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SPACES OF CONTINUOUS LATTICE-HOMOMORPHISMS FROM $C_p(X)$ TO R

By

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Abstract. For a Tychonoff space X, the space $C_p(X)$ of all real-valued continuous functions with the topology of pointwise convergence is considered as a topological lattice. We study about the space $\diamond_p C_p(X)$ of all continuous lattice-homomorphisms from $C_p(X)$ to R with the topology of pointwise convergence. For example, the subspace of $\diamond_p C_p(X)$ consisting of open continuous lattice-homomorphisms is homeomorphic to the product space $X \times C_p^{si}(R)$, where $C_p^{si}(R)$ is the space of all strictly increasing continuous functions from R to itself with the topology of pointwise convergence.

1. Introduction

All topological spaces considered here are Tychonoff. For a space X, the set of all real-valued continuous functions of X is denoted by $C(X)$. We can consider various mathematical structures on this set $C(X)$. In this paper, we study about the topological lattice structure obtained by combining an order structure and a topological structure on $C(X)$. The order on $C(X)$ is defined as follows: $f \leq g$ if and only if $f(x) \leq g(x)$ at every point $x \in X$. Then it is well-known that $C(X)$ becomes a lattice under this ordering. The topololy on $C(X)$ considered here is the topology of pointwise convergence. That is, for $f \in C(X)$, basic open neighborhoods of f is given by

$$\langle f, \{x_1, \ldots, x_n\}, \varepsilon \rangle = \{g \in C(X) : |g(x_i) - f(x_i)| < \varepsilon,\ i = 1, \ldots, n\},$$

where $\{x_1, \ldots, x_n\}$ is an arbitrary finite subset of X and an arbitrary $\varepsilon > 0$. Concerning the topology of pointwise convergence on $C(X)$, we can consult [1]. The lattice $C(X)$ with this topology is denoted by $C_p(X)$. Usually $C_p(X)$ is considered as a linear topological space, but $C_p(X)$ is a topological lattice in this paper. Since the operations $\vee$ and $\wedge$ are continuous as maps from $C_p(X) \times C_p(X)$ to $C_p(X)$, we can call $C_p(X)$ to be a topological lattice. In the topological lattice $C_p(X)$ there are important subsets called open prime ideals. A subset I of the

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algebraic lattice $C(X)$ is said to be a prime ideal if the following conditions are satisfied:

1) If $f \in I$ and $g \leq f$, then $g \in I$.
2) If $f, g \in I$, then $f \vee g \in I$.
3) If $f \wedge g \in I$, then $f \in I$ or $g \in I$.
4) $I \neq \emptyset$, $I \neq C(X)$.

In case a prime ideal I of $C_p(X)$ is an open set, we call I to be an open prime ideal.

A prime ideal I of $C(X)$ is said to be associated with a point x in X if $f \in I$, $g \in C(X)$ and $g(x) < f(x)$ imply $g \in I$. The following lemma is useful in our argument.

LEMMA 1. *If I is an open prime ideal of $C_p(X)$, then there exists a unique point x in X such that I is associated with x.*

This result is found in [4] or derived from the similar argument used in the proof of the Kapansky Theorem in [2], [3].

Let L and M be lattices. A map $F : L \to M$ is called to be a lattice-homomorphism if $F(a \vee b) = F(a) \vee F(b)$ and $F(a \wedge b) = F(a) \wedge F(b)$ are satisfied for any $a, b \in L$. Since $C_p(X)$ and the real line $\boldsymbol{R}$ are topological lattices, we can consider the set of all continuous lattice-homomorphisms from $C_p(X)$ into $\boldsymbol{R}$. We denote this set by $\diamond C_p(X)$. The symbol $\diamond$ is obtained by joining $\vee$ with $\wedge$. Further, if the topology of pointwise convergence (i.e. the relative topology of $C_p(C_p(X))$) is given on $\diamond C_p(X)$, then this set becomes a topological space. This topological space is denoted by $\diamond_p C_p(X)$. The subspace of $\diamond_p C_p(X)$ consisting of all open continuous lattice-homomorphisms will be denoted by $\diamond_p^o C_p(X)$.

