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3D simulation of superconducting microwave devices with an electromagnetic-field simulator

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Abstract

High-frequency microwave applications, such as filters, delay lines, and resonators, are quite important for superconducting electronic devices. In order to design the superconducting microwave devices, circuit parameters should be precisely extracted from the physical structure of the devices. A 3-dimentional electromagnetic-field simulators is very useful for designing microwave devices. However, designing of superconducting microwave devices using a conventional 3D electromagnetic-field simulator is difficult because most of commercially available 3D electromagnetic-field simulators can’t exactly characterize electromagnetic phenomena of superconductors. In this study, a novel calculation method of superconducting microwave devices, which can be applicable to a conventional 3D electromagnetic-field simulator, has been proposed. Calculation results of characteristic impedance of superconducting microstrip lines show very good agreements with the theoretically calculated vales. The frequency response of a superconducting Nb microwave filter designed by the proposed calculation method agrees well with the experimental results. This calculation method enables us to precisely estimate microwave characteristics of superconducting devices.

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1. Introduction

Superconducting microwave devices, such as filters, delay lines and resonators are utilized for many applications due to their extremely low losses and dispersion [1] - [3]. Especially, high-temperature-superconductor (HTS) band-pass filters are used for telecommunication due to its high frequency selectivity [4], [5], and low-temperature-superconductor (LTS) band-pass filters are used for a quantum computation system using single-flux-quantum (SFQ) circuits [6], [7]. A 3D electromagnetic-field simulation, which extracts circuit parameters and calculates the electromagnetic characteristics from the physical structures of the devices, is indispensible for designing microwave devices. However, it is difficult to simulate characteristics of superconducting devices using conventional 3D electromagnetic-field simulators because electromagnetic phenomena of superconductors are quite different from those of normal conductors and conventional electromagnetic-field simulator cannot characterize the electromagnetic properties in superconductors. In this study, we propose a novel calculation model to simulate superconducting microwave devices, and discuss the validity of the calculation method. The design of a superconducting low pass filter using the Nb fabrication process and its experimental results are also represented.

2. Calculation model of superconducting transmission lines

The differences of electromagnetic phenomena between in a superconductor and in a normal conductor originate from two unique characteristics of a superconductor. One is that penetration depth of electromagnetic fields of a superconductor doesn't depend on frequency, the other is that a superconductor has kinetic inductance caused by inertia of Cooper pairs. Therefore, 3D electromagnetic-field calculations, where superconductors are simply replaced with perfect conductors, cannot exactly simulate superconducting microwave devices.

Assuming a superconducting microstrip line with the line width of w, and the thickness of b_1, on an insulated ground plane with the thickness of b_2, internal magnetic inductance per unit length L_{int} and kinetic inductance per unit length L_{K} are respectively expressed as

$$L_{\mathrm{int}} = \frac{\mu_0}{2Kw}\left[\frac{\frac{\lambda_{\mathrm{L}}}{2}\sinh\left(\frac{2b}{\lambda_{\mathrm{L}}}\right) - b}{\left[\sinh\left(\frac{b}{\lambda_{\mathrm{L}}}\right)\right]^2}\right] \tag{1}$$

and

$$L_{\mathrm{K}} = \frac{\mu_0}{2Kw}\left[\frac{\frac{\lambda_{\mathrm{L}}}{2}\sinh\left(\frac{2b}{\lambda_{\mathrm{L}}}\right) + b}{\left[\sinh\left(\frac{b}{\lambda_{\mathrm{L}}}\right)\right]^2}\right], \tag{2}$$

where λ_L is the London penetration depth, K is a fringe factor, d is the thickness of the insulator between the microstrip line and the ground plane, and b is b_1 or b_2. If the relation, $b / \lambda_L \geq 3$, is satisfied, L_K is almost equal to L_{int}. In this study, we discuss our calculation model assuming this relation is fulfilled. The designed and measured superconducting filter fabricated by the Nb fabrication process, described later, adequately satisfies this relation.

