YOKOHAMA MATHEMATICAL
JOURNAL VOL. 42, 1994

SOME GENERALIZATIONS OF SURJECTIVITY THEOREMS FOR COMPACT PERTURBATIONS OF *M*-ACCRETIVE OPERATORS

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(Received October 3, 1993)

Abstract. In this paper, we show some conditions that the range of compact perturbation of an m-accretive operator contains $B_r(o)$ for some $r>0$.

1. Introduction

Let E be a Banach space, let $A\subset E\times E$ be an m-accretive operator on E, and let C be a compact mapping. Several authors studied surjectivity problem for compact perturbation $A-C$ of the m-accretive operator A [2, 4-8, 11]. The following theorem [2] is one of them.

Theorem A (Chen). *Let E be a uniformly convex Banach space, let $A\subset E\times E$ be an m-accretive operator and let $C:\overline{D(A)}\to E$ be a compact mapping. Suppose that there exist a, b, $r>0$ such that*

$$\inf_{x\in D(A),\|x\|\geqq b}(|Ax-Cx|-a\|x\|)\geqq r \quad \text{and} \quad \sup_{x\in D(A),\|x\|\geqq b}\frac{\|Cx\|}{\|x\|}<a\,.$$

If $(A-C)(D(A)\cap\overline{B_b(o)})$ is closed, then $B_r(o)\subset R(A-C)$.

The object of this paper is to obtain some generalizations of the above theorem. In our results, the region, $\{x\in D(A): \|x\|\geqq b\}$ in the above theorem, where the infimum of some expression is taken, will be a bounded set $\{x\in D(A): \|x\|=b\}$. In Section 2, we give preliminary definitions. In Section 3, we study the surjectivity for compact perturbations of m-accretive operators under several conditions. Our proofs are very simple by using Browder-Potter's fixed point theorem, Rothe's fixed point theorem and Borsuck's theorem.

1991 Mathematics Subject Classification: Primary 47H10, 47H17; Secondary 35B20, 47B07, 47H06.

Key words and phrases: surjectivity theorems, compact perturbations, accretive operators, fixed point theorems.

2. Preliminaries

Let E be a Banach space and let E^* be its dual, that is, the space of all continuous linear functionals on E. The value of $f\in E^*$ at $x\in E$ will be denoted by $\langle x, f\rangle$. The duality mapping J from E into E^* is defined by

$$Jx=\{x^*\in E^* : \langle x, x^*\rangle=\|x\|\|x^*\|=\|x\|^2\}.$$

Let $A\subset E\times E$ be a (multivalued) operator. Then we define

$$D(A)=\{x\in E : Ax\neq\emptyset\}, \qquad R(A)=\cup\{Ax : x\in D(A)\},$$

and

$$|Ax|=\inf\{\|y\| : y\in Ax\} \qquad \text{for each} \quad x\in D(A).$$

A is said to be accretive if for each $x, y\in D(A)$ and for each $u\in Ax$, $v\in Ay$, there exists $j\in J(x-y)$ such that

$$\mathrm{Re}\langle u-v, j\rangle\geqq 0.$$

An accretive operator A is said to be m-accretive if

$$R(I+\lambda A)=E, \qquad \forall\lambda>0.$$

It is well known that if E is a uniformly convex Banach space and $A\subset E\times E$ is an m-accretive operacor then $\overline{D(A)}$ is a convex subset of E. Let A be an m-accretive operator on E. Then for any $\lambda>0$, we can define a single valued operator $(\lambda I+A)^{-1} : E\to D(A)$. It is known that $(\lambda I+A)^{-1}$ is Lipschitz continuous with Lipschitz constant $1/\lambda$. Let K be a subset of E. The symbols ∂K, $\overline{K}$, $\mathrm{co}K$, $\overline{\mathrm{co}}K$ denote the boundary of K, the closure of K, the convex hull of K, and the closed convex hull of K, respectively. A mapping $C : K\to E$ is said to be compact if it is continuous and it maps every bounded subset of K to relatively compact subset of E. A mapping $T : K\to E$ is said to be nonexpansive if