A function $f : \boldsymbol{R} \to \boldsymbol{R}$ is called to be increasing if $r \leq s$ implies $f(r) \leq f(s)$ for any $r, s \in \boldsymbol{R}$. In case $f(r) < f(s)$ is satisfied whenever $r < s$, the function f is called to be strictly increasing. The set of all increasing continuous functions and the set of all strictly increaring continuous functions are denoted by $C^i(\boldsymbol{R})$ and $C^{si}(\boldsymbol{R})$ respectively. When the relative topologies of $C_p(\boldsymbol{R})$ are given for these sets, we will denote these spaces by $C_p^i(\boldsymbol{R})$ and $C_p^{si}(\boldsymbol{R})$ respectively.

2. Open Continuous Lattice-homomorphisms

Let X be a topological space. For any point x in X, let $\Phi_x : C_p(X) \to \boldsymbol{R}$ be the map defined by $\Phi_x(f) = f(x)$ for $f \in C_p(X)$.

PROPOSITION 1. *The map Φ_x satisfies the following.*

(1) *Φ_x is continuous.*

(2) Φ_x *is open.*
(3) $\Phi_x(f \vee g) = \Phi_x(f) \vee \Phi_x(g)$ *and* $\Phi_x(f \wedge g) = \Phi_x(f) \wedge \Phi_x(g)$ *for any* $f, g \in C_p(X)$.

Generally we can show the following.

PROPOSITION 2. *A map* $F : C_p(X) \to \boldsymbol{R}$ *is an open continuous lattice-homomorphism if and only if there are a point* x *in* X *and an order-isomorphic (= lattice-isomorphic) homeomorphism* ϕ *from* $\boldsymbol{R}$ *into* $\boldsymbol{R}$ *which satisfy* $F = \phi \circ \Phi_x$. *Further the point* x *is uniquely determined by* F.

Proof. Let $F : C_p(X) \to \boldsymbol{R}$ be an open continuous lattice-homomorphism. Since $C_p(X)$ is connected, the image $F(C_p(X))$ is connected and hence $F(C_p(X))$ must be an open interval (a, b), where $a = -\infty$ or $b = \infty$ are possible. Let c be an arbitrary number in (a, b). Let

$$I_c = \{f \in C_p(X) : F(f) < c\}.$$

Then the following are obviously satisfied:

1) if $f \in I_c$ and $g \leq f$, then $g \in I_c$,
2) if $f, g \in I_c$, then $f \vee g \in I_c$,
3) if $f \wedge g \in I_c$, then $f \in I_c$ or $g \in I_c$,
4) $I_c \neq \emptyset$ and $I_c \neq C_p(X)$.

Since I_c is open in $C_p(X)$, I_c is an open prime ideal of $C_p(X)$. Then there must be a unique point $x \in X$ such that I_c is associated with x by the above lemma. Further this point x is not depend on the choice of the value c (see [4] or [2], [3]). So we can express this point x like x_F.

Now, let us define the map $\phi : \boldsymbol{R} \to \boldsymbol{R}$ which satisfies the condition $F = \phi \circ \Phi_{x_F}$. For an arbitrary number r in $\boldsymbol{R}$, let c_r be the constant real-valued function of X with the value r. Then the map ϕ is defined by

$$\phi(r) = F(c_r).$$

Since F is a lattice-homomorphism, ϕ is obviously increasing. The continuity of ϕ is also obvious because ϕ can be expressed as the composition of a continuous map from $\boldsymbol{R}$ into $C_p(X)$ and the continuous map F. Hence it suffices to show that ϕ is strictly increasing.

(1) if $f, g \in C(X)$ and $f(x_F) < g(x_F)$, then $F(f) \leq F(g)$.

In fact, assume that $F(f) > F(g)$. Let $c = F(f)$. If we take the open prime ideal $I_c = \{h \in C_p(X) : F(h) < c\}$, then $f \notin I_c$. But, since $g \in I_c$ and $f(x_F) < g(x_F)$, f must be a member of I_c. This is a contradiction.

(2) If $r_1 < r_2$, then $\phi(r_1) < \phi(r_2)$.