Fig. 1 shows a proposed calculation model of a superconducting transmission line to simulate the electromagnetic characteristics using a conventional electromagnetic-field simulator. In this calculation model, superconducting films are replaced with perfect conductors completely coated by normal conductor layers with the thickness of λ_L. The perfect conductor is defined as a normal metal which has infinite conductivity. Because magnetic field cannot penetrate the perfect conductor, this two-conductor-layer structure is a good model for the Meissner effect and the effect of the London penetration depth of superconductors.

We discuss the validity of this calculation model. In this model, the skin depth of the normal conductor, δ must be set to the appropriate value to calculate the electromagnetic phenomena in a superconductor. It is well known that the skin depth of a normal conductor is denoted as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}, \tag{3}$$

where f is frequency, μ is permeability and σ is conductivity of a conductor. Magnetic field as a function of the depth z from the surface of a normal conductor, $B(z)$ is represented as

$$B(z) = B_S \exp\left(-\frac{z}{\delta}\right), \tag{4}$$

where B_S is the flux density at the surface, z = 0. We assume that δ is large enough so that $B(z)$ is considered to be constant within a normal conductor. Therefore, σ is set to satisfy $\delta / \lambda_L \geq 10$ at the highest analyzing frequency. Fig. 2 shows the magnetic field distribution in a superconductor and in a two-conductor-layer structure. As shown in Fig. 2, there is a difference of magnetic field distribution in a superconductor and a two-conductor-layer structure. This means that the internal magnetic inductance of our model is different from that of a superconductor.

Here, we calculate inductances of the two systems. For a superconductor, $B(z)$ is represented as

$$B(z) = B_S \exp\left(-\frac{z}{\lambda_L}\right), \tag{5}$$

and magnetic energy density ω is

$$\begin{aligned}\omega &= \frac{1}{2\mu}B^2(z) \\ &= \frac{1}{2\mu}B_S{}^2\exp\left(-\frac{2z}{\lambda_L}\right)\end{aligned} \quad (6)$$

W_1, magnetic energy of the superconducting strip per unit length is

$$\begin{aligned}W_1 &= \int_0^b \omega \cdot \mathrm{w}\mathrm{d}z \\ &= \frac{1}{4\mu}B_S{}^2 w\lambda\left\{1-\exp\left(-\frac{2b}{\lambda_L}\right)\right\}. \\ &\approx \frac{1}{4\mu}B_S{}^2 w\lambda \quad (\because b/\lambda_L \geq 3)\end{aligned} \quad (7)$$

For a two-conductor-layer structure, a constant magnetic field penetrates the external normal conductor layer and doesn't penetrate the internal prefect conductor layer. $B(z)$ and ω are then

$$B(z) = \begin{cases} B_S & \text{for } z < \lambda_L \\ 0 & \text{for } z > \lambda_L \end{cases} \quad (8)$$

$$\begin{aligned}\omega &= \frac{1}{2\mu}B^2(z) \\ &= \begin{cases} \frac{1}{2\mu}B_S{}^2 & \text{for z} < \lambda_L \\ 0 & \text{for z} > \lambda_L \end{cases}\end{aligned} \quad (9)$$

W_2, magnetic energy of the two-conductor-layer structure per unit length is

$$\begin{aligned}W_2 &= \int_0^b \omega \cdot w\mathrm{d}z \\ &= \frac{1}{2\mu}B_S{}^2 w\lambda_L\end{aligned} \quad (10)$$

Let $L_{1,\mathrm{int}}$ be internal magnetic inductance of the superconductor and $L_{2,\mathrm{int}}$ be internal magnetic inductance of the two-conductor-layer structure, we obtain

(11)

$$L_{\mathrm{int},2}/L_{\mathrm{int},1} = W_2/W_1 = 2\,.$$

This shows that the internal magnetic inductance of a two-conductor-layer structure is twice as large as that of a superconducting strip line.

The total internal inductance of a superconducting film, $L_{\mathrm{film},1}$ is then given by

$$\begin{aligned} L_{\mathrm{film},1} &= L_{\mathrm{int},1} + L_{\mathrm{K},1} \\ &= L_{\mathrm{int},1} + L_{\mathrm{int},1} \quad , \\ &= 2L_{\mathrm{int},1} \end{aligned} \tag{12}$$

and that of a two-conductor-layer structure, $L_{\mathrm{film},2}$ is

$$\begin{aligned} L_{\mathrm{film},2} &= L_{\mathrm{int},2} + L_{\mathrm{K},2} \\ &= 2L_{\mathrm{int},1} + 0 \quad . \\ &= 2L_{\mathrm{int},1} \end{aligned} \tag{13}$$

One can find that the internal inductance of our model is exactly equal to that of a superconductor. Therefore, we can calculate the electromagnetic characteristics of superconducting strip lines using our two-conductor-layer model.

