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CHARACTERIZATIONS OF GEODESIC HYPERSPHERES IN A COMPLEX PROJECTIVE SPACE IN TERMS OF RICCI TENSORS

By

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§0. Introduction

Let $P_n(C)$ be an n-dimensional complex projective space with Fubini-Study metric of constant holomorphic sectional curvature 4, and let M be a real hypersurface of $P_n(C)$. M has an almost contact metric structure (ϕ, ξ, η, g) induced from the complex structure J of $P_n(C)$ (see, §1). Many differential geometers have studied M (cf. [3], [9], [12] and [13]) by using the structure (ϕ, ξ, η, g).

Typical examples of real hypersurfaces in $P_n(C)$ are homogeneous ones. R. Takagi ([11]) showed that all homogeneous real hypersurfaces in $P_n(C)$ are realized as the tubes of constant radius over compact Hermitian symmetric spaces of rank 1 or 2. Namely, he showed the following:

Theorem A ([11]). *Let M be a homogeneous real hypersurface of $P_n(C)$. Then M is a tube of radius r over one of the following Kaehler submanifolds:*
(A_1) *hyperplane $P_{n-1}(C)$, where $0<r<\pi/2$,*
(A_2) *totally geodesic $P_k(C)$ $(1\leqq k\leqq n-2)$, where $0<r<\pi/2$,*
(B) *complex quadric Q_{n-1}, where $0<r<\pi/4$,*
(C) *$P_1(C)\times P_{(n-1)/2}(C)$, where $0<r<\pi/4$ and $n(\geqq 5)$ is odd,*
(D) *complex Grassmann $G_{2,5}(C)$, where $0<r<\pi/4$ and $n=9$,*
(E) *Hermitian symmetric space $SO(10)/U(5)$, where $0<r<\pi/4$ and $n=15$.*

Due to his classification we find the number of distinct constant principal curvatures of a homogeneous real hypersurface is 2, 3 or 5.

It is well-known that there does not exist a totally umbilic real hypersurface M in $P_n(C)$ (cf. [13]). We here recall Cecil and Ryan's work. The

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statement is as follows:

Theorem B ([1]). *Let M be a real hypersurface in $P_n(\boldsymbol{C})$, $n \geqq 3$, with at most two distinct principal curvatures at each point. Then M is locally congruent to a homogeneous real hypersurface of type* A_1.

Theorem B tells us that a geodesic hypersphere M in $P_n(\boldsymbol{C})$ (that is, a homogeneous real hypersurface M of type A_1 in $P_n(\boldsymbol{C})$) can be considered as the simplest one in the class of real hypersurfaces.

The main purpose of this paper is to provide some characterizations of geodesic hyperspheres in $P_n(\boldsymbol{C})$ in terms of Ricci tensors S. Now note that $P_n(\boldsymbol{C})$ ($n \geqq 3$) does not admit a real hypersuface M with parallel Ricci tensor S (cf. [2]). We characterize geodesic hyperspheres in $P_n(\boldsymbol{C})$ in terms of the derivative of S (cf. Theorem 1). In consequence of Theorem 1, we obtain an estimate of $\|\nabla S\|$ (that is, the length of the derivative of the Ricci tensor S), which characterizes geodesic hyperspheres in $P_n(\boldsymbol{C})$ (cf. Theorem 2).

Here we review the work of Kon, Cecil and Ryan. They determined η-Einstein real hypersurfaces M in $P_n(\boldsymbol{C})$. As a matter of course the condition "η-Einstein" is weaker than "Einstein". The statement is as follows:

Theorem C ([1], [7]). *Let M be a connected real hypersurface in $P_n(\boldsymbol{C})$, $n \geqq 3$, whose Ricci tensor S satisfies the identity $SX = aX + b\eta(X)\xi$, for some smooth functions a and b on M. Then M is locally congruent to one of the following:*

(1) *a geodesic hypersphere,*

(2) *a tube of radius r over a totally geodesic $P_k(\boldsymbol{C})$, $0 < k < n-1$, where $0 < r < \pi/2$ and $\cot^2 r = k/(n-k-1)$,*

(3) *a tube of radius r over a complex quadric Q_{n-1}, where $0 < r < \pi/4$ and $\cot^2 2r = n-2$.*

In § 3, we characterize η-Einstein real hypersurfaces in $P_n(\boldsymbol{C})$ by using an estimate of the length of the Ricci tensor S (cf. Theorem 3).

