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REMARKS ON ISOTROPIC IMMERSIONS

By

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0. Introduction

Recently, Ferus ([2], [3]) classified connected Riemannian manifolds with parallel second fundamental form in a real space form of constant curvature $\tilde{c}\geqq 0$.

In this note we may restrict our attention to isotropic submanifolds with parallel second fundamental form in the Euclidean sphere $S^m(k)$ of constant curvature k. Due to Ferus, we find that an isotropic submanifold M with parallel second fundamental form in $S^m(k)$ is locally congruent to one of compact symmetric spaces of rank one and the immersion is locally equivalent to the second or the first standard immersion according as M is a sphere or not. In Section 2, we characterize the first standard immersion of a quaternion projective space into a sphere in terms of isotropic immersions. We have

Theorem. *Let M be a real $4n$-dimensional connected quaternionic Kaehler manifold of constant Q-sectional curvature and $\tilde{M}$ be a $(4n+p)$-dimensional real space form of curvature $\tilde{c}>0$. If $p<2n^2+2n-1$ and M is an isotropic submanifold of $\tilde{M}$, then $p=2n^2-n-1$ or $p=2n^2-n$. Moreover, M is one of the following*:

(i) *M is locally congruent to a quaternion projective space which is immersed in $\tilde{M}$ through the first standard minimal immersion.*

(ii) *M is locally congruent to a quaternion projective space which is immersed in some totally umbilical hypersphere of $\tilde{M}$ through the first standard minimal immersion.*

In Section 3, we construct constant mean curvature submanifolds of a euclidean space.

1. Preliminaries

A Riemannian manifold of constant curvature is called a *real space form*. Let M be an n-dimensional submanifold of $\tilde{M}^{n+p}$ with metric g. We denote by ∇ and $\tilde{\nabla}$ the covariant differentiations on M and $\tilde{M}$, respectively. Then the second fundamental form B of the immersion is defined by $B(X, Y)=\tilde{\nabla}_X Y-\nabla_X Y$, where X and Y are vector fields tangent to M. We call $\mathfrak{h}=(1/n)(\mathrm{tr}\, B)$ the *mean curvature vector* of M in $\tilde{M}$. The *mean curvature* H of M in $\tilde{M}$ is the length of $\mathfrak{h}$. If $\mathfrak{h}$ is identically zero, the submanifold M is said to be *minimal*. The submanifold M is *totally umbilic* provided that $B(X, Y)=g(X, Y)\mathfrak{h}$ for all vector fields X and Y on M. In particular, if B vanishes identically, M is said to be a *totally geodesic* submanifold of $\tilde{M}$. For a vector field ξ normal to M, we write $\tilde{\nabla}_X\xi=-A_\xi X+D_X\xi$, where $-A_\xi X$ (resp. $D_X\xi$) denotes the tangential (resp. the normal) component of $\tilde{\nabla}_X\xi$. A normal vector field ξ is said to be *parallel* if $D_X\xi=0$ for each vector field X tangent to M. We define the covariant differentiation $\bar{\nabla}$ of the second fundamental form B with respect to the connection in (tangent bundle) + (normal bundle) as follows:

$$(\bar{\nabla}_X B)(Y, Z)=D_X(B(Y, Z))-B(\nabla_X Y, Z)-B(Y, \nabla_X Z)\,.$$

The second fundamental form B is said to be *parallel* if $(\bar{\nabla}_X B)(Y, Z)=0$ for all tangent vector fields X, Y and Z on M. Let $\xi_1, \cdots, \xi_p$ be an orthonormal basis of the normal bundle $T^\perp(M)$ and A_α be the second fundamental form with respect to ξ_α: $g(A_\alpha X, Y)=g(B(X, Y), \xi_\alpha)$. $\|B\|$ is the length of the second fundamental form B of the immersion so that $\|B\|^2=\sum_{\alpha=1}^p \operatorname{tr} A_\alpha^2$. The manifold M is said to be a (λ-)*isotropic* submanifold of $\tilde{M}$ provided that $\|B(X, X)\|$ is equal to a constant ($=\lambda$) for all unit tangent vectors X at each point. Let R and $\tilde{R}$ be the curvature tensors of M and $\tilde{M}$, respectively. For later use, we write Gauss and Codazzi equations respectively:

$$g(R(X, Y)Z, W)=g(\tilde{R}(X, Y)Z, W)+g(B(X, W), B(Y, Z)) -g(B(X, Z), B(Y, W))\,, \tag{1.1}$$

$$\{\tilde{R}(X, Y)Z\}^\perp=(\bar{\nabla}_X B)(Y, Z)-(\bar{\nabla}_Y B)(X, Z)\,, \tag{1.2}$$

where X, Y, Z and W are vector fields tangent to M and $\{*\}^\perp$ means the normal component of $\{*\}$.

