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VIABLE SOLUTIONS FOR EVOLUTION INCLUSIONS

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1. Introduction

The purpose of this paper is to prove a viability theorem for a class of nonlinear evolution inclusions of parabolic type. Then we use that theorem to establish the existence of admissible "state-control" pairs for a class of nonlinear, distributed parameter, feedback control systems with state constraints.

We approach the solution to the viability problem through Galerkin approximations. This allows us to make use of well known finite dimensional results. This method was first used by Williamson [10] and later extended to a larger class of semilinear problems by Shuzhong [7]. Here we consider nonlinear systems extending this way both the above mentioned works.

We will be using mathematical setting of Lions [5]. Thus (X, H, X^*) will be a Gelfand triple of spaces, i.e. H is a separable Hilbert space and X a subspace of H carrying the structure of a separable, reflexive Banach space, which is continuously and densely embedded in H. Identifying H with its dual (pivot space), we get $X \hookrightarrow H \hookrightarrow X^*$, with all embeddings being continuous and dense. We will also assume that they are compact. To have an example in mind take $H=L^2(0, 1)$ and $X=H_0^1(0, 1)$. By $\|\cdot\|$ (resp. $|\cdot|$, $\|\cdot\|_*$), we will denote the norm of X(resp. of H, X^*). Also by $\langle\cdot, \cdot\rangle$ we will denote the duality brackets for the pair (X, X^*), while by $(\cdot, \cdot)$ we will denote the inner product in H. Recall that the two are compatible in the sense that if $x \in X \subseteq H$ and $h \in H \subseteq X^*$, then we have $\langle x, h\rangle=(x, x^*)$.

Given a Banach space Y, by $P_{fc}(Y)$ we will denote the family of nonempty, closed, (convex) subsets of Y. If V, W are Hausdorff topological spaces, then a multifunction $G: V \to 2^W \setminus \{\emptyset\}$ is said to be upper semicontinuous (u.s.c.) if and only if for every $\mathcal{V} \subseteq W$ open, $G^+(\mathcal{V})=\{v \in V : G(v) \subseteq \mathcal{V}\}$ is open.

A "projection scheme" for a Banach space Y is a sequence of finite-dimensional subspaces $Y_n \subseteq Y$ and a sequence of continuous linear projections $P_n : Y \to Y_n$ s.t. $P_n y \xrightarrow{s} y$ for every $y \in Y$. Evidently such a space is separable, since $Y=\overline{\bigcup_{n\geq 1} Y_n}$ and we have $\sup_{n\geq 1}|P_n|=k<\infty$ (see for example Deiming [3], p. 257).

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If K is a nonempty subset of Y and $x\in\overline{K}$, then by $T_K(x)$ we will denote the Bouligand tangent cone to K at x, which is defined by:

$$T_K(x)=\left\{h\in Y: \lim_{\lambda\downarrow 0}\frac{d_K(x+\lambda h)}{\lambda}=0\right\}.$$

(recall that for any $A\subseteqq Y$ and $x\in Y$, $d_A(x)=\inf\{\|x-\alpha\|:\alpha\in A\}$-the distance of x from the set A). This set is a nonempty cone in Y, not necessarily convex and $T_K(x)=T_{\overline{K}}(x)$. However if K is convex, then $T_K(x)$ is convex too.

Finally for $B\subseteqq Y$ nonempty, by $\sigma(\cdot, B)$ we will denote the support function of B i.e. $\sigma(x^*, B)=\sup\{(x^*, b): b\in B\}$ for all $x^*\in Y^*$.

2. Viable trajectories.

Let $T=[0, b]$ and (X, H, X^*) a Gelfand triple of spaces as in the introduction. The evolution inclusion under consideration is the following:

$$\left\{\begin{array}{l}\dot{x}(t)+Ax(t)\in F(x(t)) \quad \text{a.e.}\\ x(0)=x_0\in K,\ x(t)\in K\end{array}\right\} \quad (*)$$

We will make the following hypotheses concerning the data of problem $(*)$:

H_1: $A: X\to X^*$ is an operator s.t.

