ON SOME NORMAL SUBGROUPS OF WEYL GROUPS

By

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Introduction.

Let G be a connected, compact Lie group and K a connected closed subgroup of G of maximal rank. Then, K contains a maximal torus T of G. A. Borel and J. Siebenthal found all these subgroups for simple Lie groups in [1]. By $W(G)$ and $W(K)$ we shall denote their Weyl groups, i.e., $W(G)=N_G(T)/T$, $W(K)=N_K(T)/T$ where $N_G(T)$ and $N_K(T)$ are the normalizers of T in G and K respectively. It is well known that these Weyl groups don't depend on the choice of T. Hence $W(K)$ is a subgroup of $W(G)$. In this paper we shall determine such pairs $(G;K)$ that $W(K)$ is especially a normal subgroup of $W(G)$. It is known that $(B_n;D_n)$, $(C_n;\underbrace{A_1\times A_1\times\cdots\times A_1}_{n})$, $(G_2;A_2)$ and $(F_4;D_4)$ are such examples.(*) We shall show that there is no pair but these four classes if G is a simple Lie group.

In §1, we shall reduce our problem to find W-invariant sharp-systems (see Definition). In §2, we shall give two theorems about W-invariant sharp-systems. In §3, we shall decide all W-invariant sharp-systems for complex simple Lie algebras.

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§1. Reformation of the problem.

Let G be a connected compact Lie group with the center Z. Since G is decomposed into semi-direct product of a semi-simple subgroup G' and Z, the Weyl group of G is consist with the one of G'. On the other hand, we put $K'=K\cap G'$. Then $K=K'\times Z$ (semi-direct), for a subgroup of G of maximal rank always contains the center Z. Hence the Weyl group of K is also consist with the one of K'. Therefore we may assume that G is semi-simple. Furthermore, we may assume that G is simple, for the Weyl group of a semi-simple Lie group is decomposed into the direct product of the ones of simple subgroups.

Proposition 1. *Let G be a compact connected Lie group, and $K_i\,(i=1,2)$*

(*) For $(B_n;C_n)$ and $(G_2;A_2)$, consider the index of subgroup; for $(C_n;\underbrace{A_1\times\cdots\times A_1}_{n})$, recall the method of construction of the pair by [1]; and for $(F_4;D_4)$, see [3], [4].

connected closed subgroups of maximal rank. If K_1 is isomorphic to K_2 under an inner automorphism of G, then there is an automorphism σ of $W(G)$ such that $\sigma(W(K_2))=W(K_1)$.

Proof. From the assumptions there is an element g in G such that $K_2=gK_1g^{-1}$, so gT_1g^{-1} and T_2 are maximal tori of K_2 where $T_i\,(i=1,2)$ are maximal tori of G contained in K_i respectively. By the conjugacy of maximal tori of a compact connected Lie group, there exists such $k\in K_2$ that $kgT_1g^{-1}k^{-1}=T_2$. Put $h=kg$, then $h^{-1}N_G(T_2)h=N_G(T_1)$ and $h^{-1}T_2h=T_1$, hence h induces an isomorphism σ from $N_G(T_2)/T_2$ to $N_G(T_1)/T_1$, i.e., an automorphism of $W(G)$. Furthermore, since $h^{-1}K_2h=K_1$, $\sigma(N_{K_2}(T_2)/T_2)=N_{K_1}(T_1)/T_1$. (Q. E. D.)

This proposition means that we may determine the pairs $(G;K)$ up to conjugacy for our problem. Therefore, for a given maximal torus T of G we may find such pairs $(G;K)$ that $W(K)$ is a normal subgroup of $W(G)$ and K contains T.