Assume that $\phi(r_1) = \phi(r_2)$. Let

$$U = \{f \in C_p(X) : r_1 < f(x_F) < r_2\}.$$

Then U is an open subset of $C_p(X)$. Since

$$c_{r_1}(x_F) < g(x_F) < c_{r_2}(x_F)$$

for any $g \in U$ and $F(c_{r_1}) = \phi(r_1) = \phi(r_2) = F(c_{r_2})$, $F(U)$ must be a one-point set in $\boldsymbol{R}$ by (1). But this is a contradiction since F is an open map.

It has been already proved that ϕ is a strictly increasing function. Further the above (1) shows that $F(f) = \phi(f(x_F))$ for any $f \in C(X)$. That is,

(3) $F = \phi \circ \Phi_{x_F}$.

The reverse implication is obvious since the composition of any two open continuous lattice-homomorphisms is an open continuous lattice-homomorphism.

THEOREM 1. *The space $\diamond_p^o C_p(X)$ is homeomorphic to the product space $X \times C_p^{si}(\boldsymbol{R})$.*

Proof. By the above proposition, we know that there is a one-to-one correspondence between $\diamond_p^o C_p(X)$ and $X \times C_p^{si}(\boldsymbol{R})$. That is,

$$F \longleftrightarrow (x, \phi)$$

where $F = \phi \circ \Phi_x$. We will show that this correspondence is a homeomorphism. It suffices to show the following two claims:

(1) For any subbasic open neighborhood

$$\langle F; \{f\}, \varepsilon\rangle = \{G \in \diamond_p^o C_p(X) : |G(f) - F(f)| < \varepsilon\}$$

of F in $\diamond_p^o C_p(X)$, there are an open neighborhood U of x in X and an open neighborhood V of ϕ in $C_p^{si}(\boldsymbol{R})$ such that $U \times V$ corresponds to a subset of $\langle F; \{f\}, \varepsilon\rangle$.

(2) For any open neighborhood U of x in X and any subbasic open neighborhood $\langle \phi; \{r\}, \varepsilon\rangle = \{\psi \in C_p^{si}(\boldsymbol{R}) : |\psi(r) - \phi(r)| < \varepsilon\}$ of ϕ in $C_p^{si}(\boldsymbol{R})$, there exists an open neighborhood W of F in $\diamond_p^o C_p(X)$ such that W corresponds to a subset of $U \times \langle \phi; \{r\}, \varepsilon\rangle$.

First, we show that (1) is true. Let r^-, r^+ be real numbers which satisfy the following:

$$r^- < f(x) < r^+$$
$$\phi(f(x)) - \varepsilon/2 < \phi(r^-), \phi(r^+) < \phi(f(x)) + \varepsilon/2.$$

Let $U = \{y \in X : r^- < f(y) < r^+\}$ and $V = \langle \phi; \{r^-, f(x), r^+\}, \varepsilon/2\rangle$.

In order to prove that $U \times V$ corresponds to a subset of $\langle F; \{f\}, \varepsilon\rangle$, we take an arbitrary $(y, \psi) \in U \times V$. Since $y \in U$,

$$r^- < f(y) < r^+.$$

Then

$$\psi(r^-) < \psi(f(y)) < \psi(r^+).$$

Since $\phi(r^-) - \varepsilon/2 < \psi(r^-)$ and $\psi(r^+) < \phi(r^+) + \varepsilon/2$, we have

$$\phi(r^-) - \varepsilon/2 < \psi(f(y)) < \phi(r^+) + \varepsilon/2.$$

Therefore, from $\phi(f(x)) - \varepsilon/2 < \phi(r^-)$ and $\phi(r^+) < \phi(f(x)) + \varepsilon/2$ it follows that

$$\phi(f(x)) - \varepsilon < \psi(f(y)) < \phi(f(x)) + \varepsilon.$$

Hence the lattice-homomorphism G corresponding to (y, ψ) satisfies that $|G(f) - F(f)| = |\psi(f(y)) - \phi(f(x))| < \varepsilon$. This means that $G \in \langle F; \{f\}, \varepsilon\rangle$.

Next, we show the claim (2). In this case we can assume that the image of ϕ includes the interval $(\phi(r) - \varepsilon, \phi(r) + \varepsilon)$. Let us take $r^- \in \boldsymbol{R}$ which satisfies

$$\phi(r^-) = \phi(r) - 2\varepsilon/3.$$

Then there exists a continuous function $f : X \to [r^-, r]$ such that

(a) $f(x) = r$,
(b) $f(y) = r^-$ for any $y \in X - U$.