§ 1. Preliminaries

Let M be an orientable real hypersurface of $P_n(\boldsymbol{C})$ and let N be a unit normal vector field on M. The Riemannian connections $\tilde{\nabla}$ in $P_n(\boldsymbol{C})$ and ∇ in M are related by the following formulas for any vector fields X and Y on M:

$$\tilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \tag{1.1}$$

$$\tilde{\nabla}_X N = -AX, \tag{1.2}$$

where g denotes the Riemannian metric of M induced from the Fubini-Study metric G of $P_n(\boldsymbol{C})$ and A is the shape operator of M in $P_n(\boldsymbol{C})$. An eigenvector X of the shape operator A is called a *principal curvature vector*. Also an eigenvalue λ of A is called a *principal curvature*. In what follows, we denote by V_λ the eigenspace of A associated with eigenvalue λ. It is known that M has an almost contact metric structure induced from the complex structure J on $P_n(\boldsymbol{C})$, that is, we define a tensor field ϕ of type (1.1), a vector field ξ and a 1-form η on M by $g(\phi X, Y)=G(JX, Y)$ and $g(\xi, X)=\eta(X)=G(JX, N)$. Then we have

$$\phi^2 X=-X+\eta(X)\xi, \qquad g(\xi, \xi)=1, \qquad \phi\xi=0. \tag{1.3}$$

It follows from (1.1) that

$$(\nabla_X\phi)Y=\eta(Y)AX-g(AX, Y)\xi, \tag{1.4}$$

$$\nabla_X\xi=\phi AX. \tag{1.5}$$

Let $\tilde{R}$ and R be the curvature tensors of $P_n(\boldsymbol{C})$ and M, respectively. Since the curvature tensor $\tilde{R}$ has a nice form, we have the following Gauss and Codazzi equations:

$$\begin{aligned} g(R(X, Y)Z, W)=\;&g(Y, Z)g(X, W)-g(X, Z)g(Y, W)+g(\phi Y, Z)g(\phi X, W)\\ &-g(\phi X, Z)g(\phi Y, W)-2g(\phi X, Y)g(\phi Z, W)\\ &+g(AY, Z)g(AX, W)-g(AX, Z)g(AY, W), \end{aligned} \tag{1.6}$$

$$(\nabla_X A)Y-(\nabla_Y A)X=\eta(X)\phi Y-\eta(Y)\phi X-2g(\phi X, Y)\xi. \tag{1.7}$$

From (1.3), (1.5), (1.6) and (1.7) we get

$$SX=(2n+1)X-3\eta(X)\xi+hAX-A^2X, \tag{1.8}$$

$$\begin{aligned}(\nabla_X S)Y=\;&-3(g(\phi AX, Y)\xi+\eta(Y)\phi AX)+(Xh)AY\\ &+(hI-A)(\nabla_X A)Y-(\nabla_X A)AY, \end{aligned} \tag{1.9}$$

where $h=\operatorname{trace} A$, S is the Ricci tensor of type (1, 1) on M and I is the identity map.

We here recall the notion of an *η-parallel* Ricci tensor S of M, which is defined by $g((\nabla_X S)Y, Z)=0$ for any X, Y and Z orthogonal to ξ.

In the following, we use the same terminology and notations as above unless otherwise stated. Now we prepare without proof the following in order to prove our results:

Theorem D ([10]). *Let M be a real hypersurface of $P_n(\boldsymbol{C})$. Then the Ricci tensor of M is η-parallel and ξ is principal if and only if M is locally congruent to one of homogeneous real hypersurfaces of type* A_1, A_2 *and* B.

