ALMOST f-3-STRUCTURE

By

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Summary: In this paper, I have studied almost contact 3-structures and almost Quaternion structures in a differentiable manifold and generalised them to an almost f-3-structure.

1. Almost contact 3-structure.

Definition (1.1). *An odd dimensional differential manifold* V_n, $(n=2m+1)$ *is said to be an almost contact manifold if there exist in* V_n *a tensor field* F *of the type* (1, 1), *a vector field* T, *and a* 1-*form* A *satisfying*

$$F^2(X) \overset{\text{def}}{=} F(F(X) = -X + A(X)T \,, \tag{1.1}$$

$$F(T)=0 \,, \tag{1.2)a}$$

$$\text{rank}\,(F)=2m \,, \tag{1.2)b}$$

$$A(F(X))=0 \,, \tag{1.2)c}$$

$$A(T)=1 \,. \tag{1.2)d}$$

The structure $\{F, T, A\}$ *is called an almost contact structure.*

Mishra (1972) showed that (1.2) are the consequences of (1.1). Hence (1.2) *are redundant in the Definition* (1.1) *of an almost contact manifold.*

Let $\{F_1, T_1, A_1\}$ be an almost contact structure in V_n. Let μ be a non-singular tensor field of the type (1, 1) in V_n. Let us define

$$\mu(F_2(X)) \overset{\text{def}}{=} F_1(\mu(X)) \,, \tag{1.3)a}$$

$$A_2(X) \overset{\text{def}}{=} A_1(\mu(X)) \,, \tag{1.3)b}$$

$$T_2 \overset{\text{def}}{=} \text{def}^{\,-1}\mu(T_1) \,. \tag{1.3)c}$$

Then it can be easily verified that $\{F_2, T_2, A_2\}$ is also an almost contact structure.

Definition (1.2). *Let* $\{F_1, T_1, A_1\}$, $\{F_2, T_2, A_2\}$ *be two almost contact structures in* V_n *and satisfy*

$$F_1(F_2(X)) + F_2(F_1(X)) = A_1(X)T_2 + A_2(X)T_1 \,, \tag{1.4}$$

(1.5)a $$F_1(T_2)+F_2(T_1)=0\ ,$$

(1.5)b $$A_1(F_2(X))+A_2(F_1(X))=0\ ,$$

(1.5)c $$A_1(T_2)=A_2(T_1)=0\ ,$$

Then $\{F_1, T_1, A_1\}$, $\{F_2, T_2, A_2\}$ *are said to define an almost contact* 3*-structure in* V_n (Yano, Ishihara and Konishi (1973)).

Theorem (1.1). *The equations* (1.1) *and* (1.4) *imply* (1.5)a, b.

Proof. Substituting $F_2(X)$ for X in (1.4) and using (1.2)c, we get

$$F_1(F_2^2(X))+F_2(F_1(F_2(X)))=A_1(F_2(X))T_2$$

Using (1.4) and (1.2) in this equation we get

$$F_1(F_2^2(X))-F_2^2(F_1(X))+A_2(X)F_2(T_1)=A_1(F_2(X))T_2\ .$$

In consequence of (1.1) and (1.2)c this equation takes the form

$$A_2(X)\{F_1(T_2)+F_2(T_1)\}=\{A_1(F_2(X))+A_2(F_1(X))\}T_2\ .$$

This equation holds for n linearly independent vector fields X. Since $A_2(X)\neq 0$, we have (1.5)a,b.

Remark (1.1). Substituting T_1 and T_2 for X in (1.5)b, we incidentally have

(1.6)a $$A_1(F_2(T_1))=0\ ,\qquad \text{b)}\quad A_2(F_1(T_2))=0\ .$$

Theorem (1.2). *The equations* (1.1) *and* (1.4) *imply*

(1.7) $$A_1(T_2)+A_2(T_1)=0\ .$$

Proof. Putting T_1 for X in (1.4) and using (1.2)a, d we get

$$F_1(F_2(T_1))=T_2+A_2(T_1)T_1$$

whence, in consequence of (1.2)c, we have (1.7).

Remark (1.2). From the above we see that we need not have (1.5)c. (1.7) will suffice instead of (1.5)c. The conditions (1.5)c are additional requirements, which will be seen further.

