YOKOHAMA MATHEMATICAL
JOURNAL VOL. 28, 1980

NOTES ON PSEUDO-CONFORMAL MANIFOLDS

By

HIRO-O MATSUDA

(Received Jan. 16, 1980)

§ 0. Introduction

A real submanifold M of a complex manifold generically inherits what is called a Cauchy-Riemann (or CR-) structure; that is, there is a subbundle HM of holomorphic tangent vectors of the tangent bundle of M having a complex vector space structure in each fibre H_pM for $p \in M$. When M is a real hypersurface with non-degenerate Levi form, this CR-structure is called a pseudo-conformal structure. (See [13].)

Pseudo-conformal structures are developed by Burns-Diederich-Shnider [2], Burns-Shnider [3] [4], Chern-Moser [5], Tanaka [10] [11], Webster [13] [14] [15] [16] [17] and so on.

S. Ishihara [6] studied the manifold of dimension $2n+1$ ($\geqq 3$) endowed with $(\mathscr{D}, J)$ where $\mathscr{D}$ is the subbundle of rank $2n$ (of the tangent bundle TM) with complex structure J. The method of his school ([6], [9]) depends on an almost contact structure (f, ξ, θ) associated with $\mathscr{D}$. In the point of view from a foliated structure, the vector field ξ defines a 1-dimensional foliation, so we shall have some similar results using a connection analogous to the Vaisman's second connection ([12]) on a Riemannian foliated manifold.

In § 1, we review the facts which was studied in [6], and in § 2, we construct the connection.

The author wishes to express his hearty thanks to Professor S. Ishihara who has given many valuable suggestions on the CR-manifolds.

§ 1. Preliminaries

Let M be a connected differentiable manifold of dimension $2n+1$ ($\geqq 3$) admitting with $(\mathscr{D}, J)$ where $\mathscr{D}$ is the subbundle of rank $2n$ of the tangent bundle TM of M with complex structure J. The pair $(\mathscr{D}, J)$ or $\mathscr{D}$, is called a hyperdistribution with complex structure J. Let (f, ξ, θ) be an almost contact structure associated with $\mathscr{D}$, that is, (f, ξ, θ) is an almost contact structure such that the 1-form θ anihilates $\mathscr{D}$ and the restriction of f to $\mathscr{D}$ coincides with J. Note that there are many choices of ξ, but if $[\xi, \Gamma(\mathscr{D})]\subset\Gamma(\mathscr{D})$ and $d\theta$ is non-degenerate on $\mathscr{D}$, ξ is uniquely determined such that $\theta(\xi)=1$. A tensor field S of type (1, 2) is defined by

$$S(X, Y)=[fX, fY]-f[fX, Y]-f[X, fY]+f^2[X, Y]+2d\theta(X, Y)\xi,$$

for $X, Y\in\Gamma(TM)$, where $\Gamma(TM)$ denotes the set of cross-sections of TM. Note that

$$S(X, Y)=[JX, JY]-[X, Y]-f[JX, Y]-f[X, JY],$$

for $X, Y\in\Gamma(\mathscr{D})$. $\mathscr{D}$ is called torsionless if $S(X, Y)=0$ for any $X, Y\in\Gamma(\mathscr{D})$. Set $\omega=-2d\theta$, and $G(X, Y)=\omega(fX, Y)$ for $X, Y\in\Gamma(TM)$. G is called the Levi tensor. Then, if $\mathscr{D}$ is torsionless, we have

$$G(X, Y)=G(Y, X) \quad \text{and} \quad G(JX, JY)=G(X, Y),$$

for any $X, Y\in\Gamma(\mathscr{D})$. In fact, using the formula (see [6])

$$G(X, Y)-G(Y, X)=\theta(S(fX, \xi)\theta(Y)+S(X, fY)),$$

the equality $G(X, Y)-G(Y, X)=0$ follows from $S(X, Y)=0$ and $\theta(Y)=0$. And,

$$\begin{aligned} G(JX, JY)&=-2d\theta(J(JX), JY)=-2d\theta(-X, JY)\\ &=-2d\theta(JY, X)=G(Y, X)=G(X, Y). \end{aligned}$$

We denotes the restriction of G to $\mathscr{D}$ by g. $\mathscr{D}$ is called to be non-degenerate if g is non-degenerate.