Theorem E ([6]). *Let M be a real hypersurface with constant mean curvature in $P_n(\boldsymbol{C})$. Suppose that ξ is a principal curvature vector and the corresponding principal curvature is non-zero. If $\nabla_\xi S=0$ (that is, the Ricci tensor S is parallel in the direction of ξ), then M is a tube of radius r over one of the following Kaehler submanifolds:*
(A_1) *hyperplane $P_{n-1}(\boldsymbol{C})$, where $0<r<\pi/2$ and $r\neq\pi/4$,*
(A_2) *totally geodesic $P_k(\boldsymbol{C})$ $(1\leqq k\leqq n-2)$, where $0<r<\pi/2$ and $r\neq\pi/4$,*
(B) *complex quadric Q_{n-1}, where $0<r<\pi/4$ and $\cot^2 2r=n-2$,*
(C) *$P_1(\boldsymbol{C})\times P_{(n-1)/2}(\boldsymbol{C})$, where $0<r<\pi/4$, $\cot^2 2r=1/(n-2)$ and $n(\geqq 5)$ is odd,*
(D) *complex Grassmann $G_{2,5}(\boldsymbol{C})$, where $0<r<\pi/4$, $\cot^2 2r=3/5$ and $n=9$,*
(E) *Hermitian symmetric space $SO(10)/U(5)$ where $0<r<\pi/4$, $\cot^2 2r=5/9$ and $n=15$.*

Theorem F ([8]). *Let M be a real hypersurface of $P_n(\boldsymbol{C})$. Then the following are equivalent:*
(i) *M is locally congruent to one of homogeneous ones of type A_1 and A_2,*
(ii) *$(\nabla_X A)Y=-\eta(Y)\phi X-g(\phi X, Y)\xi$ for any $X, Y\in TM$.*

Proposition A ([8]). *Assume that ξ is a principal curvature vector and the corresponding principal curvature is α. If $AX=rX$ for $X\perp\xi$, then we have*

$$A\phi X=((\alpha r+2)/(2r-\alpha))\phi X\,.$$

§2. Characterizations of geodesic hyperspheres in $P_n(\boldsymbol{C})$

We have the following

Theorem 1. *Let M be a real hypersurface of $P_n(\boldsymbol{C})$. Then the following are equivalent:*
(i) *The Ricci tensor S of M satisfies*

$$(2.1)\qquad (\nabla_X S)Y=c(g(\phi X, Y)\xi+\eta(Y)\phi X) \qquad \textit{for any } X, Y\in TM,$$

where c is a non-zero constant.
(ii) *M is locally congruent to a geodesic hypersphere in $P_n(\boldsymbol{C})$.*

Proof. Suppose that the condition (i) holds. First of all we shall show that the vector ξ is principal. From (2.1), (1.4) and (1.5), we have

$$(2.2)\quad (\nabla_W(\nabla_X S))Y-(\nabla_{\nabla_W X}S)Y=c\{\eta(X)g(AW, Y)\xi-2\eta(Y)g(AW, X)\xi + g(\phi X, Y)\phi AW+g(\phi AW, Y)\phi X+\eta(X)\eta(Y)AW\}.$$

Exchanging X and W in (2.2), we see

(2.3) $$(\nabla_X(\nabla_W S))Y-(\nabla_{\nabla_X W}S)Y=c\{\eta(W)g(AX, Y)\xi-2\eta(Y)g(AX, W)\xi + g(\phi W, Y)\phi AX+g(\phi AX, Y)\phi W+\eta(W)\eta(Y)AX\}.$$

It follows from (2.2) and (2.3) that

(2.4) $$(R(W, X)S)Y=c\{\eta(X)g(AW, Y)\xi-\eta(W)g(AX, Y)\xi + g(\phi X, Y)\phi AW-g(\phi W, Y)\phi AX+g(\phi AW, Y)\phi X - g(\phi AX, Y)\phi W+\eta(Y)(\eta(X)AW-\eta(W)AX)\},$$

where R is the curvature tensor of M.

