Spin structure function $g_2(x,Q^2)$ and twist-3 operators in large-N_C QCD

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It is shown in the framework of operator product expansion and the renormalization group method that the twist-3 part of flavor nonsinglet spin structure function $g_2(x,Q^2)$ obeys a simple Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation in the large N_C limit even in the case of massive quarks (N_C is the number of colors). There are four different types of twist-3 operators which contribute to g_2, including quark-mass-dependent operators and the ones proportional to the equation of motion. They are not all independent, but are constrained by one relation. A new choice of the independent operator bases leads to a simple form of the evolution equation for g_2 at large N_C. [S0556-2821(98)01519-7]

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I. INTRODUCTION

In experiments of polarized deep inelastic leptoproduction, we can obtain information on the spin structures of the nucleon, which are described by the two functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$. The QCD effects on g_1 and g_2 have been extensively studied [1] since earlier papers [2–4]. Increasingly accurate measurements of g_1 have been performed at SLAC, CERN, and DESY [5], while the g_2 measurements still have limited statistical precision [6].

In the language of operator product expansion (OPE), the twist-2 operators contribute to g_1 in the leading order of $1/Q^2$. As for the structure function g_2, on the other hand, both twist-2 and twist-3 operators participate in the leading order. Moreover, the number of participating twist-3 operators grows with spin (moment of g_2). Because of the increase of the number of operators and the mixing among these operators, the analysis of the twist-3 part of g_2 turns out to be rather complicated [7–14]. In other words, the Q^2 evolution equation for the moments of the twist-3 part of g_2 cannot be written in a simple form, but in a sum of terms, the number of which increases with spin.

For the case of the twist-3 flavor nonsinglet g_2, it has been observed by Ali, Braun, and Hiller (ABH) [15] that in the large N_C limit, g_2 obeys a simple Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [16]. In their formalism of working directly with the nonlocal operator contributing to the twist-3 part of g_2, they showed that local operators involving gluons effectively decouple from evolution equation for large N_C. In fact their analysis has been made with massless quarks.

In this paper, I reanalyze the Q^2 evolution of $\bar{g}_2$, the flavor nonsinglet twist-3 part of g_2, in the framework of the standard OPE and the renormalization group (RG) with massive quarks. Actually, the OPE analysis of $\bar{g}_2$ has been performed already and the anomalous dimensions of the relevant twist-3 operators have been calculated [8,9,11,17,18]. However, to the best of my knowledge, the large N_C limit of $\bar{g}_2$ has not been thoroughly studied so far in OPE and RG.

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There are four different types of twist-3 operators which contribute to $\bar{g}_2$, including quark-mass-dependent operators and the ones proportional to the equation of motion. They are not all independent, but are constrained by one relation. It was pointed out recently by Kodaira, Uematsu, and Yasui [17] that any choice of the independent operator bases leads to a unique prediction for the moments. Taking a new basis of the independent operators, I will show that the Q^2 evolution of $\bar{g}_2$ obeys a simple DGLAP equation in the $N_C\to\infty$ limit and thus the ABH result on $\bar{g}_2$ is reproduced even with massive quarks.

In the next section, we choose a new basis for the independent operators which contribute to $\bar{g}_2$ and derive a formal expression for the moments of $\bar{g}_2$ in the formalism of OPE and RG. In Sec. III we obtain the anomalous dimensions for this new set of the independent operators and show that in the large N_C limit, the Q^2 evolution of $\bar{g}_2$ obeys a simple DGLAP equation even in the case of massive quarks. Section IV is devoted to summary and discussion.

II. THE OPE ANALYSIS OF $\bar{g}_2$

The spin structure function g_2 receives contributions from both twist-2 and twist-3 operators. However, the twist-2 part of g_2 can be extracted once g_1 is measured [19]:

$$g_2^{tw.2}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(y,Q^2)}{y} dy. \tag{1}$$

Thus the difference

$$\bar{g}_2(x,Q^2) = g_2(x,Q^2) - g_2^{tw.2}(x,Q^2) \tag{2}$$

contains the twist-3 contributions only.