Almost contact structures (f, ξ, θ) and (f', ξ', θ') associated with $\mathscr{D}$ are said to be equivalent to each other if there exist a non-vanishing function α and vector field $A\in\Gamma(\mathscr{D})$ such that $f'=f-\theta\otimes A$, $\xi'=\frac{1}{\alpha}(\xi-f(A))$ and $\theta'=\alpha\theta$.

Remark 1. Let (f, ξ, θ) be an almost contact structure associated with $\mathscr{D}$. If $\mathscr{D}$ is torsionless and non-degenerate, there exists an almost contact structure (f', ξ', θ') such that $[\xi', \Gamma(\mathscr{D})]\subset\Gamma(\mathscr{D})$ associated with $\mathscr{D}$ which is equivalent to (f, ξ, θ) (see [9]). Note that $[\xi, \Gamma(\mathscr{D})]\subset\Gamma(\mathscr{D})$ is equivalent to $\mathscr{L}_\xi\theta=0$ or $\omega(\xi, X)=0$ for any $X\in\Gamma(\mathscr{D})$, where $\mathscr{L}$ denotes the Lie derivative operator.

In [6], S. Ishihara proved the following theorem.

Theorem. *Let M be a $2n+1$ $(\geqq 3)$ dimensional manifold admitting a hyperdistribution $\mathscr{D}$ with complex structure J. Then $\mathscr{D}$ determines a pseudo-conformal structure in M if it is torsionless and non-degenerate.*

From now on, we consider only manifolds $(M; \mathscr{D}, J)$ such that $\mathscr{D}$ is torsionless and non-degenerate, and assume that $[\xi, \Gamma(\mathscr{D})]\subset\Gamma(\mathscr{D})$.

§ 2. Construction of Connections

In the first, we shall recall the second connection of Vaisman on a Riemannian foliated manifold (see Vaisman [12]). Let (N, h) be a Riemannian foliated manifold with a Riemannian metric h and an integrable subbundle D_1. In this case, there is the subbundle D_2 of the subspaces of the tangent space of N which are orthogonal to those of D_1 with respect to h. Then Vaisman showed that there is a connection ∇ uniquely defined by the conditions:

(a) if $Y \in \Gamma(D_i)$, then $\nabla_X Y \in \Gamma(D_i)$ for every $X \in \Gamma(TN)$;

(b) if $X, Y, Z \in \Gamma(D_i)$, then $(\nabla_X h)(Y, Z)=0$;

(c) $T_i(X, Y)=0$ if at least one of the arguments is in $\Gamma(D_i)$, where T_i is the D_i-component of the torsion of ∇ $(i=1, 2)$.

Now, we consider $(M; \mathscr{D}, J)$ as above. Let Ξ be the subbundle of rank one defined by ξ. We define h by

$$h(X, Y)=g(X_{\mathscr{D}}, Y_{\mathscr{D}})+\theta(X)\theta(Y),$$

for $X, Y \in \Gamma(TM)$, where $Z_{\mathscr{D}}$ denotes the $\mathscr{D}$-component of $Z \in \Gamma(TM)$. Since M is a foliated manifold, there exists a local coordinate system $\{U, (x^0, x^1, \dots, x^{2n})\}$ such that

(i) $|x^A| \leqq 1$, $A=0, 1, \dots, 2n$;

(ii) the integral manifolds of Ξ are given locally by $x^1=c^1, \dots, x^{2n}=c^{2n}$ for constants c^a satisfying $|c^a| \leqq 1$, $a=1, \dots, 2n$ (cf. [8]).