Let $e_1, \cdots, e_{2n-1}$ be local fields of orthonormal vectors on M. From (2.4) and (1.3), we find

(2.5) $$\sum_{i=1}^{2n-1} g(R(e_i, X)S)Y, e_i)=c\{\eta(X)\eta(AY)-2\eta(Y)\eta(AX) - g(A\phi Y, \phi X)+h\eta(X)\eta(Y)\},$$

where $h=\text{trace}\,A$.

Now note that the left hand side of (2.5) is symmetric with respect to X, Y. (In fact, we see that

$$(\text{the left hand side of (2.5)})=\sum g(R(e_i, X)(SY), e_i)-\sum g(R(e_i, X)Y, Se_i) = g(SX, SY)-\sum g(R(e_i, X)Y, Se_i)$$

and

$$-\sum g(R(e_i, X)Y, Se_i)=\sum g(R(X, Y)e_i, Se_i)+\sum g(R(Y, e_i)X, Se_i) = \text{trace}\,(S\cdot R(X, Y))-\sum g(R(e_i, Y)X, Se_i) = -\sum g(R(e_i, Y)X, Se_i).)$$

And hence Equation (2.5) yields

$$c\{\eta(X)\eta(AY)-2\eta(Y)\eta(AX)\}=c\{\eta(Y)\eta(AX)-2\eta(X)\eta(AY)\}.$$

Since $c\neq 0$, the above equation shows

(2.6) $$\eta(X)\eta(AY)=\eta(Y)\eta(AX) \quad \text{for any } X, Y\in TM.$$

Equation (2.6) implies that ξ is principal. (In fact, let $A\xi=\alpha\xi+\beta U$, where U is a unit vector orthogonal to ξ. Putting $X=U$ and $Y=\xi$ in (2.6), we get $\beta=0$.)

Moreover, Equation (2.1) shows that the Ricci tensor of our real hypersurface M is η-parallel. Therefore Theorem D asserts that M is one of homogeneous real hypersurfaces of type A_1, A_2 and B. Next we shall check (2.1) for homogeneous real hypersurfaces of type A_1, A_2 and B one by one:

Let M be of type A_1. Setting $t=\cot r$ $(0<r<\pi/2)$, so that the shape operator A is as (cf. [12]):

(2.7) $$AX=tX-(1/t)\eta(X)\xi \quad \text{for } X\in TM.$$

Substituting the condition (ii) in Theorem F and (2.7) into (1.9), we can see that our real hypersurface M satisfies (2.1), that is,

(2.8) $$(\nabla_X S)Y=-2nt\{g(\phi X, Y)\xi+\eta(Y)\phi X\}.$$

Let M be of type A_2. Setting $t=\cot r$ $(0<r<\pi/2)$, so that our real hypersurface M has three distinct constant principal curvatures t (with multiplicity $2p$), $-1/t$ (with multiplicity $2q$) and $t-(1/t)$ (with multiplicity 1), where $p+q=n-1$ and $pq\neq 0$. Let X be a principal curvature (unit) vector orthogonal to ξ with principal curvature t. Note that $\phi X\in V_t$ (see, Proposition A). Substituting the condition (ii) in Theorem F into (1.9), we find

(2.9) $$(\nabla_X S)\phi X=\{-2(p+1)t+2q/t\}\xi.$$

Now let Y be a principal curvature (unit) vector orthogonal to ξ with principal curvature $-1/t$. By a similar computation we see

(2.10) $$(\nabla_Y S)\phi Y=\{-2pt+2(q+1)/t\}\xi.$$

From (2.9) and (2.10), we conclude that our manifold does not satisfy (2.1).