3. Comparison with an analytical formula

We have calculated the characteristic impedances of superconducting microstrip lines and compared then with theoretical values based on Chang's equation [9]. The characteristic impedance of a superconducting microstrip line is denoted as

$$Z_0 = \frac{d}{wK(w,d,b_1)}\sqrt{\frac{\mu_0\lambda_L}{\varepsilon_r\varepsilon_0}\left\{\frac{d}{\lambda_L}+\coth\left(\frac{b_1}{\lambda_L}\right)+\coth\left(\frac{b_2}{\lambda_L}\right)\right\}}, \tag{14}$$

where b_1 is the thickness of a signal line, b_2 is the thickness of a ground plane and $K(w,d,b_1)$ is fringe factor. We have calculated the characteristic impedances of Nb microstrip lines with line widths of 5 μm – 50 μm using the two-conductor-layer model and HFSS, a 3D EM simulation software tool [9]. In this calculation, we assumed that the penetration depth of Nb thin films, λ_L is 80 nm, which is the measured in Nb thin films of the SRL 2.5 kA/cm^2 Nb standard process [10] and <u>we set frequency and conductivity of the external conductor layer, σ to satisfy $\delta / \lambda_L \geq 10$.</u> Fig. 3 shows the comparison of theoretical and calculated characteristics impedances. <u>The calculated characteristic impedance based on our model perfectly agrees with theoretical impedances for all line widths where the maximum discrepancy was 0.5 %.</u> It should be noted that if we use uniform perfect conductors for the microstrip lines, 3D electromagnetic simulation gives Z_0 = 1.6 Ω for w = 34 μm, whereas the analytical calculation and the proposedk model give 2.0 Ω. It means that the discrepancy in the perfect conductor model is <u>20 %</u>. These results indicates our calculation model can simulate electromagnetic phenomena of superconducting transmission lines precisely.

4. Design and experimental results of a superconducting filter

We designed a 9th Chebyshev low-pass filter (LPF) with cut-off frequency of 7 GHz, ripple of 0.001 dB and impedance of 2 Ω using the SRL 2.5 kA/cm^2 Nb standard process [10] based on our calculation model. In order to convert its impedance to 50 Ω, we inserted impedance matching networks (IMNs) at the input and the output ports of the LPF. Fig. 4 shows equivalent circuits of the 9th LPF and the IMN, and a microphotograph of the LPF fabricated by the SRL Nb process. We measured the propagation characteristics of the LPF at 4.2 K, where the chip is mounted on a BCP-2 cryoprobe, a wide-bandwidth probe manufactured by American Cryoprobe Inc. The signal attenuation in the probe is estimated to be approximately -3.5 dB up to 10 GHz. Fig. 5 shows the measured and calculated $|S_{21}|$ as a function of the microwave frequency. Measurement results agree well with simulation results using the two-conductor-layer model, though there is an unexpected peak at 10 GHz due to the resonance in the probes.

These results indicate that our calculation method is very accurate to calculate characteristics of superconducting microwave devises. This is because our calculation model can simulate both 3D electromagnetic characteristics and superconducting phenomena using conventional 3D electromagnetic-field simulators.

5. Conclusion

In order to simulate superconducting microwave devices using a conventional 3D electromagnetic field simulator, we proposed a two-conductor-layer model, which treat two unique characteristics of a superconductor: the constant penetration depth and the kinetic inductance. Characteristic impedances of superconducting microstrip lines based on our calculation model perfectly agrees with theoretical values. Scattering parameters $|S_{21}|$ of the LPF based on our calculation model coincides well with measured results. These results indicate the validity of our calculation model.

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Figure Captions

Fig. 1. A two-conductor-layer model of a superconducting microstrip line. The external layer is composed of a conductor with very small conductivity. The internal layer is a perfect conductor with infinite conductivity.

