Dynamic Model of Decentralized Systems with Informational Connection

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Abstract— **This paper presents a design method of decentralized systems with informational connection. The informational connection in this paper denotes a event link which establish a signal links among physical controlled plants via communication network. A dynamic transition of informational connection among decentralized systems is considered and the mathematical structures are discussed using a concept of *eigenvalues* and *eigen-connections* over the Galois field $GF(2)$. The global system has variable structure characteristics due to the transition of informational connection. Examples of decentralized variable structure systems are shown. In an industry field, there are many engineering systems that have dynamic transition of informational connection. The mathematical model would be useful for analysis and synthesis of various informationally connected systems.**

I. Introduction

RECENT years, many industrial systems are becoming too complex to understand how they work. Since these systems are designed by hierarchical and distributed methods, it is difficult to guarantee the stability and the safety of the global systems. The difficulty comes from an unexpected interaction among independently designed subsystems. There are several works related to distributed systems[1]–[6].

Fig. 1 shows a general concept of hierarchical distributed systems that has a event link layer and a signal link layer. The signal links among physical controlled plants are established via communication network due to event links among connection controllers. In general, many industrial systems are implicitly designed so that the information flow will change due to conditions of the physical plant and the controller. Namely, the information flow is determined by their logical variables of the conditions. This double-layer model has the potential advantage of clearness, performance, and reuse effectiveness of the modules. Explicitly considering the double-layer concept, the model is expected to settle the difficulties of large complex systems. Similar concepts are found in *Responsive Processor*[7] and IEC61499 *Function Blocks*[8]. They, however, just provide frame works.

In this paper, analysis and design method for the logic layer of the information flow is proposed because it is important to systematically handle transitions of information flow depending on the logical condition variable. A dynamic transition of informational connection is considered and the mathematical structure is discussed using a concept of *eigenvalues* and *eigen-connections* over the Galois field $GF(2)$. The global system has variable structure characteristics due to the transition of informational connection.

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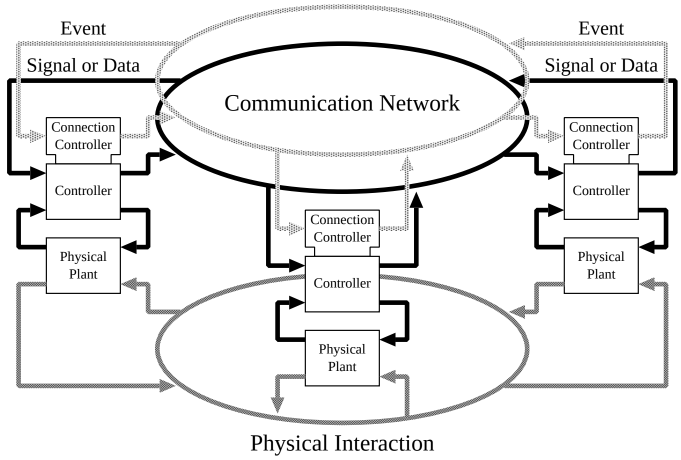


Fig. 1. Basic concept of informationally connected systems

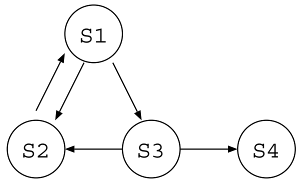


Fig. 2. An example of an informationally connected system

II. Dynamic Model of Connection Systems

Consider informationally connected systems which consist of n sub-systems. A connection matrix $X \in GF(2)^{n\times n}$ is defined as:

$$x_{ij} = \begin{cases} 1 & \text{if there is connection from} \\ & j\text{th system to } i\text{th system} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where x_{ij} is the (i, j) element of the matrix X. $GF(2)$ represents the Galois field having two elements $\{0, 1\}$. Fig. 1 shows an example of an informationally connected system which consists of sub-systems $\{S_1, S_2, S_3, S_4\}$. The connection matrix of this system becomes:

$$X = \begin{array}{c} \\ S_{1R} \\ S_{2R} \\ S_{3R} \\ S_{4R} \end{array} \begin{array}{c} \begin{array}{cccc} S_{1D} & S_{2D} & S_{3D} & S_{4D} \end{array} \\ \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array} \tag{2}$$

where the subscript D and R represent *Donator* and *Receptor* of information[2].

The operations on the Galois field $GF(2)$ are defined as follows[9]:

$$\begin{gathered} 0+0=0,\ 0+1=1,\ 1+0=1,\ 1+1=0, \\ 0-0=0,\ 0-1=1,\ 1-0=1,\ 1-1=0, \\ 0\times 0=0,\ 0\times 1=0,\ 1\times 0=0,\ 1\times 1=1, \\ 0\div 0=\text{Indeterminate},\ 0\div 1=0, \\ 1\div 0=\text{Indeterminate},\ 1\div 1=1. \end{gathered}$$

In this paper, a vector space in a set $GF(2)^{n\times n}$ is called *a connection space*. A norm of a connection $X \in GF(2)^{n\times n}$ is defined as $GF(2)^{n\times n} \to N$ by $|X| = \sum_{i=1}^{n}\sum_{j=1}^{n} X_{ij}$, where N denotes a set of all natural numbers. Note that the addition in the summation takes on N. A distance between two connected systems X_1 and X_2 is expressed as $|X_1 - X_2|$. Distance represents how different two informationally connected systems are. An inner product of two connections $X, Y \in GF(2)^{n\times n}$ is defined as $GF(2)^{n\times n} \times GF(2)^{n\times n} \to N$ by $\langle X, Y\rangle = \sum_{i=1}^{n}\sum_{j=1}^{n} X_{ij}Y_{ij}$. The connection space becomes metric space by these definitions. The metric space is needed to express a dynamic transition of an informational connection because a fluctuation is based on a concept of a distance.

Conventionally, a concept of a distance was not considered in a connection analysis because it uses a Boolean algebra. Boolean algebra is applicable not to a dynamic transition analysis but to a kinematics structure analysis.

