PHYSICAL REVIEW D VOLUME 50, NUMBER 11 1 DECEMBER 1994

Background field method: Alternative way of deriving the pinch technique's results

Shoji Hashimoto, Jiro Kodaira, and Yoshiaki Yasui
Department of Physics, Hiroshima University, Higashi-Hiroshima 724, Japan

Ken Sasaki*
Department of Physics, Yokohama National University, Yokohama 240, Japan
(Received 10 June 1994)

We show that the background field method (BFM) is a simple way of deriving the same gauge-invariant results which are obtained by the pinch technique (PT). For illustration we construct gauge-invariant self-energy and three-point vertices for gluons at the one-loop level by the BFM and demonstrate that we get the same results which were derived via the PT. We also calculate the four-gluon vertex in the BFM and show that this vertex obeys the same Ward identity that was found with the PT.

PACS number(s): 11.15.Bt, 11.30.−j, 12.38.Bx

I. INTRODUCTION

The formulation of a gauge theory begins with a gauge-invariant Lagrangian. However, except for lattice gauge theory, when we quantize the theory in the continuum we are under compulsion to fix a gauge. Consequently, the corresponding Green's functions, in general, will not be gauge invariant. These Green's functions in the standard formulation do not directly reflect the underlying gauge invariance of the theory but rather obey complicated Ward identities. If there is a method in which we can construct systematically gauge-invariant Green's functions, then it will make the computations much simpler and may have many applications.

Two approaches along this line exist: One is the pinch technique and the other the background field method. The pinch technique (PT) was proposed some time ago by Cornwall [1,2] for a well-defined algorithm to form new gauge-independent proper vertices and new propagators with gauge-invariant self-energies. Using this technique Cornwall and Papavassiliou obtained the one-loop gauge-invariant self-energy and vertex parts in QCD [3,4]. Later it was shown [5] that the PT works also in spontaneously broken gauge theories, and since then it has been applied to the standard model to obtain a gauge-invariant electromagnetic form factor of the neutrino [5], one-loop gauge-invariant WW and ZZ self-energies [6], and γWW and ZWW vertices [7].

On the other hand, the background field method (BFM) was first introduced by DeWitt [8] as a technique for quantizing gauge field theories while retaining explicit gauge invariance. In its original formulation, DeWitt worked only for one-loop calculations. The multiloop extension of the method was given by 't Hooft [10], DeWitt [9], Boulware [11], and Abbott [12]. Using these extensions of the background field method, explicit two-loop calculations of the β function for pure Yang-Mills theory was made first in the Feynman gauge [12,13], and later in the general gauge [14].

Both the PT and BFM have the same interesting feature. The Green's functions (gluon self-energies and proper gluon-vertices, etc.) constructed by the two methods retain the explicit gauge invariance; thus obey the naive Ward identities. As a result, for example, a computation of the QCD β-function coefficient is much simplified. The only thing we need to do is to construct the gauge-invariant gluon self-energy in either method and to examine its ultraviolet-divergent part. Either method gives the same correct answer [3,12]. Thus it may be plausible to anticipate that the PT and BFM are equivalent and that they produce exactly the same results.

In this paper we show that the BFM is an alternative and simple way of deriving the same gauge-invariant results which are obtained by the PT. Although the final results obtained by both methods are gauge invariant, we have found, in particular, that the BFM in the Feynman gauge corresponds to the intrinsic PT. In fact we explicitly demonstrate, for the cases of the gauge-invariant gluon self-energy and three-point vertex, that both methods with the Feynman gauge produce the same results which are equal term by term. We also give the gauge-invariant four-gluon vertex calculated in the BFM and show explicitly that this vertex satisfies the same simple Ward identity that was found with the PT.

The paper is organized as follows. In Sec. II we review the intrinsic PT and explain how the gauge-invariant proper self-energy and three-point vertex for the gluon were derived in the PT. In Sec. III we write down the Feynman rule for QCD in the BFM and compute the gauge-invariant gluon self-energy at the one-loop level in the BFM with the Feynman gauge. The result is shown to be the same, term by term, as the one obtained by the intrinsic PT. The BFM is applied, in Sec. IV, to the calculation of the three-gluon vertex. The result is shown to coincide, again term by term, with the one derived by the intrinsic PT. In Sec. V we compute the gauge-invariant four-gluon vertex at one-loop level in the

*Electronic address: a121004@c1.ed.ynu.ac.jp

BFM. We present each contribution to the vertex from the individual Feynman diagram. Then we show that the acquired four-gluon vertex satisfies the same naive Ward identity that was found with the PT and is related to the gauge-invariant three-gluon vertex obtained previously by the PT and BFM.

II. INTRINSIC PINCH TECHNIQUE

There are three equivalent versions of the pinch technique: the S matrix PT [1–3], the intrinsic PT [3], and the Degrassi-Sirlin alternative formulation of the PT [6]. To prepare for the later discussions and to establish the notation, we briefly review, in this section, the intrinsic PT and explain the way the gauge-invariant proper self-energy and three-point vertex for gluons at the one-loop level were obtained in Ref. [3].

In the S-matrix pinch technique we obtain the gauge-invariant effective gluon propagator by adding the pinch graphs in Figs. 1(b) and 1(c) to the ordinary propagator graphs [Fig. 1(a)]. The gauge dependence of the ordinary graphs is canceled by the contributions from the pinch graphs. Since the pinch graphs are always missing one or more propagators corresponding to the external legs, the gauge-dependent parts of the ordinary graphs must also be missing one or more external propagator legs. So if we extract systematically from the proper graphs the parts which are missing external propagator legs and simply throw them away, we obtain the gauge-invariant results. This is the intrinsic PT introduced by Cornwall and Papavassiliou [3].

We will now derive the gauge-invariant proper self-energy for gluons of the gauge group SU(N) using the intrinsic PT. Since we know that the PT successfully gives gauge-invariant quantities, we use the Feynman gauge. Then the ordinary proper self-energy whose corresponding graphs are shown in Fig. 2 is given by

$$\Pi^0_{\mu\nu} = \frac{iNg^2}{2}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2(k+q)^2} \times[\Gamma_{\alpha\mu\lambda}(k,q)\Gamma_{\lambda\nu\alpha}(k+q,-q) -k_\mu(k+q)_\nu - k_\nu(k+q)_\mu], \quad (1)$$

where we have symmetrized the ghost loop in Fig. 2(b) and omitted a trivial group-theoretic factor δ^{ab}. We assume dimensional regularization in $D = 4 - 2\varepsilon$ dimensions. The three-gluon vertex $\Gamma_{\alpha\mu\lambda}(k,q)$ has the expression [15]

$$\begin{aligned}\Gamma_{\alpha\mu\lambda}(k,q) &\equiv \Gamma_{\alpha\mu\lambda}(k,q,-k-q)\\ &= (k-q)_\lambda g_{\alpha\mu} + (k+2q)_\alpha g_{\mu\lambda}\\ &\quad -(2k+q)_\mu g_{\lambda\alpha}.\end{aligned} \quad (2)$$

Here and in the following we make it a rule that whenever the external momentum appears in the three-gluon vertex, we put it in the middle of the expression, that is, like q_μ in Eq. (2). Now we decompose the vertices into two pieces: a piece Γ^F which has terms with external momentum q and a piece Γ^P (P for pinch) which carries the internal momenta only:

$$\begin{aligned}\Gamma_{\alpha\mu\lambda}(k,q) &= \Gamma^F_{\alpha\mu\lambda} + \Gamma^P_{\alpha\mu\lambda},\\ \Gamma^F_{\alpha\mu\lambda}(k,q) &= -(2k+q)_\mu g_{\lambda\alpha} + 2q_\alpha g_{\mu\lambda} - 2q_\lambda g_{\alpha\mu},\\ \Gamma^P_{\alpha\mu\lambda}(k,q) &= k_\alpha g_{\mu\lambda} + (k+q)_\lambda g_{\alpha\mu}.\end{aligned} \quad (3)$$