Remark (1.3). *Yano, Ishihara* and *Konishi* (1973) had defined almost contact 3-structures in terms of (1.4) and (1.5)a,b,c. From the discussion it follows that (1.4) and (1.5)c suffice to define an almost contact 3-structure. The equations (1.5)a,b are consequences of (1.4). The justification for the assumption of (1.5)c comes from (1.4) through (1.7) which is implied by (1.4).

Theorem (1.3). *The structure* $\{F, T, A, \mu\}$ *defines an almost contact* 3-*structure if* $\{F, T, A\}$ *is an almost contact structure and if*

$$\mu(F({}^{-1}\mu(F(\mu(X)))))+F(\mu(F(X)))=A(X)T+A(\mu(X))\mu(T)\ . \tag{1.8}$$

Proof. Substituting from (1.3) in (1.4) and writing $\{F, T, A\}$ for $\{F_1, T_1, A_1\}$ we get (1.8).

Theorem (1.4). *If* $\{F, T, A, \mu\}$ *defines an almost contact* 3-*structure in* V_n, *then*

(1.9)a $$\mu(F({}^{-1}\mu(T)))+F(\mu(T))=0$$

(1.9)b $$A({}^{-1}\mu(F(\mu(X))))+A(\mu(F(X)))=0$$

(1.9)c $$A({}^{-1}\mu(T))=A(\mu(T))=0\ .$$

Proof. Substituting from (1.3) in (1.5) and writing $\{F, T, A\}$ for $\{F_1, T_1, A_1\}$ we get (1.9).

It is well known (Yano, Ishihara and Konishi, 1973) that if we put

(1.10)a $$F_3(X)\overset{\text{def}}{=}F_1(F_2(X))-A_2(X)T_1\overset{\text{def}}{=}-F_2(F_1(X))+A_1(X)T_2\ ,$$

(1.10)b $$T_3\overset{\text{def}}{=}F_1(T_2)\overset{\text{def}}{=}-F_2(T_1)$$

(1.10)c $$A_3(X)\overset{\text{def}}{=}A_1(F_2(X))\overset{\text{def}}{=}-A_2(F_1(X))\ ,$$

then any two of the three structures $\{F_\lambda, T_\lambda, A_\lambda, \lambda=1, 2, 3\}$ define an almost contact 3-structure.

Remark (1.4). From (1.10)b and (1.2)c, we easily see that

$$A_1(T_3)=A_2(T_3)=0\ . \tag{1.11}$$

Corollary (1.1). *When any two of the three structures* $\{F_\lambda, T_\lambda, A_\lambda, \lambda=1, 2, 3\}$ *related by* (1.4) *and* (1.10) *define an almost contact* 3-*structure in* V_n

(1.12) $$\text{a)}\ A_1(T_2)=0\ , \qquad \text{b)}\ A_2(T_1)=0\ .$$

Proof. Putting T_1 for X in (1.10)a and using (1.10)b, we get

$$F_3(T_1)+F_1(T_3)=-A_2(T_1)T_1\ .$$

But

$$F_3(T_1)+F_1(T_3)=0\ .$$

Therefore

$$A_2(T_1)=0$$

Similarly we can prove that

$$A_1(T_2)=0\ .$$

Remark (1.5). From the above discussion we see that the conditions (1.5) are redundant. When $\{F_1,\ T_1,\ A_1\}$, $\{F_2,\ T_2,\ A_2\}$ define an almost contact 3-structure then

$$A_1(T_2)+A_2(T_1)=0\ .$$

When any two of the three structures $\{F_1,\ T_1,\ A_1\}$, $\{F_2,\ T_2,\ A_2\}$, $\{F_3,\ T_3,\ A_3\}$ define an almost contact 3-structure and $\{F_3,\ T_3,\ A_3\}$ are given by (1.10), then

$$A_1(T_2)=A_2(T_1)=0\ .$$

Theorem (1.5). *Let μ be a non-singular tensor of the type* (1, 1). *Let $\{F,\ T,\ A\}$ be an almost contact structure in V_n. Then any two of the following*

$$\{F,\ T,\ A\}\ ,$$

$$\{{}^{-1}\mu(F(\mu))\ ,\quad {}^{-1}\mu(T)\ ,\quad A(\mu)\}\ ,$$

$$\{F({}^{-1}\mu(F(\mu)))-A(\mu)\otimes T\ ,\quad F({}^{-1}\mu(T))\ ,\quad A({}^{-1}\mu(F(\mu)))\}\ ,$$

define the same almost contact 3-structure, provided

$$F({}^{-1}\mu(F(\mu)))+{}^{-1}\mu(F(\mu(F)))=A\otimes{}^{-1}\mu(T)+A(\mu)\otimes T\ . \tag{1.13}$$

The proof is obvious.