The twist-3 operators which enter the OPE for the flavor nonsinglet $\bar{g}_2$ are the following (I follow the notation and conventions of Refs. [17,18] and omit the flavor matrices λ_i):

$$R_F^{\sigma\mu_1\cdots\mu_{n-1}}=\frac{i^{n-1}}{n}\left[(n-1)\bar{\psi}\gamma_5\gamma^\sigma D^{\{\mu_1}\cdots D^{\mu_{n-1}\}}\psi-\sum_{l=1}^{n-1}\bar{\psi}\gamma_5\gamma^{\mu_l}D^{\{\sigma}D^{\mu_1}\cdots D^{\mu_{l-1}}D^{\mu_{l+1}}\cdots D^{\mu_{n-1}\}}\psi\right]-(\text{traces}), \quad (3)$$

$$R_l^{\sigma\mu_1\cdots\mu_{n-1}}=\frac{1}{2n}(V_l-V_{n-1-l}+U_l+U_{n-1-l}), \quad (l=1,\ldots,n-2), \quad (4)$$

$$R_m^{\sigma\mu_1\cdots\mu_{n-1}}=i^{n-2}mS'\bar{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi-(\text{traces}), \quad (5)$$

$$R_E^{\sigma\mu_1\cdots\mu_{n-1}}=i^{n-2}\frac{n-1}{2n}S'[\bar{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}(i\not{D}-m)\psi+\bar{\psi}(i\not{D}-m)\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi]-(\text{traces}), \quad (6)$$

where { } means complete symmetrization over the Lorentz indices and m represents the quark mass. The symbol S' denotes symmetrization on the indices $\mu_1\mu_2\cdots\mu_{n-1}$ and antisymmetrization on $\sigma\mu_i$. The operators in Eq. (4) contain the gluon field strength $G_{\mu\nu}$ and its dual tensor $\tilde{G}_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ and they are given by

$$V_l=-i^n gS'\bar{\psi}\gamma_5 D^{\mu_1}\cdots G^{\sigma\mu_l}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi-(\text{traces}), \quad (7)$$

$$U_l=i^{n-1}gS'\bar{\psi}D^{\mu_1}\cdots\tilde{G}^{\sigma\mu_l}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi-(\text{traces}), \quad (8)$$

where g is the QCD coupling constant. The operator R_E^n in Eq. (6) is proportional to the equation of motion (EOM operator). The above twist-3 operators are not all independent, but they are constrained by the following relation [7,12]:

$$R_F^{\sigma\mu_1\cdots\mu_{n-1}}=\frac{n-1}{n}R_m^{\sigma\mu_1\cdots\mu_{n-1}}+\sum_{l=1}^{n-2}(n-1-l)R_l^{\sigma\mu_1\cdots\mu_{n-1}}+R_E^{\sigma\mu_1\cdots\mu_{n-1}}. \quad (9)$$

Thus in total there are n independent operators contributing to the $(n-1)$th moment of $\bar{g}_2$. But we will see later that in the $N_C\to\infty$ limit, the $(n-1)$th moment is expressed in terms of one operator $R_F^{\sigma\mu_1\cdots\mu_{n-1}}$.

In all the analyses of $\bar{g}_2$ performed so far in the framework of OPE and RG, operators R_l,R_m,R_E of Eqs. (4)–(6) have been taken as independent bases. In this paper I choose R_F,R_l,R_E as independent operators, replacing R_m with R_F of Eq. (3). The advantage of this choice of operator basis is that the coefficient functions take simple forms at the tree-level. In fact we have [17]

$$E_F^n(\text{tree})=1,\quad E_l^n(\text{tree})=0 \quad \text{for } l=1,\ldots,n-2, \quad (10)$$

since the antisymmetric part of the short distance expansion for the product of two electromagnetic currents can be written at the tree level as