Assume that a transition of an informational connection is affected by a current state of the connection and environmental event inputs, then we have informationally connected systems of the general form

$$X(t+1) = f(X(t), u_1(t), u_2(t), \ldots, u_m(t), t) \tag{3}$$

where t is a unit time and u_i is event input from environment and controller. Each element of the connection matrix X represents not only the connection itself but also the condition of the transition of the information flow structure. Also event input u_i represents the condition of the transition depending on status of the low layer controller and the physical plant.

A. Dynamic Model affected by Donator Connection

As a sub-class of the system (3), we have a linear dynamic model of informational connection affected by a current state of the *Donator* connection.

$$X(t+1) = A_D(t)X(t) + b_D(t)u_D(t)^T \tag{4}$$

where $A_D(t) \in GF(2)^{n\times n}$ represents *Donator matrix*, $u_D(t) \in GF(2)^n$ represents a external event input to change *Donator* connection, and $b_D(t) \in GF(2)^n$ represents its coefficient matrix. Note that ith column of the connection matrix $X(t)$ represents *Donator* connection of ith subsystem. The multiplication of $X(t)$ by $A_D(t)$ from the left affects each column of $X(t)$, i. e., *Donator* connection of each subsystem.

Example II-1

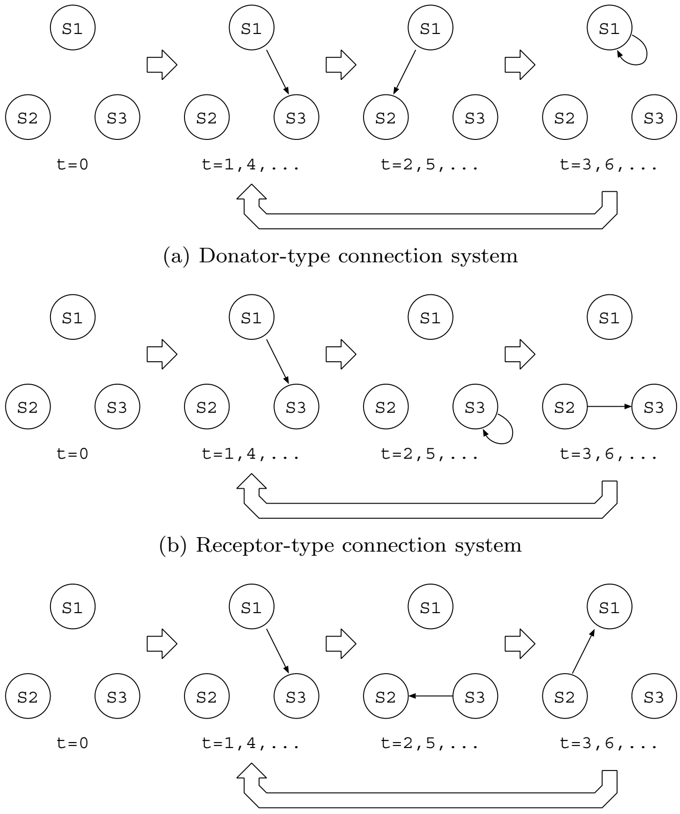
Consider the *Donator-type* informationally connected system

$$\{A_D, b_D, X(0)\} = \left\{ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \tag{5}$$

and the external event input $u_D = [u_{D1}, u_{D2}, u_{D3}]^T$. When an impulsive event input is imposed on the first subsystem,

$$u_{D1}(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

the impulse response of the transition of the connection becomes as shown in Fig. 3 (a). In this case, a periodic transition can be observed.



(a) Donator-type connection system

(b) Receptor-type connection system

(c) Donator-receptor-type connection system

Fig. 3. Impulse transition of connection system

B. Dynamic Model affected by Receptor Connection

As a dual system of the donator model (4), we have a linear dynamic model of informational connection affected by the *Receptor* connection

$$X(t+1) = X(t)A_R(t)^T + u_R(t)b_R(t)^T \tag{7}$$

where $A_R(t) \in GF(2)^{n\times n}$ represents *Receptor matrix*, $u_R(t) \in GF(2)^n$ represents a external event input to change *Receptor* connection, and $b_R(t) \in GF(2)^n$ represents its coefficient matrix. Since ith row of the connection matrix $X(t)$ represents *Receptor* connection of ith subsystem, the multiplication of $X(t)$ by $A_R(t)$ from the right affects each row of $X(t)$, i. e., *Receptor* connection of each subsystem.

Example II-2

Consider the *Receptor-type* informationally connected system

$$\{A_R, b_R, X(0)\} = \left\{ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \tag{8}$$

and the external event input $u_R = [u_{R1}, u_{R2}, u_{R3}]^T$. When an impulsive event input is imposed on the first subsystem, the impulse response of the transition of the connection becomes as shown in Fig. 3 (b). Also in this case, a periodic transition can be observed.

C. Dynamic Model affected by Donator and Receptor Connection

In General, both *Donator* and *Receptor* connection have influence on the next state of the connection. A Combination of (4) and (7) yields the general linear dynamic model:

$$\begin{aligned} X(t+1) = A_D(t)X(t)A_R(t)^T + b_D(t)u(t)b_R(t)^T \\ + b_D(t)u_D(t)^T A_R(t)^T + A_D(t)u_R(t)b_R(t)^T \end{aligned} \tag{9}$$

where $X(t), A_D(t), A_R(t) \in GF(2)^{n\times n}$, $b_D(t), b_R(t) \in GF(2)^n$. $u(t) \in GF(2)$ represents a external event input to change the connection directly and $u_D(t), u_R(t) \in GF(2)^n$ represent external event inputs to change *Donator* and *Receptor* connection, respectively. The model can express transmission of information from a subsystem to others. This is one representation of decentralized system.

Example II-3

Consider the *Donator-Receptor-type* informationally connected system

$$\{A_D, A_R, b_D, b_R, X(0)\} = \left\{ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}. \tag{10}$$

The impulse response from the event input $u(t)$ becomes as shown in Fig. 3 (c).