Then $\{\theta, dx^1, \dots, dx^{2n}\}$ and $\{\partial/\partial x^0, v_1, \dots, v_{2n}\}$ are dual bases for the cotangent and tangent spaces at each point of U. Here $\theta=dx^0+\sum_{a=1}^{2n} t_a dx^a$ and $v_a=\partial/\partial x^a - t_a \cdot \partial/\partial x^0 \in \Gamma(\mathscr{D}|U)$ for suitably chosen functions t_a $(a=1, \dots, 2n)$. With respect to such a basis $\{\partial/\partial x^0, v_1, \dots, v_{2n}\}$, we have

$$(h_{AB})_{A,B=0,1,\dots,2n}=\begin{pmatrix} 1 & 0 \\ 0 & g_{ab} \end{pmatrix}$$

where $g_{ab}=g(v_a, v_b)$, $(a, b=1, \dots, 2n)$.

Since $h(\Gamma(\mathscr{D}), \Gamma(\Xi))=0$, we have that $TM=\mathscr{D}\oplus\Xi$. By this decomposition, we denote the $\mathscr{D}$-component (resp. Ξ-component) of $Z \in \Gamma(TM)$ by $Z_{\mathscr{D}}$ (resp. Z_{Ξ}).

Proposition 1. *There exists a connection ∇ uniquely defined by the conditions*:

(1) $\nabla_X Y \in \Gamma(\mathscr{D})$ *and* $\nabla_X V \in \Gamma(\Xi)$, *for any* $Y \in \Gamma(\mathscr{D})$, *any* $V \in \Gamma(\Xi)$ *and any* $X \in \Gamma(TM)$;

(2) $(\nabla_X h)(Y, Z)=0$ *for any* $X, Y, Z \in \Gamma(\mathscr{D})$ *or* $\Gamma(\Xi)$;

(3) $\tau_{\mathscr{D}}(X, Y)=0$, *if* $X\in\Gamma(\mathscr{D})$ *or* $Y\in\Gamma(\mathscr{D})$, *and* $\tau_{\Xi}(X, Y)=0$, *if* $X\in\Gamma(\Xi)$ *or* $Y\in\Gamma(\Xi)$, *where* τ *is the torsion tensor of connection* ∇.

Proof. If there exists a connection ∇ satisfying (1), (2) and (3), then, because of the linearity of $\nabla_X Y$ in its two arguments, it suffices to obtain the covariant derivative only when X and Y belong either to $\Gamma(\mathscr{D})$ or $\Gamma(\Xi)$, or $X\in\Gamma(\mathscr{D})$, $Y\in\Gamma(\Xi)$, or, finally $X\in\Gamma(\Xi)$, $Y\in\Gamma(\mathscr{D})$.

Let $X\in\Gamma(\mathscr{D})$ and $V\in\Gamma(\Xi)$. We have

$$\tau(X, V)=\nabla_X V-\nabla_V X-[X, V]_{\mathscr{D}}-[X, V]_{\Xi},$$

whence, with (1) and (2),

$$0=\tau_{\Xi}(X, V)=\nabla_X V-[X, V]_{\Xi},$$

which defines the value of $\nabla_X V$. Similarly, we have

$$0=\tau_{\mathscr{D}}(X, V)=-\nabla_V X-[X, V]_{\mathscr{D}},$$

which defines the value of $\nabla_V X$.

Next, let $X, Y, Z\in\Gamma(\mathscr{D})$. Condition (2) gives

$$0=(\nabla_X g)(Y, Z)=X\cdot g(Y, Z)-g(\nabla_X Y, Z)-g(Y, \nabla_X Z).$$

By the same procedure as the construction of Levi-Civita connection and using (3), we get

$$2g(\nabla_X Y, Z)=X\cdot g(Y, Z)+Y\cdot g(X, Z)-Z\cdot g(X, Y)$$
$$-g(X, [Y, Z]_{\mathscr{D}})+g(Y, [Z, X]_{\mathscr{D}})+g(Z, [X, Y]_{\mathscr{D}}),$$

whence, taking into account the condition (1), $\nabla_X Y$ is defined.

In the same way, we have $\nabla_V U$ for $V, U\in\Gamma(\Xi)$. Hence the given conditions define the connection uniquely.