$$i\int d^4x e^{iq\cdot x}T[J_\mu(x)J_\nu(0)]|_{\text{antisymmetric}}=-i\varepsilon_{\mu\nu\lambda\sigma}q^\lambda\sum_{n=1,3,\cdots}\left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_{n-1}}\times\{R_q^{\sigma\mu_1\cdots\mu_{n-1}}+R_F^{\sigma\mu_1\cdots\mu_{n-1}}\}+\cdots, \quad (11)$$

where dots $\cdots$ stands for nonleading terms and

$$R_q^{\sigma\mu_1\cdots\mu_{n-1}}=i^{n-1}\bar{\psi}\gamma_5\gamma^{\{\sigma}D^{\mu_1}\cdots D^{\mu_{n-1}\}}\psi-(\text{traces}) \quad (12)$$

are twist-2 operators which contribute to g_1 and $g_2^{tw.2}$. It is true that due to the relation, Eq. (9), $R_F^{\sigma\mu_1\cdots\mu_{n-1}}$ can be expressed in terms of other operators. When eliminating R_F^n, we obtain a different set of coefficient functions. In other words, the (tree-level) coefficient functions are dependent upon the choice of the independent operators [17].

The renormalization constants for this new set of independent operators are written in the matrix form as

$$\begin{pmatrix} R_F^n \\ R_l^n \\ R_E^n \end{pmatrix}_B=\begin{pmatrix} \tilde{Z}_{FF} & \tilde{Z}_{Fj} & \tilde{Z}_{FE} \\ \tilde{Z}_{lF} & \tilde{Z}_{lj} & \tilde{Z}_{lE} \\ 0 & 0 & \tilde{Z}_{EE} \end{pmatrix}\begin{pmatrix} R_F^n \\ R_j^n \\ R_E^n \end{pmatrix}_R, \quad (l,j=1,\ldots,n-2), \quad (13)$$

where the suffix $R(B)$ denotes renormalized (bare) quantities.

Now we proceed to the moment sum rule for $\bar{g}_2$. Define the matrix elements of composite operators between nucleon states with momentum p and spin s by

$$\langle p,s|R_F^{\sigma\mu_1\cdots\mu_{n-1}}|p,s\rangle=-\frac{n-1}{n}d_n(s^\sigma p^{\mu_1}-s^{\mu_1}p^\sigma)p^{\mu_2}\cdots p^{\mu_{n-1}}, \tag{14}$$

$$\langle p,s|R_l^{\sigma\mu_1\cdots\mu_{n-1}}|p,s\rangle=-f_n^l(s^\sigma p^{\mu_1}-s^{\mu_1}p^\sigma)p^{\mu_2}\cdots p^{\mu_{n-1}}, \tag{15}$$

$$\langle p,s|R_E^{\sigma\mu_1\cdots\mu_{n-1}}|p,s\rangle=0. \tag{16}$$

Normalization is such that for free quark target, we have $d_n=1$ and $f_n^l=\mathcal{O}(g^2)$. It is recalled that physical matrix elements of the EOM operators vanish [20]. Using Eqs. (14)–(16), we can write down the moment sum rule for $\bar{g}_2$ as

$$M_n\equiv\int_0^1 dx x^{n-1}\bar{g}_2(x,Q^2)$$
$$=\frac{n-1}{2n}d_nE_F^n(Q^2)+\frac{1}{2}\sum_{l=1}^{n-2}f_n^lE_l^n(Q^2). \tag{17}$$

The coefficient functions $E_F^n(Q^2)$ and $E_l^n(Q^2)$ satisfy the following renormalization group equation:

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma_m(g)m\frac{\partial}{\partial m}\right)E_i=\tilde{\gamma}_{ji}E_j \quad \text{for } i,j=F,1,\ldots,n-2, \tag{18}$$

where $\beta(g)$ and $\gamma_m(g)$ are the QCD β function and the anomalous dimension of mass operator, respectively. The anomalous dimension matrix $\tilde{\gamma}_{ij}$ of the composite operators R_F^n and R_l^n with $l=1,\ldots,n-2$ is defined as

$$\tilde{\gamma}_{ij}=\left[\tilde{Z}^{-1}\mu\frac{\partial\tilde{Z}}{\partial\mu}\right]_{ij} \quad \text{for } i,j=F,1,\ldots,n-2. \tag{19}$$

Note that the anomalous dimension matrix which appears in Eq. (18) is a transposed one. This comes from our convention of defining renormalization constants and anomalous dimensions of the operators in Eqs. (13) and (19).