III. Structure of Connection Systems

In this section, the mathematical structure of the connection space is discussed based on the concept of *eigenvalue* and *eigen-connection*.

A. Eigenvalues

It is well known that eigenvalues $\lambda \in GF(2^\ell)$ and eigenvectors $v \in GF(2^\ell)^n$ exist for any matrix $A \in GF(2)^{n\times n}$ which satisfy $\lambda v = Av$. ℓ is an adequate positive integer mentioned below. Then eigenvalues λ can be obtained by solving a characteristic equation $\psi(\lambda) = \det(\lambda I - A) = 0$. The equation $\psi(\lambda) = 0$ is solvable when we consider an extension field of $GF(2)$.

Let characteristic polynomials of *Donator matrix* A_D and *Receptor matrix* A_R be given by

$$\begin{aligned} \psi_D(\lambda) &= \det(\lambda I - A_D) \\ &= \lambda^n + a_{Dn}\lambda^{n-1} + \ldots + a_{D2}\lambda + a_{D1} \quad (11) \\ \psi_R(\lambda) &= \det(\lambda I - A_R) \\ &= \lambda^n + a_{Rn}\lambda^{n-1} + \ldots + a_{R2}\lambda + a_{R1}. \quad (12) \end{aligned}$$

where $a_{Di}, a_{Ri} \in GF(2)$ for $1 \le i \le n$. The characteristics polynomials $\psi_D(\lambda)$ and $\psi_R(\lambda)$ are elements in a set $GF(2)[\lambda]$ which is the ring of univariate polynomials over $GF(2)$. The factorizations of (11) and (12) over $GF(2)[\lambda]$ are represented by

$$\begin{aligned} \psi_D(\lambda) &= \psi_{D1}(\lambda)^{n_1}\psi_{D2}(\lambda)^{n_2}\cdots\psi_{Dp_D}(\lambda)^{n_{p_D}} \quad (13) \\ \psi_R(\lambda) &= \psi_{R1}(\lambda)^{n_1}\psi_{R2}(\lambda)^{n_2}\cdots\psi_{Rp_R}(\lambda)^{n_{p_R}} \quad (14) \end{aligned}$$

where $\psi_{Di}(\lambda)$ and $\psi_{Rj}(\lambda)$ are irreducible polynomials over $GF(2)[\lambda]$.

Let the degrees of irreducible polynomials $\psi_{Di}(\lambda)$ and $\psi_{Rj}(\lambda)$ be ℓ_{Di} and ℓ_{Ri} for $1 \le i \le p_D$ and $1 \le j \le p_R$, respectively. Then characteristic polynomials $\psi_D(\lambda)$ and $\psi_R(\lambda)$ can be factorized into the form

$$\psi_D(\lambda) = (\lambda - \lambda_{D1})^{n_1}(\lambda - \lambda_{D2})^{n_2}\cdots(\lambda - \lambda_{D_{p_D}})^{n_{p_D}} \tag{15}$$

$$\psi_R(\lambda) = (\lambda - \lambda_{R1})^{n_1}(\lambda - \lambda_{R2})^{n_2}\cdots(\lambda - \lambda_{R_{p_R}})^{n_{p_R}} \tag{16}$$

over an extension field $GF(2^\ell)$, where

$$\ell = \mathrm{LCM}(\ell_{D1}, \ldots, \ell_{Dp_D}, \ell_{R1}, \ldots, \ell_{Rp_R}) \tag{17}$$

$\lambda_{Di}, \lambda_{Rj} \in GF(2^\ell)$ correspond to eigenvalues of A_D and A_R. In this paper, they are named *Donator eigenvalue* and *Receptor eigenvalue.*

Example III-1

For the system $\{A_D, A_R, b_D, b_R, X(0)\}$ of *Example II-3*, the characteristic polynomial of A_D and A_R can be factorized into irreducible polynomials:

$$\psi_D(\lambda) = \psi_R(\lambda) = \lambda^3 + 1 = (\lambda+1)(\lambda^2+\lambda+1). \tag{18}$$

Since the degrees of irreducible polynomials are $\ell_{D1} = \ell_{R1} = 1$, $\ell_{D2} = \ell_{R2} = 2$, the order of the extension field is obtained from (17) as $\ell = \mathrm{LCM}(1, 2, 1, 2) = 2$. Let the root of one of second degree irreducible polynomial $\lambda^2 + \lambda + 1 = 0$ be α. (18) is further factorized on polynomial modulo $\alpha^2 + \alpha + 1$.

$$\psi_D(\lambda) = \psi_R(\lambda) = (\lambda+1)(\lambda+\alpha)(\lambda+\alpha^2). \tag{19}$$

Therefore the eigenvalues of A_D and A_R are $\lambda_{D1} = \lambda_{R1} = 1,\ \lambda_{D2} = \lambda_{R2} = \alpha,\ \lambda_{D3} = \lambda_{R3} = \alpha^2$.

Fig. 4 illustrates the pole plot of (19). We shall call this pole plot *GF-plane* for convenience. In *GF-plane*, the origin denotes the element of zero. The points on the unit circle denote all elements of $GF(2^\ell)$ except 0. In this case, the pole α operates a state of connection to rotate by $2\pi/3$ [rad] on *GF-plane*. Also the operation of α^2 is by $4\pi/3$ [rad]. Therefore, we can see the period of this system is three.