Remark 2. Note that $\nabla_\xi\xi=0$. In fact,

$$0=(\nabla_\xi h)(\xi, \xi)=2h(\nabla_\xi\xi, \xi)=2\theta(\nabla_\xi\xi).$$

On the other hand, $\nabla_\xi\xi$ belongs to $\Gamma(\Xi)$, hence $\nabla_\xi\xi=0$.

Now, we define a connection D by

$$D_X Z=\nabla_X Z,\quad D_V U=\nabla_V U\quad\text{and}\quad D_V X=\nabla_V X-\frac{1}{2}J((\nabla_V J)X)$$

for any $X\in\Gamma(\mathscr{D})$, any $U, V\in\Gamma(\Xi)$ and any $Z\in\Gamma(TM)$, where ∇ is the connection in Proposition 1. Then, we have

Lemma. *The connection D satisfies the following properties:*
(1) $D_Z\Gamma(\mathscr{D})\subset\Gamma(\mathscr{D})$ *for any* $Z\in\Gamma(TM)$;
(2) *the tensor fields* ξ, J *and* ω *are all parallel;*
(3) *the torsion tensor* T *of* D *satisfies the equalities* $T(X, Y)=-\omega(X, Y)\xi$, $T(\xi, JX)=-J(T(\xi, X))$ *for any* $X, Y\in\Gamma(\mathscr{D})$.

Proof. (1) and $D\xi=0$ are trivial. For any $X\in\Gamma(\mathscr{D})$, we have that

$$\begin{aligned}(D_\xi J)X&=D_\xi(JX)-J(D_\xi X)\\&=\nabla_\xi(JX)-\frac{1}{2}J((\nabla_\xi J)JX)-J\nabla_\xi X+\frac{1}{2}J^2((\nabla_\xi J)X)\\&=\nabla_\xi(JX)-J(\nabla_\xi X)+J(\nabla_\xi X)-\nabla_\xi(JX)\\&=0,\end{aligned}$$

hence $D_\xi J=0$. To prove that $D_X J=0$ for any $X\in\Gamma(\mathscr{D})$, we note that

$$\begin{aligned}2g((D_XJ)Y, Z)&=2g(D_X(JY), Z)-2g(J(D_XY), Z)\\&=2g(\nabla_X(JY), Z)-2g(J(\nabla_XY), Z)\\&=2g(\nabla_X(JY), Z)+2g(\nabla_XY, JZ),\end{aligned}$$

for any $X, Y, Z\in\Gamma(\mathscr{D})$. By the definition of the connection ∇, we have

$$\begin{aligned}2g(\nabla_XY, Z)=&X\cdot\omega(JY, Z)+Y\cdot\omega(JX, Z)-Z\cdot\omega(JX, Y)\\&-\omega(f[Y, Z], X)+\omega(f[Z, X], Y)+\omega(f[X, Y], Z),\end{aligned}$$

because of the equalities

$$g(X, Y)=\omega(JX, Y),\quad [X, Y]_{\mathscr{D}}=[X, Y]-\theta([X, Y])\xi,$$
$$f[X, Y]=J([X, Y]-\theta([X, Y])\xi),$$

and so on. Using the formula

$$\begin{aligned}3d\omega(X, Y, Z)=&X\cdot\omega(Y, Z)+Y\cdot\omega(Z, X)+Z\cdot\omega(X, Y)\\&-\omega([X, Y], Z)-\omega([Y, Z], X)-\omega([Z, X], Y),\end{aligned}$$

we have that

$$\begin{aligned}2g((D_XJ)Y, Z)=&3d\omega(X, JY, JZ)-3d\omega(X, Y, Z)+\omega(S(Y, Z), X)\\&+\omega(X, Z)\cdot\omega(\xi, Y)-\omega(X, Y)\cdot\omega(\xi, Z)\\&+\omega(X, JY)\cdot\omega(\xi, JZ)-\omega(X, JZ)\cdot\omega(\xi, JY).\end{aligned}$$

On the other hand, $d\omega=-2d^2\theta=0$ and $S(Y, Z)=0$ because $\mathscr{D}$ is torsionless. Further, $\omega(\xi, X)$ vanishes for any $X \in \Gamma(\mathscr{D})$ (see Remark 1). Therefore, we obtain that $g((D_X J)Y, Z)=0$. Since g is non-degenerate, we have $D_X J=0$.