In the leading-logarithmic approximation, the solutions of the RG equations in Eq. (18) are given as follows [21]:

$$E_i^n(Q^2)=\left[\exp\left\{\frac{\tilde{\gamma}^{(0)n}}{2\beta_0}\ln\left(\frac{\alpha(Q^2)}{\alpha(\mu^2)}\right)\right\}\right]_{Fi} \quad \text{for } i=F,1,\ldots,n-2, \tag{20}$$

where $\alpha(Q^2)$ is the QCD running coupling constant, β_0 and $\tilde{\gamma}^{(0)n}$ are, respectively, one-loop coefficients of the β function and anomalous dimension matrix,

$$\beta(g)=-\beta_0g^3+\mathcal{O}(g^5),\ \ \beta_0=\frac{1}{(4\pi)^2}\frac{11N_c-2n_f}{3}, \tag{21}$$

$$\tilde{\gamma}_{ij}^n(g)=\tilde{\gamma}_{ij}^{(0)n}g^2+\mathcal{O}(g^4), \tag{22}$$

with n_f being the number of flavors, and we have used the fact that $E_F^n(\mu^2)=1$ and $E_l^n(\mu^2)=0$ (for $l=1,\ldots,n-2$) at the lowest-order.

III. MOMENT SUM RULE FOR $\bar{g}_2$ IN THE LARGE N_C LIMIT

Now we need the information on the anomalous dimensions $(\tilde{\gamma}^{(0)n})_{Fi}$ (for $i=F,1,\ldots,n-2$). We can get it without embarking on a new calculation of the relevant Feynman diagrams. We utilize the existing results on the anomalous dimension matrix for the operators R_l,R_m and R_E. In the case of the conventional choice of R_l,R_m and R_E as independent operators, the renormalization constant matrix takes a triangular form

$$\begin{pmatrix} R_l^n \\ R_m^n \\ R_E^n \end{pmatrix}_B=\begin{pmatrix} Z_{lj} & Z_{lm} & Z_{lE} \\ 0 & Z_{mm} & 0 \\ 0 & 0 & Z_{EE}\end{pmatrix}\begin{pmatrix} R_j^n \\ R_m^n \\ R_E^n \end{pmatrix}_R, \quad (l,j=1,\ldots,n-2). \tag{23}$$

In the minimal subtraction (MS) renormalization scheme, Z_{ij} is expressed as

$$Z_{ij}=\delta_{ij}-\frac{g^2}{16\pi^2\varepsilon}X_{ij} \quad (i,j=1,\ldots,n-2,m,E), \tag{24}$$

where $\varepsilon=(4-d)/2$ with d the space-time dimension, and the components X_{ij} have been calculated [8,9,11,18]. The following is the result on X_{ij} taken from Ref. [18]:

$$X_{lj}=C_G\frac{(j+1)(j+2)}{(l+1)(l+2)(l-j)}+(2C_F-C_G)\left((-1)^{l+j}\frac{{}_{n-2}C_{j-1}}{{}_{n-2}C_{l-1}}\frac{(n-1+l-j)}{(n-1)(l-j)}+\frac{2(-1)^j}{l(l+1)(l+2)}{}_lC_j\right) \quad (1\leq j\leq l-1), \tag{25}$$

$$X_{ll}=C_G\left(\frac{1}{l}-\frac{1}{l+1}-\frac{1}{l+2}-\frac{1}{n-l}-S_l-S_{n-l-1}\right)+(2C_F-C_G)\left(\frac{1}{n-1}+\frac{2(-1)^l}{l(l+1)(l+2)}-\frac{(-1)^l}{n-l}\right)$$
$$+C_F(3-2S_l-2S_{n-l-1}), \tag{26}$$