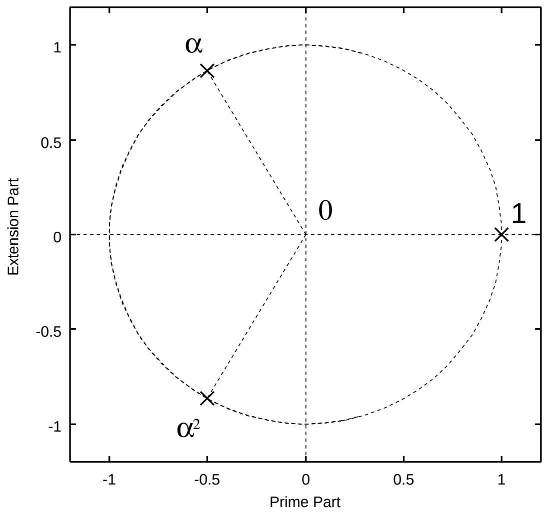


Fig. 4. Pole plot in *GF*-plane

B. Eigen-connections

Let ith generalized eigenvector of A_D be v_{Di}, jth generalized eigenvector of A_R be v_{Rj}. One of *eigen-connections* W_{ij} is defined by

$$W_{ij} = v_{Di} {v_{Rj}}^T. \tag{20}$$

If there is no duplicate eigenvalues in each of A_D and A_R, the *eigen-connection* satisfies

$$\lambda_{ij} W_{ij} = A_D W_{ij} {A_R}^T \tag{21}$$

where $\lambda_{ij} = \lambda_{Di}\lambda_{Rj}$ corresponds to the eigenvalue of connection systems. The *eigen-connections* are special connections whose shapes are invariant during dynamic transition. The norms are, however, variant due to the corresponding eigenvalues.

Since the number of *eigen-connections* in a connection system is n^2 and they are independent each other, any connection X can be expressed by the sum of them

$$X = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} W_{ij} \tag{22}$$

where $a_{ij} \in GF(2^\ell)$. The *eigen-connections* are basis which span the connection space.

Example III-2

For the system $\{A_D, A_R, b_D, b_R, X(0)\}$ of *Example III-A-1*, the eigenvectors of A_D and A_R are given by $v_{D1} = v_{R1} = [1,1,1]^T$, $v_{D2} = v_{R2} = [1,\alpha,\alpha^2]^T$, $v_{D3} = v_{R3} = [1,\alpha^2,\alpha]^T$. From (20), the eigen-connections are given by

$$\begin{aligned}
&\{W_{11}, W_{12}, W_{13}, W_{21}, W_{22}, W_{23}, W_{31}, W_{32}, W_{33}\} \\
&= \left\{ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha & \alpha^2 \end{bmatrix}, \begin{bmatrix} 1 & \alpha^2 & \alpha \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha^2 & \alpha \end{bmatrix}, \right. \\
&\quad \begin{bmatrix} 1 & 1 & 1 \\ \alpha & \alpha & \alpha \\ \alpha^2 & \alpha^2 & \alpha^2 \end{bmatrix}, \begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & \alpha^2 & 1 \\ \alpha^2 & 1 & \alpha \end{bmatrix}, \begin{bmatrix} 1 & \alpha^2 & \alpha \\ \alpha & 1 & \alpha^2 \\ \alpha^2 & \alpha & 1 \end{bmatrix}, \\
&\quad \left. \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha^2 & \alpha^2 \\ \alpha & \alpha & \alpha \end{bmatrix}, \begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha^2 & 1 & \alpha \\ \alpha & \alpha^2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & \alpha^2 & \alpha \\ \alpha^2 & \alpha & 1 \\ \alpha & 1 & \alpha^2 \end{bmatrix} \right\}.
\end{aligned} \tag{23}$$

Then their eigenvalues are given by

$$\begin{aligned}
&\{\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{21}, \lambda_{22}, \lambda_{23}, \lambda_{31}, \lambda_{32}, \lambda_{33}\} \\
&= \{\ 1,\ \alpha,\ \alpha^2, \\
&\quad \alpha,\ \alpha^2,\ 1, \\
&\quad \alpha^2,\ 1,\ \alpha\ \}.
\end{aligned} \tag{24}$$

There are three steady eigen-connections W_{11}, W_{23}, W_{32} and other six periodic eigen-connections.

C. Conjugate Connections

The connection space representation using eigen-connections includes element α of the extension field $GF(2^\ell)$. In this section, another basis of connection space excluding α is introduced using sets of conjugate connections.

Let one of the eigenvalues be β. Then a set of the eigenvalues $\{\beta, \beta^2, \beta^4, \ldots, \beta^{2^{d-1}}\}$ exist and they become conjugate, where d is a minimum positive integer satisfying $\beta^{2^d} = \beta$. For example, two eigenvalues α, α^2 in *Example III-A-1* are conjugate. We can examine it by $(\lambda+\alpha)(\lambda+\alpha^2) = \lambda^2+\lambda+1$.

Let ith set of conjugate eigenvalues of A_D be $\{\beta_i, \beta_i^2, \beta_i^4, \ldots, \beta_i^{2^{d_i-1}}\}$ and their eigenvectors be $\{v_{\beta_i 1}, v_{\beta_i 2}, \ldots, v_{\beta_i d_i}\}$. Consider $n \times n$ matrix $G_D \in GF(2^\ell)$

$$G_D = \text{block diag}(G_{D1}, G_{D2}, \ldots, G_{Di}, \ldots) \tag{25}$$

where

$$G_{Di} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ \beta_i & \beta_i^2 & \beta_i^4 & & \beta_i^{2^{d_i-1}} \\ \beta_i^2 & \beta_i^4 & \beta_i^8 & & \beta_i^{2^{d_i}} \\ \vdots & & & & \vdots \\ \beta_i^{d_i-1} & \beta_i^{2(d_i-1)} & \beta_i^{4(d_i-1)} & \cdots & \beta_i^{2^{d_i-1}(d_i-1)} \end{bmatrix}. \tag{26}$$

Then let the inverse matrix of G_D be

$$H_D = {G_D}^{-1} = \text{block diag}(H_{D1}, H_{D2}, \ldots, H_{Di}, \ldots) \tag{27}$$

where

$$H_{Di} = {G_{Di}}^{-1} = \begin{bmatrix} h_{Di11} & h_{Di12} & \cdots & h_{Di1d} \\ h_{Di21} & h_{Di22} & & h_{Di2d} \\ \vdots & & & \vdots \\ h_{Did1} & h_{Did2} & \cdots & h_{Didd} \end{bmatrix} \tag{28}$$