The full vertex $\Gamma_{\alpha\mu\lambda}(k,q)$ satisfies the Ward identities

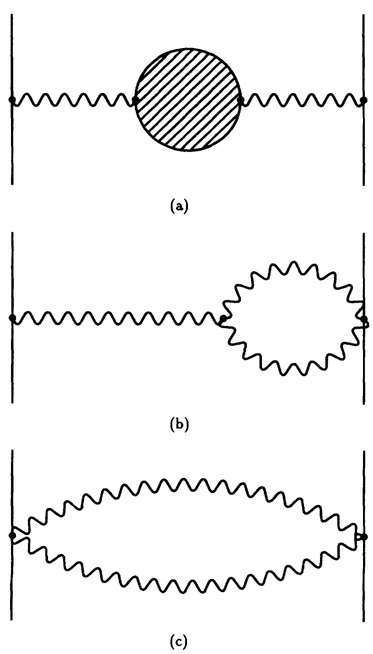

FIG. 1. S-matrix pinch technique applied for the elastic scattering of two fermions. Graphs (b) and (c) are pinch parts, which, when added to the ordinary propagator graphs (a), yield the gauge-invariant effective gluon propagator.

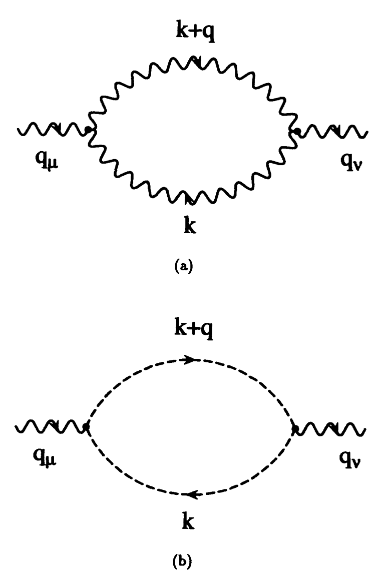

FIG. 2. Graphs for the ordinary proper self-energy $\Pi^0_{\mu\nu}$. (a) Gluon loop. (b) Ghost loop. Momenta and Lorentz indices are indicated.

$$k^\alpha \Gamma_{\alpha\mu\lambda}(k,q) = P_{\mu\lambda}(q)d^{-1}(q) - P_{\mu\lambda}(k+q)d^{-1}(k+q),$$
$$(k+q)^\lambda \Gamma_{\alpha\mu\lambda}(k,q) = P_{\alpha\mu}(q)d^{-1}(q) - P_{\alpha\mu}(k)d^{-1}(k), \quad (4)$$

where we have defined

$$P_{\mu\nu}(q) = -g_{\mu\nu} + q_\mu q_\nu q^{-2}, \qquad d^{-1}(q) = q^2. \quad (5)$$

The rules of the intrinsic PT are to let the pinch vertex Γ^P act on the full vertex and to throw out the $d^{-1}(q)$ terms thereby generated. We rewrite the product of the two full vertices $\Gamma_{\alpha\mu\lambda}\Gamma_{\lambda\nu\alpha}$ as

$$\Gamma_{\alpha\mu\lambda}\Gamma_{\lambda\nu\alpha} = \Gamma^F_{\alpha\mu\lambda}\Gamma^F_{\lambda\nu\alpha} + \Gamma^P_{\alpha\mu\lambda}\Gamma_{\lambda\nu\alpha} + \Gamma_{\alpha\mu\lambda}\Gamma^P_{\lambda\nu\alpha} - \Gamma^P_{\alpha\mu\lambda}\Gamma^P_{\lambda\nu\alpha}. \quad (6)$$

Using the Ward identities in Eq. (4) we find that the sum of the second and third terms of Eq. (6) turns out to be

$$\Gamma^P_{\alpha\mu\lambda}\Gamma_{\lambda\nu\alpha} + \Gamma_{\alpha\mu\lambda}\Gamma^P_{\lambda\nu\alpha} = 4P_{\mu\nu}(q)d^{-1}(q) - 2P_{\mu\nu}(k)d^{-1}(k) - 2P_{\mu\nu}(k+q)d^{-1}(k+q). \quad (7)$$

We drop the first term on the right-hand side (RHS) of (7) following the intrinsic PT rule. Now we use the dimensional regularization rule (which we adhere to throughout this paper)

$$\int d^D k k^{-2} = 0, \quad (8)$$

and discard the parts which disappear after integration; then the second and third terms can be written as

$$-2P_{\mu\nu}(k)d^{-1}(k) - 2P_{\mu\nu}(k+q)d^{-1}(k+q) = -2k_\mu k_\nu - 2(k+q)_\mu(k+q)_\nu. \quad (9)$$

Also applying the dimensional regularization rule Eq. (8) to the fourth term on the RHS of Eq. (6), we find

$$-\Gamma^P_{\alpha\mu\lambda}\Gamma^P_{\lambda\nu\alpha} = -2k_\mu k_\nu - (k_\mu q_\nu + q_\mu k_\nu). \quad (10)$$

Now combining the first term on the RHS of Eq. (6) with Eqs. (9) and (10), and inserting them into Eq. (1), we arrive at the following expression for the gauge-invariant self-energy [3]:

$$\widehat{\Pi}_{\mu\nu} = \frac{iNg^2}{2}\int \frac{d^D k}{(2\pi)^D}\frac{1}{k^2(k+q)^2} \times [\Gamma^F_{\alpha\mu\lambda}(k,q)\Gamma^F_{\lambda\nu\alpha}(k+q,-q) - 2(2k+q)_\mu(2k+q)_\nu]. \quad (11)$$

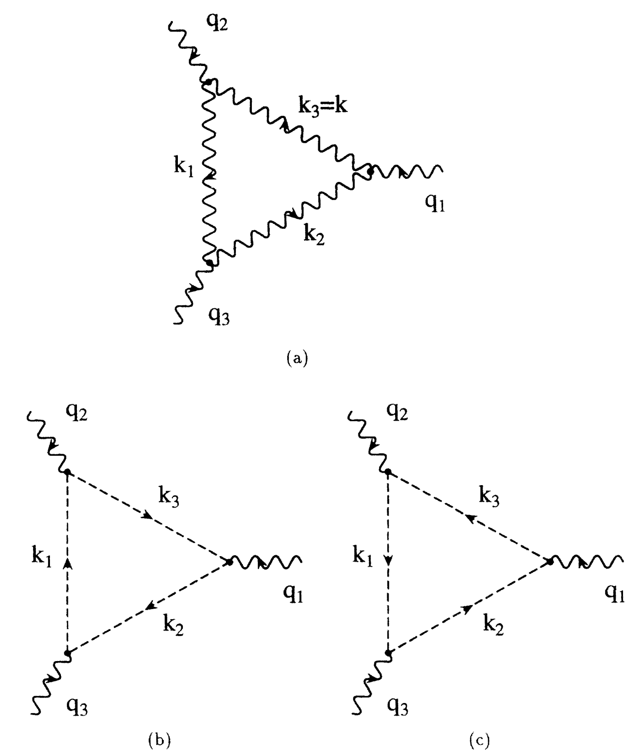

FIG. 3. Graphs for the ordinary proper three-gluon vertex $\Gamma^0_{\mu\nu\lambda}$. (a) Gluon loop. (b), (c) Ghost loops. Momenta and Lorentz indices are indicated.