To prove that $D\omega=0$, we note that $\omega(\xi, X)=0$ for any $X \in \Gamma(\mathscr{D})$. Then, for any $X, Y, Z \in \Gamma(\mathscr{D})$ and any $W \in \Gamma(TM)$, we have

$$
\begin{aligned}
(D_W\omega)(\xi, \xi) &= 0, \\
(D_W\omega)(\xi, X) &= W\cdot \omega(\xi, X) - \omega(D_W\xi, X) - \omega(\xi, D_W X) \\
&= 0, \qquad \text{(From (1) and } D\xi=0.)
\end{aligned}
$$

and

$$
\begin{aligned}
(D_X\omega)(Y, Z) &= X\cdot\omega(Y, Z) - \omega(D_X Y, Z) - \omega(Y, D_X Z) \\
&= X\cdot g(-JY, Z) - g(D_X(-JY), Z) - g(-JY, D_X Z) \\
&\qquad\qquad\qquad\qquad \text{(From } DJ=0.) \\
&= -(D_X g)(JY, Z) \\
&= -(\nabla_X g)(JY, Z) \\
&= 0. \qquad \text{(From } \nabla_X g=0.)
\end{aligned}
$$

Noting that $D_\xi X=\nabla_\xi X-\frac{1}{2}J((\nabla_\xi J)X)=\frac{1}{2}([\xi, X]-J[\xi, JX])$ and $\omega(JX, JY)=\omega(X, Y)$ for any $X, Y \in \Gamma(\mathscr{D})$, we have that for any $X, Y \in \Gamma(\mathscr{D})$,

$$
\begin{aligned}
(D_\xi\omega)(X, Y) &= \xi\cdot\omega(X, Y) - \omega(D_\xi X, Y) - \omega(X, D_\xi Y) \\
&= \xi\cdot\omega(X, Y) - \frac{1}{2}\{\omega([\xi, X], Y) + \omega(X, [\xi, Y]) \\
&\quad - \omega(J[\xi, JX], Y) - \omega(X, J[\xi, JY])\} \\
&= \frac{1}{2}\{\xi\cdot\omega(X, Y) - \omega([\xi, X], Y) - \omega(X, [\xi, Y])\} \\
&\quad + \frac{1}{2}\{\xi\cdot\omega(JX, JY) - \omega([\xi, JX], JY) - \omega(JX, [\xi, JY])\} \\
&= \frac{1}{2}\{(\mathscr{L}_\xi\omega)(X, Y) + (\mathscr{L}_\xi\omega)(JX, JY)\} \\
&= 0. \qquad \text{(From that } \mathscr{L}_\xi\omega = -2d\mathscr{L}_\xi\theta = 0.)
\end{aligned}
$$

For any $X, Y \in \Gamma(\mathscr{D})$, we have that

$$
\begin{aligned}
T(X, Y) &= D_X Y - D_Y X - [X, Y]_{\mathscr{D}} - [X, Y]_{\Xi} \\
&= T_{\mathscr{D}}(X, Y) - [X, Y]_{\Xi} \\
&= -\theta([X, Y])\xi \\
&= -\omega(X, Y)\xi,
\end{aligned}
$$

and

$$\begin{aligned}T(\xi, JX)&=D_\xi(JX)-D_{JX}\xi-[\xi, JX]\\&=[\xi, JX]-\frac{1}{2}J((\nabla_\xi J)(JX))-[\xi, JX]\\&=\frac{1}{2}(J[\xi, X]-[\xi, JX]),\end{aligned}$$

$$\begin{aligned}-J(T(\xi, X))&=-J(D_\xi X-D_X\xi-[\xi, X])=-J(D_\xi X-[\xi, X])\\&=-J([\xi, X]-\frac{1}{2}J([\xi, JX]-J[\xi, X])-[\xi, X])\\&=\frac{1}{2}(J[\xi, X]-[\xi, JX]).\end{aligned}$$

Thus (3) is proved.

