

$$x^{act}(F) = \arg\max_{x\in\{O,F\}} \min(u(F,x),\pi^{act}(x)) = F \qquad (2.34)$$

So for an active husband, by choosing F, his (imagined) satisfaction is:

$$u(F,x^{act}(F)) = u(F,F) = 1 \qquad (2.35)$$

However, by choosing O, his/her satisfaction is 0 or 0.67, both of which are below 1.

Based on the above analysis, for an active husband, he chooses F independent of the initial beliefs.

Assume the belief of a passive husband is $p^{pas}(x) \quad x \in \{F,O\}$, after the normalization procedure, the relatively likelihood function is then given as $\pi^{pas}(x)$, let us examine the passive focus point of F.

It is easily to verify the following equalities:

$$\max(u(F,F),1-\pi^{pas}(F)) = u(F,F) = 1 \qquad (2.36)$$

$$\max(u(F,O),1-\pi^{pas}(O)) = 1-\pi^{pas}(O) < 1 \qquad (2.37)$$

So the passive focus point of F is given as follows:

$$x^{pas}(F) = \arg\min_{x\in\{O,F\}} \max(u(F,x),1-\pi^{pas}(x)) = O \qquad (2.38)$$

So for a passive husband, by choosing F, his focus point is O.

The proof is completed. □

Analysis

From the above result, we can see in the battle of sex, two active players lead to the worst result. As a passive husband (or wife) tends to choose O (or F), two passive players also bears a relatively higher risk to miss each other. In contrast, an active player and a passive player probably achieve an ideal (also equilibrium) result.

2.4 Summary

In this section, we make a detailed description of the One-Shot Game Model. Generally speaking, it is a non-equilibrium approach and consists of two steps: belief formulation and one-shot decision making. When formulating belief, a player takes his/her opponent payoff into account and deduces that his/her opponent should choose an action which can generate a higher payoff (maximum, minimum or average) with a higher probability. In deciding his/her optimal action, a player evaluates his/her each alternative focusing on only one action possibly chosen by his/her opponent (the focus point), which is the one-shot decision based thinking. Players with different personalities have different focus points, which results in different optimal choices. Although a player formulates different belief based on different belief formulation criteria, we show that in some simple 2×2 games, players' choices are independent of formulated beliefs and is determined only by whether the player is active or passive. The optimal actions of players in those games possess strong robustness and the obtained solutions within our framework is intuitively acceptable.

Chapter 3

Capacity Allocation Game

3.1 Preliminary in Capacity Allocation Game

In a supply chain, the capacity shortage refers to the situation that retailers' demand is higher than the supplier's inventory, as modifications are infeasible in a short term, the supplier has to divide the limited inventory to each retailer. Different allocation mechanisms have been proposed and examined so far (Cachon & Lariviere, 1999(a) (b) (c); Hall & Liu, 2010; Lu & Lariviere, 2012), among them, the proportional allocation method, which says the supplier provides each retailer the quantity proportional to his/her order, is the most intuitive and prevalent in practice. Under such an allocation rule and the assumption that each retailer is completely informed of the states (including the demand, the shortage and wastage cost, etc.), such a problem can be formulated as a static game with complete information among retailers (Cachon & Lariviere, 1999(a)). Although simple, the Nash equilibrium of such a game is that each retailer submits an order of infinity, which is in contrary to people's intuition as well as deviates from experimental findings (Chen et al., 2012; Cui & Zhang, 2016). What is worse, such an unacceptable equilibrium is rather robust to the change of the shape of the utility function and model's parameters, which obviously increases the difficulty of solving such an abnormality.

In this section, we firstly make a brief review of the capacity allocation game. For simplicity, we consider a supply chain with one supplier and two retailers, indexed by i,

$i = 1,2$. The supplier can provide at most K units' commodities to retailers and each retailer's demand is D units. The capacity shortage refers to the situation that the supplier can't meet both retailers' demands, i.e. $K < 2D$. The wholesale price is c per unit and the market price is p per unit. It is assumed that all the parameters mentioned above are exogenous and common knowledge. Under this assumption, the supplier's problem can be ignored here and this problem is a simple static game with complete information between two retailers. During this game, the following events occur sequentially. First, each retailer submits an order quantity q_i with an upper bound Q to the supplier, the upper bound Q can be regarded as a regulation in such a competition. Second, the supplier divides his/her capacity to each retailer according to the following proportion allocation mechanism:

If the supplier has sufficient capacity, i.e. $K \ge q_1 + q_2$, then each retailer's order will be filled;

If the supplier doesn't have sufficient capacity, i.e. $K < q_1 + q_2$, then each retailer will receive an allocation proportional to her order quantity, specifically, retailer 1 will receive $\frac{q_1}{q_1 + q_2}K$ and retailer 2 will receive $\frac{q_2}{q_1 + q_2}K$. Mathematically, if we denote the capacity allocated to retailer i as z_i, then

$$z_i = \min(q_i, \frac{q_i K}{q_i + q_{-i}}). \tag{3.1}$$

For retailer i, the material payoff function is given as follows:

$$v_i(q_i, q_{-i}) = \begin{cases} pD - cz_i & z_i \ge D \\ (p-c)z_i & z_i < D \end{cases}. \tag{3.2}$$

The unique Nash equilibrium of the above game is that each retailer orders the upper bound Q independent of p and c. Such an equilibrium is practically problematic and doesn't match players' behaviors in experiment (Chen et al., 2012; Cui & Zhang, 2016). Speaking in detail, Chen, Su and Zhao (2012) find that retailers only submit a large order quantity (or the regulated upper bound in the experiment) when the cost of the commodity is very low while decrease their orders when the cost of the commodity is higher. Cui and Zhang (2016) find that retailers don't order the upper bound at first stages of the games and that their order quantities increase to the upper bound with the same game being repeated. Those abnormalities raise an interesting question why such a deviation occurs. The general answer in the two papers are that the classical game theory suppose each retailer is perfect rational, while a retailer in reality is not perfect rational. Specifically, Chen Su and Zhao (2012) argue that retailers are bounded rational in deciding his/her optimal order quantity and Cui and Zhang (2016) rationalize the experimental findings by utilizing the Cognitive Hierarchy Model, the core argument of which is that an involved player may wrongly predict his/her opponents' actions.

In the following, we utilize the One-Shot Game Model to describe the decision-making process of an individual retailer involved in such a situation. Generally speaking, two steps are thought to be required for a retailer to accomplish his/her decisions. First, a retailer formulates a subjective belief about the action of his/her rival; second, a retailer chooses an order quantity best fitting his/her objective. In the next section, we will discuss the belief formulation process in detail.

3.2 Belief Formulation Process

In this section, we consider retailer i's belief formulation process. In retailer i's eyes, which order quantity q_{-i} is more likely to be chosen by his/her rival? To answer this question, let us firstly verify the following dominating relation:

$$q_{-i} = D \text{ dominates any } q_{-i} = d < D. \tag{3.3}$$

The above domination says whatever order quantity retailer i chooses, his/her rival choosing D is always better than choosing an order quantity less than D. This fact can be verified by considering the following two cases:

(a) If $q_i \le K - D$, then the allocated capacity to his/her rival is as follows:

$$z_{-i} = \begin{cases} D, & \text{if} \quad q_{-i} = D \quad ; \\ d, & \text{if} \quad q_{-i} = d \quad . \end{cases} \tag{3.4}$$

It is obvious that $v_{-i}(q_i, D) > v_{-i}(q_i, d)$ for $q_i \le K - D$;

(b) If $q_i > K - D$, then the allocated capacity to his/her rival is as follows:

$$z_{-i} = \begin{cases} \dfrac{DK}{D+q_i}, & \text{if} \quad q_{-i} = D \quad ; \\ \dfrac{dK}{d+q_i}, & \text{if} \quad q_{-i} = d \quad and \quad q_i > K - d \quad ; \\ d, & \text{if} \quad q_{-i} = d \quad and \quad q_i \le K - d \quad . \end{cases} \tag{3.5}$$

On one hand, noticing that $d \le \dfrac{dK}{d+q_i}$ for $q_i \le K - d$, we know the following relation holds:

$$z_{-i} \le \frac{dK}{d+q_i} \quad \text{for} \quad q_{-i} = d < D. \tag{3.6}$$

On the other hand, we have the following relation:

$$z_{-i} = \frac{DK}{D+q_i} \quad \text{for} \quad q_{-i} = D. \tag{3.7}$$

Further, considering $q_i > K - D$ and $d < D$, we can verify the following relation holds:

$$\frac{dK}{d+q_i} < \frac{DK}{D+q_i} < D. \tag{3.8}$$

The above relations imply that $v_{-i}(q_i, D) > v_{-i}(q_i, d)$ for $q_i > K - D$. Combining (a) and (b), we know $q_{-i} = D$ dominates any $q_{-i} = d < D$. In the same way, we know for $0 \le d_1 < d_2 \le D$, $q_{-i} = d_2$ dominates $q_{-i} = d_1$.

In summary, within the interval $q_{-i} \in [0, D]$, the smaller q_{-i} is, the more order quantity it is dominated by. In other words, it can be said that a smaller order quantity is more easily to be identified as a dominated one by retailer i's rival, so it is reasonable for retailer i to deduce that the smaller an order quantity is, the lower probability for it to be chosen by his/her rival.

However, domination only occurs within the interval $q_{-i} \in [0, D]$, then how should retailer i deal with q_{-i} beyond $[0, D]$? To solve this question, let us firstly clarify the following statement:

$$\text{If } q_i \in [D, Q], \; q_{-i} = \frac{D^2}{K-D} \text{ dominates any } q_{-i} = d \in [D, \frac{D^2}{K-D}). \tag{3.9}$$

Bearing in mind that $D > \frac{K}{2}$, we know $\frac{D^2}{K-D} > D$. In this case, the allocated capacity to his/her rival is as follows:

$$z_{-i} = \begin{cases} \dfrac{D^2 K}{D^2 + (K-D)q_i}, & if \quad q_{-i} = \dfrac{D^2}{K-D} \quad ; \\ \dfrac{dK}{d+q_i}, & if \quad q_{-i} = d \in [D, \dfrac{D^2}{K-D}) \quad . \end{cases} \tag{3.10}$$

For $q_i \geq D$ and $d \in [D, \frac{D^2}{K-D})$, we have the following relations:

$$\frac{dK}{d+q_i} < \frac{D^2 K}{D^2 + (K-D)q_i} \leq \frac{D^2 K}{D^2 + (K-D)D} = D. \qquad (3.11)$$

from (3.11) we know $v_{-i}(q_i, \frac{D^2}{K-D}) > v_{-i}(q_i, d)$ for $q_i \geq D$ and $d \in [D, \frac{D^2}{K-D})$, which is the dominating relation (3.9).