The same rules are applied to obtain the gauge-invariant three-gluon vertex at the one-loop level. The contributions of the graphs depicted in Fig. 3 to the ordinary proper three-gluon vertex are summarized as [15]

$$\Gamma^0_{\mu\nu\alpha} = -\frac{iNg^2}{2}\int \frac{d^D k}{(2\pi)^D}\frac{1}{k_1^2 k_2^2 k_3^2} N_{\mu\nu\alpha}, \quad (12)$$

$$N_{\mu\nu\alpha} = \Gamma_{\sigma\mu\lambda}(k_2,q_1)\Gamma_{\lambda\nu\rho}(k_3,q_2)\Gamma_{\rho\alpha\sigma}(k_1,q_3) + k_{1\nu}k_{2\alpha}k_{3\mu} + k_{1\alpha}k_{2\mu}k_{3\nu}, \quad (13)$$

where the momenta and Lorentz indices are defined in Fig. 3(a) and the overall group-theoretic factor gf^{abc} is omitted.

Decomposing Γ into $\Gamma^F + \Gamma^P$ and dropping the terms involving $d^{-1}(q_i)$ which are generated by application of Γ^P to the full vertices, Cornwall and Papavassiliou obtained the following expression for gauge-invariant proper three-gluon vertex:

$$\widehat{\Gamma}_{\mu\nu\alpha}(q_1,q_2,q_3) = -\frac{iNg^2}{2}\left\{\int \frac{d^D k}{(2\pi)^D}\frac{1}{k_1^2k_2^2k_3^2}[\Gamma^F_{\sigma\mu\lambda}(k_2,q_1)\Gamma^F_{\lambda\nu\rho}(k_3,q_2)\Gamma^F_{\rho\alpha\sigma}(k_1,q_3) + 2(k_2+k_3)_\mu(k_3+k_1)_\nu(k_1+k_2)_\alpha] - 8(q_{1\alpha}g_{\mu\nu} - q_{1\nu}g_{\mu\alpha})\widetilde{A}(q_1) - 8(q_{2\mu}g_{\alpha\nu} - q_{2\alpha}g_{\mu\nu})\widetilde{A}(q_2) - 8(q_{3\nu}g_{\mu\alpha} - q_{3\mu}g_{\nu\alpha})\widetilde{A}(q_3)\right\}, \quad (14)$$

where $\widetilde{A}(q_i)$ is defined by

$$\widetilde{A}(q_i) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 (k+q_i)^2}. \tag{15}$$

III. BACKGROUND FIELD METHOD AND THE GAUGE-INVARIANT GLUON SELF-ENERGY

In this section we write down the Feynman rules for QCD in the background field calculations and compute the gluon self-energy in the Feynman gauge. Then we will see that the result coincides, term by term, with the gauge-invariant gluon self-energy derived via the intrinsic PT.

In the BFM, the field in the classical Lagrangian is written as $A+Q$, where A (Q) denotes the background (quantum) field. The Feynman diagrams with A on external legs and Q inside loops need to be calculated. The relevant Feynman rules [12] are given in Fig. 4. It is noted that the Feynman rule for the ghost-A vertex is similar to the one which appears in the scalar QED. Now let us write a three-point vertex with one A^b_μ field as

$$\widetilde{\Gamma}^{abc}_{\lambda\mu\nu}(p,q,r) = g f^{bac} \widetilde{\Gamma}_{\lambda\mu\nu}(p,q,r), \tag{16}$$

$$\widetilde{\Gamma}_{\lambda\mu\nu}(p,q,r) = \left(p - q + \frac{1}{\xi} r\right)_\nu g_{\lambda\mu} + \left(q - r - \frac{1}{\xi} p\right)_\lambda g_{\mu\nu} + (r-p)_\mu g_{\nu\lambda}. \tag{17}$$

Then we find that in the Feynman gauge $\xi = 1$, $\widetilde{\Gamma}_{\alpha\mu\lambda}(k,q,-k-q)$ turns out to be

$$\widetilde{\Gamma}_{\alpha\mu\lambda}(k,q,-k-q)|_{\xi=1} = -2q_\lambda g_{\alpha\mu} + 2q_\alpha g_{\mu\lambda} - (2k+q)_\mu g_{\lambda\alpha}, \tag{18}$$

which coincides with the expression of $\Gamma^F_{\alpha\mu\lambda}$ in Eq. (3). This fact gives us a hint that the BFM may reproduce the same results which are obtained by the intrinsic PT (and we find later that it is true in fact).

Now we calculate the gluon self-energy in the BFM with the Feynman gauge. The relevant diagrams are depicted in Fig. 5. Diagram 5(a) gives a contribution

$$\widehat{\Pi}^{(a)}_{\mu\nu} = \frac{iNg^2}{2} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 (k+q)^2} \Gamma^F_{\alpha\mu\lambda}(k,q) \times \Gamma^F_{\lambda\nu\alpha}(k+q,-q), \tag{19}$$

where we have used the fact $\widetilde{\Gamma}_{\alpha\mu\lambda}|_{\xi=1} = \Gamma^F_{\alpha\mu\lambda}$ and $\widetilde{\Gamma}_{\lambda\nu\alpha}|_{\xi=1} = \Gamma^F_{\lambda\nu\alpha}$. On the other hand, through the scalar QED-like coupling for the background field and ghost vertices, diagram 5(b) gives

$$\widehat{\Pi}^{(b)}_{\mu\nu} = \frac{iNg^2}{2} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 (k+q)^2} [-2(2k+q)_\mu (2k+q)_\nu]. \tag{20}$$

It is interesting to note that the contributions of diagrams 5(a) and 5(b), respectively, correspond to the first and

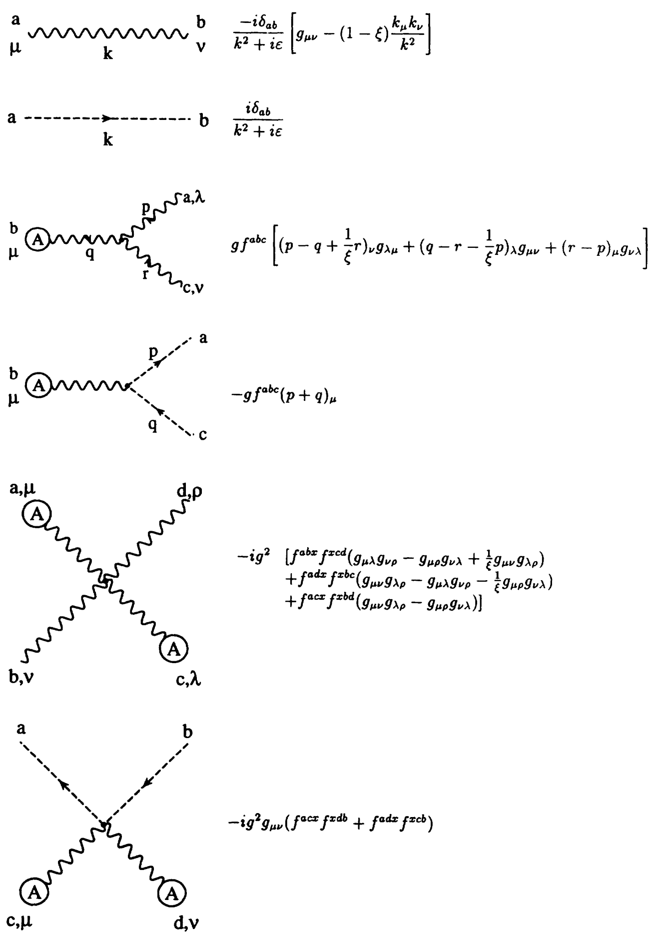

FIG. 4. Feynman rules for background field calculations in QCD. The wavy lines terminating in an A represent external gauge fields. The other wavy lines and dashed lines represent Q fields and ghost fields, respectively. Only shown are rules which are requisite for calculations in this paper.