Fig. 2. Distribution of magnetic fields in a superconductor and a two-conductor-layer structure.

Fig. 3. Comparison of the characteristic impedances of superconducting microstrip lines calculated from the analytical formula and the two-conductor-layer model. $b_1 = b_2 = d =$ 0.3 μm, λ_L = 80 nm.

Fig. 4. (a) Equivalent circuit of the 9th Chebyshev LPF. $L_1 = L_9$ = 27.7 pH, $L_3 = L_7$ = 73.2 pH, L_5 = 79.3 pH, $C_2 = C_8$ = 14.7 pF, $C_4 = C_6$ = 18.9 pF. (b) Equivalent circuit of the IMN. L = 318 pH, $C_S = C_L$ = 3.18 pF. (c) A microphotograph of the LPF and IMNs.

Fig. 5. Measurement and simulated results of scattering parameters $|S_{21}|$ of the LPF and the IMNs. b_1 = 0.4 μm, b_2 = 0.3 μm, d = 0.7 μm, λ_L = 80 nm and $\sigma = 3.96 \times 10^7$ S/m

which satisfies $\delta / \lambda_L = 10$ at 10 GHz.

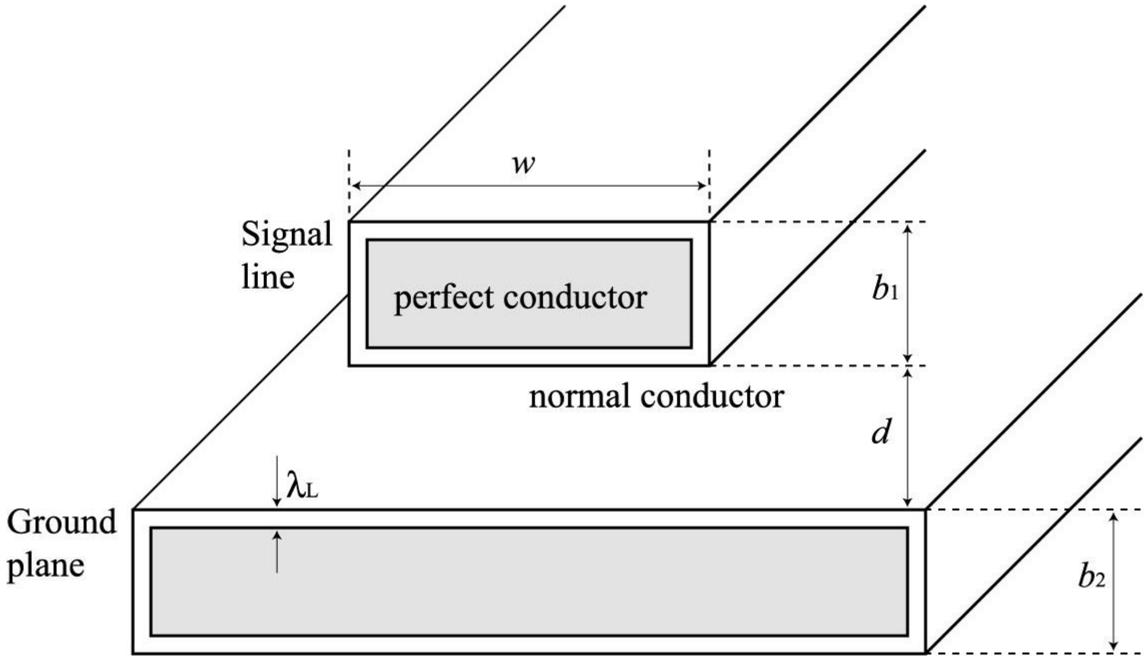
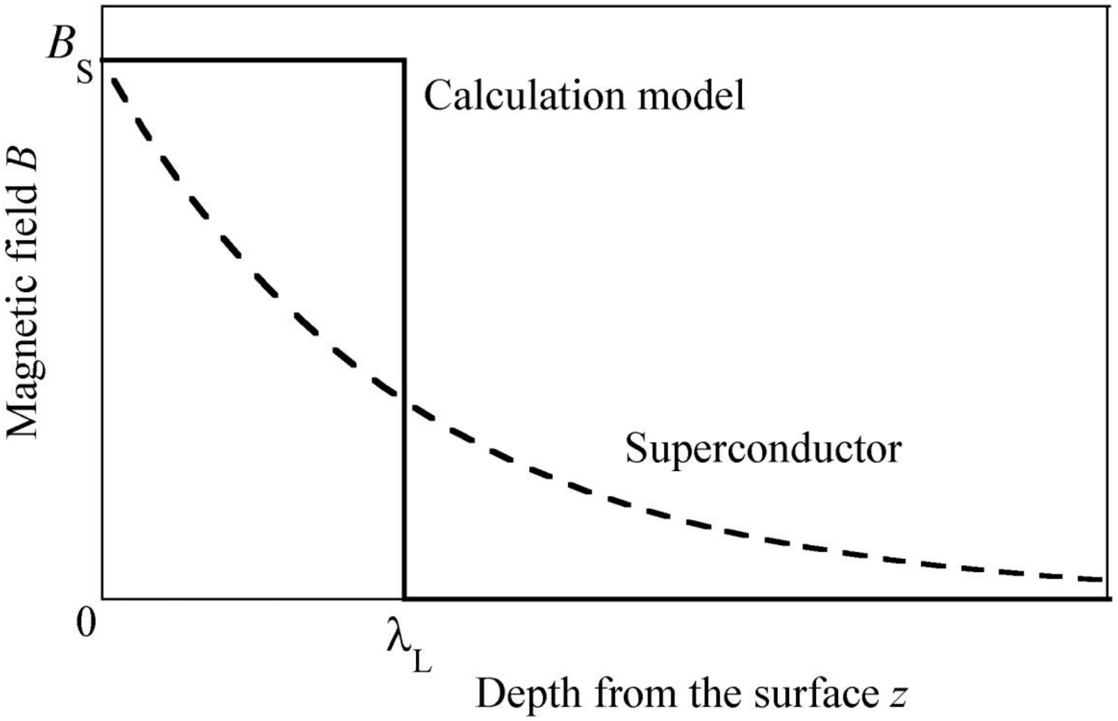