Definition (1.3). *Let $\{F_1,\ T_1,\ A_1\}$, $\{F_2,\ T_2,\ A_2\}$ define an almost contact 3-structure in V_n. Let there be defined in V_n a metric tensor g such that*

$$g(F_1(X)\ ,\ F_1(Y))=g(X,\ Y)-A_1(X)A_1(Y)\ , \tag{1.14)a}$$

$$g(F_2(X)\ ,\ F_2(Y))=g(X,\ Y)-A_2(X)A_2(Y)\ . \tag{1.14)b}$$

Then $\{\{F_1,\ T_1,\ A_1\},\ \{F_2,\ T_2,\ A_2\},\ g\}$ is said to define an almost contact Riemannian 3-structure in V_n

Theorem (1.6). $\{\{F_1,\ T_1,\ A_1\},\ \{F_3,\ T_3,\ A_3\},\ g\}$, $\{\{F_2,\ T_2,\ A_2\},\ \{F_3,\ T_3,\ A_3\},\ g\}$ *separately define the same almost contact Riemannian 3-structure in V_n*

Proof. In consequence of (1.10)a,c (1.2)c, (1.14)a,b we have

$$\begin{aligned} g(F_3(X),\ F_3(Y)) &= g(F_1(F_2(X))-A_2(X)T_1,\ F_1(F_2(Y))-A_2(Y)T_1) \\ &= g(F_1(F_2(X)),\ F_1(F_2(Y)))+A_2(X)A_2(Y) \\ &= g(F_2(X),\ F_2(Y))-A_3(X)A_3(Y)+A_2(X)A_2(Y) \\ &= g(X,\ Y)-A_3(X)A_3(Y)\ . \end{aligned}$$

Remaining part of the proof is obvious.

Corollary (1.2). *We have*

(1.15)a $$g(F_1(X),\ F_3(Y))=g(X,\ F_2(Y))-A_1(X)A_3(Y)=-g(F_2(X),\ Y) -A_1(X)A_3(Y)\ ,$$

(1.15)b $$g(F_2(X),\ F_3(Y))=-g(X,\ F_1(Y))-A_2(X)A_3(Y)=g(F_1(X),\ Y) -A_2(X)A_3(Y)\ .$$

Proof. In consequence of (1.10)a, (1.10)c

$$\begin{aligned} g(F_1(X),\ F_3(Y)) &= g(F_1(X),\ F_1(F_2(Y)))-A_2(Y)g(F_1(X),\ T_1) \\ &= g(X,\ F_2(Y))-A_1(X)A_1(F_2(Y)) \\ &= g(X,\ F_2(Y))-A_1(X)A_3(Y) \\ &= -g(f_2(X),\ Y)-A_1(X)A_3(Y) \end{aligned}$$

$$\begin{aligned} g(F_2(X),\ F_3(Y)) &= g(F_2(X),\ -F_2(F_1(Y)))+A_1(Y)g(F_2(X),\ T_2) \\ &= -g(X,\ F_1(Y))+A_2(X)A_2(F_1(Y)) \\ &= -g(X,\ F_1(Y))-A_2(X)A_3(Y) \end{aligned}$$

Definition (1.4). *Let* $\{\{F_1,\ T_1,\ A_1\},\ \{F_2,\ T_2,\ A_2\},\ g\}$ *define an almost contact Riemannian 3-structure in* V_n. *Let*

(1.16)a $$'F_1(X,\ Y)\overset{\text{def}}{=}g(F_1(X),\ Y)=\tfrac{1}{2}(dA_1)(X,\ Y)\ ,$$

(1.16)b $$'F_2(X,\ Y)\overset{\text{def}}{=}g(F_2(X),\ Y)=\tfrac{1}{2}(dA_2)(X,\ Y)\ .$$

Then $\{\{F_1,\ T_1,\ A_1\},\ \{F_2,\ T_2,\ A_2\},\ g\}$ *is called contact 3-structure.*

Theorem (1.7). *When* $\{\{F_1,\ T_1,\ A_1\},\ \{F_2,\ T_2,\ A_2\},\ g\}$ *is a contact 3-structure in* V_n, *we have*