(a) $x\to Ax$ is weakly sequentially continuous, monotone,

(b) $\|Ax\|_*\leqq a(1+\|x\|)$, $a>0$,

(c) $\langle Ax, x\rangle\geqq b\|x\|^2$, $b>0$.

H_2: $K\subseteqq H$ is nonempty, bounded, closed and convex,

H_3: $F: K\to P_{fc}(X^*)$ is a multifunction s.t.

(a) $x\to F(x)$ is u.s.c.,

(b) $|F(x)|\leqq M$.

H_4: There exists $\{X_n^*, P_n\}_{n\geq 1}$ a projection scheme for X^* s.t. $P_nK=K\cap P_nX$.

H_5: For every $x\in K\cap X$, we have $(-Ax+F(x))\cap T_K(x)\neq\emptyset$.

By a solution of $(*)$ ("viable trajectory"), we mean a function $x(\cdot)\in W(T)=\{x(\cdot)\in L^2(X): \dot{x}(\cdot)\in L^2(X^*)\}$ s.t. $x(t)\in K$ for all $t\in T$. Recall (see Lions [5], theorem 1.1, p. 102 and Tanabe [8], lemma 5.5.1, p. 151) that $W(T)\subseteqq C(T, H)$.

Our viability theorem, reads as follows:

Theorem 1. *If hypotheses $H_1\to H_5$ hold and $x_0\in K\cap X$, then $(*)$ admits a viable trajectory.*

Proof. As we indicated in the introduction, our proof employs the Galerkin approximation technique. So using the projection scheme $(X_n^*, P_n)_{n\geq 1}$ for X^* existing by hypothesis H_4, we obtain the following sequences of finite dimensional viability problems (projected problems):

$$\left\{\begin{array}{ll}\dot{x}_n(t)+P_nAx_n(t)\in P_nF(x_n(t)) & \text{a. e.}\\ x_n(0)=P_nx_0=x_{0n},\ x_n(t)\in P_nK & \end{array}\right\}\quad (*)_n$$

From proposition 14, p. 173 of Aubin-Ekeland [1], we know that for every $x_n\in P_nK=K\cap P_nX$, we have $T_{P_nK}(x_n)=\overline{P_nT_K(x_n)}$. From hypothesis H_5, for every $x_n\in P_nK=K\cap P_nX$ we have:

$$\begin{aligned}&(-Ax_n+F(x_n))\cap T_K(x_n)\neq\emptyset\\ &\Longrightarrow P_n[(-Ax_n+F(x_n))\cap T_K(x_n)]\neq\emptyset\\ &\Longrightarrow(-P_nAx_n+P_nF(x_n))\cap P_nT_K(x_n)\neq\emptyset\\ &\Longrightarrow(-P_nAx_n+P_nF(x_n))\cap T_{P_nK}(x_n)\neq\emptyset\end{aligned}$$

So for the finite dimensional problem $(*)_n$, the tangential (Nagumo type) condition is satisfied and so we can apply corollary I-1 of Haddad [4] and get a viable trajectory $x_n(\cdot)$, $n\geqq 1$. Note that $x_n(\cdot)\in W(T)\subseteqq C(T, H)$ (see Barbu [2], theorem 4.2, p. 167). Our claim is that $\{x_n(\cdot)\}_{n\geq 1}$ is sequentially compact in $C(T, X_w)$. To this end, we have:

$$\langle\dot{x}_n(t), x_n(t)\rangle+\langle P_nAx_n(t), x_n(t)\rangle\leqq\sigma(x_n(t), P_nF(x_n(t)))\ \text{a.e.}$$

$$\Longrightarrow\frac{d}{dt}|x_n(t)|^2+2\langle P_nAx_n(t), x_n(t)\rangle\leqq 2k\|x_n(t)\|\,|F(x_n(t))|\ \text{a.e.}$$

$$\Longrightarrow\frac{d}{dt}|x_n(t)|^2+2\langle Ax_n(t), x_n(t)\rangle\leqq 2k\|x_n(t)\|\,|F(x_n(t))|\ \text{a.e.}$$

$$\Longrightarrow|x_n(b)|^2-|x_{0n}|^2+2b\|x_n(\cdot)\|^2_{L^2(X)}\leqq\int_0^b 2k\|x_n(t)\|\,|F(x_n(t))|\,dt$$

$$\leqq 2k\left[\int_0^b\|x_n(t)\|^2dt\right]^{1/2}\cdot\left[\int_0^b|F(x_n(t))|^2dt\right]^{1/2}$$