Remark 3. The conditions (1) and (2) in lemma imply that g and θ are parallel. Because, for any $X\in\Gamma(\mathscr{D})$ and $Z\in\Gamma(TM)$, we have that

$$(D_Z\theta)(\xi)=Z\cdot\theta(\xi)-\theta(D_Z\xi)=0, \qquad (\text{From } D\xi=0 \text{ and } \theta(\xi)=1)$$
$$(D_Z\theta)(X)=Z\cdot\theta(X)-\theta(D_ZX)=0, \qquad (\text{From (1) and } \theta(X)=0)$$

whence θ is parallel.

For any X, $Y\in\Gamma(\mathscr{D})$ and $Z\in\Gamma(TM)$, we have

$$\begin{aligned}(D_Zg)(X, Y)&=Z\cdot\omega(JX, Y)-\omega(J(D_ZX), Y)-\omega(JX, D_ZY)\\&=Z\cdot\omega(JX, Y)-\omega(D_Z(JX), Y)-\omega(JX, D_ZY) \quad (\text{From } DJ=0)\\&=(D_Z\omega)(JX, Y)\\&=0,\end{aligned}$$

whence g is parallel.

Theorem. *There exists a unique connection D on M satisfying the conditions (a), (b) and (c):*

(a) $D_Z\Gamma(\mathscr{D})\subset\Gamma(\mathscr{D})$ *for any* $Z\in\Gamma(TM)$;

(b) J, ξ *and* ω *are all parallel;*

(c) *the torsion tensor T of D satisfies the equalities* $T(X, Y)=-\omega(X, Y)\xi$, $T(\xi, JX)=-J(T(\xi, X))$, *for any* X, $Y\in\Gamma(\mathscr{D})$.

Proof. The existence of such a connection D is proved in above. We assume that there exist two connections D and D' satisfying (a), (b) and (c). Let T (resp. T') be the torsion tensor of D (resp. D'). From (b) and Remark 3, we have that

$$(D_Xg)(Y, Z)=0=(D'_Xg)(Y, Z), \qquad \text{for any } X, Y, Z\in\Gamma(\mathscr{D})$$

and from (c), we have that

$$T_{\mathscr{D}}(X, Y)=0=T'_{\mathscr{D}}(X, Y) \qquad \text{for any} \quad X, Y\in\Gamma(\mathscr{D}).$$

By those facts and (a) and non-degeneracy of g, we have $D_X Y=D'_X Y$ for any X, $Y\in\Gamma(\mathscr{D})$. It is evident that $D_Z V=D'_Z V$ for any $Z\in\Gamma(TM)$ and $V\in\Gamma(\Xi)$, from $D\xi=0=D'\xi$. For any $X\in\Gamma(\mathscr{D})$, we have that

$$D_\xi X=D_X\xi+[\xi, X]+T(\xi, X)=[\xi, X]+T(\xi, X),$$

and

$$\begin{aligned} D_\xi X&=-JD_\xi(JX)=-J[\xi, JX]-J(T(\xi, JX))\\ &=-J[\xi, JX]-T(\xi, X), \end{aligned}$$

whence $D_\xi X=\frac{1}{2}([\xi, X]-J[\xi, JX])$. Thus we have $D_\xi X=D'_\xi X$ for any $X\in\Gamma(\mathscr{D})$.

Therefore we complete the proof.

We shall give a geometric interpretation of the vanishing of the torsion of D. We recall the concept of pseudo-conformal vector fields (see [6]).