Noticing that the above dominating relation holds under the condition that retailer i doesn't choose his/her dominated order quantities, we can say that in order to recognize this dominating relation, retailer i's rival should at least think one step more, which obviously requires more computation and higher reasoning ability, so the dominated order quantities within the interval $[D, \frac{D^2}{K-D})$ is more difficult to be abandoned than those within the interval $[0, D)$, and it is natural for retailer i to deduce his/her rival adopts an order quantity within $[D, \frac{D^2}{K-D})$ with higher probability than adopting one within $[0, D)$. Also, in the same way, we can obtain the following relations:

If $q_i \in [D, Q]$ and $D \leq d_1 < d_2 \leq \frac{D^2}{K-D}$, $q_{-i} = d_2$ dominates $q_{-i} = d_1$. (3.12)

Utilizing the same logic, within the interval $[D, \frac{D^2}{K-D})$, a higher order quantity is more difficult to be identified as a dominated one, so the higher an order quantity is, the higher probability for it to be chosen. Likewise, we can verify the following sequence of dominating relations:

$$q_{-i} = \frac{D^3}{(K-D)^2} \text{ dominates any } q_{-i} = d \in [\frac{D^2}{K-D}, \frac{D^3}{(K-D)^2}),$$

$$q_{-i} = \frac{D^4}{(K-D)^3} \text{ dominates any } q_{-i} = d \in [\frac{D^3}{(K-D)^2}, \frac{D^4}{(K-D)^3}),$$

……

$$q_{-i} = \frac{D^n}{(K-D)^{n-1}} \text{ dominates any } q_{-i} = d \in [\frac{D^{n-1}}{(K-D)^{n-2}}, \frac{D^n}{(K-D)^{n-1}}). \quad (3.13)$$

As $\lim_{n\to\infty} \frac{D^n}{(K-D)^{n-1}} = \infty$, there exists an N such that $Q \in [\frac{D^{N-1}}{(K-D)^{N-2}}, \frac{D^N}{(K-D)^{N-1}})$. Also, within each interval, a higher order quantity corresponds to a larger dominating set, so a higher order quantity is chosen with a higher probability.

In summary, retailer i 's probabilistic belief $p_i(q_{-i})$ should satisfies the following two principles:

(1) $p_i(q_{-i})$ increases within each sub-interval,

(2) $p_i(q_{-i})$ increases over sub-intervals.

Without loss of generality and for the simplicity of analysis, we use an increasing continuous function $p_i(x)$ to serve as the probabilistic belief of retailer i in this situation.

So far we have finished the belief formulation procedure. In the next section, let us consider retailer i 's decision making process.

3.3 Decision Making Process

In this section, we utilize the one-shot decision theory thinking to character a retailer's decision making process. As one and only one order quantity of his/her rival will be realized after the game, it is natural for retailer i to imagine only one order quantity submitted by his/her rival before deciding whether to adopt an order quantity or not. In other words, when retailer i evaluates an order quantity, he/she doesn't regard this order quantity as a lottery whose value equals its expectation, instead, he/she imagines a specific order quantity possibly chosen by his/her rival and evaluates this order quantity as if his/her imagined scenario comes true. The above argument is in line with human intuitive cognitive phycology as well as matches the one-shot feature of retailer i's decision making situation. We also consider two kinds of retailers, namely the active retailer and the passive retailer, which correspond to different attitudes in decision making process. Speaking in detail, for each order quantity q_i, an active retailer i contemplates a q_{-i} making him/her better off with a relatively high likelihood, mathematically, such an imagined q_{-i} can be interpreted as a solution of the following bi-objective optimization problem:

$$\max_{q_{-i}} v_i(q_i, q_{-i}), \max_{q_{-i}} p_i(q_{-i}). \tag{3.14}$$

In this problem, the satisfaction function $u_i(q_i, q_{-i})$ can be obtained by a simple linear transformation as follows:

$$u_i(q_i, q_{-i}) = \frac{v_i(q_i, q_{-i}) - L}{U - L}, \tag{3.15}$$

where L is the lowest possible material payoff and U is the highest one. Obviously, for retailer i, he/she earns the most when receiving exactly his/her demanding quantities, so the highest possible material payoff is $U = (p-c)D$; as for the lowest one, retailer i

is the worst off when he/she receives all the capacity K or when he/she receives nothing, the material payoff is $pD - cK$ in the former case and 0 in the latter case. For simplicity, we first make the assumption that $pD - cK > 0$ here and thus the lowest material payoff is $L = 0$. Based on the above analysis and the expression of $v_i(q_i, q_{-i})$, $u_i(q_i, q_{-i})$ can be explicitly given as follows:

$$u_i(q_i, q_{-i}) = \begin{cases} \dfrac{pD - cz_i}{(p-c)D} & z_i \geq D \\ \dfrac{z_i}{D} & z_i < D \end{cases}, \tag{3.16}$$

where z_i is given in (3.1).

As the probability density function $p_i(q_{-i})$ is an increasing function with $p_i(0) = 0$, the relative likelihood function $\pi_i(q_{-i})$ is given in the following form:

$$\pi_i(q_{-i}) = \frac{p_i(q_{-i})}{p_i(Q)}. \tag{3.17}$$

Based on (3.15)-(3.17), problem (3.14) can be equally written in the following form:

$$\max_{q_{-i}} u_i(q_i, q_{-i}), \max_{q_{-i}} \pi_i(q_{-i}) \tag{3.18}$$

Further we know $u_i(q_i, q_{-i})$ and $\pi_i(q_{-i})$ both range over $[0,1]$, we utilize the following operator to find out an un-dominated solution of problem (3.14) as well as problem (3.18):

$$q_{-i}^1(q_i) \in \arg\max_{q_{-i}} \min_{q_i} [u_i(q_i, q_{-i}), \pi_i(q_{-i})] \tag{3.19}$$

In the above formula, as $u_i(q_i, q_{-i})$ and $\pi_i(q_{-i})$ are both canonical numbers, making comparison between them is acceptable. $\min_{q_i} [u_i(q_i, q_{-i}), \pi_i(q_{-i})]$ stands for the lower bound of the relative material payoff and the relative likelihood, and increasing the lower bound of them means increasing them simultaneously, resulting in picking out a scenario with relatively high likelihood and relatively high satisfaction, the chosen scenario is called the active focus point, which reflects the tendency of an active retailer. Denoting the set of active focus points as $Q^1_{-i}(q_i)$, the optimal order quantity for an active retailer is the one bringing him/her the highest material payoff (or the highest satisfaction) under the focus point, which can be expressed as follows:

$$q_i^{1,*} \in \arg\max_{q_i} \max_{q^1_{-i}(q_i)\in Q^1_{-i}(q_i)} u_i(q_i, q^1_{-i}(q_i)) \tag{3.20}$$

In (3.20), when multiple active focus points exist, we choose the one leading to the largest satisfaction, which is $\max_{q^1_{-i}(q_i)\in Q^1_{-i}(q_i)} u_i(q_i, q^1_{-i}(q_i))$, to reflect the optimistic attitude of an active retailer.

Besides the active retailer with an optimistic attitude, we also consider another kind of retailer with pessimistic attitude, which we call the passive retailer. Different from the active retailer, for each order quantity q_i, a passive retailer i focuses on a q_{-i} bringing him/her a relatively bad result with a relatively high likelihood. Also, we can use the following bi-objective optimization problem to identify the focus point of a passive retailer:

$$\min_{q_{-i}} v_i(q_i, q_{-i}), \max_{q_{-i}} p_i(q_{-i}) \quad . \tag{3.21}$$

The above problem also equals to:

$$\min_{q_{-i}} u_i(q_i, q_{-i}), \max_{q_{-i}} \pi_i(q_{-i}) \quad . \tag{3.22}$$

Also, we utilize the following formula to find out an un-dominated solution of problem (3.21) as well as problem (3.22):

$$q^2_{-i}(q_i) \in \arg\min_{q_{-i}} \max_{q_i} [u_i(q_i, q_{-i}), 1-\pi_i(q_{-i})] \,. \tag{3.23}$$

In the above formula, $\max_{q_i}[u_i(q_i, q_{-i}), 1-\pi_i(q_{-i})]$ stands for the upper bound of the relative material payoff and the inverse of relative likelihood, and decreasing the upper bound of them means decreasing them simultaneously, or decreasing the satisfaction level and increasing the relatively likelihood simultaneously. The scenario obtained in this way is likely to happen in the future and will bring retailer i an undesirable result upon happening. As a passive retailer can be imagined as a conservative one, such a scenario reflects his/her concern well. Denoting the set of passive focus points as $Q^2_{-i}(q_i)$, the optimal choice for a passive retailer is the one with the highest satisfaction level associating its focus point, which can be expressed as follows:

$$q_i^{2,*} \in \arg\max_{q_i} \min_{q^2_{-i}(q_i)\in Q^2_{-i}(q_i)} u_i(q_i, q^2_{-i}(q_i)) \tag{3.24}$$

In (3.24), when multiple passive focus points exist, we choose the one leading to the lowest satisfaction, which is $\min_{q^2_{-i}(q_i)\in Q^2_{-i}(q_i)} u_i(q_i, q^2_{-i}(q_i))$, to reflect the pessimistic attitude of a passive retailer.

In summary, in this section, we describe a retailer's decision making process by seizing the one-time feature of the problem as well as considering the impact of decision makers' personalities. Different from the existing expected theory, a retailer makes his/her decision relying on only one specific imagined scenario in the future, it is closer to the real decision making process, also, the obtained optimality conditions show the reasonability and effectiveness of our framework and we will show them in the following section.

3.4 Analytical Results and Explanation

For an active retailer, we have the following result.

Theorem 1. For an active retailer, his/her optimal order quantity is $q_i^{1,*} = Q$.

Proof. In the first step, let us examine the focus point for each $q_i \in [0, Q]$. For a fixed q_i, let us consider $\max_{q_{-i}} \min(\pi_i(q_{-i}), u_i(q_i, q_{-i}))$ within $[0, \frac{K}{D} q_i - q_i]$ and $[\frac{K}{D} q_i - q_i, Q]$, respectively. As $D < K < 2D$ and $0 \le q_i \le Q$, we know $0 \le \frac{K}{D} q_i - q_i \le Q$, which guarantees the feasibility of such a division.