(1.17) $$(dA_1)(F_2(X),\ Y)+(dA_2)(F_1(X),\ Y)=2\{A_1(X)A_2(Y)+A_2(X)A_1(Y)\}\ .$$

Proof. In consequence of (1.10)a, (1.16)a we have

$$\begin{aligned} 'F_3(X,\ Y) &= g(F_3(X),\ Y)=g(F_1(F_2(X))-A_2(X)T_1,\ Y) \\ &= 'F_1(F_2(X),\ Y)-A_2(X)A_1(Y) \\ &= \tfrac{1}{2}(dA_1)(F_2(X),\ Y)-A_2(X)A_1(Y) \end{aligned}$$

Similarly we have

$$'F_3(X,\ Y)=-\tfrac{1}{2}(dA_2)(F_1(X),\ Y)+A_1(X)A_2(Y)\ .$$

From the last two equations, we have (1.17).

2. Almost Quaternion structure.

Definition (2.1). *Let there be given in a 4m-dimensional differential manifold V_n, $(n=4m)$ tensor fields F_1, F_2 of the type* (1, 1) *satisfying*

$$(2.1)\qquad \text{a) } F_1^2(X)+X=0, \qquad \text{b) } F_2^2(X)+X=0\,,$$

$$(2.2)\qquad F_1(F_2(X))=-F_2(F_1(X))\,.$$

Then $\{F_1, F_2\}$ is called an almost Quaternion structure in V_n.

Theorem (2.1). *Let us put*

$$(2.3)\qquad F_3(X)\overset{\text{def}}{=}F_1(F_2(X))=-F_2(F_1(X))\,.$$

Then

$$(2.4)\qquad F_3^2(X)+X=0\,,$$

$$(2.5)\text{a}\qquad F_1(X)=F_2(F_3(X))=-F_3(F_2(X))\,.$$

$$(2.5)\text{b}\qquad F_2(X)=F_3(F_1(X))=-F_1(F_3(X))\,.$$

Consequently any two of the three structures F_1, F_2, F_3 define an almost Quaternion structure.

Proof. In consequence of (2.3) and (2.1)a, b, we have

$$F_3^2(X)=-F_1(F_2^2(F_1(X)))=F_1^2(X)=-X\,.$$

Also from (2.3) and (2.1)b

$$F_2(F_3(X))=-F_2^2(F_1(X))=F_1(X)\,,$$

$$F_3(F_2(X))=F_1(F_2^2(X))=-F_1(X)\,.$$

We can similarly prove (2.5)b.

Remaining part of the proof is obvious.

Corollary (2.1). *Let F be an almost complex structure in V_{4m}. Let μ be a non-singular tensor field of the type* (1, 1). *Then any two of the three structures*

$$F,\quad {}^{-1}\mu(F(\mu))\,,\quad F({}^{-1}\mu(F(\mu)))\,,$$

define an almost Quaternion structure in V_{4m}, provided

$$(2.6)\qquad F({}^{-1}\mu(F(\mu)))+{}^{-1}\mu(F(\mu(F)))=0\,.$$

Proof. The proof is obvious.

Definition (2.2). *Let $\{F_1, F_2\}$ define an almost Quaternion structure in*

V_n, $(n=m)$. *Let there be defined in* V_n *a metric tensor* g *satisfying*

(2.7)a $$g(F_1(X),\ F_1(Y))=g(X,\ Y)\ ,$$

(2.7)b $$g(F_2(X),\ F_2(Y))=g(X,\ Y)\ .$$

Then $\{F_1,\ F_2,\ g\}$ *are said to define an almost Quaternion Riemannian structure in* V_n

Theorem (2.2). $\{F_1,\ F_3,\ g\}$, $\{F_2,\ F_3,\ g\}$ *separately define the same almost Quaternion Riemannian structure in* V_n.

Proof. In consequence of (2.3) and (2.7)a, b

$$\begin{aligned} g(F_3(X),\ F_3(Y)) &= g(F_1(F_2(X)),\ F_1(F_2(Y))) \\ &= g(F_2(X),\ F_2(Y)) \\ &= g(X,\ Y)\ . \end{aligned}$$

Also, since we have assumed in (1.1)a that

$$F_1^2(X)+X=0\ ,$$

and proved in (2.4) and (2.5)b that

$$F_3^2(X)+X=0$$

and

$$F_1(F_3(X))=-F_3(F_1(X))\ ,$$

the structure $\{F_1,\ F_3,\ g\}$ defines the same almost Quaternion Riemannian structure in V_n.

