ON A PAPER OF SHIUE AND CHAO

By

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Let p be a prime number, and let m, n be positive integers. A ring $R(\neq 0)$ will be called a $(p; m, n)$-ring if $pR=0$ and $x^{p^m+p^n}=x$ for all $x\in R$. When $p=2$, the fact $2R=0$ follows from the assumption $x^{p^m+p^n}=x$. If R is a $(p; m, n)$-ring, then R is a commutative reduced ring by Jacobson's theorem to which a brief elementary proof has been given in [1]. Moreover, if we set $h=p^m$ and $k=p^n$ then for any non-negative integer i we have

$$x+x^{i+1}=(x+x^{i+1})^{h+k}=(x^h+x^{(i+1)h})(x^k+x^{(i+1)k})=x+x^{ih+1}+x^{ik+1}+x^{i+1},$$

whence it follows

$$(*)\qquad x^{ih+1}=-x^{ik+1}.$$

Especially, we have $2x=0$, which means that p must be 2. Now, the main results of [2] can be proved with notable economy of effort as follows:

Proposition. *Let R be a $(2; m, n)$-ring, and $n=(m+1)q+r$, $0\leq r<m+1$. Then $x^{2^r+1}=x$ for all $x\in R$.*

Proof. Let $h=2^m$, and $k=2^n=(2h)^q2^r$. Then by (*) we have $x^{h+1}=x^{k+1}$. Accordingly we obtain

$$x=x^{k+1}x^{h-1}=x^{h+1}x^{h-1}=x^{2h}$$

and similarly

$$x=x^{2k}=x^{(2h)^q2^r+1}=x^{2^r+1}.$$

Corollary. *Let R be a $(2; m, n)$-ring, and $n=(m+1)q+r$, $0\leq r<m+1$. If $r=0$ then R is a Boolean ring, in particular, if $m=1$ and n is even then R is a Boolean ring.*

REFERENCES

[1] T. Nagahara and H. Tominaga: *Elementary proofs of a theorem of Wedderburn and a theorem of Jacobson.* Abh. Math. Sem. Univ. Hamburg, 41 (1974) 72–74.

[2] J.-S. Shiue and W.-M. Chao: *On the Boolean rings.* Yokohama Math. J., 24 (1976) 93–96.

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