We now recall a notion of CR-submanifolds M of a Kaehler manifold $\tilde{M}$ (cf. [1]). A submanifold M of a Kaehler manifold $\tilde{M}$ with complex structure J is called a *CR-submanifold* if there exists a C^∞-distribution $D: p\to D_p\subseteqq T_pM$ on M satisfying the following conditions:

(a) D is holomorphic (i.e., $JD_p=D_p$ at each point $p\in M$) and

(b) the complementary orthogonal distribution $D^\perp: p\to D_p^\perp\subseteqq T_pM$ is totally real (i.e., $JD_p^\perp\subseteqq T_p^\perp M$ at each point $p\in M$).

It follows from definition that all holomorphic submanifolds, totally real submanifolds and real hypersurfaces are necessarily CR-submanifolds. Here we prepare without proof the following Proposition 1.1 in order to prove our Theorem. For orthonormal vectors $X, Y\in T_XM$, we denote by $K(X, Y)$ (resp. $\tilde{K}(X, Y)$) the sectional curvature of the plane spanned by X and Y for M (resp. for $\tilde{M}$) and we put $\Delta_{XY}=K(X, Y)-\tilde{K}(X, Y)$.

We call Δ the discriminant at $x\in M$.

Proposition 1.1 ([7]). *Let M^n be a λ (>0)-isotropic submanifold in a Riemannian manifold $\tilde{M}^{n+p}$. Assume that the discriminant Δ at $x\in M$ is constant. Then the following inequalities hold at x:*

$$-((n+2)/2(n-1))\lambda^2 \leqq \Delta \leqq \lambda^2 .$$

Let N_x^1 be the first normal space at x of the above immersion, that is, the vector space spanned by all vectors $B(X, Y)$. Then we have

(1) $\Delta=\lambda^2 \Leftrightarrow M$ *is umbilic at* $x \Leftrightarrow \dim N_x^1=1$,

(2) $\Delta=-((n+2)/2(n-1))\lambda^2 \Leftrightarrow M$ *is minimal at* $x \Leftrightarrow \dim N_x^1=n(n+1)/2-1$

(3) $-((n+2)/2(n-1))\lambda^2<\Delta<\lambda^2 \Leftrightarrow \dim N_x^1=n(n+1)/2$.

A quaternionic Kaehler manifold of constant Q-sectional curvature is called a *quaternionic space form*. As is well-known (cf. [4]), the curvature tensor R of a quaternionic space form M of constant Q-sectional curvature $4c$ is given by

$$\begin{aligned}(1.3)\quad R(X, Y)Z=c\{&g(Y, Z)X-g(X, Z)Y+g(IY, Z)IX-g(IX, Z)IY\\ &+g(JY, Z)JX-g(JX, Z)JY+g(KY, Z)KX-g(KX, Z)KY\\ &+2g(X, IY)IZ+2g(X, JY)JZ+2g(X, KY)KZ\}\end{aligned}$$

for all vector fields X, Y and Z tangent to M, where $\{I, J, K\}$ is a canonical local basis of M.

Finally we establish the following Proposition 1.2 in order to prove our Theorem.