Applying Cauchy's inequality with $\varepsilon=b/k$ ($k=\sup_{n\geq 1}\|P_n\|\geqq 1$), we have:

$$|x_n(b)|^2-|x_{0n}|^2+2b\|x_n(\cdot)\|^2_{L^2(X)}\leqq b\|x_n(\cdot)\|^2_{L^2(X)}+k^2M^2$$

$$\Longrightarrow\|x_n(\cdot)\|^2_{L^2(X)}\leqq\frac{k^2M^2+|x_{0n}|^2}{b}$$

$\Longrightarrow\{x_n(\cdot)\}_{n\geq 1}$ is bounded, hence sequentially w-compact in $L^2(X)$

(because of the reflexivity of $L^2(X)$ and the Eberlein-Smulian theorem).

Next let $v(\cdot)\in L^2(X)=[L^2(X^*)]^*$. For every $n\geqq 1$ we have:

$$\begin{aligned}\int_0^b\langle \dot{x}_n(t), v(t)\rangle dt &= \int_0^b\langle -P_nAx_n(t), v(t)\rangle dt+\int_0^b\sigma(v(t), P_nF(x(t)))dt \\ &\leq \int_0^b\|P_nAx_n(t)\|_*\|v(t)\|dt+\int_0^b k\|v(t)\|\,|F(x_n(t))|\,dt \\ &\leq k\int_0^b\|Ax_n(t)\|_*\|v(t)\|dt+k\int_0^b\|v(t)\|\,|F(x_n(t))|\,dt \\ &\leq k\left[\int_0^b(\|Ax_n(t)\|_*+|F(x_n(t))|)^2dt\right]^{1/2}\cdot\|v\|_{L^2(X)} \\ &\leq \left[k\left(\int_0^b\|Ax_n(t)\|_*^2dt\right)^{1/2}+k\left(\int_0^b|F(x_n(t))|^2dt\right)^{1/2}\right]\cdot\|v\|_{L^2(X)} \\ &\leq k(ab^{1/2}+a\|x_n(\cdot)\|_{L^2(X)}+Mb^{1/2})\cdot\|v\|_{L^2(X)}\end{aligned}$$

But we saw that $\sup_{n\geq 1}\|x_n(\cdot)\|_{L^2(X)}=M_1<\infty$. Hence we have:

$$\|\dot{x}_n(\cdot)\|_{L^2(X^*)}\leq k(ab^{1/2}+aM_1+Mb^{1/2})=M_2 \quad n\geq 1$$

$\Rightarrow\{\dot{x}_n(\cdot)\}_{n\geq 1}$ is bounded in $L^2(X^*)$.

Next consider the space $R\subseteq L^2(X^*)$ defined by:

$$R=\left\{y(\cdot)\in L^2(X^*): \int_A y(s)ds\in X, \text{ for all } A\subseteq T \text{ Lebesgue measurable}\right\}.$$

Clearly $R(\cdot)$ is a linear subspace. Also yet $y_n\xrightarrow{s}y$ in $L^2(X^*)$, $y_n\in R$. Then for every $x\in X$ and $A\subseteq T$ Lebesgue measurable, we have:

$$\int_0^b\langle\chi_A(t)x, y_n(t)\rangle dt \longrightarrow \int_0^b\langle\chi_A(t)x, y(t)\rangle dt$$

$$\Longrightarrow\left\langle x, \int_A y_n(t)dt\right\rangle\longrightarrow\left\langle x, \int_A y(t)dt\right\rangle$$

Since $X\hookrightarrow X^*$ continuously and densely, we deduce that $\int_A y_n(t)dt\xrightarrow{w}\int_A y(t)dt$ in X and recall that X being reflexive is weakly sequentially complete. So for all $A\subseteq T$ Lebesgue measurable, we have $\int_A y(t)dt\in X\Rightarrow y\in R\Rightarrow R\subseteq L^2(X^*)$ is a reflexive, separable Banach space with the induced $L^2(X^*)$-norm topology.