$$X_{lj}=C_G\frac{(n-1-j)(n-j)}{(n-1-l)(n-l)(j-l)}+(2C_F-C_G)\left((-1)^{l+j}\frac{{}_{n-2}C_j}{{}_{n-2}C_l}\frac{(n-1-l+j)}{(n-1)(j-l)}+(-1)^{n-j}\frac{{}_{n-2-l}C_{n-2-j}}{n-l}\right) \quad (l+1\leqslant j\leqslant n-2), \tag{27}$$

$$X_{lm}=\frac{4C_F}{nl(l+1)(l+2)},\quad X_{mm}=-4C_FS_{n-1}. \tag{28}$$

If we impose that the renormalized and bare operators, respectively, satisfy the constraint Eq. (9), we find from Eqs. (13) and (23) that $\tilde{Z}$'s are related to the conventional Z's as follows:

$$\tilde{Z}_{FF}=Z_{mm}+\frac{n}{n-1}\sum_{l=1}^{n-2}(n-1-l)Z_{lm}, \tag{29}$$

$$\tilde{Z}_{Fj}=-(n-1-j)\tilde{Z}_{FF}+\sum_{l=1}^{n-2}(n-1-l)Z_{lj}, \tag{30}$$

$$\tilde{Z}_{lF}=\frac{n}{n-1}Z_{lm}, \tag{31}$$

$$\tilde{Z}_{lj}=Z_{lj}-\frac{n}{n-1}(n-1-j)Z_{lm}, \tag{32}$$

where $l,j=1,\ldots,n-2$. Using MS scheme rule, $1/\varepsilon\to\ln\mu^2$, we obtain, from Eqs. (19) and (22),

$$-8\pi^2\tilde{\gamma}_{FF}^{(0)n}=X_{mm}+\frac{n}{n-1}\sum_{l=1}^{n-2}(n-1-l)X_{lm}, \tag{33}$$

$$-8\pi^2\tilde{\gamma}_{Fj}^{(0)n}=-(n-1-j)\left(X_{mm}+\frac{n}{n-1}\times\sum_{l=1}^{n-2}(n-1-l)X_{lm}\right)+\sum_{l=1}^{n-2}(n-1-l)X_{lj}, \tag{34}$$

$$-8\pi^2\tilde{\gamma}_{lF}^{(0)n}=\frac{n}{n-1}X_{lm}, \tag{35}$$

$$-8\pi^2\tilde{\gamma}_{lj}^{(0)n}=X_{lj}-\frac{n}{n-1}(n-1-j)X_{lm}, \tag{36}$$

$$\cdots.$$

It is straightforward to calculate the above $\tilde{\gamma}_{ij}^{(0)n}$ using the expressions X_{ij} in Eqs. (25)–(28). Especially, we obtain

$$8\pi^2\tilde{\gamma}_{FF}^{(0)n}=4C_F\left(S_{n-1}-\frac{1}{4}+\frac{1}{2n}\right), \tag{37}$$

$$\begin{aligned}8\pi^2\tilde{\gamma}_{Fj}^{(0)n}=-(2C_F-C_G)\Bigg[&(n-1-j)\left(2S_{n-1}-S_j-S_{n-j-1}+1+\frac{1}{n}\right)\\&+\sum_{l=1}^{j-1}(n-1-l)\left((-1)^{l+j}\frac{{}_{n-2}C_j}{{}_{n-2}C_l}\frac{(n-1-l+j)}{(n-1)(j-l)}+(-1)^{n-j}\frac{{}_{n-2-l}C_{n-2-j}}{n-l}\right)\\&+(n-1-j)\left(\frac{1}{n-1}+\frac{2(-1)^j}{j(j+1)(j+2)}-\frac{(-1)^j}{n-j}\right)\\&+\sum_{l=j+1}^{n-2}(n-1-l)\left((-1)^{l+j}\frac{{}_{n-2}C_{j-1}}{{}_{n-2}C_{l-1}}\frac{(n-1+l-j)}{(n-1)(l-j)}+\frac{2(-1)^j}{l(l+1)(l+2)}\,{}_lC_j\right)\Bigg]\end{aligned}$$

for $j=1,\ldots,n-2$. (38)