$$\|Tx-Ty\|\leqq\|x-y\|, \qquad \forall x, y\in K.$$

It is well known [1, Theorem 8.4] that if K is a bounded closed convex subset of a uniformly convex Banach space E and $T : K\to E$ is a nonexpansive mapping $I-T$ is demiclosed, i.e., $x_n\to x$ weakly and $x_n-Tx_n\to y$ strongly imply $x-Tx=y$. It is also well known that in a uniformly convex Banach space, an m-accretive operator is demiclosed. As generalized demiclosedness conditions concerning accretive operators, we know the following [11, Theorem 1], [8, Lemma 1].

Proposition 1 (Takahashi and Zhang). *Let* E *be a Banach space and* $f\in E$.

Let $A \subset E \times E$ be an accretive operator such that $\overline{co}D(A)+f \subset R(I+A)$ and each bounded closed convex subset of $\overline{co}D(A)$ has the fixed point property for non-expansive self-mappings. If there exist $(x_n, y_n) \in A$ such that $\{x_n\}$ is bounded and $y_n \to f$ strongly, then there exists $v \in D(A)$ such that $f \in Av$.

Proposition 2 (Kartsatos). *Let E be a uniformly convex Banach space, let $A \subset E \times E$ be an m-accretive operator and let $C: \overline{D(A)} \to E$ be a continuous mapping from the weak topology of E to the strong topology of E. Let $y \in E$ and let $b>0$. If there are a positive sequence $\{\beta_n\}$ with $\beta_n \to 0$ and a sequence $\{x_n\} \subset D(A) \cap \overline{B_b(o)}$ such that*

$$Cx_n + y \in \beta_n x_n + Ax_n, \qquad n=1, 2, \cdots,$$

then there exists $x_0 \in D(A) \cap \overline{B_b(o)}$ such that $y \in (A-C)x_0$.

The following is a special case of Dugundji's extension theorem (cf. [3, Theorem 7.2]), which is used in our proof.

Proposion 3. *Let X and Y be normed linear spaces, let $K \subset X$ be a closed subset and let $F: K \to Y$ be a continuous mapping. Then F has a continuous extension $\tilde{F}: X \to Y$ such that $\tilde{F}(X) \subset \mathrm{co}F(K)$.*

Throughout this paper, o denotes the orgin of a Banach space and $B_r(o)$ denotes the open ball with center o and radius $r>0$.

3. Main results

In the following theorem, we show several conditions which guarantee surjectivity for compact perturbations of m-accretive operators.

Theorem 1. *Let E be a Banach space, let $A \subset E \times E$ be an m-accretive operator and let $C: \overline{co}D(A) \to E$ be a compact mapping. Suppose that one of the following conditions is satisfied:*

(i) *$o \in D(A)$, and there exist positive constants b and r such that*

$$|Ao| < r \leqq \inf_{\substack{x \in D(A) \\ \|x\|=b}} \left(\max_{0 \leq \alpha \leq 1} |Ax - \alpha Cx| - \|Cx\|\right);$$

(ii) *$o \in D(A)$, and there exist positive constants b and r such that*

$$|Ao| < r \leqq \inf_{\substack{x \in D(A) \\ \|x\|=b}} (|Ax| - \|Cx\|);$$

(iii) *there exist positive constants a, b and r such that*

$$\inf_{\substack{x \in D(A) \\ \|x\|=b}} \left(\max_{0 \leq \alpha \leq 1} |Ax - \alpha Cx| - \|Cx\|\right) \geqq r \quad \text{and} \quad \inf_{\substack{x \in D(A) \\ \|x\|>b}} (|Ax| - a\|x\|) \geqq 0;$$

(iv) *$D(A)$ is symmetric, A and C are odd, and there exist positive constants b and r such that*

$$r \leqq \inf_{\substack{x\in D(A)\\ \|x\|=b}} |Ax-Cx| ;$$

(v) *$x\in D(A)$ implies $\mu x\in D(A)$ for any $\mu\in(0, 1)$, and there exist positive constants b and r such that*

$$\inf_{\substack{x\in D(A)\\ \|x\|=b\\ j\in Jx}} \sup_{w\in Ax} \mathrm{Re}\langle w-Cx, j\rangle \geqq rb .$$

Then for any $y\in B_r(o)$ and for any sufficiently small $\beta>0$, there exists $x\in D(A)\cap\overline{B_b(o)}$ such that $y-\beta x\in(A-C)x$.