$q_{-i} \in [0, \frac{K}{D} q_i - q_i]$: Bearing in mind that $\pi_i(q_{-i})$ is an increasing function, it is obvious that the following relation holds:

$$\max_{q_{-i} \in [0, \frac{K}{D} q_i - q_i]} \min(\pi_i(q_{-i}), u_i(q_i, q_{-i})) \le \max_{q_{-i} \in [0, \frac{K}{D} q_i - q_i]} \pi_i(q_{-i}) = \pi_i(\frac{K}{D} q_i - q_i). \tag{3.25}$$

$q_{-i} \in [\frac{K}{D} q_i - q_i, Q]$: Firstly, it can be verified that $u_i(q_i, q_{-i})$ is a decreasing function of q_{-i} within this interval and that $u_i(q_i, \frac{K}{D} q_i - q_i) = 1$. Secondly, $\pi_i(q_{-i})$ is an increasing function and $\pi_i(Q) = 1$. We can then obtain the following relations:

$$1 = u_i(q_i, \frac{K}{D} q_i - q_i) > \pi_i(\frac{K}{D} q_i - q_i), \tag{3.26}$$

$$u_i(q_i, Q) < \pi_i(Q) = 1. \tag{3.27}$$

Considering the monotonicity of $u_i(q_i, q_{-i})$ and $\pi_i(q_{-i})$, we know

$$\max_{q_{-i} \in [\frac{K}{D} q_i - q_i]} \min(\pi_i(q_{-i}), u_i(q_i, q_{-i})) = \pi_i(\overline{q_{-i}}) = u_i(q_i, \overline{q_{-i}}), \tag{3.28}$$

where $\overline{q_{-i}}$ is the unique solution of $\pi_i(q_{-i}) = u_i(q_i, q_{-i})$ within $q_{-i} \in (\frac{K}{D} q_i - q_i, Q)$. As $\overline{q_{-i}} \in (\frac{K}{D} q_i - q_i, Q)$ and $\pi_i(q_{-i})$ is an increasing function, we know

$\pi_i(\overline{q_{-i}}) > \pi_i(\frac{K}{D}q_i - q_i)$, from which we can obtain

$$\max_{q_{-i}\in[0,Q]} \min(\pi_i(q_{-i}), u_i(q_i, q_{-i})) = \pi_i(\overline{q_{-i}}) = u_i(q_i, \overline{q_{-i}}), \tag{3.29}$$

$$\arg\max_{q_i\in[0,Q]} \min(\pi_i(q_{-i}), u_i(q_i, q_{-i})) = \overline{q_{-i}} \ . \tag{3.30}$$

As $\overline{q_{-i}}$ is the unique solution of $\pi_i(q_{-i}) = u_i(q_i, q_{-i})$, specifically, $\overline{q_{-i}}$ satisfies the following equation:

$$\pi_i(\overline{q_{-i}}) = \frac{Kq_i}{D(q_i + \overline{q_{-i}})} \ . \tag{3.31}$$

Differentiating both sides with respect to q_i, we know

$$\overline{q_{-i}}'(q_i) = \frac{KD\overline{q_{-i}}}{KDq_i + D^2(q_i + \overline{q_{-i}})^2 \pi_i'(\overline{q_{-i}})} > 0 \ . \tag{3.32}$$

The above relation shows that $\overline{q_{-i}}$ increases with increasing q_i, as $\pi_i(\overline{q_{-i}})$ is an increasing function of $\overline{q_{-i}}$, $u_i(q_i, \overline{q_{-i}})$ is an increasing function of q_i, we know the optimal order quantity in this case is $q_i^{1,*} = Q$. It proves theorem 1. □

Comment 1

From the proof of Theorem 1, we know that for an order quantity q_i, an active retailer ignores his/her rival's order quantities with relatively low likelihood (i.e. $q_{-i} \in [0, \frac{K}{D}q_i - q_i]$); within the interval $q_{-i} \in [\frac{K}{D}q_i - q_i, Q]$, retailer i 's satisfaction decreases while the likelihood of q_{-i} increases with increasing q_{-i}, as an active retailer i subconsciously seeks a more possible and more beneficial state, making a balance between the two factors , or mathematically choosing the intersection point of those two functions as the focus point, is intuitively acceptable in this situation.

For a passive retailer, we have the following theorem.

Theorem 2. For a passive retailer, his/her optimal order quantity $q_i^{2,*}$ satisfies

$$\frac{q^{2,*}K}{(q^{2,*}+Q)D}=1-\pi_i\left(\frac{cKq_i^{2,*}(q_i^{2,*}+Q)}{pD(q_i^{2,*}+Q)-(p-c)Kq_i^{2,*}}-q_i^{2,*}\right). \tag{3.33}$$

Proof. Firstly, for $q_i\in[0,D]$, utilizing the monotonicity of $\pi_i(q_{-i})$ and $u_i(q_i,q_{-i})$, we know the following relation hold:

$$Q=\arg\max_{q_{-i}\in[0,Q]}\pi_i(q_{-i}), \tag{3.34}$$

$$Q=\arg\min_{q_{-i}\in[0,Q]}u_i(q_i,q_{-i}). \tag{3.35}$$

Based on (3.21) and (3.22), we know in this case, the focus point of q_i is $q_{-i}^2(q_i)=Q$ and $u_i(q_i,q_{-i}^2(q_i))=u_i(q_i,Q)=\dfrac{Kq_i}{(q_i+Q)D}$. Considering the monotonicity of $u_i(q_i,Q)$, we know the optimal order quantity within $[0,D]$ is D and the satisfaction is $\dfrac{K}{(D+Q)}$.

Secondly, for $q_i\in(D,Q]$, let us consider the focus point for each q_i within $[0,\frac{K}{D}q_i-q_i]$ and $[\frac{K}{D}q_i-q_i,Q]$, respectively.

$q_{-i}\in[0,\frac{K}{D}q_i-q_i]$: Firstly, it can be verified that $u_i(q_i,q_{-i})$ is a strictly increasing function of q_{-i} within this interval and that $u_i(q_i,\frac{K}{D}q_i-q_i)=1$. Secondly, $1-\pi_i(q_{-i})$ is a decreasing function and $1-\pi_i(0)=1$. We can then obtain the following relations:

$$u_i(q_i,0)<1-\pi_i(0)=1\ , \tag{3.36}$$

$$u_i(q_i,\frac{K}{D}q_i-q_i)=1>\pi_i(\frac{K}{D}q_i-q_i). \tag{3.37}$$

Considering the monotonicity of $u_i(q_i,q_{-i})$ and $1-\pi_i(q_{-i})$, we know

$$\min_{q_{-i}\in[0,\frac{K}{D}q_i-q_i]} \max(1-\pi_i(q_{-i}),u_i(q_i,q_{-i})) = 1-\pi_i(\hat{q}_{-i}) = u_i(q_i,\hat{q}_{-i}), \quad (3.38)$$

where $\hat{q}_{-i}$ is the unique solution of $1-\pi_i(q_{-i}) = u_i(q_i,q_{-i})$ within $1-\pi_i(q_{-i}) = u_i(q_i,q_{-i})$.

$q_{-i}\in[\frac{K}{D}q_i-q_i,Q]$: Within this interval, both $u_i(q_i,q_{-i})$ and $1-\pi_i(q_{-i})$ are decreasing function of q_{-i}, combining the fact that $1-\pi_i(Q)=0$, we can obtain the following relation:

$$\min_{q_{-i}\in[\frac{K}{D}q_i-q_i,Q]} \max(1-\pi_i(q_{-i}),u_i(q_i,q_{-i})) = \max(1-\pi_i(Q),u_i(q_i,Q)) = u_i(q_i,Q). \quad (3.39)$$

Based on the above analysis, we know the focus point of an order quantity q_i can be expressed in the following form:

$$q_{-i}^2(q_i) = \begin{cases} \hat{q}_{-i} \quad , & if \quad u_i(q_i,\hat{q}_{-i}) < u_i(q_i,Q) \\ Q \quad , & if \quad u_i(q_i,\hat{q}_{-i}) > u_i(q_i,Q) \\ \{\hat{q}_{-i},Q\} \quad , & if \quad u_i(q_i,\hat{q}_{-i}) = u_i(q_i,Q) \end{cases}, \quad (3.40)$$

where $u_i(q_i,q_{-i}^2(q_i)) = \min(u_i(q_i,\hat{q}_{-i}),u_i(q_i,Q))$. As $\hat{q}_{-i}$ is the unique solution of $1-\pi_i(q_{-i}) = u_i(q_i,q_{-i})$, specifically, $\hat{q}_{-i}$ satisfies the following equation:

$$1-\pi_i(\hat{q}_{-i}) = \frac{p}{p-c} - \frac{cKq_i}{(p-c)D(q_i+\hat{q}_{-i})}. \quad (3.41)$$

Differentiating both sides with respect to q_i, we know

$$(\hat{q}_{-i})'(q_i) = \frac{p(p-c)KD\hat{q}_{-i}}{p(p-c)KDq_i + (p-c)^2D^2(q_i+q_{-i})^2\pi_i'(\hat{q}_{-i})} > 0 \quad (3.42)$$

The above relation shows that $\hat{q}_{-i}$ increases with increasing q_i, as $1-\pi_i(\hat{q}_{-i})$ is a decreasing function of $\hat{q}_{-i}$, $u_i(q_i,\hat{q}_{-i})$ is a decreasing function of q_i.

As $u_i(q_i,Q) = \dfrac{Kq_i}{D(q_i+Q)}$, it is obvious that $u_i(q_i,Q)$ is an increasing function of q_i.

So $u_i(q_i, q^2_{-i}(q_i)) = \min(u_i(q_i, \hat{q}_{-i}), u_i(q_i, Q))$ attains its minimum at the intersection point of the two functions $u_i(q_i, \hat{q}_{-i})$ and $u_i(q_i, Q)$, denoting the optimal order quantity as $q_i^{2,*}$, it should satisfy the following condition:

$$u_i(q_i^{2,*}, \hat{q}_{-i}) = u_i(q_i^{2,*}, Q). \tag{3.43}$$

From (3.16), we know $u_i(q_i^{2,*}, \hat{q}_{-i})$ and $u_i(q_i^{2,*}, Q)$ are as follows:

$$u_i(q_i^{2,*}, \hat{q}_{-i}) = \frac{pD - c\dfrac{Kq_i^{2,*}}{q_i^{2,*} + \hat{q}_{-i}}}{(p-c)D} \tag{3.44}$$

$$u_i(q_i^{2,*}, Q) = \frac{\dfrac{Kq_i^{2,*}}{q_i^{2,*} + Q}}{D} \tag{3.45}$$

(3.43) (3.44) (3.45) leads to the following relation:

$$\hat{q}_{-i} = \frac{cKq_i^{2,*}(q_i^{2,*} + Q)}{pD(q_i^{2,*} + Q) - (p-c)Kq_i^{2,*}} - q_i^{2,*} \tag{3.46}$$

Further, as $\hat{q}_{-i}$ is the unique solution of $1 - \pi_i(q_{-i}) = u_i(q_i, q_{-i})$, we have the following relation:

$$1 - \pi_i(\hat{q}_{-i}) = u_i(q_i^{2,*}, \hat{q}_{-i}). \tag{3.47}$$

(3.43) (3.46) (3.47) together lead to (3.33).