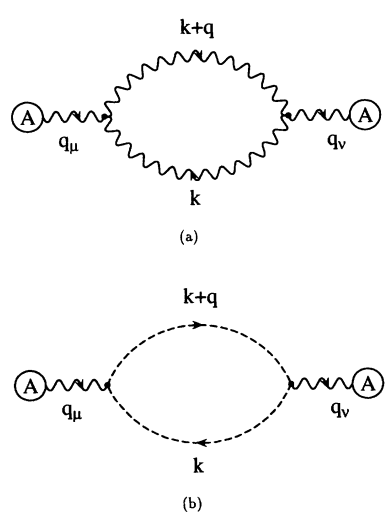

FIG. 5. Graphs for a calculation of the gauge-invariant self-energy $\widehat{\Pi}_{\mu\nu}$ in the BFM. (a) Gluon loop. (b) Ghost loop. Momenta and Lorentz indices are indicated.

second terms in the parentheses of Eq. (11) and the sum of the two contributions coincides with the expression of the gauge-invariant self-energy $\widehat{\Pi}_{\mu\nu}$ which was derived in Sec. II by the method of the intrinsic PT.

IV. GAUGE-INVARIANT THREE-GLUON VERTEX

The success in deriving the gauge-invariant PT result for the gluon self-energy by the BFM gives us momentum to study, for the next step, the gauge-invariant three-gluon vertex at the one-loop level. The relevant diagrams are shown in Fig. 6, where momenta and Lorentz and color indices are displayed. With the fact that an AQQ vertex in the Feynman gauge, $\widetilde{\Gamma}_{\xi=1}$, is equivalent to Γ^F in Eq. (3), it is easy to show that the contribution of diagram 6(a) is

$$^{(a)}\Gamma^{abc}_{\mu\nu\alpha}(q_1,q_2,q_3) = -\frac{iNg^3f^{abc}}{2}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k_1^2k_2^2k_3^2} \times[\Gamma^F_{\sigma\mu\lambda}(k_2,q_1)\Gamma^F_{\lambda\nu\rho}(k_3,q_2) \times\Gamma^F_{\rho\alpha\sigma}(k_1,q_3)]. \qquad (21)$$

The contribution of diagram 6(b) (and the similar one with the ghost running the other way) is

$$^{(b)}\Gamma^{abc}_{\mu\nu\alpha}(q_1,q_2,q_3) = -\frac{iNg^3f^{abc}}{2}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k_1^2k_2^2k_3^2} \times[2(k_2+k_3)_\mu(k_3+k_1)_\nu(k_1+k_2)_\alpha]. \qquad (22)$$

When we calculate diagram 6(c), again we use the Feynman gauge ($\xi = 1$) for the four-point vertex with two background fields. Remembering that diagram 6(c) has a symmetric factor $\frac{1}{2}$ and adding the two other similar diagrams, we find

$$^{(c)}\Gamma^{abc}_{\mu\nu\alpha}(q_1,q_2,q_3) = \frac{iNg^3f^{abc}}{2}[8(q_{1\alpha}g_{\mu\nu}-q_{1\nu}g_{\mu\alpha})\widetilde{A}(q_1) +8(q_{2\mu}g_{\alpha\nu}-q_{2\alpha}g_{\mu\nu})\widetilde{A}(q_2) +8(q_{3\nu}g_{\mu\alpha}-q_{3\mu}g_{\nu\alpha})\widetilde{A}(q_3)]. \qquad (23)$$

Finally, the contribution of diagram 6(d) (and two other similar diagrams) turns out to be null because of the group-theoretical identity for the structure constants f^{abc} such as

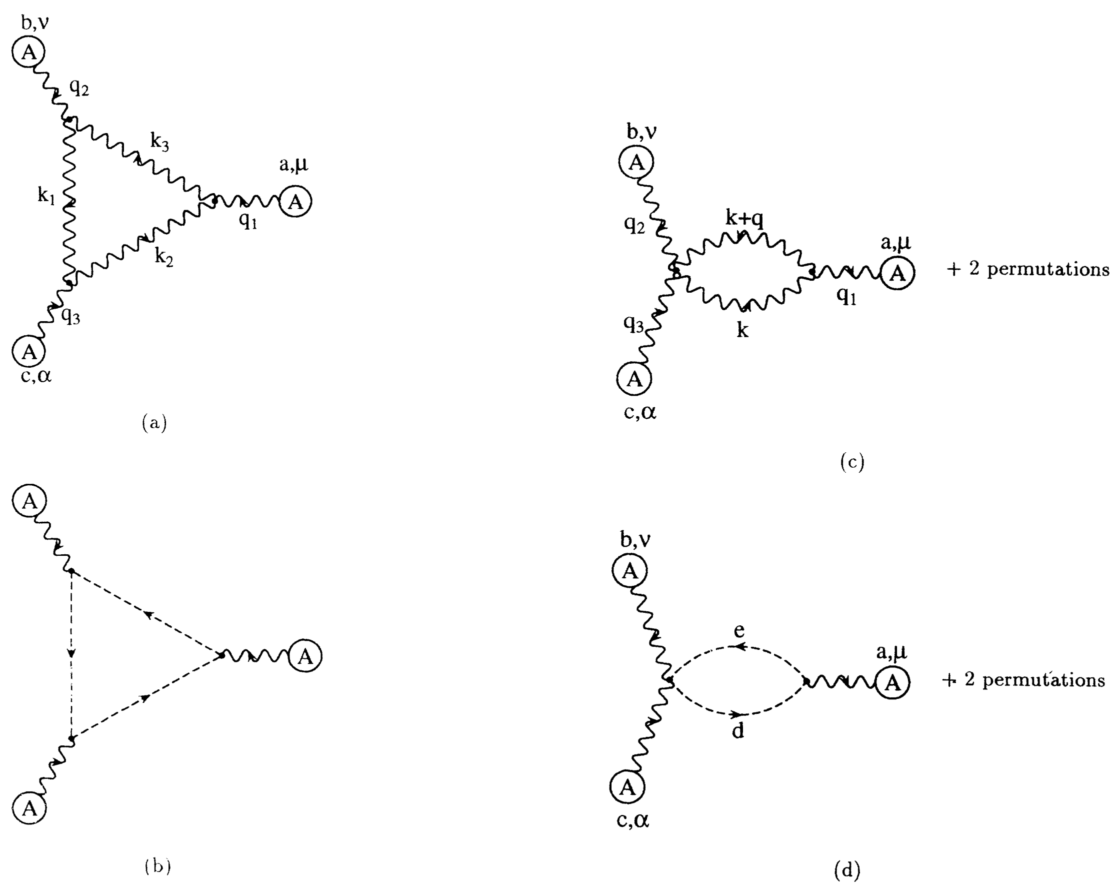

FIG. 6. Graphs for a calculation of the gauge-invariant three-gluon vertex $\widehat{\Gamma}_{\mu\nu\lambda}$ in the BFM. (a), (c) Gluon loops. (b),(d) Ghost loops. Momenta and Lorentz indices are indicated.

$$f^{ead}(f^{dbx}f^{xce} + f^{dcx}f^{xbe}) = 0. \qquad (24)$$

Now adding the contributions from diagrams (a)–(c) in Fig. 6 and omitting the overall group-theoretic factor gf^{abc}, we find that the result coincides with the expression of Eq. (14) which was obtained by the intrinsic PT. Also we note that each contribution from diagrams 6(a)–6(c), respectively, corresponds to a particular term in Eq. (14).