Now we see that the mixing anomalous dimension $\tilde{\gamma}_{Fj}^{(0)n}$ turns out to be proportional to $(2C_F-C_G)$. Since

$$C_F=\frac{N_C^2-1}{2N_C},\quad C_G=N_C, \tag{39}$$

we have $2C_F=C_G$ and thus $\tilde{\gamma}_{Fj}^{(0)n}=0$ in the $N_C\to\infty$ limit. Then Eq. (20) gives

$$E_F^n(Q^2)=\left(\frac{\alpha(Q^2)}{\alpha(\mu^2)}\right)^{\tilde{\gamma}_{FF}^{(0)n}/2\beta_0}, \tag{40}$$

$$E_l^n(Q^2)=0 \quad \text{for } l=1,\ldots,n-2. \tag{41}$$

Returning to Eq. (17), we find that, at N_C going to infinity, the moment sum rule for $\bar{g}_2$ takes a simple form as follows:

$$\int_0^1 dx x^{n-1}\bar{g}_2(x,Q^2)=\frac{n-1}{2n}d_n\left(\frac{\alpha(Q^2)}{\alpha(\mu^2)}\right)^{\tilde{\gamma}_{FF}^{(0)n}/2\beta_0} \tag{42}$$

with

$$\frac{\tilde{\gamma}_{FF}^{(0)n}}{2\beta_0}=\frac{2N_C(S_{n-1}-1/4+1/2n)}{(1/3)(11N_C-2n_f)}. \tag{43}$$

In other words, at large N_C, the operators $R_l^{\sigma\mu_1\cdots\mu_{n-1}}$ involving the gluon fields decouple from the evolution equation of $\bar{g}_2$ and the whole contribution is represented by one type of operators $R_F^{\sigma\mu_1\cdots\mu_{n-1}}$. With the substitution $C_F=N_C/2$ and $n=j+1$, the anomalous dimension $8\pi^2\tilde{\gamma}_{FF}^{(0)n}$ coincides with Eq. (18) of Ref. [15]. This completes the reproduction, in the framework of OPE and RG, of the ABH result on $\bar{g}_2$.

IV. SUMMARY AND DISCUSSION

It should be emphasized that we have reproduced the ABH result without assuming massless quarks. A question expected to come up immediately is that the replacement of the mass-dependent operator R_m^n with R_F^n may be equivalent to working with massless quarks. The answer is no. Indeed it can be shown that even when we include the mass-dependent operator R_m^n among the independent operator bases, we reach the same conclusion. Let us take, for an example, R_F^n, R_l^n (with $l=2,\ldots,n-2$), R_m^n and R_E^n as independent operators replacing one quark-gluon operator $R_{l=1}^n$ with R_F^n. With this choice of new operator bases, the moment sum rule for $\bar{g}_2$ is written in terms of the coefficient functions $\hat{E}_F^n(Q^2)$, $\hat{E}_l^n(Q^2)$ with $l=2,\ldots,n-2$, and $\hat{E}_m^n(Q^2)$. The renormalization constants for these operators are written as

$$\begin{pmatrix} R_F^n \\ R_l^n \\ R_m^n \\ R_E^n \end{pmatrix}_B=\begin{pmatrix} \hat{Z}_{FF} & \hat{Z}_{Fj} & \hat{Z}_{Fm} & \hat{Z}_{FE} \\ \hat{Z}_{lF} & \hat{Z}_{lj} & \hat{Z}_{lm} & \hat{Z}_{lE} \\ 0 & 0 & \hat{Z}_{mm} & 0 \\ 0 & 0 & 0 & \hat{Z}_{EE} \end{pmatrix}\begin{pmatrix} R_F^n \\ R_j^n \\ R_m^n \\ R_E^n \end{pmatrix}_R, \quad (l,j=2,\ldots,n-2). \tag{44}$$