Based on the above analysis, we know the optimal order quantity within $(D, Q]$ is $q_i^{2,*}$, and the satisfactions is $\dfrac{Kq_i^{2,*}}{(q_i^{2,*} + Q)D}$, which is larger than the highest satisfaction by choosing an order quantity within $[0, D]$. It proves theorem 2. □

Comment 2

From the proof of Theorem 2, we know that for an order quantity q_i, a passive retailer i possibly focuses on two situations, one is that his/her rival orders too much (i.e. $q_{-i} = Q$), in this case, retailer i receives too little and suffers from the largest opportunity cost, so it is an undesirable situation. Considering the character of a passive retailer, worrying about this situation is understandable. The other situation that may draw retailer i's attention is that his/her rival orders too less, which will cause retailer i a larger wastage loss. However, as the likelihood that his/her rival orders too less is not high, considering the low possibility, retailer i may ignore the a very low q_{-i}, instead, retailer i tends to concentrate on a low q_{-i} relative probably chosen by his/her rival, which is the $\hat{q}_{-i}$ in theorem 2. Pessimistically, a passive retailer makes a comparison between the two situations and chooses a worse state as the focus point.

Based on Theorem 2, we can also obtain the following results.

Proposition 3 For a passive retailer, his/her optimal order quantity $q_i^{2,*}$ is an increasing function of the upper bound Q and his/her demand D; his/her optimal order quantity $q_i^{2,*}$ is a decreasing function of the capacity K.

Proof. Bearing in mind that $\hat{q}_{-i} = \dfrac{cKq_i^{2,*}(q_i^{2,*}+Q)}{pD(q_i^{2,*}+Q)-(p-c)Kq_i^{2,*}} - q_i^{2,*}$, the optimal condition (3.33) can be written as follows:

$$\frac{q^{2,*}K}{(q^{2,*}+Q)D} = 1 - \pi_i(\hat{q}_{-i}) \tag{3.48}$$

Fixed D and K, differentiating both sides of (3.48) with respect to $q_i^{2,*}$, utilizing the implicit function theorem, we know the derivative of Q with respect to $q_i^{2,*}$ is as follows:

$$Q'(q_i^{2,*}) = \frac{QKD + D^2(q+Q)^2\pi_i'(\hat{q}_{-i})\hat{q}'_{-i}(q_i^{2,*})}{qKD} \tag{3.49}$$

As $\pi_i(\hat{q}_{-i})$ is an increasing function, $\pi_i'(\hat{q}_{-i}) > 0$; from (3.42), we know $\hat{q}'_{-i}(q_i^{2,*}) > 0$, those two relations leads to the result that $Q'(q_i^{2,*}) > 0$, in other word, Q is an increasing function of $q_i^{2,*}$, which is equal to say that $q_i^{2,*}$ is an increasing function of Q.

Similarly, by fixing the other two parameters, we can obtain the derivatives of D and K with respect to $q_i^{2,*}$ are as follows:

$$D'(q_i^{2,*}) = \frac{QKD + D^2(q+Q)^2\pi_i'(\hat{q}_{-i})\hat{q}'_{-i}(q_i^{2,*})}{qK(Q+q)} \tag{3.50}$$

$$K'(q_i^{2,*}) = -\frac{QKD + D^2(q+Q)^2\pi_i'(\hat{q}_{-i})\hat{q}'_{-i}(q_i^{2,*})}{qD(Q+q)} \tag{3.51}$$

Based on the analysis of $Q'(q_i^{2,*})$ and (3.50), (3.51), it is easy to check that $D'(q_i^{2,*}) > 0$ and $K'(q_i^{2,*}) < 0$, which imply that $q_i^{2,*}$ is an increasing function of D and a decreasing function of the K.The proof is completed. □

Comment 3

The results in Proposition 3 are also intuitive. Firstly, when the capacity is low, it is natural for a retailer to raise his/her order quantity to gain a larger share of the total capacity. Secondly, when the demand is high, a retailer should increase his/her order quantity to guarantee a larger allocation. Thirdly, when the upper bound is high, as the opponent can order more, a retailer should also raise his/her order quantity to make him/her more competitive.

Explanations of the experiment findings

Utilizing the above analytical results, we can provide a unified explanation for the experimental findings in the literature.

Explanation of the experimental findings of Chen, Su and Zhao

In the work of Chen, Su and Zhao (2012), the experiments show that when the wholesale price is very low ($c = 2$ in the experiment), most retailers' choices are close to Q. Theorem 1 suggests that regardless of π_i, an active retailer's optimal order quantity is Q. As it is acceptable that a retailer adopts an active attitude when facing a commodity of low cost, our result is in consistent with the experimental findings. When the wholesale price is very high ($c = 20$ in the experiment), retailers' order quantities are distributed in the interval $[D, Q]$, in other words, in this case, a retailer also orders more than his/her demand, but don't order the highest Q for a fear of possible large loss. From Theorem 2, we know the optimal order quantity of a passive retailer $q_i^{2,*}$ lies in the interval $[D, Q]$, further, by assuming different initial beliefs $p_i(q_{-i})$, $q_i^{2,*}$ can be different values within $[D, Q]$. It is also acceptable that a retailer is passive when facing a commodity of high cost, so we can say our results match the experimental data to some extent.

Explanation of the experimental findings of Cui and Zhang

The experiments conducted by Cui and Zhang (2016) also show some tendencies of retailers' behaviors in the capacity allocation game. Firstly, a retailer orders more when the capacity is more restricted (i.e. K is low). Secondly, a retailer doesn't order the upper bound at first, however, when the game is repeated, a retailer's order quantity increases to the upper bound. These phenomena can be also explained by our model. Firstly, in Proposition 3, we show that the optimal order quantity is increasing with decreasing the capacity, what is more, we also show that the optimal order quantity is increasing with increasing the demand and the upper bound, which are both consistent with human's intuition. Secondly, at first stage, retailers have no experience of such a game, in other word, a retailer finds him/herself in an unfamiliar environment. In such an environment, it is probable that the retailer is passive (or less confident), so the retailer chooses an order quantity less than the upper bound, which is consistent with the choice of a passive retailer in our model; however, with repeating playing the same game, a retailer gains more experiences and becomes more confident (or active), and the order quantity gradually increases to the upper bound, which is the choice of the active retailer in our model.

3.5 Summary

In this section, we utilize the One Shot Game Model to examine the capacity allocation game. Firstly, each retailer forms the belief based on strategy dominance. Secondly, optimal choices are made based on the form belief. As there is one and only one chance for a retailer to submit an order quantity and one and only one order quantity will be chosen by his/her rival, it is a typical one-shot decision problem for each retailer, so we utilize the one-shot decision theory to analyze the decision making process of an retailer. The obtained analytical results are more intuitive than the classical Nash Equilibrium and match the experimental findings in literature.

Moreover, the proposed framework in this section can be potentially extended to investigate games resembling the capacity allocation game in the following two aspects: firstly, the unique equilibrium is obtained by iteration of elimination of dominated strategies; secondly, the equilibrium is an implausible extreme value.

One example is the Traveler's Dilemma proposed by Kaushik Basu in 1994. The story is that two travelers fly home, each bring a souvenir of the same price. Their luggage is lost and the airline company asks them to make independent claims for compensations. If their claims differ, each will get the minimum of the two claims. In addition, the traveler making the lower claim will be paid a fixed reward, which will be imposed as penalty to the other traveler. The unique Nash equilibrium obtained after iteration of elimination of dominated strategies is each traveler submit a 0 claim, which is implausible but robust.

The other example is the $p-$Beauty Contest Game studied by Nagel in 1995. The rule is simple: a large number of players simultaneously choose a number between 0 and 100, the more closer a player's chosen number is to the p $(0 < p < 1)$ times the average of all chosen numbers, the more profit he/she gets from the game. When the chosen number is restricted to integers, it is one kind of integer games. The $p-$beauty contest game is also a dominance solvable game with the unique Nash equilibrium that every player chooses 0, which is a prediction far from experimental findings.

Both the two abnormalities mentioned above can be potentially solved by the proposed framework in this section. On one hand, as dominated strategies can be eliminated iteratively, a monotone belief can reflect the dominance relation of such games better; on the other hand, considering the one-time feature of such games, we utilize the one-shot decision theory to handle a player's decision process. Players' action set as well as utility function in both games are different from those in the capacity allocation game, and further analysis is left for the future work.

Chapter 4

First Price Sealed Bid Auction

4.1 Anomalies in First Price Sealed Bid Auction

The classical solution concept in first price sealed bid auctions is the risk-neutral Bayesian Nash Equilibrium (hereafter RNBNE) (Vickrey, 1961). However, experimental evidence shows that real bidders don't follow it at most time. Generally speaking, there exist two major tendencies. One is that bidders with low valuation tend to bid randomly: on one hand, Cox, Smith and Walker (1988) point out the 'throw away' phenomenon, which says that some subjects in first price auction experiments, upon drawing a low value, enter a bid at (or near) zero, or less frequently, a bid at (or near) the value. On the other hand, overbidding (Pezanis-Christou, 2002) and underbidding (Kirchkamp & Philipp, 2004), which respectively mean bidding above and below the RNBNE, are also observed. The other one is that bidders with high valuation tend to overbid (Cox et al., 1982).

Several explanations have been considered in the literature. For example, the throw away phenomenon is thought caused by the fact that bidders with low evaluation don't take the auction seriously (Cox et al., 1992). Overbidding is explained by theoretical models such as risk aversion (Cox et al., 1983), joy of winning (Cox et al., 1992), ad hoc bidding strategy (Pezanis-Christou, 2002), quantal response equilibrium (Goeree et al., 2002), the level-k model (Crawford & Iriberri, 2007), regret averse (Hayashi & Yoshimoto, 2014), etc. Underbidding is rationalized by the anticipated emotions model

(Roider & Schmitz, 2007). However, to the best of our knowledge, until now, there exists no theoretical model providing a unified explanation for those phenomena. In the following, we utilize the One-Shot Game Model to analyze the first price sealed bid auction. Instead of treating an involved bidder as a decision making machine, we try to narrate the real decision making process of him/her.