A transformation ϕ of M is called a pseudo-conformal mapping if ϕ satisfies that $\phi_*(X)$ belongs to $\Gamma(\mathscr{D})$ for any $X\in\Gamma(\mathscr{D})$ and $\phi_* J=J\phi_*$, where ϕ_* denotes the differential mapping of ϕ. A vector field X is called a pseudo-conformal vector field if any local transformation ϕ_t of M generated by X is always a pseudo-conformal transformation. By definition, X is a pseudo-conformal vector field if and only if $\mathscr{L}_X\theta=\alpha\theta$ and $\mathscr{L}_X J=0$ where α is a function.

Proposition 2 (see [13]). The vector field ξ is a pseudo-conformal vector field if and only if $T(\xi, X)=0$ for any $X\in\Gamma(\mathscr{D})$.

Proof. We have $\mathscr{L}_\xi\theta=0$ and $(\mathscr{L}_\xi J)(X)=[\xi, JX]-J[\xi, X]$ for any $X\in\Gamma(\mathscr{D})$. On the other hand, we have

$$\begin{aligned} T(\xi, X)&=D_\xi X-D_X\xi-[\xi, X]\\ &=[\xi, X]-\frac{1}{2}J([\xi, JX]-J[\xi, X])-[\xi, X]\\ &=-\frac{1}{2}J([\xi, JX]-J[\xi, X]). \end{aligned}$$

Thus $T(\xi, X)$ vanishes if and only if $(\mathscr{L}_\xi J)(X)$ vanishes. Therefore we have the assertion.

References

[1] D. E. Blair, *Contact Manifolds in Riemannian Geometry,* Springer Math. Lecture Notes, No. 509, Springer Verlag, 1976.

[2] D. Burns, Jr., K. Diederich and S. Shnider, *Distinguished curves in pseudoconvex boundaries,* Duke Math. J. **44** (1977), 407–431.

[3] D. Burns, Jr. and S. Shnider, *Pseudoconformal geometry of hypersurfaces in* C^{n+1}, Proc. Nat. Acad. Sci. U.S.A., **72** (1975), 2433–2436.

[4] D. Burns, Jr. and S. Shnider, *Real hypersurfaces in complex manifolds,* Proc. Symp. Pure Math., **30**, Part 2, 141–168, Amer. Math. Soc., Providence, R.I., 1977.

[5] S. S. Chern and J. Moser, *Real hypersurfaces in complex manifolds,* Acta Math. **133** (1974), 219–271.

[6] S. Ishihara, *Distributions with complex structure,* Kōdai Math. J. **1** (1978), 264–278.

[7] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry,* Vol. 1 (1963), Vol. 2 (1969), John Wiley and Sons, New York.

[8] B. L. Reinhart, *Foliated manifolds with bundle-like metrics,* Ann. of Math., **69** (1959), 119–132.

[9] K. Sakamoto and Y. Takemura, *On almost contact structures belonging to a CR-structure,* to appear Kōdai Math. J..

[10] N. Tanaka, *On the pseudo-conformal geometry of hypersurfaces of the space of n complex variables,* J. Math. Soc. of Japan **14** (1962), 397–429.

[11] N. Tanaka, *A Differential Geometric Study on Strongly Pseudoconvex Manifolds,* Lectures in Math., Kyoto Univ., Kinokuniya Book Stores, 1975.

[12] I. Vaisman, *Variétés riemanniennes feuilletées,* Czechosl. Math. J., **21** (1971), 46–75.

[13] S. M. Webster, *Real hypersurfaces in complex spaces,* Thesis, Univ. of California, Berkley, 1975.

[14] S. M. Webster, *On the pseudo-conformal geometry of Kähler manifold,* Math. Zeit. **157** (1977), 265–270

[15] S. M. Webster, *Kähler metrics associated to a real hypersurface,* Comment. Math. Helv. **52** (1977), 235–250.

[16] S. M. Webster, *On the transformation group of a real hypersurfaces,* Trans. of the A.M.S. **231** (1977), 179–190.

[17] S. M. Webster, *Pseudo-hermitian structures on a real hypersurfaces,* J. of Differential Geometry **13** (1978), 25–41.

Department of Mathematics
Kanazawa Medical University
Ishikawa, 920–02 Japan