Next for $A\subseteq T$ Lebesgue measurable, consider the operator $K(A): R\to X$ defined by:

$$K(A)(y)=\int_0^b\chi_A(t)y(t)dt$$

Observe that for every $x^*\in X^*$, $v(x^*)(y)=\langle x^*, K(A)y\rangle$ is continuous, linear on R. So by the Riesz representation theorem, there exists $g(x^*)(\cdot)\in L^2(X)=$

$[L^2(X^*)]^*$ s.t.

$$\langle x^*, K(A)(y)\rangle=\int_0^b \chi_A(x)\langle g(x^*)(t), y(t)\rangle dt$$

$$\leqq\left(\int_A \|g(x^*)(t)\|^2 dt\right)^{1/2}\cdot\|y\|_{L^2(X^*)}$$

Now recall (see for example Barbu [2], p. 167) that $x_n(\cdot)$ is X^*-absolutely continuous. So we have:

$$x_n(t+h)-x_n(t)=\int_t^{t+h}\dot{x}_n(s)ds,\ n\geqq 1,$$

$$\Longrightarrow x_n(\cdot)\in R,\ n\geqq 1$$

Let $A=[t, t+h]$. We have:

$$|\langle x^*, x_n(t+h)-x_n(t)\rangle|\leqq\left(\int_t^{t+h}\|g(x^*)(t)\|^2 dt\right)^{1/2}\cdot M_2$$

$$\Longrightarrow \{x_n(\cdot)\}_{n\geq 1}\subseteq C(T, X_w) \text{ is a } w\text{-equicontinuous set.}$$

Furthermore, given $x^*\in X^*$, we have for all $n\geqq 1$:

$$|\langle x^*, x_n(t)\rangle|\leqq\|x_{0n}\|\cdot\|x^*\|+\left(\int_0^b\|g(x^*)(t)\|^2 dt\right)^{1/2}$$

$$\leqq k\|x_0\|\cdot\|x^*\|+\left(\int_0^b\|g(x^*)(t)\|^2 dt\right)^{1/2}=M_3(x^*)$$

Thus from the uniform boundedness principle, we deduce that

$$\|x_n(t)\|\leqq M_4\quad n\geqq 1,\ t\in T$$

and recall that $B(0, M_4)=\{x\in X: \|x\|\leqq M_4\}$ is w-compact, since X is reflexive. So $\overline{\{x_n(t)\}_{n\geq 1}}^w$ is w-compact in X. Invoking the Arzela-Ascoli theorem, we conclude that $\{x_n(\cdot)\}_{n\geq 1}$ is relatively sequentially compact in $C(T, X_w)$. Hence by passing to a subsequence if necessary, we may assume that $x_n\to x$ in $C(T, X_w)$. Also since $\{\dot{x}_n(\cdot)\}_{n\geq 1}$ is bounded in $L^2(X^*)$, it is sequentially w-compact. Thus we may assume that $\dot{x}_n(\cdot)\xrightarrow{w} z(\cdot)$ in $L^2(X^*)$. It is clear that $z=\dot{x}\in L^2(X^*)$. Then we have:

$$\dot{x}_n(t)\in -P_nAx_n(t)+P_nF(x_n(t))\quad \text{a.e. } n\geqq 1.$$

For every $v\in X$ we have:

$$\langle -P_nAx_n(t), v\rangle=\langle -Ax_n(t), P_n^*v\rangle$$

But from Deimling [3] (p. 258), we know that $P_n^*v\xrightarrow{s}v$, while from the weak sequential continuity of $A(\cdot)$ (see hypothesis $H_1(a)$), we have:

$$-Ax_n(t) \xrightarrow{w} -Ax(t) \text{ in } X^* \Longrightarrow \langle -P_n Ax_n(t), v\rangle \longrightarrow \langle -Ax(t), v\rangle$$

$$\Longrightarrow P_n Ax_n(t) \xrightarrow{w} Ax(t) \text{ in } X^*$$

On the other hand we have:

$$\begin{aligned}\sigma(v, P_n F(x_n(t))) &= \sigma(P_n^* v, F(x_n(t))) \\ &= \sigma(P_n^* v, F(x_n(t))) - \sigma(v, F(x_n(t))) + \sigma(v, F(x_n(t))) \\ &\leqq \sigma(P_n^* v - v, F(x_n(t))) + \sigma(v, F(x_n(t))) \\ &\leqq M \cdot \|P_n^* v - v\| + \sigma(v, F(x_n(t)))\end{aligned}$$

$$\Longrightarrow \overline{\lim}\, \sigma(v, P_n F(x_n(t))) \leqq \overline{\lim}\, \sigma(v, F(x_n(t)))$$

Since $X \hookrightarrow H$ compactly, we have that $x_n(t) \xrightarrow{s} x(t)$ in H. Recall that by hypothesis $H_3(b)$, $F(\cdot)$ is u.s.c. on H. So $z \to \sigma(v, F(z))$ is u.s.c. Hence we get:

$$\overline{\lim}\, \sigma(v, P_n F(x_n(t))) \leqq \sigma(v, F(x(t)))$$

Then invoking proposition 4.1 of [6], we deduce that:

$$w - \overline{\lim}\, P_n F(x_n(t)) \subseteqq F(x(t)) \text{ a.e.}$$

Now through theorem 3.1 of [6], we have that:

$$\dot{x}(t) \in \overline{\text{conv}}\, w - \overline{\lim}\, (-P_n Ax_n(t) + P_n F(x_n)t))) \text{ a.e.}$$

$$\Longrightarrow \dot{x}(t) \in -Ax(t) + F(x(t)) \text{ a.e.}$$

and $x(t) \in K$ since by hypothesis $H_2 K \subseteqq H$ is closed.

Thus $x(\cdot)$ is the desired viable trajectory of (*). Q.E.D.

3. Trajectories of feedback systems with state constraints.

We can use the viability result proved in the previous section, to establish the existence of trajectories for a class of nonlinear, distributed parameter, feedback (closed loop) systems with state constraints.

So the closed loop, infinite dimensional control system, is governed by the following evolution equation:

$$\left\{\begin{array}{l} \dot{x}(t) + Ax(t) = B(x(t))u(t) \quad \text{a.e.} \\ x(0) = x_0,\ u(t) \in U(x(t)) \quad \text{a.e.},\ x(t) \in K \end{array}\right\} \quad (**)$$

Here Z is a separable, reflexive Banach space modelling the control space. We will need some new hypotheses concerning (**).

H'_3: $B : K \to \mathcal{L}(Z, X^*)$ s.t. $B(K)$ is bounded and $B^*(\cdot)$ is continuous from K into $\mathcal{L}(X, Z^*)$ with the strong operator topology,

H'_5: $U: K \to P_{fc}(Z)$ is an u.s.c. multifunction s.t. $|U(x)| \leqq \gamma$.
H'_6: For every $x \in K \cap X$, $C(x) = \{u \in U(x): -Ax + f(x, u) \in T_K(x)\} \neq \emptyset$.

We have the following existence result concerning (**).