Remark. If $\{x\in D(A): \|x\|=b\}=\phi$, then the conditions (i), (ii), (iv) and (v) are satisfied for any sufficiently large r.

Proof. Let I: $\overline{\mathrm{co}}D(A)\to E$ be the identity mapping. By Proposition 3, I has a continuous extension $\tilde{I}$: $E\to E$ such that $\tilde{I}(E)\subset\overline{\mathrm{co}}D(A)$. We denote $C\tilde{I}$ by $\tilde{C}$.

Case (i): Let $y\in B_r(o)$. Choose $s>0$ such that

$$\max\{|Ao|, \|y\|\}<s<r .$$

Let β be a positive number with $\beta<(r-s)/b$. From the definition of $|\cdot|$, there exists a point $z\in Ao$ with $\|z\|<s$. Let S_1: $\overline{B_b(o)}\times[0, 1]\to E$ be a compact mapping defined by

$$S_1(x, t)=(\beta I+A)^{-1}(t\tilde{C}x+ty+(1-t)z), \qquad (x, t)\in\overline{B_b(o)}\times[0, 1] .$$

First we show that

$$S_1(x, 0)=o, \qquad \forall x\in\partial\overline{B_b(o)} .$$

In fact, $z\in Ao$ implies $z\in(\beta I+A)o$, and hence for any $x\in\partial\overline{B_b(o)}$,

$$S_1(x, 0)=(\beta I+A)^{-1}z=o .$$

Next we show that for any $x\in\partial\overline{B_b(o)}$,

$$S_1(x, t)\neq x, \qquad \forall t\in[0, 1] .$$

Suppose that there exists a point $(x, t)\in\partial\overline{B_b(o)}\times[0, 1]$ such that $S_1(x, t)=x$. Then we have $\|x\|=b$, $x\in D(A)$ and

$$tCx+ty+(1-t)z\in\beta x+Ax . \qquad (*)$$

On the other hand, from (i), we get

$$
\begin{aligned}
s &\leqq \max_{0\leq\alpha\leq 1} |Ax-\alpha Cx| - \|Cx\| + s - r \\
&\leqq |\beta x + Ax - tCx| + \beta\|x\| + \max_{0\leq\alpha\leq 1} |t-\alpha|\,\|Cx\| - \|Cx\| + s - r \\
&\leqq |\beta x + Ax - tCx| + \beta b + s - r \\
&< |\beta x + Ax - tCx| .
\end{aligned}
$$

Since $\|ty+(1-t)z\|<s$, the inequality contradicts $(*)$. Hence, by Browder-Potter's fixed point theorem [9, Theorem 4.3.3], there exists a point $x\in\overline{B_b(o)}$ such that $S_1(x, 1)=x$. This implies that $x\in D(A)\cap\overline{B_b(o)}$ and $y+Cx\in\beta x+Ax$.

Case (ii): The condition (ii) is a special case of the condition (i).