From now, let G be a connected compact semi-simple Lie group, T a fixed maximal torus of G, and let K be a closed subgroup of G containing T. Let $\mathfrak{g}_0$ be the Lie algebra of G and $\mathfrak{k}_0, \mathfrak{t}_0$ subalgebras of $\mathfrak{g}_0$ corresponding to K and T respectively. By $\mathfrak{g}, \mathfrak{k}$ and $\mathfrak{h}$ we denote the complexifications of $\mathfrak{g}_0, \mathfrak{k}_0$ and $\mathfrak{t}_0$. As $\mathfrak{h}$ is a Cartan subalgebra of $\mathfrak{g}$ (cf [5], Exposé 23), let Δ denote the set of roots of $\mathfrak{g}$ with respect to $\mathfrak{h}$. For each root α we put

$$\mathfrak{g}_\alpha=\{X\in\mathfrak{g}\,;\,[H,X]=\alpha(H)X \text{ for all } H\in\mathfrak{h}\}.$$

Now let η denote the complex conjugation of $\mathfrak{g}$ with respect to $\mathfrak{g}_0$, i.e., $\eta(X)=X_1-iX_2$ whenever $X=X_1+iX_2$ $(X_1, X_2\in\mathfrak{g}_0)$, where the letter "$i$" is the imaginary unit. Though the next lemma is well known, (for example, cf. [2], p. 220) we shall give it with proof.

Lemma 1. 1) $\eta(\mathfrak{k})=\mathfrak{k}$,

2) $\eta(\mathfrak{g}_\alpha)=\mathfrak{g}_{-\alpha}$.

Proof. 1) This is trivial.

2) Since $\mathfrak{t}_0$ is compact, we can consider as $\mathrm{ad}(\mathfrak{t}_0)\subset O(n)$, where $O(n)$ is the group of all orthogonal matrices of degree n. Hence $\alpha(H)$ is a pure-imaginary number for any root α and any element H in $\mathfrak{t}_0$. For any element X in $\mathfrak{g}_\alpha$ and any element H in $\mathfrak{h}$, $[H,X]=\alpha(H)X$. Applying η on the both sides, we have $[\eta H, \eta X]=\overline{\alpha(H)}X$ where the bar on the right-hand side means the complex conjugation. Since $H=H_1+iH_2$ $(H_1, H_2\in\mathfrak{t}_0)$ and $\overline{\alpha(H)}=\overline{\alpha(H_1)+i\alpha(H_2)}=-\alpha(H_1)+i\alpha(H_2)=-\alpha(\eta H)$, so we have $\eta(\mathfrak{g}_\alpha)\subset\mathfrak{g}_{-\alpha}$. As $\eta^2=I$, $\eta(\mathfrak{g}_\alpha)=\mathfrak{g}_{-\alpha}$. (Q. E. D.)

Definition. A subset Θ of Δ is called *a sharp-system* if the next two conditions are satisfied,

1) $-\Theta=\Theta$,

2) Θ is additively closed, i.e., if α, β are elements in Θ and $\alpha+\beta$ is a root, then $\alpha+\beta$ is in Θ.

Theorem 1. *Let G be a connected, compact semi-simple Lie group and T a fixed maximal torus of G. Let $\mathfrak{g}_0$ be the Lie algebra of G, $\mathfrak{t}_0$ the subalgebra of $\mathfrak{g}_0$ corresponding to T, and let $\mathfrak{g}, \mathfrak{h}$ be their complexifications respectively. By Δ we denote the set of roots of $\mathfrak{g}$ with respect to $\mathfrak{h}$. Then, there exists a bijection between the set of connected closed subgroups of G containing T and the set of sharp-systems of Δ.*

Proof. Let K be a connected closed subgronp of G containing T, $\mathfrak{k}_0$ its Lie algebra, and $\mathfrak{k}$ the complexification of $\mathfrak{k}_0$. Since $\mathfrak{h}$ is contained in $\mathfrak{k}$, we have a subset Θ of Δ such that $\mathfrak{k}=\mathfrak{h}+\sum_{\alpha\epsilon\Theta}\mathfrak{g}_\alpha$. Now we shall show that Θ is a sharp-system of Δ. For any root α in Θ, $\mathfrak{g}_\alpha\subset\mathfrak{k}$, so $\eta(\mathfrak{g}_\alpha)\subset\eta(\mathfrak{k})$, i.e., $\mathfrak{g}_{-\alpha}\subset\mathfrak{k}$ by Lemma 1. Hence, $-\alpha$ is contained in Θ. Obviously, Θ is additively closed as $\mathfrak{k}$ is a subalgebra. These imply that Θ is a sharp-system of Δ. Thus we have a mapping from the set of connected closed subgroups of G containing T to the set of sharp-systems of Δ. Let $\Theta_i\,(i=1,2)$ be the sharp-systems corresponding to connected closed subgroups K_i containing T. If $\Theta_1=\Theta_2$, then $\mathfrak{k}_1\cap\mathfrak{g}_0=\mathfrak{k}_2\cap\mathfrak{g}_0$ where $\mathfrak{k}_i$ are the complexifications of the Lie algebras of K_i. Thus we obtain $K_1=K_2$ by the connectedness of K_i. This means that the mapping is an injection. Next, for any sharp-system Θ, we put $\mathfrak{k}=\mathfrak{h}+\sum_{\alpha\epsilon\Theta}\mathfrak{g}_\alpha$ and $\mathfrak{k}_0=\mathfrak{k}\cap\mathfrak{g}_0$, then $\mathfrak{k}_0$ is a subalgebra of $\mathfrak{g}_0$ such that $\mathfrak{k}_0\supset\mathfrak{t}_0$. Let K denote an analytic subgroup of G corresponding to $\mathfrak{k}_0$, then $K\supset T$, and K is closed. (Q. E. D.)