Finally we close this section with a mention that the constructed $\widehat{\Gamma}_{\mu\nu\alpha}(q_1, q_2, q_3)$ is related to the gauge-invariant self-energy $\widehat{\Pi}_{\mu\nu}$ of Eq. (11) through a Ward identity [3]

$$q_1^\mu \widehat{\Gamma}_{\mu\nu\alpha}(q_1, q_2, q_3) = -\widehat{\Pi}_{\nu\alpha}(q_2) + \widehat{\Pi}_{\nu\alpha}(q_3), \qquad (25)$$

which is indeed a naive extension of the tree-level one.

V. GAUGE-INVARIANT FOUR-GLUON VERTEX AND ITS WARD IDENTITY

The gauge-invariant four-gluon vertex has been constructed by Papavassiliou [4] using the S-matrix PT. As he stated in Ref. [4], the construction was a nontrivial task because of the large number of graphs and certain subtleties of the PT. Although he did not report the exact closed form of the gauge-invariant four-gluon vertex, he showed that the new four-gluon vertex is related to the previously constructed $\widehat{\Gamma}_{\mu\nu\alpha}$ in Eq. (14) through a simple Ward identity. In this section we apply the BFM with the Feynman gauge to obtain the gauge-invariant four-gluon vertex at the one-loop level. We give the closed form of this vertex and show that it satisfies the same Ward identity which was proved by Papavassiliou.

The bare four-gluon vertex in Fig. 7(a) is expressed as $-ig^2\Gamma^{abcd}_{\mu\nu\alpha\beta}$ with

$$\begin{aligned}\Gamma^{abcd}_{\mu\nu\alpha\beta} = {} & f^{abx}f^{cdx}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) \\ & + f^{acx}f^{dbx}(g_{\mu\beta}g_{\alpha\nu} - g_{\mu\nu}g_{\alpha\beta}) \\ & + f^{adx}f^{bcx}(g_{\mu\nu}g_{\beta\alpha} - g_{\mu\alpha}g_{\beta\nu}),\end{aligned} \qquad (26)$$

while the bare three-gluon vertex in Fig. 7(b) is expressed as $g\Gamma^{abc}_{\mu\nu\lambda}(k_1, k_2, k_3)$ with

$$\Gamma^{abc}_{\mu\nu\lambda}(k_1, k_2, k_3) = f^{abc}\Gamma_{\mu\nu\lambda}(k_1, k_2, k_3) \qquad (27)$$

and $\Gamma_{\mu\nu\lambda}(k_1, k_2, k_3)$ is given by Eq. (2). Now acting with q_1^μ on $\Gamma^{abcd}_{\mu\nu\alpha\beta}$, we get

$$\begin{aligned}q_1^\mu\Gamma^{abcd}_{\mu\nu\alpha\beta} = {} & f^{abx}f^{cdx}(q_{1\alpha}g_{\nu\beta} - q_{1\beta}g_{\nu\alpha}) \\ & + f^{acx}f^{dbx}(q_{1\beta}g_{\alpha\nu} - q_{1\nu}g_{\alpha\beta}) \\ & + f^{adx}f^{bcx}(q_{1\nu}g_{\beta\alpha} - q_{1\alpha}g_{\beta\nu}).\end{aligned} \qquad (28)$$

Next with the help of the Jacobi identity

$$f^{abx}f^{cdx} + f^{acx}f^{dbx} + f^{adx}f^{bcx} = 0, \qquad (29)$$

we add

$$\begin{aligned}0 = {} & (f^{abx}f^{cdx} + f^{acx}f^{dbx} + f^{adx}f^{bcx}) \\ & \times[(q_4 - q_3)_\nu g_{\alpha\beta} + (q_2 - q_4)_\alpha g_{\beta\nu} + (q_3 - q_2)_\beta g_{\nu\alpha}]\end{aligned} \qquad (30)$$

to the RHS of Eq. (28) and we obtain the tree-level Ward identity [15]

$$\begin{aligned}q_1^\mu\Gamma^{abcd}_{\mu\nu\alpha\beta} = {} & -f^{abx}\Gamma^{cdx}_{\alpha\beta\nu}(q_3, q_4, q_1 + q_2) \\ & -f^{acx}\Gamma^{dbx}_{\beta\nu\alpha}(q_4, q_2, q_1 + q_3) \\ & -f^{adx}\Gamma^{bcx}_{\nu\alpha\beta}(q_2, q_3, q_1 + q_4).\end{aligned} \qquad (31)$$

The bare three- and four-gluon vertices are manifestly gauge independent. However, if we consider the *usual* one-loop corrections to these vertices, they become gauge dependent and do not satisfy Eq. (31) any more.

We now apply the BFM to the case of the four-gluon vertex and show that the constructed gauge-invariant vertex satisfies the generalized version of the Ward identity in Eq. (31). The diagrams for the four-gluon vertex at the one-loop level are shown in Fig. 8. It is noted that the two-gluon loop diagrams 8(e) have a symmetric factor $\frac{1}{2}$. For later convenience let us introduce the group-theoretic quantities

$$f(abcd) \equiv f^{alm}f^{bmn}f^{cne}f^{del}, \qquad (32)$$

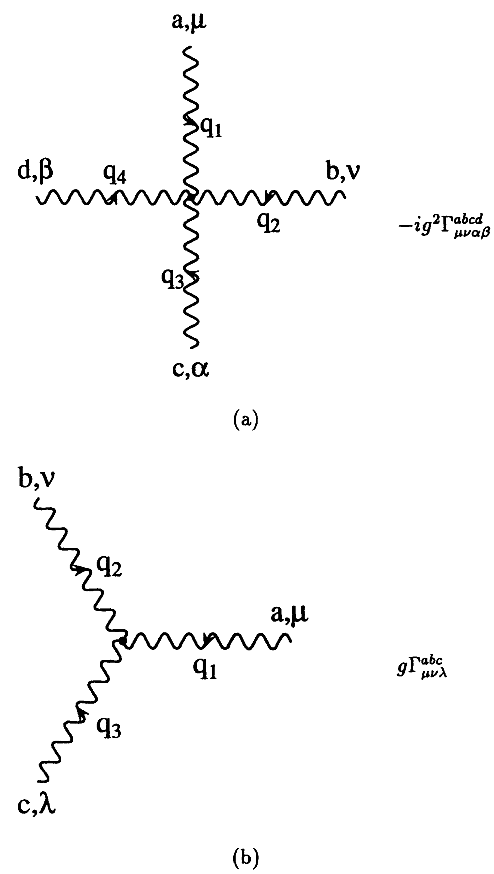

FIG. 7. The bare four-gluon vertex (a) and the bare three-gluon vertex (b). Momenta and color and Lorentz indices are indicated.