The fact that $\{F_2,\ F_3,\ g\}$ defines the same almost Quaternion Riemannian structure in V_n, can be proved similarly.

Definition (2.3). *Let* $\{F_1,\ F_2,\ g\}$ *define an almost Quaternion Riemannian structure in* V_n. *Then if*

(2.8)a $$(D_XF_1)(Y)=0\ , \quad \text{b)}\ (D_XF_2)(Y)=0\ ;$$

(2.9)a $$(D_XF_1)(Y)+(D_YF_1)(X)=0\ , \quad \text{b)}\ (D_XF_2)(Y)+(D_YF_2)(X)=0:$$

(2.10)a $$(D_X{}'F_1)(Y,\ Z)+(D_Y{}'F_1)(Z,\ X)+(D_Z{}'F_1)(X,\ Y)=0\ ,$$

(2.10)b $$(D_X{}'F_2)(Y,\ Z)+(D_Y{}'F_2)(Z,\ X)+(D_Z{}'F_2)(X,\ Y)=0\ ;$$

where

(2.10)c $${}'F(X,\ Y)\overset{\text{def}}{=}g(F(X),\ Y)\ ,$$

(2.11)a $$\operatorname{div}(\nabla F_1)(X)=0\ , \quad \text{b)}\ \operatorname{div}(\nabla F_2)(Y)=0\ ;$$

(2.12)a $$(D_X F_1)(Y)+(D_{F_1(X)}F_1)(F_1(Y))=0\ ,$$

(2.12)b $$(D_X F_2)(Y)+(D_{F_2(X)}F_2)(F_2(Y))=0\ .$$

the structure $\{F_1,\ F_2,\ g\}$ *is said to be almost Quaternion Kähler, almost Quaternion Tachibana, almost Quaternion almost Kähler, almost Quaternion semi-Kähler and almost Quaternion almost O-structure respectively.*

Theorem (2.3). *Let the structure* $\{F_1,\ F_2,\ g\}$ *define an almost Quaternion Kähler, almost Quaternion Tachibana, almost Quaternion almost Kähler, almost quaternion semi-Kähler or almost Quaternion almost O-structure, then the structures* $\{F_1,\ F_3,\ g\}$ *and* $\{F_2,\ F_3,\ g\}$ *separately define the same almost Quaternion Kähler, almost Quaternion Tachibana, almost Quaternion almost Kähler, almost Quaternion semi or almost Quaternion O-structure.*

Proof. The structure $\{F_1,\ F_2,\ g\}$ defines almost Quaternion Riemannian structure in V_n. Therefore in consequence of (2.1)a, (2.4) and Theorem (2.2).

(2.13)a $$F_1^2(X)+X=0\ ,$$

(2.13)b $$F_3^2(X)+X=0\ ,$$

(2.13)c $$g(F_3(X),\ F_3(Y))=g(X,\ Y)\ .$$

Now in consequence of (2.8)a

$$(D_X F_3)(Y)=+F_1(D_X F_2)(Y)+(D_X F_1)(F_2(Y))=0\ .$$

Hence if the structure $\{F_1,\ F_2,\ g\}$ defines an almost Quaternion Kähler manifold the structure $\{F_1,\ F_3,\ g\}$ defines an almost Quaternion Kähler manifold.

The proof of the remaining cases follows the same pattern.

3. Almost f-3-structure.

We will now have the following definition

Definitoin (3.1). *Let* $F_\lambda(\lambda=1,\ 2,\ 3)$ *be an f-structure in* V_n *that is*

(3.1) $$F_\lambda^3(X)+F_\lambda(X)=0$$

and rank $(F)=r$ *everywhere. Let*

(3.2)a $$F_1(F_3(X))\overset{\text{def}}{=}F_1^2(F_2(X))\ ,$$

(3.2)b $$F_2(F_3(X))\overset{\text{def}}{=}-F_2^2(F_1(X))\ ,$$

(3.2)c $$F_3(F_1(X))\overset{\text{def}}{=}-F_2(F_1^2(X))\ ,$$

(3.2)d $$F_3(F_2(X))\overset{\text{def}}{=}F_1(F_2^2(X))\ ,$$

be satisfied at every point of V_n. Then $\{F_\lambda\}$ is called an almost f-3-structure.

Remark (3.1). F_λ is a vector valued linear function on V_n. We do not put any further restriction on F_λ regarding its symmetry or skew-symmetry.