Now let $c_{r^-}, c_r \in C_p(X)$ be the constant functions with the values r^-, r respectively. Then $f < c_r$ and

$$F(c_{r^-}) = \phi(c_{r^-}(x)) = \phi(r^-),$$
$$F(c_r) = \phi(c_r(x)) = \phi(r).$$

Let us take $\langle F; \{c_{r^-}, f, c_r\}, \varepsilon/3\rangle$ as an open neighborhood W of F in $\diamond_p^o C_p(X)$. Then for any $G \in W$, we can show that the corresponding element $(y, \psi) \in X \times C_p^{si}(\boldsymbol{R})$ satisfies

$$(y, \psi) \in U \times \langle \phi; \{r\}, \varepsilon\rangle.$$

Note that (y, ψ) is determined by $G(h) = \psi(h(y))$ for any $h \in C_p(X)$. Since $G(c_{r^-}) \in (F(c_{r^-}) - \varepsilon/3, F(c_{r^-}) + \varepsilon/3)$, it is satisfied that

$$G(c_{r^-}) \in (\phi(r^-) - \varepsilon/3, \phi(r^-) + \varepsilon/3).$$

Similarly we can show the following;

$$G(f) \in (\phi(r) - \varepsilon/3, \phi(r) + \varepsilon/3),$$
$$G(c_r) \in (\phi(r) - \varepsilon/3, \phi(r) + \varepsilon/3).$$

Therefore

$$G(c_{r^-}) < G(f) \leq G(c_r)$$

since $\phi(r^-) = \phi(r) - 2\varepsilon/3$. Hence

$$\psi(c_{r^-}(y)) < \psi(f(y)) \leq \psi(c_r(y)).$$

This means that $r^- < f(y)$ since ψ is strictly increasing. It follows that $y \in U$.

It remains to show that $\psi \in \langle \phi; \{r\}, \varepsilon \rangle$. Since $G(c_r) < F(c_r) + \varepsilon/3$, we have $\psi(r) < \phi(r) + \varepsilon/3$. On the other hand, from $\phi(r^-) - \varepsilon/3 < \psi(r^-)$, it follows that $\phi(r) - \varepsilon < \psi(r^-)$. Hence we conclude that $\phi(r) - \varepsilon < \psi(r) < \phi(r) + \varepsilon$. This shows that $\psi \in \langle \phi; \{r\}, \varepsilon \rangle$.

3. Continuous Lattice-homomorphisms

In this section, we will study about the space $\diamond_p C_p(X)$ of all continuous lattice-homomorphisms from $C_p(X)$ into $\boldsymbol{R}$, with the topology of pointwise convergence. In this case, the part of all constant lattice-homomorphisms has a peculiar quality. We denote this set of all constant lattice-homomorphisms by $Con(C_p(X))$.

First, we consider the subspace $\diamond_p C_p(X) - Con(C_p(X))$. By modifying the proof of Proposition 2, it is not difficult to show the following.

PROPOSITION 3. *A map $F : C_p(X) \to \boldsymbol{R}$ is a non-constant, continuous lattice-homomorphism if and only if there are a point $x \in X$ and a non-constant, increasing (= lattice-homomorphic) continuous function $\phi : \boldsymbol{R} \to \boldsymbol{R}$ which satisfy $F = \phi \circ \Phi_x$. Further the point x is uniquely determined by F.*

Then we can prove the following theorem by the similar argument as the proof of Theorem 1. Here the set of all constant functions from $\boldsymbol{R}$ into itself is denoted by $Con(\boldsymbol{R})$.

THEOREM 2. *The space $\diamond_p C_p(X) - Con(C_p(X))$ is homeomorphic to the product space $X \times (C_p^i(\boldsymbol{R}) - Con(\boldsymbol{R}))$.*