Proposition 1.2. *Let M be an n-dimensional quaternionic space form with canonical local basis $\{I, J, K\}$ which is isometrically immersed into a real space form $\tilde{M}^{4n+p}(\tilde{c})$. Suppose that $B(X, Y)=B(IX, IY)=B(JX, JY)=B(KX, KY)$ for all X, Y. Then the second fundamental form of the immersion is parallel.*

Proof. Now we recall (cf. [4])

$$\begin{aligned}\nabla_X I&=r(X)J-q(X)K ,\\ \nabla_X J&=-r(X)I+p(X)K ,\\ \nabla_X K&=q(X)I-p(X)J .\end{aligned}$$

So we have

$$\begin{aligned}(\bar{\nabla}_Z B)(IX, Y)&=D_Z(B(IX, Y))-B((\nabla_Z I)X, Y)-B(I(\nabla_Z X), Y)-B(IX, \nabla_Z Y)\\ &=-D_Z(B(X, IY))-r(Z)B(JX, Y)+q(Z)B(KX, Y)\\ &\quad -B(I(\nabla_Z X), Y)-B(IX, \nabla_Z Y)\\ &=-\{D_Z(B(X, IY))-B(\nabla_Z X, IY)-B(X, I(\nabla_Z Y))\\ &\quad -B(X, (r(Z)J-q(Z)K)Y)\}\\ &=-\{D_Z(B(X, IY))-B(\nabla_Z X, IY)-B(X, (\nabla_Z I)Y)-B(X, I(\nabla_Z Y))\}\\ &=-(\bar{\nabla}_Z B)(X, IY) .\end{aligned}$$

Then we find

$$(\bar{\nabla}_Z B)(IX, Y) = -(\bar{\nabla}_Z B)(X, IY). \tag{1.4}$$

On the other hand the Codazzi equation (1.2) is reduced to

$$(\bar{\nabla}_X B)(Y, Z) = (\bar{\nabla}_Y B)(X, Z). \tag{1.5}$$

By using (1.4) and (1.5) repeatedly, we get

$$(\bar{\nabla}_Z B)(X, Y) = -(\bar{\nabla}_Z B)(X, Y) \qquad \text{for any} \quad X, Y \text{ and } Z$$

so that $\bar{\nabla} B \equiv 0$ (for details, see [3]). Q.E.D.

2. Proof of Theorem

Let M be a real $4n$-dimensional quaternionic space form of constant Q-sectional curvature $4c$. The Gauss equation (1.1) is reduced to

$$\begin{aligned} &g(B(X, Y), B(Z, W)) - g(B(X, Z), B(Y, W)) \\ &= (c - \tilde{c})(g(W, Z)g(X, Y) - g(X, Z)g(W, Y)) \\ &\quad + c(g(IW, Z)g(IX, Y) - g(IX, Z)g(IW, Y) + g(JW, Z)g(JX, Y) \\ &\quad - g(JX, Z)g(JW, Y) + g(KW, Z)g(KX, Y) - g(KX, Z)g(KW, Y) \\ &\quad + 2g(X, IW)g(IZ, Y) + 2g(X, JW)g(JZ, Y) + 2g(X, KW)g(KZ, Y)). \end{aligned} \tag{2.1}$$

On the other hand, by the assumption that the immersion is isotropic, all normal curvature vectors at x have the same length, say, λ. Namely, we have $g(B(X, X), B(X, X)) = \lambda^2 g(X, X)g(X, X)$.

This is equivalent to

$$\begin{aligned} &g(B(X, Y), B(Z, W)) + g(B(X, Z), B(Y, W)) + g(B(X, W), B(Y, Z)) \\ &= \lambda^2(g(X, Y)g(Z, W) + g(X, Z)g(Y, W) + g(X, W)g(Y, Z)). \end{aligned} \tag{2.2}$$

From (2.1) and (2.2) we obtain (for details, see [6])

$$\begin{aligned} g(B(X, Y), B(Z, W)) &= (\lambda^2 + 2(c - \tilde{c}))/3 \cdot g(X, Y)g(Z, W) \\ &\quad + (\lambda^2 - (c - \tilde{c}))/3 \cdot (g(X, W)g(Y, Z) + g(X, Z)g(Y, W)) \\ &\quad + c(g(IX, Z)g(IY, W) + g(IY, Z)g(IX, W) + g(JX, Z)g(JY, W) \\ &\quad + g(JY, Z)g(JX, W) + g(KX, Z)g(KY, W) + g(KY, Z)g(KX, W)). \end{aligned} \tag{2.3}$$

Here we put $\Sigma = \{x \in M : \lambda(x) > 0\}$. Since a quaternionic space form cannot be immersed in a real space form as a totally geodesic submanifold, we see that the set Σ is an open dense subset of M. In the following, we study at a fixed point x of Σ. Now we investigate the first normal space by using (2.3). We choose a local field of orthonormal frame $e_1, \cdots, e_n, e_{n+1} = Ie_1, \cdots, e_{2n} = Ie_n, e_{2n+1} = Je_1, \cdots, e_{3n} = Je_n,$