w
Signal
line
perfect conductor
b1
normal conductor
d
λL
Ground
plane
b2

Fig. 1



B_S
Calculation model
Magnetic field B
Superconductor
0
λ_L
Depth from the surface z

Fig. 2

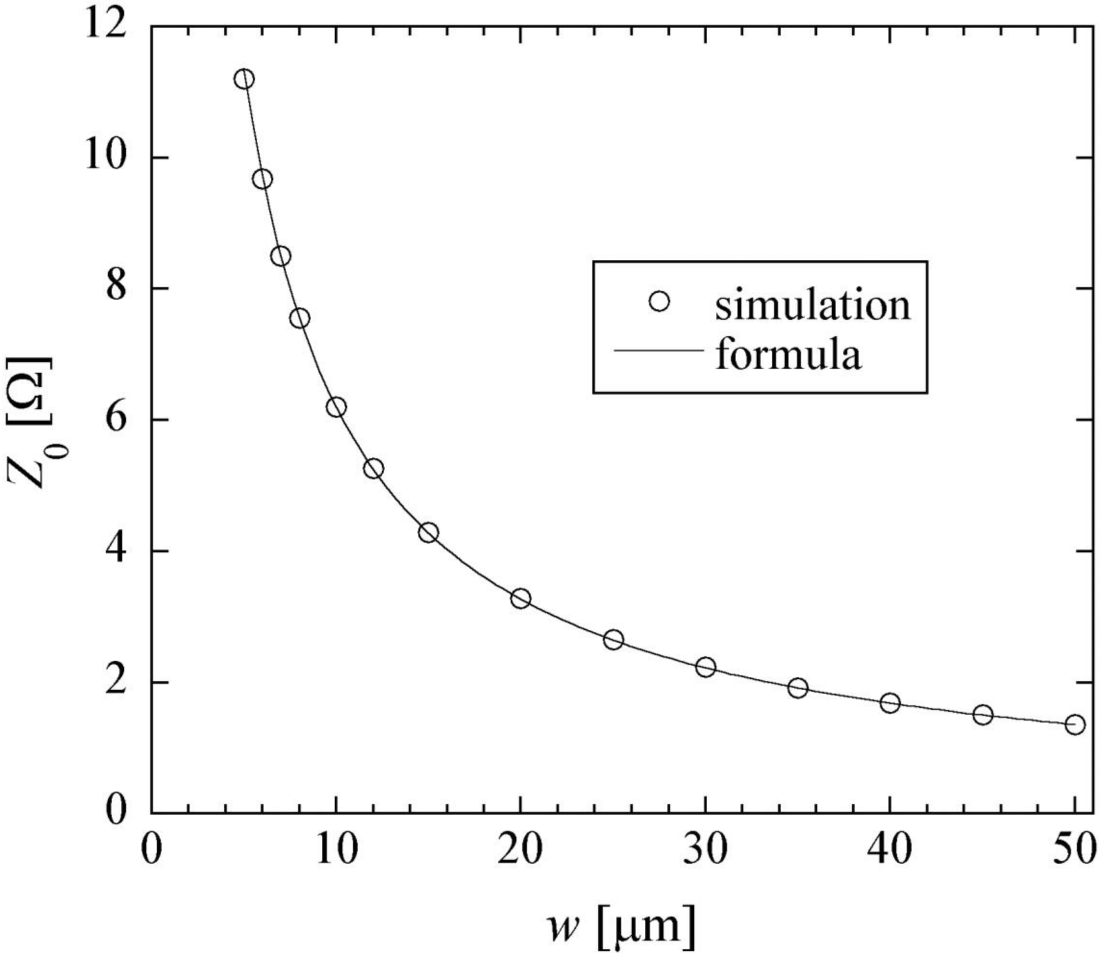
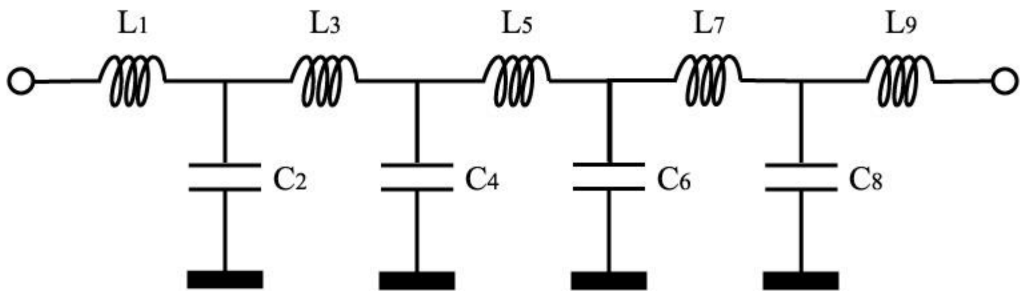
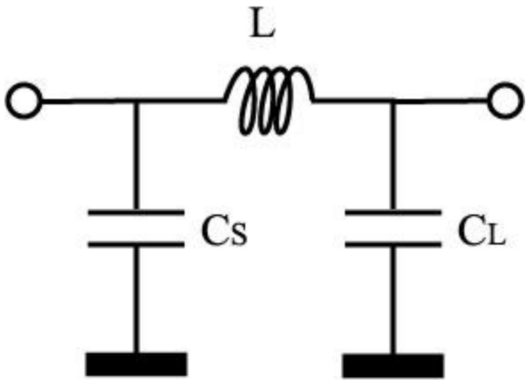


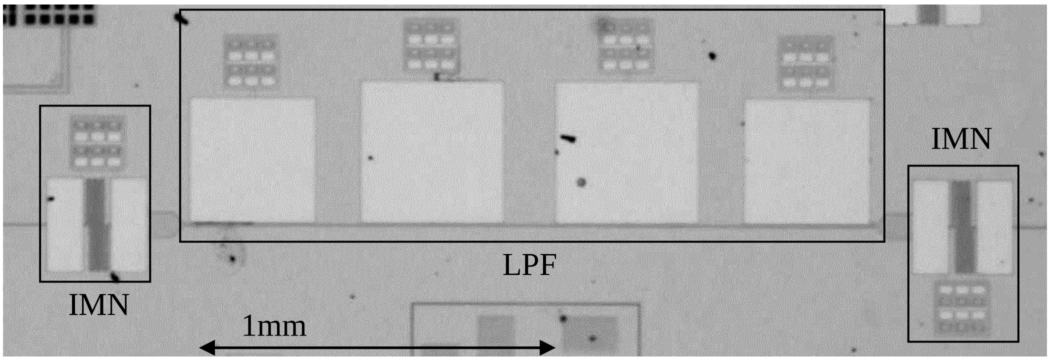
Fig. 3



(a)