4.2 Two Bidders' First Price Sealed Bid Auction

For easily understanding our models, let us begin with examining the two bidders' case. For bidder $i \in \{1,2\}$, the set of his/her bidding prices is $B_i = [0, v_i]$ where v_i is his/her valuation of the auctioned subject which is an independent private value; his/her belief – the probability density function of his/her rivals' bidding prices is p_i where $i \neq j \in \{1,2\}$.

In our models, we not only take into account the simple gain achieved by the bidding price but also the regret caused after knowing the result. The effect of regret on bidders' behaviors is initially examined by Engelbrecht-Wiggans (1989) and further studied by Filiz-Ozbay & Ozbay (2007), Engelbrecht-Wiggans & Katok (2007, 2009) and Hayashi & Yoshimoto (2014). The winner in a first-price sealed-bid auction is the bidder with the highest bidding price. However, it is always the case that the winner finds himself/herself bid too high after the revelation of all the other bidders' bidding prices. In this situation, we say that the winner suffers from 'winner's regret'. In contrast, after an auction, a loser may find that the winner's bidding price is below his/her valuation of the auctioned object. In this case, the loser actually misses an opportunity to gain and we say that the loser suffers from 'loser's regret'. The evaluation function is given as follows:

$$f_i(b_i, b_j) = \begin{cases} (v_i - b_i) - k_{i,1}(b_i - b_j) & , \quad b_i > b_j \quad ; \\ -k_{i,2}(v_i - b_j) & , \quad v_i \geq b_j \geq b_i \quad ; \\ 0 & , \quad b_j > v_i \quad ; \end{cases} \tag{4.1}$$

where $k_{i,1}$ is bidder $i's$ winning regret parameter and $k_{i,2}$ is bidder $i's$ losing regret parameter. Here we assume $k_{i,1}, k_{i,2} \in (0, C]$ where C is a positive real number. This assumption is reasonable and can be interpreted as follows: on one hand, the empirical study (Engelbrecht-Wiggans & Katok, 2007) shows both winning regret and losing regret have effect on the bidder's evaluation; on the other hand, neither winning regret nor losing regret is so large that the direct profit can be ignored in the bidder's evaluation. The evaluation function (1) involves the following three cases. Case 1 is that bidder i wins the auction ($b_i > b_j$). In this case, the evaluation value is the gain $v_i - b_i$ offsetting by the weighted winning regret $k_{i,1}(b_i - b_j)$. Case 2 is for the situation $v_i \geq b_j \geq b_i$. In this case, the bidder i feels regret because if he presents a little higher than b_j he could gain $v_i - b_j$ so that the evaluation value is $-k_{i,2}(v_i - b_j)$, it should be mentioned here that we treat the tie case as lose in (1) to reflect a relatively conservative attitude of an involved bidder; In Case 3, that is, $b_j > v_i$, bidder i loses the auction. However, there is neither regret nor gain for him/her.

The satisfaction function can be obtained through the following transformation:

$$u_i(f_i) = \frac{f_i - LBf_i}{UPf_i - LBf_i}, \tag{4.2}$$

where LBf_i and UPf_i are a lower bound and a upper bound of f_i, respectively. Since $-Cv_i \leq f_i \leq v_i$ always holds, we take v_i and $-Cv_i$ as the upper bound and lower bound of f_i, respectively, and rewrite (4.2) as follows:

$$u_i(b_i, b_j) = \begin{cases} (v_i - b_i) - k_{i,1}(b_i - b_j)/(1+C)v_i & , \quad b_i > b_j \quad ; \\ -k_{i,2}(v_i - b_j)/(1+C)v_i & , \quad v_i \geq b_j \geq b_i \quad ; \\ C/(1+C) & , \quad b_j > v_i \quad ; \end{cases} . \tag{4.3}$$

For simplicity, we set

$$((v_i - b_i) - k_{i,1}(b_i - b_j) + Cv_i)/(1+C)v_i = u_i^1(b_i, b_j), \tag{4.4}$$

$$(-k_{i,2}(v_i - b_j) + Cv_i)/(1+C)v_i = u_i^2(b_j), \tag{4.5}$$

and $u_i^1(b_i,b_j)$ and $u_i^2(b_j)$ are used henceforward.

Usually, bidder $i's$ valuation v_i is assumed to be a real number within $[0,1]$ and bidder $i's$ belief on his/her rival's bidding prices b_j is assumed to be uniformly distributed within $[0,1]$ (Engelbrecht-Wiggans, 1989; Crawford & Iriberri, 2007), that is,

$$p_i(b_j)=\begin{cases}0, & b_j<0 \ ;\\ 1, & 0\le b_j\le 1;\\ 0, & b_j>1 \ ;\end{cases}, \tag{4.6}$$

The relative likelihood function is then given as follows:

$$\pi_i(b_j)=\begin{cases}0, & b_j<0 \ ;\\ 1, & 0\le b_j\le 1;\\ 0, & b_j>1 \ ;\end{cases}. \tag{4.7}$$

We argue that a bidder determines his/her bidding price based on his/her imagined scenario which is consistent with his/her personality. Let us consider that the bidder takes a conservative attitude to the imagined scenario. Speaking in detail, for each available bidding price b_i, the bidder i contemplates a scenario b_j which can bring him/her a relatively low satisfaction level $u_i(b_i,b_j)$ with a relatively high relative likelihood degree $\pi_i(b_j)$. It can be represented by the following bi-objective optimization problem.

$$\max_{b_j} \pi_i(b_j), \ \min_{b_j} u_i(b_i,b_j), \tag{4.8}$$

where $b_i\in[0,v_i]$. Regarding $u_i(b_i,b_j)$ and $\pi_i(b_j)$ equally important, we can find out one Pareto optimal solution of (4.8) from the set of all undominated solutions as follows:

$$b_j(b_i)\in\arg\min_{b_j} \max(1-\pi_i(b_j),u_i(b_i,b_j)). \tag{4.9}$$

$b_j(b_i)$ is a focused scenario amongst all scenarios b_j when bidder i presents the bidding price b_i and called as the focus point of b_i.

A bidder aims to find out a bidding price whose focus point associates with the largest satisfaction level. Such a bidding price is regarded as an optimal one. Denoting the set of focus points of b_i as $B(b_i)$ and an optimal bidding price as b_i^*, we have

$$b_i^* \in \arg\max_{b_i} \min_{b_k \in B(b_i)} u_i(b_i, b_k) . \tag{4.10}$$

(4.10) is for the case that multiple focus points of b_i exist. In this case, different focus points bring different satisfaction levels. We take a conservative attitude to evaluate the satisfaction levels so that we take a minimum, that is, $\min_{b_k \in B(b_i)} u_i(b_i, b_k)$. If a unique focus point $b_j(b_i)$ exists for b_i, then (4.10) becomes $b_i^* \in \arg\max_{b_i} u_i(b_i, b_j(b_i))$.

Let us make a brief summary of a bidder's decision procedure described by (4.6)-(4.10). The first step is the formulation of his/her belief about his/her rival's bidding price, that is the formulas (4.6) and (4.7); the second step is determining the focus point (one imagined bidding price offered by his/her rival) of his/her each bidding price, that is the formulas (4.8) and (4.9); the third step is to evaluate his/her each bidding price by the focus point and determine his/her optimal bidding price. Clearly, different from the existing auction models where the bidding price is evaluated by the expected utility resulted by all possible bidding prices of the rival, our model evaluates the bidding price only by the satisfaction level resulted by its focus point. The following theorem is for the optimal bidding price.

Theorem 1.

The optimal bidding price of bidder i is given as follows:

$$b_i^* = \frac{(1+k_{i,2})v_i}{1+k_{i,1}+k_{i,2}}. \tag{4.11}$$

Proof.

Firstly, let us examine the focus point for each $b_i \in [0, v_i]$.

Since $\pi_i(b_j) = 1$ and $u_i(b_i, b_j) \geq 0$ always hold, we have

$$\max(1 - \pi_i(b_j), u_i(b_i, b_j)) = u_i(b_i, b_j) \tag{4.12}$$

so that we know

$$\min_{b_j \in [0,1]} \max(1 - \pi_i(b_j), u_i(b_i, b_j)) = \min_{b_j \in [0,1]} u_i(b_i, b_j). \tag{4.13}$$

Using (4.4) and (4.5), (4.3) can be rewritten as

$$u_i(b_i, b_j) = u_i^1(b_i, b_j) \quad \text{for} \quad b_j \in [0, b_i), \tag{4.14}$$

$$u_i(b_i, b_j) = u_i^2(b_j) \quad \text{for} \quad b_j \in [b_i, v_i], \tag{4.15}$$

$$u_i(b_i, b_j) = C/(1+C) \quad \text{for} \quad b_j \in (v_i, 1]. \tag{4.16}$$

Since $u_i^1(b_i, b_j)$ and $u_i^2(b_j)$ are increasing in b_j, we know

$$\min_{b_j \in [0, b_i)} u_i^1(b_i, b_j) = u_i^1(b_i, 0), \tag{4.17}$$

$$\min_{b_j \in [b_i, v_i]} u_i^2(b_j) = u_i^2(b_i) = (-k_{i,2}(v_i - b_i) + Cv_i)/(1+C)v_i < C/(1+C). \tag{4.18}$$

From (4.13), (4.17) and (4.18), we know

$$\min \max(1 - \pi_i(b_j), u_i(b_i, b_j)) = \min_{b_j \in [0,1]} u_i(b_i, b_j) = \min(u_i^1(b_i, 0), u_i^2(b_i)). \tag{4.19}$$

So for each b_i, its focus point, denoted as $b_j(b_i)$ is given as follows:

$$b_j(b_i) \in \begin{cases} \{0\} & , \quad u_i^1(b_i,0) < u_i^2(b_i) \quad ; \\ \{0,b_i\} & , \quad u_i^1(b_i,0) = u_i^2(b_i) \quad ; \\ \{b_i\} & , \quad u_i^1(b_i,0) > u_i^2(b_i) \quad ; \end{cases} \tag{4.20}$$

Secondly, let us examine the optimal bidding price. From (4.19), we know

$$\min_{b_k \in B(b_i)} u_i(b_i,b_k) = \min(u_i^1(b_i,0), u_i^2(b_i)) . \tag{4.21}$$

As $u_i^1(0,0) = 1 > (C - k_{i,2})/(1+C) = u_i^2(0)$ and $u_i^1(v_i,0) = (C - k_{i,1})/(1+C) < C/(1+C) = u_i^2(v_i)$ hold, considering that $u_i^1(b_i,0) = ((1+C)v_i - (1+k_{i,1})b_i)/(1+C)v_i$ is a decreasing function of b_i and $u_i^2(b_i) = ((C - k_{i,2})v_i + k_{i,2}b_i)/(1+C)v_i$ is an increasing function of b_i, it is obvious that $\min(u_i^1(b_i,0), u_i^2(b_i))$ attains its maximum when the following equation is satisfied:

$$u_i^1(b_i,0) = u_i^2(b_i) . \tag{4.22}$$

Solving (4.22), we can obtain the optimal bidding price as follows:

$$b_i^* = \frac{(1+k_{i,2})v_i}{1+k_{i,1}+k_{i,2}} . \tag{4.11}$$

It proves Theorem 1. □

From the proof of Theorem 1, we know that for a bidding price b_i, the following two situations draw attention of bidder i: one is that his/her rival gives a bidding price 0, and in this case, bidder i suffers from the largest winning regret; the other one is that his/her rival offers the same bidding price with his/her bidding price, that is, $b_j = b_i$, and in this case, bidder i has the largest losing regret. By making a comparison between the above two situations, bidder i eventually chooses a worse scenario which brings him/her a lower satisfaction as the focus point of b_i. The focus point is bidder i's imagining scenario which will happen when he /she offers a bidding price b_i.