Let M be of type B. Setting $t=\cot r$ $(0<r<\pi/4)$, so that our real hypersurface M has three distinct constant principal curvatures $(1+t)/(1-t)$ (with multiplicity $n-1$), $(t-1)/(t+1)$ (with multiplicity $n-1$) and $t-(1/t)$ (with multiplicity 1). Suppose that the manifold M satisfies (2.1). And hence, in particular, the manifold M satisfies the hypothesis of Theorem E. Therefore we have only to consider the case of $t=\sqrt{n-1}+\sqrt{n-2}$. Let X be a principal curvature (unit) vector orthogonal to ξ with principal curvature $(1+t)/(1-t)$. Note that $\phi X\in V_{(t-1)/(t+1)}$ (see, Prop. A). From (1.9) we find (cf. (2.4) in [5])

(2.11) $$(\nabla_X S)\phi X=\{4n-2+2\sqrt{n-1}+4(n-1)\sqrt{n-1}\}/\sqrt{n-2}\cdot\xi.$$

Now let Y be a principal curvature (unit) vector orthogonal to ξ with principal curvature $(t-1)/(t+1)$. By a similar computation we see (cf. (2.5) in [5])

(2.12) $$(\nabla_Y S)\phi Y=\{4n-2-2\sqrt{n-1}-4(n-1)\sqrt{n-1}\}/\sqrt{n-2}\cdot\xi.$$

From (2.11) and (2.12) we conclude that our manifold does not satisfy (2.1), which is a contradiction. Q.E.D.

Motivated by Theorem 1, we prove the following

Theorem 2. *Let M be a real hypersurface with constant mean curvature in $P_n(\boldsymbol{C})$, $n\geqq 3$. Then the following inequality holds:*

(2.13) $$\|\nabla S\|^2\geqq 4n/(n-1)\cdot(h-\eta(A\xi))\{n(h-\eta(A\xi))-\mathrm{trace}(\phi A\nabla_\xi A)\},$$

where S is the Ricci tensor of M and $h=\mathrm{trace}\, A$.

Moreover, the equality of (2.13) holds if and only if M is locally congruent to a geodesic hypersphere of $P_n(\boldsymbol{C})$ provided that $\eta(A\xi)$ is constant.

Proof. Equation (2.8) shows that the derivative of the Ricci tensor S of a geodesic hypersphere is as:

(2.14) $$(\nabla_X S)Y=-n/(n-1)\cdot(h-\eta(A\xi))\{g(\phi X, Y)\xi+\eta(Y)\phi X\}.$$

Let $e_1, \cdots, e_{2n-1}$ be local fields of orthonormal vectors on M. Making use of (2.14), we define the following tensor on M as:

(2.15) $$T(X, Y)=(\nabla_X S)Y+n/(n-1)\cdot(h-\eta(A\xi))\{g(\phi X, Y)\xi+\eta(Y)\phi X\}.$$

Now we shall calculate the length of T. From (1.3) we have

(2.16) $$\|T\|^2=\|\nabla S\|^2+4n^2/(n-1)\cdot(h-\eta(A\xi))^2 +2n/(n-1)\cdot(h-\eta(A\xi))\sum_{i=1}^{2n-1} g((\nabla_{e_i}S)e_j, g(\phi e_i, e_j)\xi+\eta(e_j)\phi e_i).$$

It follows from (1.3), (1.7) and (1.9) that

(2.17) $$\sum g((\nabla_{e_i}S)e_j, g(\phi e_i, e_j)\xi+\eta(e_j)\phi e_i) =-4n(h-\eta(A\xi))+2\eta(A\phi(\mathrm{grad}\, h))+2\,\mathrm{trace}\,(\phi A\nabla_\xi A).$$

Therefore Inequality (2.13) follows from (2.16) and (2.17) provided that h is constant. Now we consider the equality of (2.13), so that the derivative of the Ricci tensor S is given by (2.14). Here we suppose that $\eta(A\xi)$ is constant and $n\geqq 3$. Then $h-\eta(A\xi)$ is a nonzero constant (cf. [2]). Hence, Theorem 1 shows that the equality of (2.13) holds if and only if M is locally congruent to a geodesic hypersphere. Q.E.D.