Let $\mathfrak{g}$ be a complex semi-simple Lie algebra and $\mathfrak{h}$ a Cartan subalgebra. Since the Killing form $\varphi(X,Y)=Tr(ad(X)\,ad(Y))$ is nondegenerate, $\overline{\varphi}$ is also nondegenerate where $\overline{\varphi}$ is the restriction of φ over $\mathfrak{h}$. Let $\mathfrak{h}^*$ be the dual space of $\mathfrak{h}$, then, for each $\lambda\epsilon\mathfrak{h}^*$, there exists uniquely an element $H_\lambda\epsilon\mathfrak{h}$ such that $\lambda(H)=\overline{\varphi}(H,H_\lambda)$ for any H in $\mathfrak{h}$. In $\mathfrak{h}^*$ we shall define the inner product $(\lambda,\mu)=\overline{\varphi}(H_\lambda,H_\mu)$. Obviously this inner product is also nondegenerate. Put $\mathfrak{h}_0{}^*=\{\lambda\epsilon\mathfrak{h}^*;\lambda(H_\alpha)$ is real for each $\alpha\epsilon\Delta\}$. Then, the inner product is strictly positive definite over $\mathfrak{h}_0{}^*$ (cf. [2], p. 145). So, we can define the length of a root α by $\|\alpha\|=(\alpha,\alpha)^{1/2}$. Now given any basis in the dual space of $\mathfrak{h}_0{}^*$, we can introduce a lexicographic ordering in $\mathfrak{h}_0{}^*$. Thus Δ becomes an ordered set. The maximal root with respect to this order is called a highest root of $\mathfrak{g}$. Let ρ be a highest root, then $\|\rho\| \geqq \|\alpha\|$ for any root α.

For each root α, the linear transformation S_α of $\mathfrak{h}_0{}^*$ is defined by

$$S_\alpha(\lambda)=\lambda-\frac{2(\lambda, \alpha)}{(\alpha, \alpha)} \alpha,$$

i. e., S_α is the reflection with respect to the hyperplane $\{\lambda \in \mathfrak{h}_0{}^* ; (\lambda, \alpha)=0\}$. The Weyl group $W=W(\mathfrak{g})$ of $\mathfrak{g}$ is generated by $S_\alpha, \alpha \in \Delta$. If we identify the Weyl group $W(G)$ with W (cf. [5], Exposé 23), then $W(K)$ can be identified with $W(\mathfrak{k})$.

For any subset Σ of Δ, let W_Σ be the group generated by $S_\alpha, \alpha \in \Sigma$.

If K is a connected closed subgroup containing T and Θ is the sharp-system corresponding to K by Theorem 1, then $W(\mathfrak{k})$ is considered as W_Θ. Therefore we may determine the sharp-systems Θ such that W_Θ are normal subgroups of W. But W_Θ is normal in W if and only if Θ is W-invariant, i. e., $w(\Theta) \subset \Theta$ for any element $w \in W$. Thus we obtain the next Theorem.

Theorem 2. *Let G and T be as Theorem* 1. *Let K be a connected closed subgroup of G containing T, then $W(K)$ is a normal subgroup of $W(G)$ if and only if Θ is W-invariant where Θ is the sharp-system corresponding to K by Theorem* 1.

§ 2. W-invariant sharp-systems.