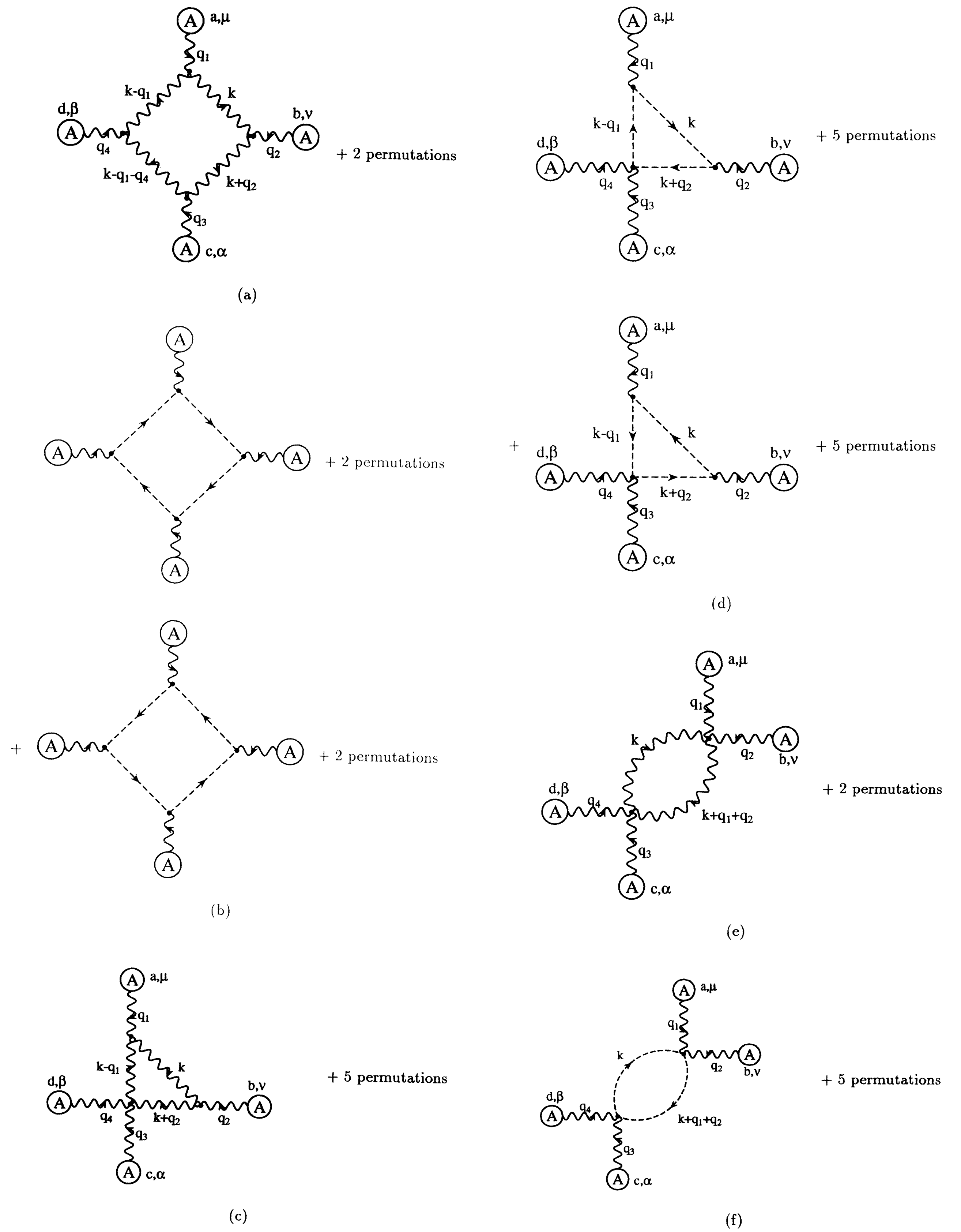

FIG. 8. Graphs for a calculation of the gauge-invariant four-gluon vertex $\widehat{\Gamma}_{\mu\nu\alpha\beta}$ in the BFM. (a),(c),(e), Gluon loops. (b),(d),(f) Ghost loops. Momenta and color and Lorentz indices are indicated.

which satisfy the relations

$$f(abcd) = f(bcda) = f(badc), \quad (33)$$

$$f(abcd) - f(abdc) = -\frac{N}{2} f^{abx} f^{cdx}. \quad (34)$$

The last relation is derived from an identity for the structure constants f^{abc},

$$f^{alm} f^{bmn} f^{cnl} = \frac{N}{2} f^{abc}, \quad (35)$$

and the Jacobi identity Eq. (29).

It is straightforward to evaluate the diagrams in Fig. 8, and the relevant momenta, color, and Lorentz indices are indicated in the graphs. The relations for color factors in Eqs. (33) and (34) are extensively used. We present each contribution to the vertex from the individual Feynman diagram, expecting that each contribution corresponds to the particular term of the intrinsic PT result once the calculation is made in the future. The results are the following.

(a) Diagrams 8(a) give

$$\begin{aligned}
{}^{(a)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = {} & ig^2 f(abcd) \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(k+q_2)^2(k-q_1-q_4)^2(k-q_1)^2} \\
& \times [\Gamma^F_{\lambda\mu\rho}(k-q_1,q_1)\Gamma^F_{\rho\nu\tau}(k,q_2)\Gamma^F_{\tau\alpha\kappa}(k+q_2,q_3)\Gamma^F_{\kappa\beta\lambda}(k-q_1-q_4,q_4)] \\
& +\{(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_3,c,\alpha) \longleftrightarrow (q_4,d,\beta)\},
\end{aligned} \quad (36)$$

where the notation $\{(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)\}$ represents a term obtained from the first one by the substitution $(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)$. The same notation applies to the third term, and also to the expressions below.

(b) Diagrams 8(b) give

$$\begin{aligned}
{}^{(b)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = {} & -ig^2 f(abcd) \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(k+q_2)^2(k-q_1-q_4)^2(k-q_1)^2} \\
& \times [2(2k-q_1)_\mu(2k+q_2)_\nu(2k-q_1-q_4+q_2)_\alpha(2k-2q_1-q_4)_\beta] \\
& +\{(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_3,c,\alpha) \longleftrightarrow (q_4,d,\beta)\}.
\end{aligned} \quad (37)$$

(c) Diagrams 8(c) give

$$\begin{aligned}
{}^{(c)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = {} & ig^2 \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(k+q_2)^2(k-q_1)^2} \\
& \times \{-[f(abcd)+f(abdc)]g_{\alpha\beta}\Gamma^F_{\lambda\mu\rho}(k-q_1,q_1)\Gamma^F_{\rho\nu\lambda}(k,q_2) \\
& \quad +N f^{abx} f^{cdx}[\Gamma^F_{\beta\mu\lambda}(k-q_1,q_1)\Gamma^F_{\lambda\nu\alpha}(k,q_2) - \Gamma^F_{\alpha\mu\lambda}(k-q_1,q_1)\Gamma^F_{\lambda\nu\beta}(k,q_2)]\} \\
& +\{(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_2,b,\nu) \longleftrightarrow (q_4,d,\beta)\} \\
& +\{(q_1,a,\mu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_1,a,\mu) \longleftrightarrow (q_4,d,\beta)\} \\
& +\{(q_1,a,\mu) \longleftrightarrow (q_3,c,\alpha), \quad (q_2,b,\nu) \longleftrightarrow (q_4,d,\beta)\}.
\end{aligned} \quad (38)$$

(d) Diagrams 8(d) give

$$\begin{aligned}
{}^{(d)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = {} & ig^2 \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(k+q_2)^2(k-q_1)^2} \\
& \times [f(abcd)+f(abdc)]2g_{\alpha\beta}(2k-q_1)_\mu(2k+q_2)_\nu \\
& +\{(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_2,b,\nu) \longleftrightarrow (q_4,d,\beta)\} \\
& +\{(q_1,a,\mu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_1,a,\mu) \longleftrightarrow (q_4,d,\beta)\} \\
& +\{(q_1,a,\mu) \longleftrightarrow (q_3,c,\alpha), \quad (q_2,b,\nu) \longleftrightarrow (q_4,d,\beta)\}.
\end{aligned} \quad (39)$$

(e) Diagrams 8(e) give

$$\begin{aligned}
{}^{(e)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = {} & ig^2 \widetilde{A}(q_1+q_2) \\
& \times \{[f(abcd)+f(abdc)]Dg_{\mu\nu}g_{\alpha\beta} + 4N f^{abx} f^{cdx}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\} \\
& +\{(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_2,b,\nu) \longleftrightarrow (q_4,d,\beta)\},
\end{aligned} \quad (40)$$

where $\widetilde{A}(q_i)$ is defined in Eq. (15).