$e_{3n+1}=Ke_1, \cdots, e_{4n}=Ke_n$ around x. From (1.3) we immediately find $g(R(e_i, e_j)e_j, e_i)=c$ for $1 \leqq i \neq j \leqq n$. So we may apply Proposition 1.1 to the linear subspace of T_xM, which is generated by $\{e_1, \cdots, e_n\}$. First we consider the case (1) of Proposition 1.1, that is, $\lambda^2=c-\tilde{c}$. From (2.2) we have

$$(2.4) \qquad 2g(B(e_i, e_j), B(e_i, e_j))+g(B(e_i, e_i), B(e_j, e_j))=(c-\tilde{c})(2\delta_{ij}\delta_{ij}+1),$$

where i and j run over the range $\{1, 2, \cdots, 4n\}$.

Hence the equation (2.4) yields

$$(2.5) \qquad 2\|B\|^2+16n^2H^2=8n(2n+1)(c-\tilde{c}),$$

where $\|B\|$ is the length of the second fundamental form B and H is the mean curvature of M. On the other hand the equation (2.1) shows

$$(2.6) \qquad \|B\|^2-16n^2H^2=4n(4n-1)\tilde{c}-16n(n+2)c.$$

As an immediate consequence of (2.5) and (2.6), we get $\|B\|^2=-4n(2c+\tilde{c})$. Since $\lambda^2=c-\tilde{c}>0$ and $\tilde{c}>0$, this is a contradiction. So we may find that $\lambda^2 \neq c-\tilde{c}$ and either the case (2) or the case (3) of Proposition 1.1 must happen at x. Moreover a straightforward calculation, by virtue of (2.3), yields the following orthogonal relations:

$$(2.7) \qquad \begin{aligned} g(B(e_i, e_j), B(e_k, Ie_l)) &= g(B(e_i, e_j), B(e_k, Je_l)) \\ &= g(B(e_i, e_j), B(e_k, Ke_l))=0 \\ &\text{for } 1 \leqq i \leqq j \leqq n \text{ and } 1 \leqq k < l \leqq n. \end{aligned}$$

$$(2.8) \qquad \begin{aligned} g(B(e_i, Ie_j), B(e_k, Je_l)) &= g(B(e_i, Je_j), B(e_k, Ke_l)) \\ &= g(B(e_i, Ke_j), B(e_k, Ie_l))=0 \\ &\text{for } 1 \leqq i < j \leqq n \text{ and } 1 \leqq k < l \leqq n. \end{aligned}$$

$$(2.9) \qquad \begin{aligned} g(B(e_i, Ie_j), B(e_k, Ie_l)) &= g(B(e_i, Je_j), B(e_k, Je_l)) \\ &= g(B(e_i, Ke_j), B(e_k, Ke_l)) \\ &= (\lambda^2-(c-\tilde{c}))/3 \cdot \delta_{ik}\delta_{jl} \\ &\text{for } 1 \leqq i < j \leqq n \text{ and } 1 \leqq k < l \leqq n. \end{aligned}$$

Then, in consideration of Proposition 1.1, (2.7), (2.8) and (2.9), we see that the codimension $p \geqq n(n+1)/2-1+3n(n-1)/2=2n^2-n-1$ at a fixed point x. Here we take $3n$ vectors $B(e_i, Ie_i)$, $B(e_i, Je_i)$ and $B(e_i, Ke_i)$ $(i=1, 2, \cdots, n)$. A similar calculation shows the following orthogonal relations:

$$(2.10) \qquad \begin{aligned} g(B(e_i, e_j), B(e_k, Ie_k)) &= g(B(e_i, e_j), B(e_k, Je_k)) \\ &= g(B(e_i, e_j), B(e_k, Ke_k))=0 \\ &\text{for } 1 \leqq i \leqq j \leqq n \text{ and } 1 \leqq k \leqq n. \end{aligned}$$

$$g(B(e_i, Ie_j), B(e_k, Ie_k)) = g(B(e_i, Je_j), B(e_k, Je_k)) = g(B(e_i, Ke_j), B(e_k, Ke_k)) = 0 \quad \text{for} \quad 1 \leqq i < j \leqq n \quad \text{and} \quad 1 \leqq k \leqq n. \tag{2.11}$$