Let us consider new vectors $v'_{\beta_i j}$ defined by $v'_{\beta_i j} = \sum_{k=1}^{d_i} h_{Dikj} v_{\beta_i k}$ i. e.

$$V'_D = V_D H_D \tag{29}$$

where

$$\begin{aligned} V_D &= [v_{\beta_1 1}, v_{\beta_1 2}, \ldots, v_{\beta_1 d_1}, v_{\beta_2 1}, v_{\beta_2 2}, \ldots, v_{\beta_2 d_2}, \ldots,] \\ V'_D &= [v'_{\beta_1 1}, v'_{\beta_1 2}, \ldots, v'_{\beta_1 d_1}, v'_{\beta_2 1}, v'_{\beta_2 2}, \ldots, v'_{\beta_2 d_2}, \ldots,] \end{aligned}$$

In a similar manner, let jth set of conjugate eigenvalues of A_R be $\{\gamma_j, \gamma_j^2, \gamma_j^4, \ldots, \gamma_j^{2^{r_j-1}}\}$ and their eigenvectors be $v_{\gamma_j 1}, v_{\gamma_j 2}, \ldots, v_{\gamma_j r_j}$. Then we compute

$$V'_R = V_R H_R \tag{30}$$

where

$$\begin{aligned} V_R &= [v_{\gamma_1 1}, v_{\gamma_1 2}, \ldots, v_{\gamma_1 r_1}, v_{\gamma_2 1}, v_{\gamma_2 2}, \ldots, v_{\gamma_2 r_2}, \ldots,] \\ V'_R &= [v'_{\gamma_1 1}, v'_{\gamma_1 2}, \ldots, v'_{\gamma_1 r_1}, v'_{\gamma_2 1}, v'_{\gamma_2 2}, \ldots, v'_{\gamma_2 r_2}, \ldots,] \end{aligned}$$

The conjugate basis of connection space is given by

$$W'_{ij} = v'_{Di} {v'_{Rj}}^T \tag{31}$$

where v'_{Di} denotes ith column vector of V'_D and v'_{Rj} denotes jth column vector of V'_R.

Example III-3

For the system of *Example III-A-1*, the conjugate connections are computed as follows. We compute

$$G_D = G_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & \alpha & \alpha^2 \end{bmatrix} \tag{32}$$

Then $H_D = {G_D}^{-1}$ and $H_R = {G_R}^{-1}$ are given by

$$H_D = H_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha^2 & 1 \\ 0 & \alpha & 1 \end{bmatrix} \tag{33}$$

From (29) and (30)

$$[v'_{D1}, v'_{D2}, v'_{D3}] = [v'_{R1}, v'_{R2}, v'_{R3}] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \tag{34}$$

Therefore, the conjugate basis of connection space is given by

$$\begin{aligned} &\{W'_{11}, W'_{12}, W'_{13}, W'_{21}, W'_{22}, W'_{23}, W'_{31}, W'_{32}, W'_{33}\} \\ &= \left\{ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \right. \\ &\quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \\ &\quad \left. \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \right\}. \end{aligned} \tag{35}$$

D. Autonomous Rhythm of Connections

In this section, the period of transition of connection systems is discussed. For ith eigenvalue λ_{Di} of A_D, the period p_{Di} is defined by the minimum positive integer which satisfies $\lambda_{Di}^{p_{Di}} = 1$. Let the generalized eigenvector corresponding the eigenvalue λ_{Di} be v_{Di} and its depth be q_{Di}. For the vector v_{Di}, the depth q_{Di} is defined by the positive number which satisfies $v_{Di} \in \mathrm{Ker}(\lambda_{Di} I - A_D)^{q_{Di}}$ and $v_{Di} \notin \mathrm{Ker}(\lambda_{Di} I - A_D)^{q_{Di}-1}$. Then the period of ith eigenvector is given by $2^{g_{Di}} p_{Di}$ where g_{Di} denotes a minimum integer satisfying $q_{Di} \leq 2^{g_{Di}}$. In a similar manner, the period of jth eigenvector of A_R is also given by $2^{g_{Rj}} p_{Rj}$.

Now we compute the period p_{ij} for an eigen-connection $W_{ij} = v_{Di} {v_{Rj}}^T$ as

$$p_{ij} = \mathrm{LCM}(2^{g_{Di}} p_{Di}, 2^{g_{Rj}} p_{Rj}). \tag{36}$$

Therefore the period P for a connection X is given by

$$\mathrm{LCM}(P),\ P = \{p_{ij} | (i,j) \in C\} \tag{37}$$

where C represents a set of subscripts of eigen-connections occupied by X given by $C = \{(i,j) | a_{ij} \neq 0, X = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} W_{ij}\}$.

When a system has duplicate eigenvalues and the deeper eigen-connection space is excited, the period is doubled, quadrupled, etc. Also when a m-degree primitive polynomial is included in the characteristic polynomial, the maximum period $2^m - 1$ is observed.

E. Steady Connection

In this section, a set of *steady connections* is derived.

When a pair of eigenvalues β and γ satisfies $\beta \times \gamma = 1$, they are defined as *contrary eigenvalues*. The system (9) has a steady connection except $X = 0$, if and only if A_D and A_R have a pair of contrary eigenvalues. Let ith eigenvalues of A_D be β_i and its contrary eigenvalue of A_R be γ_i. A_D has other conjugate eigenvalues $\beta_i^2, \beta_i^4, \ldots, \beta_i^{2^{d-1}}$ and A_R has also other conjugate eigenvalues $\gamma_i^2, \gamma_i^4, \ldots, \gamma_i^{2^{d-1}}$. Then each of them makes a pair of contrary eigenvalues, i. e., $\beta_i^{2^j} \times \gamma_i^{2^j} = 1$ for $0 \leq j \leq d-1$. Let the corresponding eigenvectors be

$$V_{Di} = [v_{Di0}, v_{Di1}, \ldots, v_{Did-1}] \tag{38}$$
$$V_{Ri} = [v_{Ri0}, v_{Ri1}, \ldots, v_{Rid-1}] \tag{39}$$

where v_{Dij} denotes the eigenvector for the eigenvalue $\beta_i^{2^j}$ and v_{Rij} denotes the eigenvector for the eigenvalue $\gamma_i^{2^j}$, i. e., $v_{Dij} \in \mathrm{Ker}(\beta_i^{2^j} I - A_D)$ and $v_{Rij} \in \mathrm{Ker}(\gamma_i^{2^j} I - A_R)$. Under these preparations, we have a following theorem.