Case (iii): Let $y\in B_r(o)$ and let β be a positive number with $\beta<\min\{a, (r-\|y\|)/b\}$. We define a compact mapping S_3: $\overline{B_b(o)}\times[0, 1]\to E$ by

$$
S_3(x, t)=(\beta I+A)^{-1}(t\tilde{C}x+ty), \qquad (x, t)\in\overline{B_b(o)}\times[0, 1].
$$

First we show that if $x\in\partial\overline{B_b(o)}$ then $\|S_3(x, 0)\|\leqq b$. Suppose that there exists a point $x\in\partial\overline{B_b(o)}$ such that $\|S_3(x, 0)\|>b$. From (iii), we get

$$
\begin{aligned}
0 &\leqq |AS_3(x, 0)| - a\|S_3(x, 0)\| \\
&\leqq |(\beta I+A)S_3(x, 0)| + (\beta-a)\|S_3(x, 0)\| \\
&< |(\beta I+A)S_3(x, 0)| .
\end{aligned}
$$

But this inequality contradicts $o\in(\beta I+A)S_3(x, 0)$. Next we show that for any $x\in\partial\overline{B_b(o)}$,

$$
S_3(x, t)\neq x, \qquad \forall t\in[0, 1].
$$

Suppose that there exists a point $(x, t)\in\partial\overline{B_b(o)}\times[0, 1]$ such that $S_3(x, t)=x$. Then we have $\|x\|=b$, $x\in D(A)$ and

$$
ty+tCx\in\beta x+Ax. \tag{**}
$$

On the other hand, from (iii), we get

$$
\begin{aligned}
\|y\| &\leqq \max_{0\leq\alpha\leq 1} |Ax-\alpha Cx| - \|Cx\| + \|y\| - r \\
&\leqq |\beta x + Ax - tCx| + \beta\|x\| + \max_{0\leq\alpha\leq 1} |t-\alpha|\,\|Cx\| - \|Cx\| + \|y\| - r \\
&\leqq |\beta x + Ax - tCx| + \beta b + \|y\| - r \\
&< |\beta x + Ax - tCx| .
\end{aligned}
$$

This inequality contradicts $(**)$. So from Browder-Potter's fixed point theorem, there exists a point $x\in\overline{B_b(o)}$ such that $S_3(x, 1)=x$. This implies that $x\in D(A)\cap\overline{B_b(o)}$ and $y+Cx\in\beta x+Ax$.

Case (iv): Let $y\in B_r(o)$, let β be a positive number with $\beta<(r-\|y\|)/b$

and let $\tilde{C}': E\to\overline{\text{co}}D(A)$ be a mapping defined by $\tilde{C}'x=(\tilde{C}x-\tilde{C}(-x))/2$ for $x\in E$. We define a mapping S_4: $\overline{B_b(o)}\times[0, 1]\to E$ by

$$S_4(x, t)=(\beta I+A)^{-1}(\tilde{C}'x+ty), \qquad (x, t)\in\overline{B_b(o)}\times[0, 1].$$

Suppose that there is a point $(x, t)\in\partial\overline{B_b(o)}\times[0, 1]$ such that $S_4(x, t)=x$. Then we have $\|x\|=b$, $x\in D(A)$ and

$$Cx+ty\in\beta x+Ax. \tag{***}$$

Since

$$\|ty-\beta x\|\leqq\|y\|+\beta\|x\|<\|y\|+r-\|y\|=r,$$

(***) contradicts (iv). Hence for any $(x, t)\in\partial\overline{B_b(o)}\times[0, 1]$, we have $S_4(x, t)\neq x$. Here we proved that $d(I-S_4(\cdot, 0), B_b(o), o)=d(I-S_4(\cdot, 1), B_b(o), o)$, where $d(I-S_4(\cdot, t), B_b(o), o)$ is the Leray-Scahuder degree of $I-S(\cdot, t)$ at o. Since $S_4(\cdot, 0)$ is odd and it has no fixed points on $\partial\overline{B_b(o)}$, by Borsuck's theorem [3, Theorem 8.3], $d(I-S_4(\cdot, 0), B_b(o), o)$ is an odd number. Hence there exists a point $x\in\overline{B_b(o)}$ such that $S_4(x, 1)=x$. This implies that $x\in D(A)\cap\overline{B_b(o)}$ and

$$y+Cx\in\beta x+Ax.$$

Case (v): Let $y\in\overline{B_r(o)}$ and let β be any positive number. Define a compact mapping S_5: $\overline{B_b(o)}\to E$ by