$$g(B(e_i, Ie_i), B(e_j, Je_j)) = g(B(e_i, Je_i), B(e_j, Ke_j)) = g(B(e_i, Ke_i), B(e_j, Ie_j)) = 0 \quad \text{for} \quad i, j = 1, \cdots, n. \tag{2.12}$$

$$\begin{aligned} g(B(e_i, Ie_i), B(e_j, Je_k)) &= g(B(e_i, Ie_i), B(e_j, Ke_k)) \\ &= g(B(e_i, Je_i), B(e_j, Ie_k)) \\ &= g(B(e_i, Je_i), B(e_j, Ke_k)) \\ &= g(B(e_i, Ke_i), B(e_j, Ie_k)) \\ &= g(B(e_i, Ke_i), B(e_j, Je_k)) = 0 \\ &\quad \text{for} \quad 1 \leqq i \leqq n \quad \text{and} \quad 1 \leqq j < k \leqq n. \end{aligned} \tag{2.13}$$

$$\begin{aligned} g(B(e_i, Ie_i), B(e_j, Ie_j)) &= g(B(e_i, Je_i), B(e_j, Je_j)) \\ &= g(B(e_i, Ke_i), B(e_j, Ke_j)) \\ &= (\lambda^2 - (4c - \tilde{c}))/3 \cdot \delta_{ij} \\ &\quad \text{for} \quad i, j = 1, 2, \cdots, n. \end{aligned} \tag{2.14}$$

Now suppose that $\lambda^2 \neq 4c - \tilde{c}$. Then, in view of (2.10), (2.11), (2.12), (2.13) and (2.14), we find that the codimension $p \geqq (2n^2 - n - 1) + 3n = 2n^2 + 2n - 1$, which contradicts the assumption $p < 2n^2 + 2n - 1$. And hence we have

$$\lambda^2 = 4c - \tilde{c}. \tag{2.15}$$

Substituting (2.15) into the right-hand side of (2.3), we obtain

$$\begin{aligned} g(B(X, Y), B(Z, W)) = (2c - \tilde{c})g(X, Y)g(Z, W) + c(&g(X, W)g(Y, Z) \\ &+ g(X, Z)g(Y, W) + g(IX, Z)g(IY, W) \\ &+ g(IY, Z)g(IX, W) + g(JX, Z)g(JY, W) \\ &+ g(JY, Z)g(JX, W) + g(KX, Z)g(KY, W) \\ &+ g(KY, Z)g(KX, W)) \end{aligned} \tag{2.16}$$

The equation (2.16) shows the following:

$$\begin{aligned} g(B(X, Y), B(X, Y)) &= g(B(IX, IY), B(IX, IY)) \\ &= g(B(JX, JY), B(JX, JY)) \end{aligned} \tag{2.17}$$

$$
\begin{aligned}
&= g(B(KX, KY), B(KX, KY)) \\
&= (2c-\tilde{c})(g(X, Y))^2 + c\{(g(X, Y))^2 \\
&\quad + g(X, X)g(Y, Y) \\
&\quad - (g(X, IY))^2 - (g(X, JY))^2 - (g(X, KY))^2\} .
\end{aligned}
$$

$$
\begin{aligned}
(2.18)\qquad g(B(X, Y), B(IX, IY)) &= g(B(X, Y), B(JX, JY)) \\
&= g(B(X, Y), B(KX, KY)) \\
&= (2c-\tilde{c})(g(X, Y))^2 + c\{(g(X, Y))^2 \\
&\quad + g(X, X)g(Y, Y) - (g(X, IY))^2 \\
&\quad - (g(X, JY))^2 - (g(X, KY))^2\} .
\end{aligned}
$$

Thus, in consideration of (2.17) and (2.18), we see $B(X, Y) = B(IX, IY) = B(JX, JY) = B(KX, KY)$ for all X, Y. And hence, from Proposition 1.2 we find that the second fundamental form of our immersion is parallel on the open dense subset Σ of M so that $\bar{\nabla} B \equiv 0$ at each point of M. Therefore, due to the classification of parallel submanifolds, we obtain our conclusion (for details, see [3]). Q.E.D.