Theorem 2. *If hypotheses H_1, H_2, H'_3, H_4, H'_5, H'_6 hold and $x_0 \in K \cap X$, then* (**) *admits a trajectory.*

Proof. Set $F(x) = B(x)U(x)$. Then $F: K \to P_{wkc}(X^*)$. For every $v \in X$, we have:

$$\sigma(v, F(x)) = \sigma(v, B(x)U(x)) = \sigma(B^*(x)v, U(x))$$

Now if $x_n \xrightarrow{s} x$ in $K \subseteqq H$, then we have:

$$\begin{aligned} &\sigma(B^*(x_n)v, U(x_n)) - \sigma(B^*(x)v, U(x_n)) + \sigma(B^*(x)v, U(x_n)) \\ &\leqq \sigma(B^*(x_n)v - B^*(x)v, U(x_n)) + \sigma(B^*(x)v, U(x_n)) \\ &\leqq M \cdot \|B^*(x_n)v - B^*(x)v\| + \sigma(B^*(x)v, U(x_n)) \end{aligned}$$

Because of hypothesis H'_3, we have that $\|B^*(x_n)v - B^*(x)v\| \to 0$ as $n \to \infty$. Also because of the upper semicontinuity hypothesis on $U(\cdot)$ (see H'_5), we have that $x \to \sigma(B^*(y)v, U(x))$ is u.s.c. for all $y \in K$, $v \in X$. So we have:

$$\begin{aligned} &\overline{\lim}\, \sigma(B^*(x)v, U(x_n)) \leqq \sigma(B^*(x)v, U(x)) \\ &\Longrightarrow \overline{\lim}\, \sigma(B^*(x_n)v, U(x_n)) \leqq \sigma(B^*(x)v, U(x)) \\ &\Longrightarrow x \to \sigma(v, B(x)U(x)) = \sigma(v, F(x)) \text{ is u.s.c.} \end{aligned}$$

Invoking theorem 10, p. 128 of Aubin-Ekeland [1], we conclude that $x \to F(x)$ is u.s.c. from $K \subseteqq H$ into X^*.

Now consider the following evolution inclusion:

$$\left. \begin{cases} \dot{x}(t) + Ax(t) \in F(x(t)) & \text{a.e.} \\ x(0) = x_0, \ x(t) \in K \end{cases} \right\} \quad (**)'$$

Note that because of H'_6, system (**)′ satisfies all the hypotheses of theorem 1. So (**)′ admits a viable trajectory $x(\cdot)$. Now let $L: T \to 2^Z$ be defined by:

$$L(t) = \{u \in U(x(t)): \dot{x}(t) + Ax(t) = B(x(t))u\}$$

From the definition of $F(x)$, $L(t) \neq \emptyset$ for all $t \in T \setminus N$, $\lambda(N) = 0$ where $\lambda(\cdot)$ is the Lebesgue measure on T. On N, redefine $L(\cdot)$ by setting $L(t) = \{0\}$, $t \in N$. So $L(t) \neq \emptyset$ for all $t \in T$. Let $g(t, u) = \dot{x}(t) + Ax(t) - B(x(t))u$ on $(T \setminus N) \times Z$ and $g(t, u) = 0$ on $N \times Z$. Clearly this is a Caratheodory function on $T \times Z$ (i.e. is Lebesgue measurable in t, continuous in u). Hence it is jointly measurable. Also since $U(\cdot)$ is u.s.c., the multifunction $t \to U(x(t))$ is graph measurable i.e. $\mathrm{Gr}\, U(x(\cdot)) = \{(t, z) \in T \times Z: z \in U(x(t))\} \in B(T) \times B(Z)$, where $B(T)$ is the Borel

σ-field of T and $B(Z)$ the Borel σ-field of Z. Then:

$$\text{Gr}\, L=\{(t, u)\in T\times Z : g(t, u)=0\}\cap \text{Gr}\, U(x(\cdot))\in \hat{B}(T)\times B(Z)$$

where $\hat{B}(T)$ is the Lebesgue σ-field of T (i.e. the completion of $B(T)$, with respect to the Lebesgue measure $\lambda(\cdot)$). Apply Aumann's selection theorem (see Wagner [9]), to get $u : T\to Z$ measurable s.t. $u(t)\in L(t)$ a.e. Clearly $(x(\cdot), u(\cdot))$ is the desired admissible pair for (**). Q.E.D.

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