(f) Diagrams 8(f) give

$$\begin{aligned}
{}^{(f)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = {} & -ig^2 \widetilde{A}(q_1+q_2)[f(abcd)+f(abdc)]2g_{\mu\nu}g_{\alpha\beta} \\
& +\{(q_2,b,\nu) \longleftrightarrow (q_3,c,\alpha)\} + \{(q_2,b,\nu) \longleftrightarrow (q_4,d,\beta)\}.
\end{aligned} \quad (41)$$

Then the final form of the gauge-invariant four-gluon vertex $\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}$ at the one-loop level is given by the sum

$$\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} = {}^{(a)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} + {}^{(b)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} + {}^{(c)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} + {}^{(d)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} + {}^{(e)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} + {}^{(f)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}. \quad (42)$$

Our next task is to show explicitly that the above four-gluon vertex $\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}$ satisfies the following generalized version of the Ward identity in Eq. (31):

$$\begin{aligned} q_1^\mu \widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} = & -f^{abx}\widehat{\Gamma}^{cdx}_{\alpha\beta\nu}(q_3, q_4, q_1+q_2) \\ & -f^{acx}\widehat{\Gamma}^{dbx}_{\beta\nu\alpha}(q_4, q_2, q_1+q_3) \\ & -f^{adx}\widehat{\Gamma}^{bcx}_{\nu\alpha\beta}(q_2, q_3, q_1+q_4), \end{aligned} \quad (43)$$

where

$$\widehat{\Gamma}^{abc}_{\mu\nu\lambda}(q_1, q_2, q_3) = f^{abc}\widehat{\Gamma}_{\mu\nu\lambda}(q_1, q_2, q_3) \quad (44)$$

and $\widehat{\Gamma}_{\mu\nu\lambda}(q_1, q_2, q_3)$ is the gauge-invariant three-gluon vertex given in Eq. (14). The Ward identity Eq. (43), first found by Papavassiliou with the PT [4], is naturally expected to hold in the BFM formalism.

We act with q_1^μ on the individual contributions to $\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}$ which are expressed in Eqs. (36)–(41). Before going through the evaluation we make some preparations. Let us introduce the following integrals for the three-point vertex with the constraint $q_1+q_2+q_3=0$:

$$\begin{aligned} B_{\mu\nu\alpha}(q_1, q_2, q_3) \equiv & \int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2(k+q_2)^2(k-q_1)^2} \\ & \times [\Gamma^F_{\lambda\mu\rho}(k-q_1, q_1)\Gamma^F_{\rho\nu\tau}(k, q_2) \\ & \times \Gamma^F_{\tau\alpha\lambda}(k+q_2, q_3)], \\ C_{\mu\nu\alpha}(q_1, q_2, q_3) \equiv & \int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2(k+q_2)^2(k-q_1)^2} \\ & \times (2k-q_1)_\mu (2k+q_2)_\nu \\ & \times (2k-q_1+q_2)_\alpha. \end{aligned} \quad (45)$$

In terms of $B_{\mu\nu\alpha}$ and $C_{\mu\nu\alpha}$, the gauge-invariant three-gluon vertex $\widehat{\Gamma}_{\mu\nu\lambda}$ in Eq. (14) is expressed as

$$\begin{aligned} \widehat{\Gamma}_{\mu\nu\lambda}(q_1, q_2, q_3) = & -\frac{iNg^2}{2}\{B_{\mu\nu\alpha}(q_1, q_2, q_3) + 2C_{\mu\nu\alpha}(q_1, q_2, q_3) \\ & -8(q_{1\alpha}g_{\mu\nu} - q_{1\nu}g_{\mu\alpha})\widetilde{A}(q_1) - 8(q_{2\mu}g_{\alpha\nu} - q_{2\alpha}g_{\mu\nu})\widetilde{A}(q_2) - 8(q_{3\nu}g_{\mu\alpha} - q_{3\mu}g_{\nu\alpha})\widetilde{A}(q_3)\}. \end{aligned} \quad (46)$$

These $B_{\mu\nu\alpha}$ and $C_{\mu\nu\alpha}$ satisfy the relations

$$\begin{aligned} B_{\mu\nu\alpha}(q_1, q_2, q_3) & = B_{\nu\alpha\mu}(q_2, q_3, q_1) \\ & = -B_{\nu\mu\alpha}(q_2, q_1, q_3), \end{aligned} \quad (47)$$

$$\begin{aligned} C_{\mu\nu\alpha}(q_1, q_2, q_3) & = C_{\nu\alpha\mu}(q_2, q_3, q_1) \\ & = -C_{\nu\mu\alpha}(q_2, q_1, q_3), \end{aligned} \quad (48)$$

which can be proved by changing integration variables under the constraint $q_1+q_2+q_3=0$ and using the fact

$$\Gamma^F_{\lambda\mu\rho}(k, q) = -\Gamma^F_{\rho\mu\lambda}(-k-q, q). \quad (49)$$

Throughout the algebraic manipulations, we often take the means of changing the integration variables under the constraint $q_1+q_2+q_3+q_4=0$ and make use of identities

$$\begin{aligned} q_1^\mu \Gamma^F_{\lambda\mu\rho}(k-q_1, q_1) & = [(k-q_1)^2 - k^2]g_{\lambda\rho}, \\ q_1^\mu (2k-q_1)_\mu & = k^2 - (k-q_1)^2 \end{aligned} \quad (50)$$

to reduce the number of propagators by 1, the relations of Eqs. (47)–(49) for $B_{\mu\nu\alpha}$, $C_{\mu\nu\alpha}$, and $\Gamma^F_{\lambda\mu\rho}$ to classify terms into groups with the same color factors. We also use the identity of Eq. (34). The results are

$$\begin{aligned} \text{(a)}\quad q_1^{\mu\ (a)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1, q_2, q_3, q_4) = & \, ig^2\Bigg\{[f(abcd)+f(abdc)]q_{1\nu}\int\frac{d^Dk}{(2\pi)^D}\frac{\Gamma^F_{\lambda\alpha\rho}(k-q_3, q_3)\Gamma^F_{\rho\beta\lambda}(k, q_4)}{(k-q_3)^2k^2(k+q_4)^2} \\ & + Nf^{abx}f^{cdx}\Bigg[\frac{1}{2}B_{\alpha\beta\nu}(q_3, q_4, q_1+q_2) \\ & + q_1^\lambda\int\frac{d^Dk}{(2\pi)^D}\frac{\Gamma^F_{\lambda\alpha\rho}(k-q_3, q_3)\Gamma^F_{\rho\beta\nu}(k, q_4) - (\nu \longleftrightarrow \lambda)}{(k-q_3)^2k^2(k+q_4)^2}\Bigg]\Bigg\} \\ & + \{\text{cyclic permutations}\}, \end{aligned} \quad (51)$$

where {cyclic permutations} represents two terms which are obtained from the first term by the substitution $\{(q_2, b, \nu) \to (q_3, c, \alpha) \to (q_4, d, \beta) \to (q_2, b, \nu)\}$ and the substitution $\{(q_2, b, \nu) \to (q_4, d, \beta) \to (q_3, c, \alpha) \to (q_2, b, \nu)\}$. The same notation applies to the expressions below:

$$\begin{aligned} \text{(b)}\quad q_1^{\mu\ (b)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1, q_2, q_3, q_4) = & \, ig^2\Bigg\{[f(abcd)+f(abdc)](-2q_{1\nu})\int\frac{d^Dk}{(2\pi)^D}\frac{(2k-q_3)_\alpha(2k+q_4)_\beta}{(k-q_3)^2k^2(k+q_4)^2} \\ & + Nf^{abx}f^{cdx}C_{\alpha\beta\nu}(q_3, q_4, q_1+q_2)\Bigg\} + \{\text{cyclic permutations}\}, \end{aligned} \quad (52)$$