3. Constant mean curvature submanifolds of higher codimension

In this section we construct constant mean curvature submanifolds in a euclidean space. Let $j: M^k \to P^n(C)$ be a minimal immersion of a CR-submanifold M^k into an n-dimensional complex projective space with Fubini-Study metric of constant holomorphic sectional curvature 2. Let $z_0, z_1, \cdots, z_n$ be homogeneous coordinates of $z \in P^n(C)$ such that $z_0\bar{z}_0 + z_1\bar{z}_1 + \cdots + z_n\bar{z}_n = 1$. Let $S^N(c)$ denote an N-dimensional sphere of curvature c. We define an isometric imbedding m of $P^n(C)$ (with complex structure J) into the round sphere $S^{n(n+2)}(1)$, in this way: $m(z)$ is the point of $E^{(n+1)^2}$ with cartesian coordinates

$$
z_i\bar{z}_i, \quad \frac{1}{\sqrt{2}}(z_i\bar{z}_j + \bar{z}_i z_j), \quad \frac{\sqrt{-1}}{\sqrt{2}}(z_i\bar{z}_j - \bar{z}_i z_j) \qquad (i, j = 0, \cdots, n; i < j) .
$$

Then, by virtue of the following Proposition, we find that the isometric immersion $m \circ j$ is of constant mean curvature (not only in $E^{(n+1)^2}$, but also in $S^{n(n+2)}(1)$).

Proposition. *Let H be the mean curvature of M in $E^{(n+1)^2}$. Then $H = \sqrt{(k^2+k+2t)/k^2}$, where t $(=\dim_C D_p$ for any $p \in M)$ is a constant.*

Proof. Let $\{e_1, \cdots, e_t, e_{t+1} = Je_1, \cdots, e_{2t} = Je_t\}$ (resp. $\{e_{2t+1}, \cdots, e_k\}$) be a local field of orthonormal frame for D (resp. for $D^\perp$). We denote by B_1 (resp. by B_2) the second fundamental form of M^k in $P^n(C)$ (resp. of $P^n(C)$ in $E^{(n+1)^2}$). We first note that the second fundamental form B_2 is parallel and is described as follows (for details, see [5]):

$$g(B_2(X, Y), B_2(Z, W)) = g(X, Y)g(Z, W) + 1/2\cdot(g(X, W)g(Y, Z) + g(X, Z)g(Y, W) + g(JX, Z)g(JY, W) + g(JY, Z)g(JX, W)), \tag{3.1}$$

where X, Y, Z and W are vector fields on $P^n(C)$.

This yields that $B_2(JX, JY) = B_2(X, Y)$ for any X, Y. Since M is minimal in $P^n(C)$, the mean curvature vector $\mathfrak{h}$ of M in $E^{(n+1)^2}$ is expressed as follows:

$$\mathfrak{h} = \frac{1}{k}\sum_{i=1}^{k} B_2(e_i, e_i) = \frac{2}{k}\sum_{i=1}^{t} B_2(e_i, e_i) + \frac{1}{k}\sum_{i=2t+1}^{k} B_2(e_i, e_i).$$

Then, from (3.1) we have

$$H^2 = \|\mathfrak{h}\|^2 = (k^2 + k + 2t)/k^2. \qquad \text{Q.E.D.}$$

Remark. Due to the above Proposition, we see that the imbedding $m \circ j$ of the submanifold M into $E^{(n+1)^2}$ is of constant mean curvature. However the mean curvature vector $\mathfrak{h}$ is not necessarily parallel. To show this fact, we finally refer the following examples:

Example 1. Let M be a real $2k$-dimensional holomorphic submanifold of $P^n(C)$, that is, $t = k$. Then $H = \sqrt{(k-1)/k}$ and $\|D\mathfrak{h}\| = \|B_1\|/k$ (cf. [5]).

Example 2. Let M be an n-dimensional totally real minimal submanifold of $P^n(C)$, that is, $t = 0$.

Then $H = \sqrt{(n+1)/n}$ and $\|D\mathfrak{h}\| = 0$.

Example 3. Let M be a minimal real hypersurface of $P^n(C)$, that is, $t = n-1$. Then $H = \sqrt{4n^2 - 2}/(2n-1)$ and $\|D\mathfrak{h}\| \neq 0$.

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