Lemma 1: A set of steady connections is given by

$$\Omega_1 = \{X | X = V_{Di} {V_{Ri}}^T, i \in C\} \tag{40}$$

where $C = \{i | \beta_i \gamma_i = 1\}$.

Proof: The definitions of V_D and V_R yield

$$A_D V_D = V_D \mathrm{diag}(\beta, \beta^2, \ldots, \beta^{2^{d-1}}) \tag{41}$$
$$A_R V_R = V_R \mathrm{diag}(\gamma, \gamma^2, \ldots, \gamma^{2^{d-1}}) \tag{42}$$

Note that each of eigenvalues is contrary, i. e., $\beta^{2^j}\gamma^{2^j} = 1$ for $0 \le j \le d-1$. Then we compute

$$\begin{aligned} A_D V_D {V_R}^T {A_R}^T &= V_D \mathrm{diag}(\beta\gamma, \beta^2\gamma^2, \ldots, \beta^{2^{d-1}}\gamma^{2^{d-1}}) {V_R}^T \\ &= V_D {V_R}^T \end{aligned} \quad (43)$$

Therefore, the connection $X = V_D {V_R}^T$ is the steady connection which holds $X = A_D X {A_R}^T$. The necessity of the condition is trivial. ■

The system (9) has a *vanishing connection* which converges on the origin $X = 0$ within finite steps if and only if A_D or A_R has eigenvalues 0.

Lemma 2: A set of the *vanishing connections* is given by

$$\begin{aligned} \Omega_0 \;=\; &\{X | X = v_D {a_R}^T + a_D {v_R}^T,\ v_D \in \mathrm{Ker}{A_D}^{m_D}, \\ &v_R \in \mathrm{Ker}{A_R}^{m_R},\ \forall a_D, a_R \in GF(2)^n\}. \end{aligned} \quad (44)$$

The connection converges on 0 within $\max(m_D, m_R)$ steps, where m_D denotes a minimum integer satisfying $\mathrm{rank}{A_D}^{m_D} = \mathrm{rank}{A_D}^n$. Also m_R is defined for A_R in a similar manner.

Proof: If either A_R or A_D has an eigenvalue 0, a non-zero eigenvector $v_D \in \mathrm{Ker}{A_D}^{m_D}$ or $v_R \in \mathrm{Ker}{A_R}^{m_R}$ exists. In a case of $m_D \ge m_R$, we compute the connection in (44) after m_D steps so that ${A_D}^{m_D} X ({A_R}^T)^{m_D} = {A_D}^{m_D}(v_D {a_R}^T + a_D {v_R}^T)({A_R}^T)^{m_D} = 0$ because ${A_D}^{m_D} v_D = 0$ and ${A_R}^{m_D} v_R = {A_R}^{m_D - m_R} \times {A_R}^{m_R} v_R = 0$. The case of $m_D < m_R$ is also computed in similar manner. ■

Thus, a set of connections which converge on a steady connection within finite steps is given by $\Omega_0 \oplus \Omega_1$

IV. Control of Connection Systems

The eigenvalues of the connection systems (9) can be designed using external event inputs u, u_D, and u_R. On (9), when we set *connection feedback*

$$u = {f_D}^T X f_R,\ u_D = X^T f_D,\ u_R = X f_R \quad (45)$$

then the following dynamics of the connection is obtained.

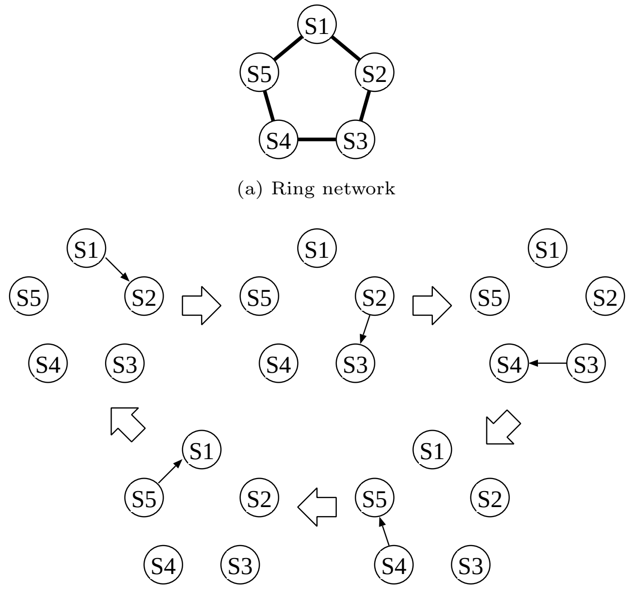
$$X(t+1) = (A_D + b_D {f_D}^T) X(t) (A_R + b_R {f_R}^T)^T \quad (46)$$

Any eigenvalues can be designed if and only if both (A_D, b_D) and (A_R, b_R) are controllable.

V. Decentralized Implementation

Let the connection matrix be $X = [x_{D1}, \ldots, x_{Dn}] = [x_{R1}, \ldots, x_{Rn}]^T$. Each subsystem consists of $\{x_{Di}, x_{Ri}, A_D, A_R\}$ where $x_{Di}, x_{Ri} \in GF(2)^n$ and $A_D, A_R \in GF(2)^{n\times n}$. x_{Di} and x_{Ri} represent local donator and receptor connections of ith subsystems, respectively. Usually, the subsystems communicate signals each other depending on their donator and receptor connections x_{Di}, x_{Ri}. The matrices A_D and A_R are homogeneous among subsystems. If an event occurs, the update procedure of connections is carried out as follows:

1. Update donator connections by $x_{Di} \leftarrow A_D x_{Di}$ for each subsystems.
2. Update receptor connection x_{Rj} to be consistent to x_{Di} for $j \ne i$.
3. Update receptor connections by ${x_{Ri}}^T \leftarrow {x_{Ri}}^T {A_R}^T$ for each subsystems.
4. Update donator connection x_{Dj} to be consistent to x_{Rj} for $j \ne i$.