Next we consider the entire space $\diamond_p C_p(X)$. The set $\diamond C_p(X)$ is the disjoint sum of $\diamond C_p(X) - Con(C_p(X))$ and $Con(C_p(X))$. And it is obvious that the subspace $Con(C_p(X))$ of $\diamond_p C_p(X)$ is homeomorphic to the real line $\boldsymbol{R}$. Then it follows that the underlying set of $\diamond_p C_p(X)$ can be considered as the disjoint sum of the product $X \times (C_p^i(\boldsymbol{R}) - Con(\boldsymbol{R}))$ and the real line $\boldsymbol{R}$ by the above theorem. Here the constant map $c_a \in \diamond_p C_p(X)$ with the value a corresponds to the number $a \in \boldsymbol{R}$. Let us consider the following topology on $X \times (C_p^i(\boldsymbol{R}) - Con(\boldsymbol{R})) \cup \boldsymbol{R}$: The topology on $X \times (C_p^i(\boldsymbol{R} - Con(\boldsymbol{R}))$ is left as it is. For a point a in $\boldsymbol{R}$, we take the family of the following sets as a neighborhood base at a:

$$\bigcup\{U_n \times \langle a; \{-n, n\}, \varepsilon\rangle : n = 1, 2, \dots\} \cup (a - \varepsilon, a + \varepsilon),$$

where ε is an arbitrary positive number,

$$\langle a; \{-n, n\}, \varepsilon\rangle = \{\sigma \in C_p^i(\boldsymbol{R}) - Con(\boldsymbol{R}) : |\sigma(-n) - a| < \varepsilon, |\sigma(n) - a| < \varepsilon\}$$

and $\{U_n : n = 1, 2, \dots\}$ is a cover of X consisting of cozero-sets of X such that there is a sequence $\{Z_n : n = 1, 2, \dots\}$ of zero-sets which satisfies that $U_n \subset Z_n \subset U_{n+1}$ for each $n = 1, 2, \dots$. Let us denote this topological space by $(X \times C_p^i(\boldsymbol{R}))^\sim$.

THEOREM 3. *For any space X, the space $\diamond_p C_p(X)$ is homeohorphic to $(X \times C_p^i(\boldsymbol{R}))^\sim$.*

Proof. Let F be a constant lattice-homomorphism from $C_p(X)$ into $\boldsymbol{R}$ with the value a. It suffices to show the following:

(1) For any neighborhood W of F in $\diamond_p C_p(X)$, there are $\varepsilon > 0$ and a sequence $\{U_n : n = 1, 2, \dots\}$ of cozero-sets of X with the required property and such that the neighborhood $V = \bigcup\{U_n \times \langle a; \{-n, n\}, \varepsilon\rangle : n = 1, 2, \dots\} \cup (a - \varepsilon, a + \varepsilon)$ of a in $(X \times C_p^i(\boldsymbol{R})^\sim$ corresponds to a subset of W.

(2) For any neighborhood $V = \bigcup\{U_n \times \langle a; \{-n, n\}, \varepsilon\rangle : n = 1, 2, \dots\} \cup (a - \varepsilon, a + \varepsilon)$ of a in $(X \times C_p^i(\boldsymbol{R}))^\sim$, there is a neighborhood W of F in $\diamond_p C_p(X)$ which corresponds to a subset of V.

First we consider the claim (1). Let

$$W = \langle F; \{f_1, f_2, \dots, f_k\}, \varepsilon\rangle$$

be an arbitrary basic open neighborhood of F in $\diamond_p C_p(X)$. If we take $f^- = f_1 \wedge f_2 \wedge \dots \wedge f_k$ and $f^+ = f_1 \vee f_2 \vee \dots \vee f_k$ and consider the basic neighborhood

$$\langle F; \{f^-, f^+\}, \varepsilon\rangle$$

of F, then this neighborhood of F is a subset of W. Hence as a basic neighborhood W of F it suffices to consider the case when $k = 2$ and $f_1 \leq f_2$. Let

$$U_n = \{x \in X : -n < f_1(x), f_2(x) < n\}$$

and $Z_n = \{x \in X : -n \leq f_1(x), f_2(x) \leq n\}$. Then it is not difficult to show that $V = \bigcup\{U_n \times \langle a; \{-n, n\}, \varepsilon\rangle : n = 1, 2, \dots\} \cup (a - \varepsilon, a + \varepsilon)$ corresponds to a subset of W.