Theorem (3.1). *Let $\{F_\lambda\}$ be an almost f-3-structure. Then the following are also satisfied:*

$$F_1^2(F_3(X))=F_3(F_2^2(X))=-F_1(F_2(X)), \tag{3.3}$$

$$F_2^2(F_3(X))=F_3(F_1^2(X))=F_2(F_1(X)). \tag{3.4}$$

Proof. Premultiplying (3.2)a, b by F_1, F_2 respectively and substituting from (3.1), we obtain

$$F_1^2(F_3(X))=-F_1(F_2(X))$$

$$F_2^2(F_3(X))=F_2(F_1(X)).$$

Putting $F_1(X)$, $F_2(X)$ for X in (3.2)c, d respectively and substituting from (3.1), we obtain

$$F_3(F_1^2(X))=F_2(F_1(X)),$$

$$F_3(F_2^2(X)=-F_1(F_2(X)).$$

Hence we have (3.3) and (3.4).

Thorem (3.2). *Let $r=n$. Then the almost f-3-structure reduces to almost Quaternion structure and $n=4m$.*

Proof. When $r=n$, inverse of (F) exists and (3.1), (3.2), (3.3) and (3.4) reduce to

$$F_\lambda^2(X)+X=0, \tag{3.5}$$

$$F_2(X)=-F_1(F_3(X))=F_3(F_1(X)), \tag{3.6)a}$$

$$F_1(X)=F_2(F_3(X))=-F_3(F_2(X)), \tag{3.6)b}$$

$$F_3(X)=F_1(F_2(X))=-F_2(F_1(X)). \tag{3.6)c}$$

These equations prove the statement.

Theorem (3.3). *Let $r=n-1$. Then the almost f-3-structure reduces to an almost contact 3-structure.*

Proof. When $r=n-1$, (3.1), (3.2), (3.3) and (3.4) reduce to

$$F_\lambda^2(X)+X=A_\lambda(X)T_\lambda, \tag{3.7}$$

where A_λ are 1-forms and T_λ are vector fields,

(3.8)a $\quad F_2(X)=-F_1(F_3(X))+A_1(F_2(X))T_1=F_3(F_1(X))+A_1(X)F_2(T_1)$,

(3.8)b $\quad F_1(X)=F_2(F_3(X))+A_2(F_1(X))T_2=-F_3(F_2(X))+A_2(X)F_1(T_2)$,

(3.8)c
$$F_3(X)=F_1(F_2(X))+A_1(F_3(X))T_1=F_1(F_2(X))+A_2(X)F_3(T_2)$$
$$=-F_2(F_1(X))+A_2(F_3(X))T_2=-F_2(F_1(X))+A_1(X)F_3(T_1) .$$

Substituting T_1, T_2, T_3 for X in (3.8) and assuming

(3.9)a $$A_1(T_2)=A_2(T_1)=A_1(T_3)=A_3(T_1)=A_2(T_3)=A_3(T_2)=0 ,$$

we get

(3.9)b $$F_1(T_2)=-F_2(T_1)=T_3 ,$$

(3.9)c_1 $$F_2(T_3)=-F_3(T_2)=T_1 ,$$

(3.9)c_2 $$F_3(T_1)=-F_1(T_3)=T_2 .$$

Since $A_1(F_1)=A_2(F_2)=A_3(F_3)=0$, the equations (3.8) yield

(3.10)a $$A_1(F_2)=-A_2(F_1)=A_3 ,$$

(3.10)b $$-A_1(F_3)=A_3(F_1)=A_2 ,$$

(3.10)c $$A_2(F_3)=-A_3(F_2)=A_1 .$$

In consequence of (3.9) and (3.10), the equations (3.8) assume the forms

(3.11)a $$F_2(X)=-F_1(F_3(X))+A_3(X)T_1=F_3(F_1(X))-A_1(X)T_3 ,$$

(3.11)b $$F_1(X)=F_2(F_3(X))-A_3(X)T_2=-F_3(F_2(X))+A_2(X)T_3 ,$$

(3.11)c $$F_3(X)=F_1(F_2(X))-A_2(X)T_1=-F_2(F_1(X))+A_1(X)T_2 .$$

The equations (3.7) and (3.11) prove the statement.

Remark (3.1). Putting T_3 in (3.8)c we immediately get

$$A_1(T_3)=A_2(T_3)=0$$

Remaining equations (3.9)a are assumed in consideration of these two equations.

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