Since bidder i's winning regret increases but losing regret decreases with increasing his/her bidding price, bidder i should make a trade-off between them when he/she decides the optimal bidding price. As a result, the optimal bidding price makes the bidder have equal satisfaction levels in winning and losing situations.

Clearly, the above explanation is intuitively acceptable and fits the psychological behavior of a conservative bidder.

From Theorem 1, the following properties can be easily obtained.

Proposition 2.

Bidder $i's$ optimal bidding price is an increasing function of his/her valuation v_i and his/her losing regret parameter $k_{i,2}$, respectively; and a decreasing function of his/her winning regret parameter $k_{i,1}$.

Proposition 2 shows that if a bidder highly evaluates the auctioned object then he/she will offer a high bidding price, which is also called the 'efficiency of auctions'; if a bidder values losing regret highly, then he/she will increase his/her bidding price to make winning more possible; if a bidder puts major emphasis on winning regret, then he/she will decrease his/her bidding price in order to avoid spending too much unnecessary money on the auctioned object. The above conclusions are quite consistent with the actual behavior of a bidder and meanwhile support the fundamental hypothesis of this research that for each bidder he/she faces a one-shot decision problem in which each bidder makes a decision based on his/her imagined scenario.

Comment. We have proposed two bidders' first-price sealed-bid auction models with the one-shot decision theory. In the proposed model, each bidder tries to obtain the optimal bidding price with conjecturing that the other bidder randomly decides his/her bidding price within $[0,1]$. This idea is the same as the level-k model, in which a level-1 bidder faces a level-0 bidder (Crawford & Iriberri, 2007). However, our models are scenario-based whereas the level-k auction models are lottery-based. Moreover, in the level-k auction model, regret is not considered and overbidding is attributed to higher level reasoning and bidding prices are raised in the following way: in order to win (or best respond to) the level-0 bidders, level-1 bidders should submit a higher price; in order to win the level-1 bidders, bidding prices of level-2 players should be even higher. Compared to it, it suffices to use level-1 bidders to rationalize overbidding in our model by considering regret. As shown in Proposition 2, overbidding can be caused by a relatively large losing regret parameter or a relatively small winning regret parameter, the same results also hold in the N-bidder's case, which will be shown in the following section.

4.3 N-bidders' First Price Sealed Bid Auction

We further extend the proposed model for the two bidders' case to the N-bidder's case. In the N-bidder's case, each bidder tries to optimize his/her bidding price with conjecturing that the other $N-1$ bidders randomly decide their bidding prices.

For bidder $i \in \{1,..., N\}$, his/her greatest concern is the highest price amongst the other bidders' bidding prices, denoted as b_j. Here we use the same symbol b_j as in the two bidders' case because for bidder i in essence there are always two prices competing, the one is his/hers and the other is the highest price of the other bidders. With the assumption that any bidder i thinks the other N-1 bidders offer the bidding prices which are mutually independently and distributed uniformly within $[0,1]$, the probability density function of b_j is as follows:

$$p_i(b_j) = \begin{cases} 0 & , \ b_j < 0 \ ; \\ (N-1)b_j^{\ N-2} & ,0 \le b_j \le 1; \\ 0 & , \ b_j > 1 \ ; \end{cases} \tag{4.23}$$

Like (4.7), we have the following normalized probability density function:

$$\pi_i(b_j) = \begin{cases} 0 & , \ b_j < 0 \ ; \\ b_j^{\ N-2} & ,0 \le b_j \le 1;. \\ 0 & , \ b_j > 1 \ ; \end{cases} \tag{4.24}$$

Knowing (4.24), the N-bidder's first-price sealed-bid auction problems become the two bidders' problems. Speaking in detail, the problem of each bidder facing $N-1$ other bidders whose bidding prices are mutually independently and uniformly distributed over $[0,1]$ can be reduced to the problem of each bidder facing another bidder with the normalized joint probability density function (4.24). Clearly, the satisfaction function (4.3) is still appropriate for the N-bidder's case. Using the same ideas as in 2-2, we have the following theorem for the optimal bidding price.

Theorem 3.

(I) If $v_i \le \sqrt[N-2]{1/(1+C)}$ $(N \ge 3)$, then bidder $i's$ optimal bidding price is $b_i^* \in [0, v_i]$;

(II) If $v_i > \sqrt[N-2]{1/(1+C)}$ $(N \ge 3)$, then bidder $i's$ optimal bidding price b_i^* satisfies the following equation:

$$\frac{-k_{i,2}(v_i - b_i^*) + Cv_i}{(1+C)v_i} = 1 - \left(\frac{(1+k_{i,1}+k_{i,2})b_i^* - (1+k_{i,2})v_i}{k_{i,1}}\right)^{N-2} . \tag{4.25}$$

Proof.

Firstly, let us also examine the focus point for each $b_i \in [0, v_i]$.

Since $u_i^2(b_j)$ is a strictly increasing continuous function and $1-(b_j)^{N-2}$ is a strictly decreasing continuous function, from the following two inequalities:

$$u_i^2(0) = (C - k_{i,2})/(1+C) < 1 = 1 - 0^{N-2}, \tag{4.26}$$

$$u_i^2(1) = (C - k_{i,2})v_i + k_{i,2})/(1+C)v_i > 0 = 1 - 1^{N-2}, \tag{4.27}$$

we know that there exists a unique solution of $u_i^2(b_j) = 1-(b_j)^{N-2}$ for $b_j \in [0,1]$ and denote it as $\overline{b_j}$. Considering the relation amongst v_i, b_i and $\overline{b_j}$, we have four cases, that is, $v_i \le \overline{b_j}$, $v_i > \overline{b_j} > b_i$, $v_i > b_i \ge \overline{b_j}$ and $v_i = b_i > \overline{b_j}$. And for each case, let us examine (4.9) for the following three subcases: (1) $b_j \in [0, b_i)$, (2) $b_j \in [b_i, v_i)$ and (3) $b_j \in [v_i, 1]$, respectively.

Case 1. $v_i \le \overline{b_j}$

For subcase (1), we can obtain

$$\min_{b_j \in [0,b_i)} \max(1-\pi_i(b_j,d), u_i(b_i,b_j)) \ge \min_{b_j \in [0,b_i)} (1-\pi_i(b_j)) = 1-(b_i)^{N-2}. \quad (4.28)$$

For subcase (2), considering the monotonicity of $1-\pi_i(b_j) = 1-(b_j)^{N-2}$ and $u_i(b_i,b_j) = u_i^2(b_j)$ and combining the fact that $\overline{b_j}$ is the unique solution of $u_i^2(b_j) = 1-(b_j)^{N-2}$, we can obtain

$$\min_{b_j \in [b_i,v_i)} \max(1-\pi_i(b_j), u_i(b_i,b_j)) = \min_{b_j \in [b_i,v_i)} (1-\pi_i(b_j)) = 1-(v_i)^{N-2} \ . \quad (4.29)$$

For subcase (3), As $u_i(b_i,b_j) \equiv C/(1+C)$ and Since $\overline{b_j}$ is the unique solution of $u_i^2(b_j) = 1-(b_j)^{N-2}$, we know the following relation holds as a result of $v_i \le \overline{b_j}$:

$$1-(v_i)^{N-2} \ge u_i^2(v_i) = C/(1+C). \quad (4.30)$$

Since $1-(b_j)^{N-2}$ is strictly decreasing and attains 0 at $b_j = 1$, with considering (4.30), we know there exists a unique $b_j^* \in [v_i, 1]$ such that the following three conditions hold:

$$1-(b_j^*)^{N-2} = C/(1+C), \quad (4.31)$$

$$1-(b_j)^{N-2} > C/(1+C) \ \text{ for } \ v_i \le b_j < b_j^*, \quad (4.32)$$

$$1-(b_j)^{N-2} < C/(1+C) \ \text{ for } \ b_j^* < b_j \le 1. \quad (4.33)$$

Let us divide $[v_i, 1]$ into $[v_i, b_j^*)$ and $[b_j^*, 1]$, then we have

$$\begin{aligned} &\min_{b_j \in [v_i,b_j^*)} \max(1-\pi_i(b_j), u_i(b_i,b_j)) = \\ &\min_{b_j \in [v_i,b_j^*)} (1-\pi_i(b_j)) = 1-(b_j^*)^{N-2} = C/(1+C) \end{aligned}, \quad (4.34)$$

$$\min_{b_j \in [b_j^*,1]} \max(1-\pi_i(b_j), u_i(b_i,b_j)) = \min_{b_j \in [b_j^*,1]} u_i(b_i,b_j) = C/(1+C). \quad (4.35)$$

(4.34) and (4.35) yield

$$\min_{b_j\in[v_i,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=C/(1+C)\,. \tag{4.36}$$

Since $b_i \le v_i$ always holds, it leads to $1-(v_i)^{N-2} \le 1-(b_i)^{N-2}$, and considering (4.28), (4.29), (4.30) and (4.36), we have

$$\min_{b_j\in[0,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=C/(1+C)\,, \tag{4.37}$$

$$b_j(b_i)\in\arg\min_{b_j\in[0,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=[b_j^*,1]\,. \tag{4.38}$$

Case 2. $v_i > \overline{b_j} > b_i$.