(a) Ring network

(b) Token passing sequence

Fig. 5. Token passing protocol

VI. Illustrative Examples

A. Token Passing Protocol on Ring Network

Token Passing Protocol is a communication mechanism for multi-node network that realizes time critical communications. To avoid collisions, the node passes a token from one to another, and it has a right to talk on the network only if it holds the token.

Consider a ring network with five nodes as shown in Fig. 5. First, the token is passed from the node S_1 to the node S_2, then it is passed from the node S_2 to the node S_3. Namely, it moves cyclically as $S_1 \to S_2 \to S_3 \to S_4 \to S_5 \to S_1$.

The delivery of token is modeled as an autonomous system with informational connection: $X(t+1) = A_D X(t) A_R^T$ where

$$\{A_D, A_R, X(0)\} = \left\{ \begin{bmatrix} 0&0&0&0&1\\ 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0 \end{bmatrix}, \begin{bmatrix} 0&0&0&0&1\\ 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0 \end{bmatrix}, \begin{bmatrix} 0&0&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \right\}. \quad (47)$$

The characteristic polynomials of A_D and A_R are computed as

$$\begin{aligned} &\psi_D(\lambda) = \psi_R(\lambda) = \lambda^5 + 1 \\ &= (\lambda+1)(\lambda^4+\lambda^3+\lambda^2+\lambda+1) \\ &= (\lambda+1)(\lambda+\alpha^3)(\lambda+\alpha^6)(\lambda+\alpha^9)(\lambda+\alpha^{12}) \end{aligned} \quad (48)$$

where $\alpha \in GF(2^4)$ is a root of 4th degree irreducible polynomial $\lambda^4 + \lambda + 1 = 0$. From (48) the Donator eigenvalues λ_D and the Receptor eigenvalues λ_R are $\lambda_D = \lambda_R = \{1, \alpha^3, \alpha^6, \alpha^9, \alpha^{12}\}$. Then eigenvalues of the system are calculated as $\{1, \alpha^3, \alpha^6, \alpha^9, \alpha^{12}\}$ by (21) for $1 \leq i \leq 5$, $1 \leq j \leq 5$. The system has five fivefold eigenvalues. Fig. 6 shows *GF-plane* of the system. It is observed that the period of four eigenvalues $\{\alpha^3, \alpha^6, \alpha^9, \alpha^{12}\}$ is five.

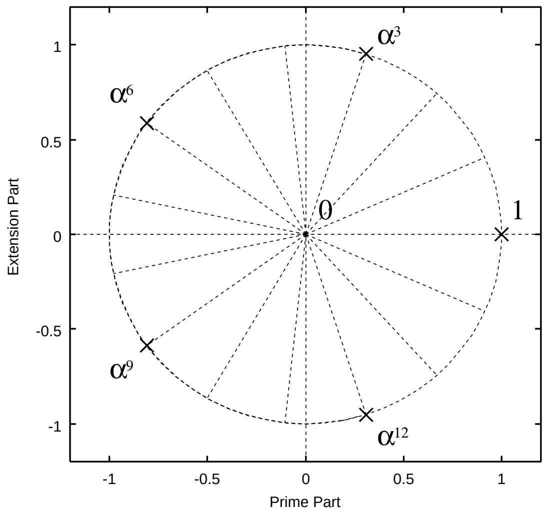


Fig. 6. Pole plot of token passing model

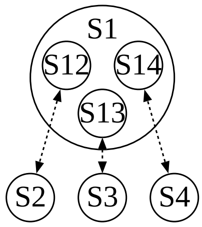


Fig. 7. Asynchronous 1-scheduler model

B. Asynchronous 1-Scheduler

The proposed modeling method is applied to an asynchronous 1-scheduler for simple parallel computers. There is a scheduler process that divides a job into multiple parts and dispatches them to the other processes. Let S_1 be the scheduler process and $S_2, S_3, S_4, \dots$ be the other sub-processes. A sequence of a request $S_1 \to S_i$ and an acknowledgement $S_1 \leftarrow S_i$ is asynchronously realized.

Assume that only three sub-precesses S_2, S_3, S_4 exist. The scheduler process S_1 is divided into three parts according to the condition of the communication as shown in Fig. 7. Then we have the connection system: $X(t+1) = A_D X(t) A_R^T + B_D \mathrm{diag}(u_1, u_2, u_3) B_R^T$ where

$$\{A_D, A_R, B_D, B_R\} = \left\{ \begin{bmatrix} 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\ 0&0&0\\ 0&0&0\\ 1&0&0\\ 0&1&0\\ 0&0&1 \end{bmatrix}, \begin{bmatrix} 1&0&0\\ 0&1&0\\ 0&0&1\\ 0&0&0\\ 0&0&0\\ 0&0&0 \end{bmatrix} \right\} \tag{49}$$

where event input u_i denotes a request flag from the scheduler to $(i+1)th$ sub-process.

Since the characteristic equations of the Donator matrix A_D and the Receptor matrix A_R are $\psi_D(\lambda) = \psi_R(\lambda) = \lambda^6 = 0$, the system has sixfold poles. The corresponding eigenvectors are calculated as $v_{D1} = [1,0,0,0,0,0]^T$, $v_{D2} = [0,0,0,1,0,0]^T$, $v_{D3} = [0,1,0,0,0,0]^T$, $v_{D4} = [0,0,0,0,1,0]^T$, $v_{D5} = [0,0,1,0,0,0]^T$, $v_{D6} = [0,0,0,0,0,1]^T$, $v_{R1} = [0,0,0,1,0,0]^T$, $v_{R2} = [1,0,0,0,0,0]^T$, $v_{R3} = [0,0,0,0,1,0]^T$, $v_{R4} = [0,1,0,0,0,0]^T$, $v_{R5} = [0,0,0,0,0,1]^T$, $v_{R6} = [0,0,1,0,0,0]^T$. Thus the system has 36 eigen-connections given by (20). From B_D and B_R, the eigen-connections excited by the external inputs $\{u_1, u_2, u_3\}$ are $\{W_{22}, W_{44}, W_{66}\}$. These eigen-connections represent the dispatches of the jobs from the scheduler to the sub-processes.