Next we show the claim (2). Let us take any neighborhood $V = \bigcup\{U_n \times \langle a; \{-n, n\}, \varepsilon\rangle : n = 1, 2, \dots\} \cup (a - \varepsilon, a + \varepsilon)$ of a in $(X \times C_p^i(\boldsymbol{R}))^\sim$. Since there are zero-sets $\{Z_n : n = 1, 2, \dots\}$ such that $U_n \subset Z_n \subset U_{n+1}$ for any n, there is a sequence $\{f_n : n = 1, 2, \dots\}$ in $C_p(X)$ such that

$$f_n^{-1}(n) = Z_n, f_n^{-1}(n+1) = X - U_{n+1}$$

and $f_n(X) \subset [n, n+1]$. For any $x \in X$, let k be the first number $k > 1$ which satisfies $x \in U_k$ and let $f^+(x) = f_{k-1}(x) + 1$. Then $f^+ \in C_p(X)$. Further let f^- be the function defined by $f^-(x) = -f^+(x)$ for any $x \in X$. Let us take $\langle F; \{f^-, f^+\}, \varepsilon\rangle$ as a neighborhood W of F. If $G \in \langle F; \{f^-, f^+\}, \varepsilon\rangle$ is non-constant, then the correspondent element (y, σ) in $X \times (C_p^i(\boldsymbol{R}) - Con(\boldsymbol{R}))$ is an element of V. In fact, there exists n such that $n \leq f^+(y) < n + 1$. Then $y \in U_n$ and $y \notin U_{n-1}$ since $f^+(y) = f_{n-1}(y) + 1$. From $f^-(y) \leq -n$ and $n \leq f^+(y)$, it follows that $G(f^-) \leq G(c_{-n})$ and $G(c_n) \leq G(f^+)$ by the same argument of (1) in Porposition 2. And hence $a - \varepsilon < \sigma(f^-(y)) \leq \sigma(-n)$ and $\sigma(n) \leq \sigma(f^+(y)) < a + \varepsilon$ are satisfied, since $G(f^-) = \sigma(f^-(y))$, $G(f^+) = \sigma(f^+)(y))$ and $a = F(f^-) = F(f^+)$. This means that $(y, \sigma) \in U_n \times \langle a; \{-n, n\}, \varepsilon\rangle$.

In case X is compact, we can show the following. Here c_a is the constant function with the value a.

Theorem 4. *Let X be a compact space. Then $\Diamond_p C_p(X)$ is homeomorphic to the quotient space $(X \times C_p^i(\boldsymbol{R}))/\{X \times \{c_a\} : a \in \boldsymbol{R}\}$ obtained by collapsing the closed subset $X \times \{c_a\}$ to a point for each $a \in \boldsymbol{R}$.*

Proof. Note that the underlying set of $(X \times C_p^i(\boldsymbol{R}))/\{X \times \{c_a\} : a \in \boldsymbol{R}\}$ corresponds canonically to the underlying set of $(X \times C_p^i(\boldsymbol{R}))^\sim$. Since the topology of $(X \times C_p^i(\boldsymbol{R}))^\sim$ is generally weaker than the quotient topology, it suffices to show the following:

$(*)$ If V is an open subset of $X \times C_p^i(\boldsymbol{R})$ including $X \times \{c_a\}$, then there are $\varepsilon > 0$ and $r^-, r^+ \in \boldsymbol{R}$ such that

$$X \times \{c_a\} \subset X \times \langle c_a; \{r^-, r^+\}, \varepsilon\rangle \subset V.$$

We can assume that $V = \bigcup\{U_x \times \langle c_a; \{r_x^-, r_x^+\}, \varepsilon_x\rangle : x \in X\}$, where $r_x^- < r_x^+$ and U_x is an open neighborhood of x in X. Then there are finite points $x_1, \dots, x_n \in X$ such that $U_{x_1} \cup \cdots \cup U_{x_n} = X$ since X is compact. Let $r^- = \min\{r_{x_1}^-, \dots, r_{x_n}^-\}$, $r^+ = \max\{r_{x_1}^+, \dots, r_{x_n}^+\}$ and $\varepsilon = \min\{\varepsilon_{x_n}, \dots, \varepsilon_{x_n}\}$. Then it is not difficult to see that $X \times \langle c_a; \{r^-, r^+\}, \varepsilon\rangle \subset V$.

The author does not know the answer of the following problem.

PROBLEM. In case $\diamond_p C_p(X)$ and $\diamond_p C_p(Y)$ are homeomorphic. Are X and Y homeomorphic?

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