The subcase (1) of this case is the same with that of **Case 1.**

In the subcase (2), considering the monotonicity of $1-\pi_i(b_j)$ and $u_i(b_i,b_j)=u_i^2(b_j)$, it can be verified that the following relation holds:

$$\min_{b_j\in[b_i,v_i)}\max(1-\pi_i(b_j),u_i(b_i,b_j))=1-(\overline{b_j})^{N-2}=u_i^2(\overline{b_j})\,. \tag{4.39}$$

In the subcase (3), it is easy to check that (4.36) also holds. Utilizing the condition that $\overline{b_j} < v_i$ and considering the monotonicity of $1-\pi_i(b_j)$ and $u_i(b_i,b_j)$, we can obtain the following relation:

$$u_i^2(\overline{b_j}) < u_i^2(v_i) = C/(1+C)\,. \tag{4.40}$$

Considering (4.28), (4.36), (4.39) and (4.40), we know

$$\min_{b_j\in[0,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=1-(\overline{b_j})^{N-2}=u_i^2(\overline{b_j})\,, \tag{4.41}$$

$$b_j(b_i)\in\arg\min_{b_j\in[0,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=\overline{b_j}\,. \tag{4.42}$$

Case 3. $v_i > b_i \ge \overline{b_j}$.

In this case, for the subcase (1), Considering the monotonicity of $1-\pi_i(b_j)$ and

$u_i(b_i,b_j)=u_i^1(b_i,b_j)$, we know $\max(u_i^1(b_i,b_j),1-(b_j)^{N-2})$ attains its minimum when $u_i^1(b_i,\hat{b}_j)=1-(\hat{b}_j)^{N-2}$ where $\hat{b}_j$ is the solution of this equation. In other words, we have

$$\min_{b_j\in[0,b_i)}\max(1-\pi_i(b_j),u_i(b_i,b_j))=1-(\hat{b}_j)^{N-2}=u_i^1(b_i,\hat{b}_j). \tag{4.43}$$

For the subcase (2), we have

$$\min_{b_j\in[b_i,v_i)}\max(1-\pi_i(b_j),u_i(b_i,b_j))=\inf_{b_j\in[b_i,v_i)}u_i(b_i,b_j)=u_i^2(b_i). \tag{4.44}$$

For the subcase (3), we have

$$\begin{aligned}&\min_{b_j\in[v_i,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=\\&\min_{b_j\in[v_i,1]}\max(1-\pi_i(b_j),C/(1+C))=C/(1+C)\end{aligned}. \tag{4.45}$$

Since $u_i^1(b_i,b_j)$ is a strictly increasing function and $\hat{b}_j<b_i$, we have

$$u_i^1(b_i,\hat{b}_j)<u_i^1(b_i,b_i). \tag{4.46}$$

Meanwhile we know

$$u_i^2(b_i)=(-k_{i,2}(v_i-b_i)+Cv_i)/(1+C)v_i<C/(1+C). \tag{4.47}$$

It follows from (4.43)-(4.47) that

$$\min_{b_j\in[0,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=\begin{cases}u_i^2(b_i)\ , & u_i^1(b_i,\hat{b}_j)\ge u_i^2(b_i);\\ u_i^1(b_i,\hat{b}_j), & u_i^1(b_i,\hat{b}_j)<u_i^2(b_i);\end{cases}, \tag{4.48}$$

$$b_j(b_i)\in\begin{cases}\{b_i\} & ,\ u_i^1(b_i,\hat{b}_j)>u_i^2(b_i)\ ;\\ \{b_i,\hat{b}_j\} & ,\ u_i^1(b_i,\hat{b}_j)=u_i^2(b_i)\ ;\\ \{\hat{b}_j\} & ,\ u_i^1(b_i,\hat{b}_j)<u_i^2(b_i)\ ;\end{cases} \tag{4.49}$$

Case 4. $v_i=b_i>\overline{b_j}$.

In this case, subcases (1) – (3) are the same as Case 3 except that (4.44) becomes

$$u_i^2(b_i)=u_i^2(v_i)=(-k_{i,2}(v_i-v_i)+Cv_i)/(1+C)v_i=C/(1+C). \tag{4.50}$$

Instead of making a comparison between $u_i^1(b_i,\hat{b}_j)$ and $u_i^2(b_i)$, we need to make a comparison between $u_i^1(b_i,\hat{b}_j)$ and $C/(1+C)$, which leads to the following conclusion:

$$\min_{b_j\in[0,1]}\max(1-\pi_i(b_j),u_i(b_i,b_j))=\begin{cases}C/(1+C), & u_i^1(b_i,\hat{b}_j)\geq C/(1+C);\\ u_i^1(b_i,\hat{b}_j), & u_i^1(b_i,\hat{b}_j)<C/(1+C);\end{cases}, \tag{4.51}$$

$$b_j(b_i)\in\begin{cases}[v_i,1] & , \quad u_i^1(b_i,\hat{b}_j)>C/(1+C) \quad ;\\ \{\hat{b}_j\} & , \quad u_i^1(b_i,\hat{b}_j)<C/(1+C) \quad ;\\ \{\hat{b}_j\}\cup[v_i,1], & \quad u_i^1(b_i,\hat{b}_j)=C/(1+C) \quad ;\end{cases}. \tag{4.52}$$

Secondly, let us examine the optimal bidding price.

If $v_i\leq\overline{b_j}$, then for any $b_i\in[0,v_i]$ $b_j(b_i)\geq v_i$ so that we have $u_i(b_i,b_j(b_i),d(b_i))\equiv\frac{1}{2}$ and $b_i^*\in[0,v_i]$ which is Theorem 3(I).

If $v_i>\overline{b_j}$, let us examine $u_i(b_i,b_j(b_i))$ for $b_i\in[0,\overline{b_j})$ and $b_i\in[\overline{b_j},v_i]$, respectively.

In the former case, we know

$$u_i(b_i,b_j(b_i))\equiv u_i^2(\overline{b_j}). \tag{4.53}$$

In the latter case, we know

$$u_i(b_i,b_j(b_i))=\min[u_i^1(b_i,\hat{b}_j(b_i)),u_i^2(b_i)]. \tag{4.54}$$

For (4.54), on one hand, we have

$$u_i^1(\overline{b_j},\hat{b}_j(\overline{b_j}))=1-\pi_i(\hat{b}_j(\overline{b_j}))>1-\pi_i(\overline{b_j})=u_i^2(\overline{b_j}), \tag{4.55}$$

where the inequality in (4.55) holds as a result of $\hat{b}_j(\overline{b_j})<\overline{b_j}$; on the other hand, we have

$$u_i^1(v_i,\hat{b}_j(v_i))=(-k_{i,1}(v_i-\hat{b}_j(v_i))+Cv_i)\big/(1+C)v_i<C/(1+C)=u_i^2(v_i). \tag{4.56}$$

Further, let us check the continuity and monotonicity of $u_i^1(b_i,\hat{b}_j(b_i))$ and $u_i^2(b_i)$.

Obviously, $u_i^2(b_i)$ is a strictly increasing continuous function of b_i. For $u_i^1(b_i,\hat{b}_j(b_i))$, as $u_i^1(b_i,\hat{b}_j(b_i)) = 1-(\hat{b}_j(b_i))^{N-2}$ holds, differentiating both sides of the equation with respect to b_i we can obtain

$$\hat{b}_j'(b_i) = \frac{1+k_{i,1}}{k_{i,1}+(N-2)(1+C)(\hat{b}_j(b_i))^{N-3}v_i} > 0\,. \tag{4.57}$$

From (4.57) we know $\hat{b}_j(b_i)$ is a strictly increasing continuous function of b_i, which implies that $1-(\hat{b}_j(b_i))^{N-2}$ (and also $u_i^1(b_i,\hat{b}_j(b_i))$) is a strictly decreasing continuous function of b_i. From (4.55), (4.56) and the monotonicity of $u_i^1(b_i,\hat{b}_j(b_i))$ and $u_i^2(b_i)$, we know $\min(u_i^1(b_i,\hat{b}_j(b_i)),u_i^2(b_i))$ attains its maximum at b_i^* where b_i^* is the solution of $u_i^1(b_i,\hat{b}_j(b_i)) = u_i^2(b_i)$. As a result, the following conditions hold:

$$\max_{b_i\in[\overline{b_j},v_i]} u_i(b_i,b_j(b_i)) = u_i^1(b_i^*,\hat{b}_j(b_i^*)) = u_i^2(b_i^*)\,, \tag{4.58}$$

$$\arg\max_{b_i\in[\overline{b_j},v_i]} u_i(b_i,b_j(b_i)) = b_i^*\,. \tag{4.59}$$

As $b_i^* > \overline{b_j}$ holds and $u_i^2(b_j)$ is a strictly increasing function, we know

$$u_i^2(b_i^*) > u_i^2(\overline{b_j})\,. \tag{4.60}$$

It follows from (4.53) and (4.58)-(4.60) that the optimal bidding price is b_i^*, and b_i^* and $\hat{b}_j(b_i^*)$ satisfy

$$u_i^1(b_i^*,\hat{b}_j(b_i^*)) = u_i^2(b_i^*)\,, \tag{4.61}$$

$$u_i^1(b_i^*,\hat{b}_j(b_i^*)) = 1-(\hat{b}_j(b_i^*))^{N-2}\,. \tag{4.62}$$

From (4.61), we can obtain

$$\hat{b}_j(b_i^*) = \frac{(1+k_{i,1}+k_{i,2})b_i^* - (1+k_{i,2})v_i}{k_{i,1}}. \tag{4.63}$$

From (4.61) and (4.62) we know

$$u_i^2(b_i^*) = 1-(\hat{b}_j(b_i^*))^{N-2}. \tag{4.64}$$

Substituting (4.63) into (4.64) leads to (4.25).

Thirdly, let us show the following equivalence conditions to make our conclusion more tractable:

$$v_i < \overline{b_j} \Leftrightarrow v_i < \sqrt[N-2]{1/(1+C)} \tag{4.65}$$

As $(-k_{i,2}(v_i - \overline{b_j}) + Cv_i)/((1+C)v_i) = 1-(\overline{b_j})^{N-2}$ holds, by algebraic transformation, we obtain the following equation:

$$(1+k_{i,2})/(1+C) - (k_{i,2}\overline{b_j})/((1+C)v_i) = (\overline{b_j})^{N-2}. \tag{4.66}$$

If $v_i < \sqrt[N-2]{1/(1+C)}$ and $v_i \geq \overline{b_j}$, then the left side of (4.66) is

$$(1+k_{i,2})/(1+C) - (k_{i,2}\overline{b_j})/((1+C)v_i) \geq 1/(1+C), \tag{4.67}$$

and the right side of (4.66) is

$$(\overline{b_j})^{N-2} \leq (v_i)^{N-2} < 1/(1+C). \tag{4.68}$$

The contradiction between (4.67) and (4.68) shows that (4.65) holds.