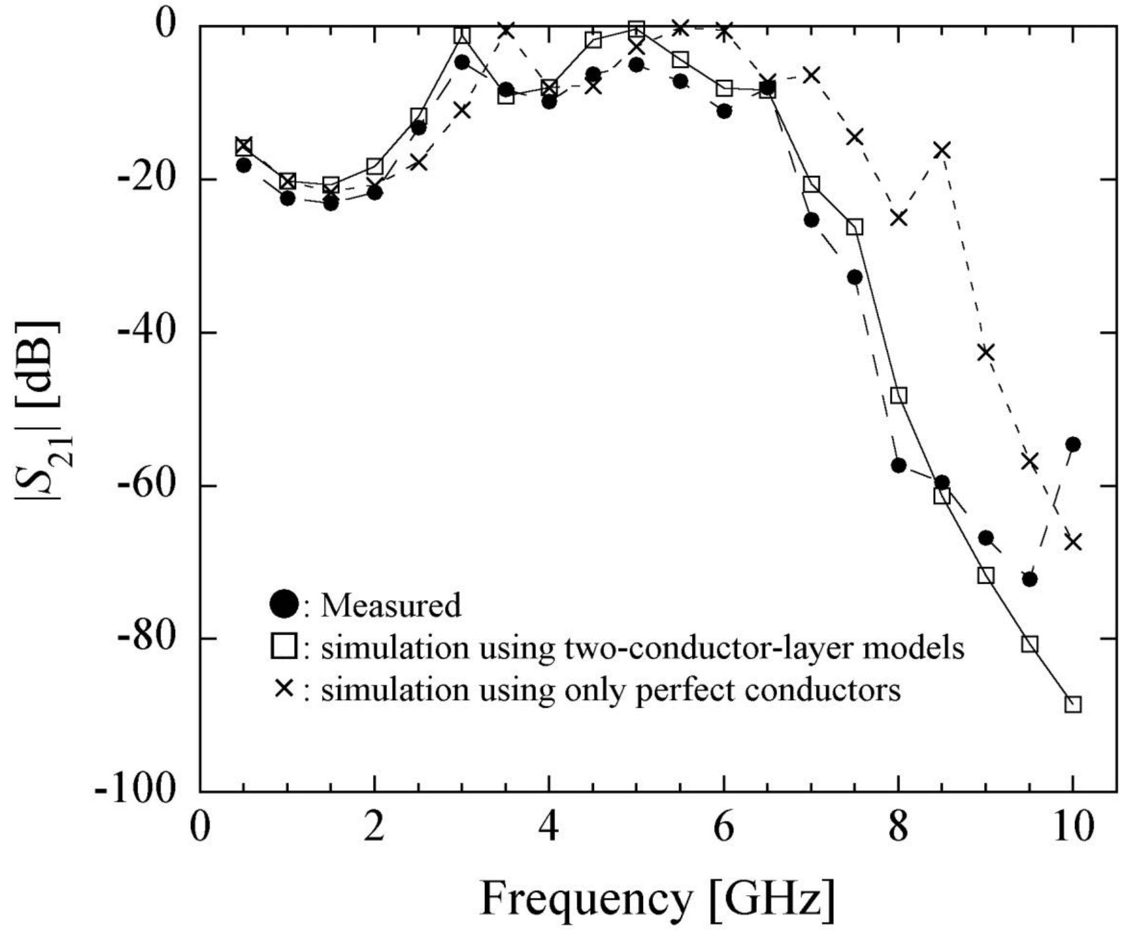


(b)



(c)

Fig. 4



0
-20
-40
-60
-80
-100
0
2
4
6
8
10
|S21| [dB]
Frequency [GHz]
: Measured
: simulation using two-conductor-layer models
: simulation using only perfect conductors

Fig. 5