$$S_5(x)=(\beta I+A)^{-1}(\tilde{C}x+y), \qquad x\in\overline{B_b(o)}.$$

We show that if $x\in\partial\overline{B_b(o)}$, then $S_5(x)\neq\lambda x$ for any $\lambda>1$. Suppose that there exist a point x and $\lambda>1$ such that $\|x\|=b$ and $S_5(x)=\lambda x$. From (v), we have $x\in D(A)$, and hence

$$Cx+y\in\beta\lambda x+A(\lambda x).$$

Then there is a point $z\in A(\lambda x)$ such that

$$\beta\lambda x+z=Cx+y.$$

By (v) and Takahashi's minimax theorem [10, Theorem 8], we have

$$\sup_{w\in Ax}\inf_{j\in Jx}\text{Re}\langle w-Cx, j\rangle=\inf_{j\in Jx}\sup_{w\in Ax}\text{Re}\langle w-Cx, j\rangle\geqq rb.$$

So, for any $\varepsilon>0$, there exists $w\in Ax$ such that

$$\inf_{j\in Jx}\text{Re}\langle w-Cx, j\rangle\geqq rb-\varepsilon.$$

Since A is accretive, there also exists $j\in Jx$ such that

$$\text{Re}\langle z-w, j\rangle\geqq 0.$$

Hence we get

$$\|y\|b \geqq \mathrm{Re}\langle y, j\rangle = \mathrm{Re}\langle \beta\lambda x + z - Cx, j\rangle$$
$$\geqq \beta\lambda b^2 + \mathrm{Re}\langle w - Cx, j\rangle \geqq \beta\lambda b^2 + rb - \varepsilon .$$

Since $\varepsilon>0$ is arbitrary, we have $\|y\| \geqq r + \beta\lambda b$. This contradicts $\|y\| \leqq r$. By Rothe's fixed point theorem [9, Theorem 4.2.3], there is a point $x \in \overline{B_b(o)}$ such that $S_b(x) = x$. This implies that $x \in D(A) \cap \overline{B_b(o)}$ and $y - \beta x \in (A-C)x$. □

The condition (v) is weaker than the relevant condition of Theorem 2 in [4]. Using Theorem 1, we obtain some results. The following is Theorem 2 in [11].

Corollary 1 (Takahashi and Zhang). *Let E be a Banach space and let $A \subset E \times E$ be an m-accretive operator such that each nonempty bounded closed convex subset of $\overline{\mathrm{co}}D(A)$ has the fixed point property for nonexpansive self-mappings. Suppose that for some $x_0 \in D(A)$ and $r>0$,*

$$|Ax_0| < r \leqq \liminf_{\substack{\|x\|\to\infty \\ x\in D(A)}} |Ax| .$$

Then $B_r(o) \subset R(A)$.

Proof. Let $B \subset E \times E$ be an m-accretive operator defined by

$$D(B) = D(A) - x_0 \quad \text{and} \quad Bx = A(x + x^0), \quad x \in D(B).$$

Then it is easy to see that $o \in D(B)$ and

$$|B(o)| < r \leqq \liminf_{\substack{\|x\|\to\infty \\ x\in D(B)}} |Bx| .$$

For any $\varepsilon>0$, there exists $b>0$ such that

$$\inf_{\substack{\|x\|\geqq b \\ x\in D(B)}} |Bx| \geqq r - \varepsilon .$$

Let $y \in B_{r-\varepsilon}(o)$. By Theorem 1 with (ii), we have that for sufficiently small $\beta>0$, there exists $x \in D(A) \cap \overline{B_b(o)}$ such that $y - \beta x \in Ax$. Hence, by Proposition 1, we get $y \in R(B) = R(A)$. Since $\varepsilon>0$ is arbitrary, we obtain the stated result. □

The following is a slight generalization of Theorem A.