It proves Theorem 3(II). □

From the proof of Theorem 3 (I) (Case 1), the following insights can be gained. When v_i is small ($v_i \leq \sqrt[N-2]{1/(1+C)}$), for any bidding price b_i, bidder i focuses on $b_j(b_i)$ (a bidding price provided by the rivals) which is larger than v_i. In this case, losing without regret is always the result for bidder i so that any $b_i \in [0, v_i]$ does not make any difference. Interestingly, it matches bidders' behaviors well. When a bidder's valuation is relatively small, several experimental findings have been reported in the literature. Firstly, the throw away phenomenon says that some subjects in first price auction experiments, upon drawing a low value enter a bid at (or near) zero, or less frequently, a bid at or near the value (Cox et al., 1992). Secondly, overbid (Pezanis-Christou, 2002) and underbid (Kirchkamp & Philipp, 2004) are also observed. In summary, we can say bidders with low valuation tend to bid randomly. However, there exists no model able to match such behaviors. Our model provides a better description of bidders' behaviors as well as an intuitive explanation.

As in two bidders' case, we examine the properties of the optimal bidding price for the N-bidder's case.

Proposition 4.

(I) The optimal bidding price b_i^* is a decreasing function of the winning regret parameter $k_{i,1}$ and an increasing function of the bidder's valuation v_i, the losing regret parameter $k_{i,2}$ and the number of bidders N, respectively.

(II) The optimal bidding price b_i^* satisfies $\frac{(1+k_{i,2})v_i}{1+k_{i,1}+k_{i,2}} < b_i^* < v_i$ when $N \geq 3$.

(III) The optimal bidding price $b_i^* \to v_i$ when $N \to \infty$.

Proof.

4(I): Denoting A as $((1+k_{i,1}+k_{i,2})b_i^* - (1+k_{i,2})v_i)/k_{i,1}$, (4.62), (4.63) together with the fact that $0 < u_i^1(b_i, b_j) < 1$ yield $A \in (0,1)$. From the proof of Theorem 3, we know that b_i^*

is the unique solution of $u_i^1(b_i, \hat{b}_j(b_i)) = u_i^2(b_i)$ within $[\overline{b_j}, v_i]$, combining the fact that $u_i^1(v_i, \hat{b}_j(v_i)) < u_i^2(v_i)$ (see (4.56)), we know $b_i^* < v_i$. Differentiating both sides of (4.14) with respect to v_i, $k_{i,1}$, $k_{i,2}$ and N, respectively, we obtain

$$\frac{\partial b_i^*}{\partial v_i} = \frac{k_{i,1}k_{i,2}b_i^* + (N-2)(1+C)v_i^2 A^{N-3}(1+k_{i,2})}{k_{i,1}k_{i,2}v_i + (N-2)(1+C)v_i^2 A^{N-3}(1+k_{i,1}+k_{i,2})} > 0, \tag{4.69}$$

$$\frac{\partial b_i^*}{\partial k_{i,1}} = \frac{(N-2)(1+C)v_i A^{N-3}(1+k_{i,2})(b_i^* - v_i)}{k_{i,1}^2 k_{i,2}v_i + (N-2)(1+C)k_{i,1}v_i A^{N-3}(1+k_{i,1}+k_{i,2})} < 0, \tag{4.70}$$

$$\frac{\partial b_i^*}{\partial k_{i,2}} = \frac{((N-2)(1+C)v_i A^{N-3}(1+k_{i,2}) + k_{i,1})(v_i - b_i^*)}{k_{i,1}k_{i,2}v_i + (N-2)(1+C)v_i A^{N-3}(1+k_{i,1}+k_{i,2})} > 0, \tag{4.71}$$

$$\frac{\partial b_i^*}{\partial N} = \frac{-b_i^*((1+k_{i,1}+k_{i,2})b_i^* - (1+k_{i,2})v_i)\ln A}{(1+k_{i,1}+k_{i,2})b_i^* - (1+k_{i,2})v_i + (N-2)b_i^*(1+k_{i,1}+k_{i,2})} > 0. \tag{4.72}$$

It proves 4 (I).

4 (II): On one hand, from the proof of Proposition 4 (I) we know $A > 0$ holds, which leads to $b_i^* > \frac{(1+k_{i,2})v_i}{1+k_{i,1}+k_{i,2}}$; on the other hand, it has been shown that $b_i^* < v_i$ in the proof of Proposition 4 (I). It proves 4 (II).

4 (III): We need to prove the following condition:

$$\forall \delta > 0, \quad \exists N^*(\delta), \quad \forall N > N^*(\delta), \quad b_i^*(N) > v_i - \delta, \tag{4.73}$$

where $b_i^*(N)$ is the solution of

$$\frac{-k_{i,2}(v_i - x) + Cv_i}{(1+C)v_i} = 1 - \left(\frac{(1+k_{i,1}+k_{i,2})x - (1+k_{i,2})v_i}{k_{i,1}}\right)^{N-2}. \tag{4.74}$$

On one hand, as $\frac{-k_{i,2}\delta + Cv_i}{(1+C)v_i} < 1$ and

$$\frac{(1+k_{i,1}+k_{i,2})(v_i-\delta)-(1+k_{i,2})v_i}{k_{i,1}}=\frac{k_{i,1}v_i-(1+k_{i,1}+k_{i,2})\delta}{k_{i,1}}<v_i\le 1,$$

$\forall\delta>0,\quad \exists N^*(\delta),\quad \forall N>N^*(\delta)$, the following condition holds:

$$\frac{-k_{i,2}(v_i-(v_i-\delta))+Cv_i}{(1+C)v_i}<1-\left(\frac{(1+k_{i,1}+k_{i,2})(v_i-\delta)-(1+k_{i,2})v_i}{k_{i,1}}\right)^{N-2};\tag{4.75}$$

On the other hand, $b_i^*(N)$ satisfies the following condition:

$$\frac{-k_{i,2}(v_i-b_i^*(N))+Cv_i}{(1+C)v_i}=1-\left(\frac{(1+k_{i,1}+k_{i,2})b_i^*(N)-(1+k_{i,2})v_i}{k_{i,1}}\right)^{N-2}.\tag{4.76}$$

Considering the monotonicity of both sides of (4.74), we know $b_i^*(N)>v_i-\delta$ from (4.75) and (4.76). It proves 4 (III). □

Proposition 4(I) provides us the same intuitive results as those in Proposition 2. From Proposition 2 and Proposition 4(I), we know whatever the number of bidders is, the bidder's optimal bidding price increases with increasing the losing regret parameter $k_{i,2}$ and decreasing the winning regret parameter $k_{i,1}$. These properties can serve as an alternative for rationalizing overbidding and underbidding behaviors. Overbidding says that bidders frequently offer a bidding price above the risk-neutral Bayesian Nash Equilibrium (BNE). Traditionally, such deviations are explained by theoretical models such as risk aversion, joy of winning, quantal response equilibrium, the level-k model, spiteful bidding etc. Not so often, bidders also underbid in some situations. Kirchkamp and Philipp argue that bidders that follow the simple rules-of-thumb may underbid at low valuations. Theoretically, such underbidding behavior is rationalized by the 'anticipated emotions' model.

In our model, we explain overbidding and underbidding as follows: overbidding and underbidding are caused by bidders' attitudes towards winning regret and losing regret. Speaking in detail, bidders will bid high if they value losing regret more and bid less if they value winning regret more. Such an explanation is straightforward and fits the mentality of the bidders in the real world.

Proposition 4(II) and 4(III) tell the effect of the bidders' number on an individual bidder's optimal bidding price. When $N \geq 3$, the bidder's optimal bidding price is higher than that in the case $N = 2$. The bidder's bidding price approaches his/her valuation with $N \to \infty$, which is consistent with the marketing law that an infinitely large number of players brings no gains to any involved player.

4.4 Summary

In this section, we provide a unified explanation for the deviations in first-price sealed-bid auction. Basically, we reformulate the auction problems as each individual bidder's decision problem under uncertainty which also takes regret into consideration. Procedurally, bidders are thought to focus on only one scenario, which is the one-shot decision theory based thinking. On one hand, when a bidder's evaluation is low, he/she focuses on a bidding price higher than his/her evaluation, which makes every bidding price indifferent to him/her and explains the throw away phenomenon; On the other hand, a bidder's bidding price is partially decided by his/her attitudes towards winning and losing regret, so overbidding and underbidding can be thought as a reflection of to what extent regret affects a bidder's decision.

Chapter 5

Conclusion

In this research, we propose the One-Shot Game Model to analyze players' behaviors in games. Generally speaking, we reformulate games as each individual player's decision making problem. Firstly, an individual player forms his/her beliefs about the other player(s)' actions, we mainly suggest three kinds of belief formulation. The first is that an involved player formulates his/her belief based on the payoff of his/her opponents. The second is that a player formulates his/her belief based on strategy dominance (capacity allocation game); the third is that a player simply assumes that the other players choose their actions with equal probability (sealed bid first price auction). Secondly, based on the formulated belief, the player undertakes his/her decision making process. Different from the Expected Utility Theory, the decision process within our framework is scenario based rather than lottery based. Simply speaking, when choosing an action, an involved player only focuses on a single scenario, under which he/she evaluates the chosen action. Within this framework, we explain the observed deviation from the predicted equilibrium in the capacity allocation game and rationalize bidders' bidding tendencies in sealed bid first price auction. We also examine the effect of players' personality on their behaviors as well as the result of the game. In some simple games, whatever an initial beliefs a player formulates, his/her decision is the same and is only determined by whether he/she is an active or a passive decision maker.

Although we utilize the One-Shot Game Model to make a reformulation of players' decision making process in games and show the efficiency of the proposed framework, two kinds of games are still unsolvable within our framework: One is the Prisoner's Dilemma, in which the dominated strategy for any prisoner is defect while cooperation is often reported; the other is dynamic games, solutions of which are obtained by backward induction but deviate from experimental findings. Inspired by those two kinds of games, we seek to improve our approach to handle games in more general horizon and leave it for the future work.

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