By Theorem 2, our problem was reduced to find all W-invariant sharp-systems of the root system of a complex semi-simple Lie algebra with respect to a given Cartan subalgebra. In this section, we shall study about W-invariant sharp-systems.

Proposition 2. *If $\mathfrak{g}$ is simple, then two roots with same length can be transformed each other by an element of the Weyl group.*

Proof. Since any root is transformed into a simple root by an element of the Weyl group, we may assume that the roots are both simple. Let α, β be simple roots with same length. If $\beta=\alpha$ or $\beta=-\alpha$, then we can choose identity or S_α respectively, as an element of the Weyl group. If $\beta \neq \pm\alpha$, $(\alpha, \beta) \neq 0$, then $S_\alpha(\beta)=\alpha+\beta$, so $S_\beta S_\alpha(\beta)=\alpha$. At last, if $(\alpha, \beta)=0$, then we can choose a sequence of simple roots $\gamma_1=\alpha, \gamma_2, \cdots, \gamma_k=\beta$ such that $(\gamma_i, \gamma_{i+1}) \neq 0$. Then it is easily proved that $\|\gamma_i\| = \|\gamma_{i+1}\|$ from the Dynkin diagram. From the result proved already, γ_i is transformed into γ_{i+1}. So we can prove the proposition for α and β by the induction with respect to k. (Q. E. D.)

For any root α, put $W(\alpha)=\{w(\alpha) ; w \in W\}$.

Theorem 3. *Let* $\mathfrak{g}$ *be a simple Lie algebra,* $\mathfrak{h}$ *a Cartan subalgebra of* $\mathfrak{g}$, *and let* Δ *denote the system of roots of* $\mathfrak{g}$ *with respect to* $\mathfrak{h}$. *If* ρ *is a highest root, then* $W(\rho)$ *is a W-invariant sharp-system.*

Proof. Since W-invariance is trivial, we may only show that it is a sharp-system. If α is an element in $W(\rho)$, then there is such element w in W that $\alpha=w(\rho)$. Hence, $-\alpha=S_\alpha(\alpha)=S_\alpha w(\rho)\in W(\rho)$. Next, we shall assume that $\alpha+\beta$ is a root for α and β in $W(\rho)$. Since $\|\alpha\|=\|\beta\|$, $\hat{\alpha\beta}=60°$, $90°$, or $120°$. If $\hat{\alpha\beta}=60°$ or $90°$, then $\|\alpha+\beta\|>\|\alpha\|=\|\rho\|$ which contradicts to the choice of ρ. Therefore $\hat{\alpha\beta}=120°$, hence $\alpha+\beta$ has the same length as α. By Proposition 2, $\alpha+\beta$ is contained in $W(\alpha)=W(\rho)$. This means that $W(\rho)$ is a sharp-system. (Q. E. D.)

Next, we shall show that any non-trivial W-invariant sharp-systems are only of this type.

Theorem 4. *Let assumptions be as Theorem* 3, *then any non-empty W-invariant sharp-system is equal to* $W(\rho)$ *or* Δ, *where* ρ *is a highest root.*

Proof. Let Θ be a W-invariant sharp-system. For any root α in Θ, $W(\alpha)$ is contained in Θ. Furthermore, if $W(\alpha)$ is consist with Θ, then α must have the same length as ρ. In fact, if $\|\alpha\|<\|\rho\|$, then there are two simple roots α_i, α_j such that $(\alpha_i, \alpha_j)\neq 0$ and $\|\alpha_i\|<\|\alpha_j\|$. So $\|\alpha\|=\|\alpha_i\|$, hence $W(\alpha_i)=W(\alpha)=\Theta$, i. e., $W(\alpha_i)$ is a sharp-system. On the other hand, since $\frac{2(\alpha_i, \alpha_j)}{(\alpha_j, \alpha_j)}=-1$, $\alpha_j=S_{\alpha_j}(\alpha_i)-\alpha_i$. Since $W(\alpha_i)$ is a sharp-system, so the right-hand side of this equality is contained in $W(\alpha_i)$. Hence $\alpha_j\in W(\alpha_i)$ and $\|\alpha_i\|=\|\alpha_j\|$, which contradicts the inequality $\|\alpha_i\|<\|\alpha_j\|$. Next, if $W(\alpha)\neq\Theta$, then we can select an element β in $\Theta-W(\alpha)$. Then $W(\alpha)$ and $W(\beta)$ are obviously disjoint, moreover $\|\alpha\|\neq\|\beta\|$ by Proposition 2. So $W(\alpha)\cup W(\beta)=\Delta$, for, if not, there is a root such that its length is different from ones of α and β, which is impossible in a simple Lie algebra. Hence $\Theta=\Delta$, which completes the proof. (Q. E. D.)