The eigen-connection W_{22} is composed of two eigenvectors v_{D2} and v_{R2}. Since the depth of these vectors is 2, dynamics of the sub-space is governed by $W_{11} = A_D W_{22} A_R^T$ instead of (21). The sub-spaces W_{44} and W_{66} are governed by $W_{33} = A_D W_{44} A_R^T$ and $W_{55} = A_D W_{66} A_R^T$ as well. The eigen-connections $\{W_{11}, W_{33}, W_{55}\}$ represent the acknowledgements from sub-processes.

Then, the sub-spaces $\{W_{11}, W_{33}, W_{55}\}$ are governed by (21), namely $0 = A_D W_{11} A_R^T$, $0 = A_D W_{33} A_R^T$, $0 = A_D W_{55} A_R^T$. They correspond to ends of the jobs.

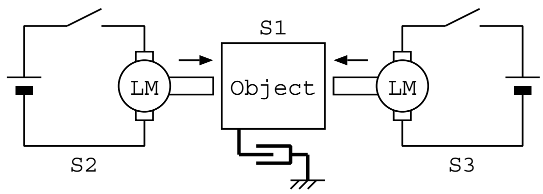


Fig. 8. Decentralized acutuation system

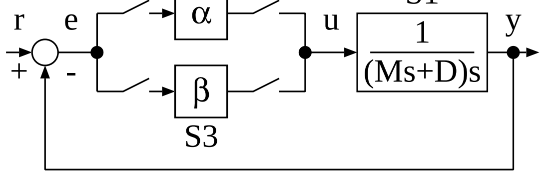


Fig. 9. Variable structure system with sliding mode

C. Variable Structure System

Fig. 8 shows an illustrative example of a decentralized positioning system with a object and two one-way linear actuators. The system is regarded as a variable structure system. Fig. 9 shows a block diagram of the system where M is inertia of object, D is dumping factor, r is reference position, y is position of object, e is error signal, u is force imposed on object, and α and β are gains for actuator #1 and #2, respectively.

Since the system has two actuators, we consider the dynamic connection system:

$$\{A_D, A_R, b_D, b_R, X(0)\} = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}. \tag{50}$$

The external event inputs are given by the connection feedback (45) and $f_D = f_R = [0, 1, 1]^T$. The event inputs are activated when $\sigma(t) \times \sigma(t - T_s) < 0$ where $\sigma(t)$ is defined by $\sigma(t) = ce(t) + \dot{e}(t)$ and T_s is the control period. $\sigma(t) = 0$ denotes sliding surface. The connection feedback yields controlled system matrices:

$$A_D + b_D f_D^T = A_R + b_R f_R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{51}$$

The dynamic transition of the informational connection is shown in Fig. 10. The unit time in this representation is defined as the minimal interval of the events. The Donator eigenvalues and the Receptor eigenvalues are calculated as $\lambda_D = \lambda_R = \{1\}$. The system has triple poles. From (37), the period of the system becomes 2 because the depth of one of the eigenvectors is 2. It corresponds to the fact that the system has two structures.

The system is implemented in decentralized subsystems connected via *Fast Ethernet* as shown in Fig. 11, where the personal computer PC1 emulates the object. The PC2 and PC3 are the controller S_1 and S_2, respectively. The protocol is implemented on the upper level over the UDP/IP. Fig. 12 shows the experimental result of the phase plane trajectory.

The proposed model gives an analytical system structure from the view point of the upper event layer. Also it gives an decentralized implementation model as shown in the section V.

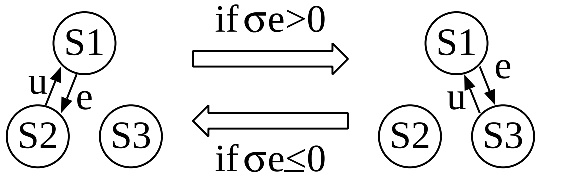


Fig. 10. Informational connection of VSS

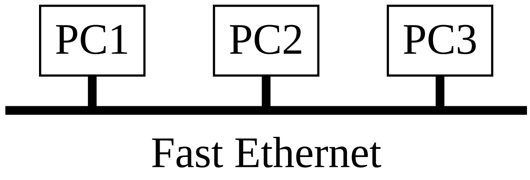


Fig. 11. System configuration

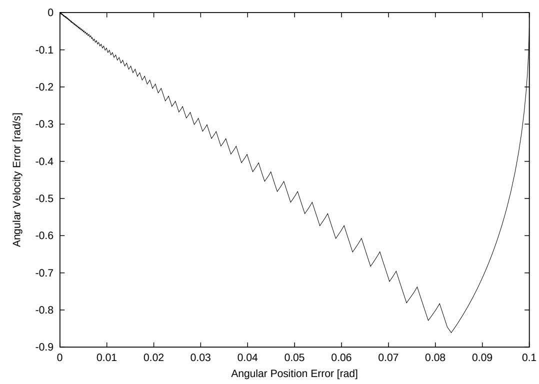


Fig. 12. Experimental results in phase plane

VII. Conclusions

A design method of decentralized systems with informational connection is proposed. The dynamic transition of informational connection among decentralized systems is considered and the mathematical structures are discussed using a concept of *eigenvalues* and *eigen-connections* over the Galois field $GF(2)$. Examples of decentralized variable structure systems are shown. In an industry field, there are many engineering systems that have dynamic transition of informational connection. The mathematical model would be useful for analysis and synthesis of various informationally connected systems.

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