Remarks.

(1) In general, "Both trace A and $\eta(A\xi)$ are constant" does not imply "ξ is a principal curvature vector" (cf. § 3 of [4]).

(2) "ξ is principal" always implies "$\eta(A\xi)$ is constant" (cf. [8]).

(3) Suppose that both trace A and $\eta(A\xi)$ are constant. Then the following holds:

$$\mathrm{trace}\,(\phi A\nabla_\xi A)=3/2\cdot\eta(A\xi)\,\mathrm{tr}\,A^2-1/2\cdot\mathrm{tr}\,A\cdot\|A\xi\|^2-g(A^3\xi, \xi) +\mathrm{tr}\,(A\phi A\phi A)+2\,\mathrm{tr}\,A-(n+1)\eta(A\xi),$$

which shows that the right hand side of (2.13) is expressed in terms of the shape operator A.

§ 3. Characterization of η-Einstein real hypersurfaces in $P_n(C)$

Our aim here is to prove the following

Theorem 3. *Let M be a real hypersurface of $P_n(\boldsymbol{C})$, $n \geqq 3$. Then the following holds:*

$$\|S\|^2 \geqq (\eta(S\xi))^2 + (\rho - \eta(S\xi))^2/2(n-1), \tag{3.1}$$

where $\|S\|$ is the length of the Ricci tensor S of M and ρ is the scalar curvature of M. The equality of (3.1) *holds if and only if M is η-Einstein.*

Proof. We first remark that the following are equivalent:

$$SX = aX + b\eta(X)\xi \quad \text{for any } X \in TM, \tag{3.2}$$

(3.3) "$g(SX, Y) = \lambda g(X, Y)$ for any $X, Y \perp \xi$" and "ξ is an eigenvector of S".

We here rewrite the condition "$g(SX, Y) = \lambda g(X, Y)$ for any $X, Y \perp \xi$" as follows:

$$g(SX, Y) = \lambda g(X, Y) \quad \text{for any } X, Y \perp \xi .$$

$$\Leftrightarrow g(SX, Y) = \rho_0 g(X, Y) \quad \text{for any } X, Y \perp \xi, \text{ where } \rho_0 = 1/(2n-2)\cdot(\rho - g(S\xi, \xi)).$$

$$\Leftrightarrow g(SX - \eta(X)S\xi, Y - \eta(Y)\xi) = \rho_0 g(X - \eta(X)\xi, Y - \eta(Y)\xi) \quad \text{for any } X, Y \in TM.$$

$$\Leftrightarrow SX - \rho_0 X - \eta(X)S\xi - \eta(SX)\xi + (\rho_0 + \eta(S\xi))\eta(X)\xi = 0 \quad \text{for any } X \in TM.$$

Now we define the tensor T as follows:

$$T(X, Y) = g(SX, Y) - \rho_0 g(X, Y) - \eta(X)g(S\xi, Y) - \eta(SX)\eta(Y)$$
$$+ (\rho_0 + \eta(S\xi))\eta(X)\eta(Y) \quad \text{for any } X, Y \in TM .$$

Calculating the length of T, we find

$$\|T\|^2 = \|S\|^2 - (\rho - \eta(S\xi))^2/2(n-1) - 2\|S\xi\|^2 + (\eta(S\xi))^2 . \tag{3.4}$$

Note that for any real hypersurface M the following inequality holds

$$\|S\xi\|^2 \geqq (\eta(S\xi))^2 . \tag{3.5}$$

Hence (3.1) follows from (3.4) and (3.5). Of course the equality of (3.5) holds if and only if ξ is an eigenvector of the Ricci tensor S. Then we assert that the equality of (3.1) holds if and only if M is η-Einstein. Q.E.D.

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