§ 3. $W(\rho)$ in simple Lie algebras.

In this section, we shall decide the $W(\rho)$ for complex simple Lie algebras. Throughout this section, we shall use the following notations.

$\mathfrak{gl}(n, C)$: The set of all complex matrices of degree n.
$\mathfrak{sl}(n, C)$: The set of all complex matrices of degree n with trace 0.
$D(h_1, h_2, \cdots, h_n)$: The diagonal matrix with diagonal elements $h_1, h_2, \cdots, h_n$.
${}^tA$: The transposed matrix of A.
I_n : The identity matrix of degree n.

$$K=\left(\begin{array}{c|c|c} 1 & \multicolumn{2}{c}{0} \\ \hline & 0 & I_n \\ \cline{2-3} 0 & I_n & 0 \end{array}\right) \qquad J=\left(\begin{array}{c|c} 0 & I_n \\ \hline -I_n & 0 \end{array}\right)$$

Type A_n, D_n, and E_i $(i=6, 7, 8)$

Since all roots are as long as the highest root ρ in these cases, we always have $W(\rho)=\Delta$ from Proposition 2.

Type B_n

$\mathfrak{g}=\{A\in\mathfrak{gl}(2n+1, C)\,;{}^tAK+KA=0\}$ is a Lie algebra of type B_n,

and

$\mathfrak{h}=\{H=D(0, h_1, \cdots, h_n, -h_1, \cdots, -h_n)\in\mathfrak{g}\}$ is a Cartan subalgebra of $\mathfrak{g}$. Let $\lambda_i\,(i=1, 2, \cdots, n)$ be the linear forms on $\mathfrak{h}$ defined by $\lambda_i(H)=h_i$ where $H=D(0, h_1, \cdots, h_n, -h_1, \cdots, -h_n)$. So the root system Δ of $\mathfrak{g}$ with respect to $\mathfrak{h}$ is expressed as follows;

$$\Delta=\{\pm\lambda_i, \pm\lambda_i\pm\lambda_k : i\neq k\}$$

Put $\alpha_i=\lambda_i-\lambda_{i+1}\,(1\leqq i\leqq n-1)$, $\alpha_n=\lambda_n$, then $\Pi=\{\alpha_1, \cdots, \alpha_n\}$ is a simple root system and the highest root is

$$\rho=\alpha_1+2\alpha_2+2\alpha_3+\cdots+2\alpha_n=\lambda_1+\lambda_2 .$$

The Weyl group W consists of all permutations of λ_i's and changes of signature of λ_i's. Hence $W(\rho)=\{\pm\lambda_i\pm\lambda_j, i\neq j\}$. The subalgebra corresponding to this sharp-system is of type D_n.

Type C_n

$\mathfrak{g}=\mathfrak{sp}(n, C)=\{A\in\mathfrak{gl}(2n, C)\,;{}^tAJ+JA=0\}$ is a Lie algebra of type C_n, in other words, $\mathfrak{g}$ consists of all complex matrices of degree $2n$ such that

$$A=\left(\begin{array}{c|c} X & Y \\ \hline Z & -{}^tX \end{array}\right), \qquad {}^tY=Y, \quad {}^tZ=Z .$$

$\mathfrak{h}=\{H=D(h_1, \cdots, h_n, -h_1, \cdots, -h_n)\in\mathfrak{g}\}$ is a Cartan subalgebra of $\mathfrak{g}$. Let λ_i be the linear forms on $\mathfrak{h}$ defined by $\lambda_i(H)=h_i$, where $H=D(h_1, \cdots, h_n, -h_1, \cdots, -h_n)$, then the root system is

$$\Delta=\{\pm\lambda_i\pm\lambda_k\}$$

Put $\alpha_i=\lambda_i-\lambda_{i+1}\,(1\leqq i\leqq n-1)$, $\alpha_n=2\lambda_n$, then, $\Pi=\{\alpha_1, \cdots, \alpha_n\}$ is a simple root system and the highest root is

$$\rho=2\alpha_1+\cdots+2\alpha_{n-1}+\alpha_n=2\lambda_1 .$$

The Weyl group of $\mathfrak{g}$ is the same as the Weyl group of type B_n. Hence, $W(\rho)=\{2\lambda_i\,;1\leqq i\leqq n\}$. The corresponding subalgebra is of type $\underbrace{A_1\times A_1\times\cdots\times A_1}_{n}$.