$$(c)\quad q_1^{\mu}\,{}^{(c)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = ig^2\Bigg([f(abcd)+f(abdc)]\Bigg\{-Dq_{1\nu}g_{\alpha\beta}\widetilde{A}(q_1+q_2) - q_{1\nu}\int\frac{d^Dk}{(2\pi)^D}\frac{\Gamma^F_{\lambda\alpha\rho}(k-q_3,q_3)\Gamma^F_{\rho\beta\lambda}(k,q_4)}{(k-q_3)^2k^2(k+q_4)^2}\Bigg\} + Nf^{abx}f^{cdx}\Bigg\{4(q_{2\alpha}g_{\beta\nu}-q_{2\beta}g_{\nu\alpha})[\widetilde{A}(q_1+q_2)-\widetilde{A}(q_2)] - q_1^{\lambda}\int\frac{d^Dk}{(2\pi)^D}\frac{\Gamma^F_{\lambda\alpha\rho}(k-q_3,q_3)\Gamma^F_{\rho\beta\nu}(k,q_4)-(\nu\longleftrightarrow\lambda)}{(k-q_3)^2k^2(k+q_4)^2}\Bigg\}\Bigg) + \{\text{cyclic permutations}\}, \tag{53}$$

$$(d)\quad q_1^{\mu}\,{}^{(d)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = ig^2[f(abcd)+f(abdc)]\Bigg\{2q_{1\nu}g_{\alpha\beta}\widetilde{A}(q_1+q_2) + 2q_{1\nu}\int\frac{d^Dk}{(2\pi)^D}\frac{(2k-q_3)_\alpha(2k+q_4)_\beta}{(k-q_3)^2k^2(k+q_4)^2}\Bigg\} + \{\text{cyclic permutations}\}, \tag{54}$$

$$(e)\quad q_1^{\mu}\,{}^{(e)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = ig^2\{[f(abcd)+f(abdc)]Dq_{1\nu}g_{\alpha\beta}\widetilde{A}(q_1+q_2) + Nf^{abx}f^{cdx}4(q_{1\alpha}g_{\beta\nu}-q_{1\beta}g_{\nu\alpha})\widetilde{A}(q_1+q_2)\} + \{\text{cyclic permutations}\}, \tag{55}$$

$$(f)\quad q_1^{\mu}\,{}^{(f)}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = ig^2[f(abcd)+f(abdc)](-2q_{1\nu})g_{\alpha\beta}\widetilde{A}(q_1+q_2) + \{\text{cyclic permutations}\}. \tag{56}$$

Adding together all the contributions [Eqs. (51)–(56)], we find

$$q_1^{\mu}\widehat{\Gamma}^{abcd}_{\mu\nu\alpha\beta}(q_1,q_2,q_3,q_4) = \frac{ig^2N}{2}f^{abx}f^{cdx}[B_{\alpha\beta\nu}(q_3,q_4,q_1+q_2)+2C_{\alpha\beta\nu}(q_3,q_4,q_1+q_2) - 8\{(q_1+q_2)_\beta g_{\nu\alpha}-(q_1+q_2)_\alpha g_{\nu\beta}\}\widetilde{A}(q_1+q_2) + 8\{q_{2\beta}g_{\nu\alpha}-q_{2\alpha}g_{\nu\beta}\}\widetilde{A}(q_2)] + \{\text{cyclic permutations}\}. \tag{57}$$

It is noted that all the terms which are proportional to factors $[f(abcd)+f(abdc)]$, $[f(acdb)+f(acbd)]$, and $[f(adbc)+f(adcb)]$ cancel out and only terms with factors $Nf^{abx}f^{cdx}$, $Nf^{acx}f^{dbx}$, and $Nf^{adx}f^{bcx}$ remain in the final result. The last step is to add

$$0 = \frac{ig^2N}{2}[f^{abx}f^{cdx}+f^{acx}f^{dbx}+f^{adx}f^{bcx}] \times\{-8(q_{2\beta}g_{\nu\alpha}-q_{2\alpha}g_{\nu\beta})\widetilde{A}(q_2) - 8(q_{3\nu}g_{\alpha\beta}-q_{3\beta}g_{\alpha\nu})\widetilde{A}(q_3) - 8(q_{4\alpha}g_{\beta\nu}-q_{4\nu}g_{\beta\alpha})\widetilde{A}(q_4)\} \tag{58}$$

to the RHS of Eq. (57) and to use Eq. (46), and we arrive at the desired result of Eq. (43).

VI. CONCLUSIONS

In this paper we demonstrated that the background field method is an alternative and simple way of deriving the same gauge-invariant results which are obtained by the pinch technique. We have found, in particular, in the cases of gauge-invariant gluon self-energy and the three-gluon vertex that both the BFM in the Feynman gauge and the intrinsic PT produce the same results which are equal term by term. We also calculated the gauge-invariant four-gluon vertex in the BFM and presented its exact form. Finally we explicitly showed that this four-gluon vertex satisfies the same simple Ward identity that was found with the PT.

Note added. After submitting this paper for publication, we learned that Denner, Weinglein, and Dittmaier [16] very recently dealt with the same topic and reached the same conclusion as we have. They have applied the BFM also to the electroweak sector of the standard model and derived various Ward identities.

ACKNOWLEDGMENTS

One of the authors (K.S.) would like to thank Professor D. Zwanziger and Joannis Papavassiliou for the hospitality extended to him at New York University where part of this work was done, and Professor T. Muta for the hospitality extended to him at Hiroshima University. He is also very grateful to Joannis Papavassiliou for informative and helpful discussions on the pinch technique. S.H. was supported in part by the Monbusho Grant-in-Aid for Scientific Research No. 040011. J.K. was supported in part by the Monbusho Grant-in-Aid Scientific Research No. C-05640351. Y.Y. was supported in part by the Monnbusho Grant-in-Aid for Scientific Research No. 050076.

[1] J. M. Cornwall, in *Proceedings of the French-American Seminar on Theoretical Aspects of Quantum Chromodynamics*, Marseille, France, 1981, edited by J. W. Dash (Centre de Physique Théorique, Marseille, 1982).
[2] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
[3] J. M. Cornwall and J. Papavassiliou, Phys. Rev. D **40**, 3474 (1989).
[4] J. Papavassiliou, Phys. Rev. D **47**, 4728 (1993).
[5] J. Papavassiliou, Phys. Rev. D **41**, 3179 (1990).
[6] G. Degrassi and A. Sirlin, Phys. Rev. D **46**, 3104 (1992).
[7] J. Papavassiliou and K. Philippides, Phys. Rev. D **48**, 4255 (1993).
[8] B. S. DeWitt, Phys. Rev. **162**, 1195 (1967); also in *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1963).
[9] B. S. DeWitt, in *Quantum Gravity 2*, edited by C. J. Isham, R. Penrose, and D. W. Sciama (Oxford University Press, New York, 1981).
[10] G. 't Hooft, in *Functional and Probabilistic Methods in Quantum Field Theory*, proceedings of the 12th Winter School of Theoretical Physics, Karpacz, Poland, 1975, edited by Bernard Jancewicz (Wroclaw University, Wroclaw, 1976).
[11] D. G. Boulware, Phys. Rev. D **23**, 389 (1981).
[12] L. F. Abbott, Nucl. Phys. **B185**, 189 (1981); Acta Phys. Pol. B **13**, 33 (1982).
[13] S. Ichinose and M. Omote, Nucl. Phys. **B203**, 221 (1982).
[14] D. M. Capper and A. MacLean, Nucl. Phys. **B203**, 413 (1982).
[15] The definition of three-gluon vertex $\Gamma_{\alpha\mu\lambda}$ and, therefore, of $\Gamma^F_{\alpha\mu\lambda}$ and $\Gamma^P_{\alpha\mu\lambda}$ in this paper is different from the corresponding ones in Ref. [3] by the overall factor -1.
[16] A. Denner, G. Weiglein, and S. Dittmaier, Phys. Lett. B **333**, 420 (1994).