Corollary 2 (Chen). *Let E be a uniformly convex Banach space and let $A \subset E \times E$ be an m-accretive operator and let $C : \overline{D(A)} \to E$ be an compact mapping. Suppose that there exist positive constants a, b and r such that*

$$\inf_{\substack{\|x\|\geq b\\ x\in D(A)}}(|Ax-Cx|-a\|x\|)\geqq r \quad \textit{and} \quad \sup_{\substack{\|x\|\geq b\\ x\in D(A)}}\frac{\|Cx\|}{\|x\|}<a\,.$$

If $(A-C)(D(A)\cap\overline{B_b(o)})$ *is closed, then* $B_{r+(a-\gamma)b}(o)\subset R(A-C)$, *where*

$$\gamma=\sup_{\substack{\|x\|=b\\ x\in D(A)}}\frac{\|Cx\|}{\|x\|}.$$

Proof. Note that in a uniformly convex Banach space, $\overline{D(A)}$ is convex. Let

$$\beta=\sup_{\substack{\|x\|\geq b\\ x\in D(A)}}\frac{\|Cx\|}{\|x\|}<a\,.$$

Then $a-\beta>0$ and

$$\begin{aligned}r&\leqq\inf_{\substack{\|x\|\geq b\\ x\in D(A)}}(|Ax-Cx|-a\|x\|)\\&\leqq\inf_{\substack{\|x\|\geq b\\ x\in D(A)}}(|Ax|+\|Cx\|-a\|x\|)\\&\leqq\inf_{\substack{\|x\|\geq b\\ x\in D(A)}}(|Ax|-(a-\beta)\|x\|)\,.\end{aligned}$$

We also have

$$\begin{aligned}r+(a-\gamma)b&\leqq\inf_{\substack{\|x\|=b\\ x\in D(A)}}(|Ax-Cx|-\gamma\|x\|)\\&\leqq\inf_{\substack{\|x\|=b\\ x\in D(A)}}(|Ax-Cx|-\|Cx\|)\,.\end{aligned}$$

Since the condition (iii) of Theorem 1 holds, we get the stated result. □

From Theorem 1 and Proposition 2, we obtain the following theorem, since in a uniformly convex Banach space, the closure of the domain of an m-accretive operator is convex.

Theorem 2. *Let E be a uniformly convex Banach space, let $A\subset E\times E$ be an m-accretive operator and let $C:\overline{D(A)}\to E$ be a continuous mapping from the weak topology of E to the strong topology of E. Suppose that one of the following conditions is satisfied:*

(i) $o\in D(A)$, *and there exist positive constants b and r such that*

$$|Ao|<r\leqq\inf_{\substack{x\in D(A)\\ \|x\|=b}}(\max_{0\leq\alpha\leq1}|Ax-\alpha Cx|-\|Cx\|)\,;$$

(ii) $o\in D(A)$, *and there exist positive constants b and r such that*

$$|Ao|<r\leqq\inf_{\substack{x\in D(A)\\ \|x\|=b}}(|Ax|-\|Cx\|)\,;$$

(iii) *there exist positive constants a, b and r such that*

$$\inf_{\substack{x\in D(A)\\ \|x\|=b}}(\max_{0\leq\alpha\leq1}|Ax-\alpha Cx|-\|Cx\|)\geqq r \quad \textit{and} \quad \inf_{\substack{x\in D(A)\\ \|x\|>b}}(|Ax|-a\|x\|)\geqq0\,;$$

(iv) *$D(A)$ is symmetric, A and C are odd, and there exist positive constants b and r such that*

$$r \leqq \inf_{\substack{x \in D(A) \\ \|x\|=b}} |Ax - Cx| ;$$

(v) *$x \in D(A)$ implies $\mu x \in D(A)$ for any $\mu \in (0, 1)$, and there exist positive constants b and r such that*

$$\inf_{\substack{x \in D(A) \\ \|x\|=b}} \sup_{w \in Ax} \mathrm{Re} \langle w - Cx, Jx \rangle \geqq rb .$$

Then

$$B_r(o) \subset (A-C)(D(A) \cap \overline{B_b(o)}) .$$

Acknowledgement The author would like to express his hearty thanks to Professor Wataru Takahashi for encouragement and many helpful comments.

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