Type G_2

The root system can be expressed as follows;

$$\Delta=\{\pm(\lambda_i-\lambda_j),\pm(\lambda_i+\lambda_j-2\lambda_k):1\leqq i,j,k\leqq 3\}$$

Put $\alpha_1=\lambda_1-\lambda_2$, $\alpha_2=-\lambda_1+2\lambda_2-\lambda_3$, then $\Pi=\{\alpha_1,\alpha_2\}$ is a simple root system and the highest root is

$$\rho=3\alpha_1+2\alpha_2=\lambda_1+\lambda_2-2\lambda_3 .$$

Hence $W(\rho)\subset\{\pm(\lambda_i+\lambda_j-2\lambda_k)\}$. As W contains the Weyl group of type A_2, $W(\rho)=\{\pm(\lambda_i+\lambda_j-2\lambda_k)\}$. Thus we can know that the Weyl group of type A_2 is a normal subgroup of the Weyl group of type G_2, for the corresponding subalgebra to $W(\rho)$ is of type A_2.

Type F_4

The root system can be expressed as follows;

$$\Delta=\{\pm\lambda_i\pm\lambda_j\,(0\leqq i<j\leqq 3),\frac{1}{2}(\pm\lambda_0\pm\lambda_1\pm\lambda_2\pm\lambda_3)\}$$

where $\lambda_0,\lambda_1,\lambda_2,\lambda_3$ are an orthonormal basis of a Euclidean space of dimension 4. Put $\alpha_1=\lambda_1-\lambda_2$, $\alpha_2=\lambda_2-\lambda_3$, $\alpha_3=\lambda_3$, $\alpha_4=\frac{1}{2}(\lambda_0-\lambda_1-\lambda_2-\lambda_3)$, then $\Pi=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$ is a simple root system and the highest root is

$$\rho=2\alpha_1+3\alpha_2+4\alpha_3+3\alpha_4=\lambda_0+\lambda_1 .$$

Since the length of root is invariant under the operations of Weyl group, $W(\rho)\subset\{\pm\lambda_i\pm\lambda_j\}$. On the other hand, since W contains the Weyl group of type B_4 as subgroup (cf. [4]), $W(\rho)=\{\lambda_i\pm\lambda_j\,(i\neq j)\}$. The subalgebra corresponding to $W(\rho)$ is of type D_4.

Thus we can reach the next Theorem.

Theorem 5. *Let G be a connected, compact simple Lie group, and K a connected, closed subgroup of G of maximal rank. Let $W(G)$ and $W(K)$ denote their Weyl groups. Then $W(K)$ is a normal subgroup of $W(G)$ if and only if $(G;K)$ belongs to next four classes;* $(B_n;D_n)$, $(C_n;\underbrace{A_1\times\cdots\times A_1}_{n})$, $(G_2;A_2)$ *and* $(F_4;D_4)$.

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REFERENCES

[1] A. Borel et J. Siebenthal; Les sous-groupes fermés connexes de rang maximum des groupes de Lie clos. Comm. Math. Helv. Vol. 23 (1949-1950), pp. 200-221.

[2] S. Helgason; Differential geometry and symmetric spaces. Academic Press, 1962.

[3] N. Iwahori; On the structure of a Hecke ring of a Chevalley group over a field. Jour. Fac. Sci., Univ. Tokyo, Vol. 10 (1964), pp. 215-236.

[4] T. Kondo; The characters of the Weyl group of type F_4. Jour. Fac. Sci., Univ. Tokyo, Vol. 11 (1965), pp. 145-153.

[5] Seminaire Sophus Lie, Theorie des algebres de Lie, Topologie des groupes de Lie. Ecole Norm. Sup., Paris, 1955.

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