Again imposing that the renormalized and bare operators, respectively, satisfy the constraint Eq. (9), we find that $\hat{Z}$'s are related to conventional Z's as follows:

$$\hat{Z}_{FF}=\frac{1}{n-2}\sum_{l=1}^{n-2}(n-1-l)Z_{l1}, \tag{45}$$

$$\hat{Z}_{Fj}=-(n-1-j)\hat{Z}_{FF}+\sum_{l=1}^{n-2}(n-1-l)Z_{lj}, \quad (j=2,\ldots,n-2), \tag{46}$$

$$\hat{Z}_{Fm}=-\frac{n-1}{n}\hat{Z}_{FF}+\frac{n-1}{n}Z_{mm}+\sum_{l=1}^{n-2}(n-1-l)Z_{lm}. \tag{47}$$

Then it is easy to obtain the following one-loop coefficients of the relevant anomalous dimensions

$$8\pi^2\hat{\gamma}_{FF}^{(0)n}=4C_F\left(S_{n-1}-\frac{1}{4}+\frac{1}{2n}\right) + \text{terms proportional to } (2C_F-C_G), \tag{48}$$

$$8\pi^2\hat{\gamma}_{Fj}^{(0)n}\propto(2C_F-C_G) \quad \text{for } j=2,\ldots,n-2, \tag{49}$$

$$8\pi^2\hat{\gamma}_{Fm}^{(0)n}\propto(2C_F-C_G). \tag{50}$$

Inserting these anomalous dimensions to the solutions of the RG equations for the coefficient functions $\hat{E}_F^n(Q^2)$, $\hat{E}_l^n(Q^2)$ $(l=2,\ldots,n-2)$ and $\hat{E}_m^n(Q^2)$,

$$\hat{E}_i^n(Q^2)=\left\{\exp\left[\frac{\hat{\gamma}^{(0)n}}{2\beta_0}\ln\left(\frac{\alpha(Q^2)}{\alpha(\mu^2)}\right)\right]\right\}_{Fi} \quad \text{for } i=F,2,\ldots,n-2,m, \tag{51}$$

we obtain in the large N_C limit

$$\hat{E}_F^n(Q^2)=\left(\frac{\alpha(Q^2)}{\alpha(\mu^2)}\right)^{\tilde{\gamma}_{FF}^{(0)n}/2\beta_0}=E_F^n(Q^2), \tag{52}$$

$$\hat{E}_l^n(Q^2)=0 \quad \text{for } l=2,\ldots,n-2, \tag{53}$$

$$\hat{E}_m^n(Q^2)=0. \tag{54}$$

Thus we reach the same conclusion Eq. (42) even when we include the mass-dependent operators among the independent operator bases.

A few comments are in order. Firstly, the twist-3 quark-gluon operators R_l^n decouple from the evolution equation for $\bar{g}_2$ at large N_C. This might be explained by an argument on quark condensate [7]. A hint is that the mixing anomalous dimensions $\tilde{\gamma}_{Fj}^{(0)n}$ turn out to be proportional to $(2C_F - C_G)$. There are two types in the products of color matrices entering into the calculation of anomalous dimensions for the flavor nonsinglet $\bar{g}_2$:

$$T^bT^aT^b=\left(C_F-\frac{1}{2}C_G\right)T^a=-\frac{1}{2N_C}T^a, \tag{55}$$

$$T^bT^bT^a=C_FT^a=\frac{1}{2}\left(N_C-\frac{1}{N_C}\right)T^a. \tag{56}$$

It is argued in Ref. [7] that the quark condensate contains all colors and at large N_C, the condensate polarization becomes small and that the combination $T^bT^aT^b$ is connected with condensate polarization effects.

Secondly, we have chosen particular sets of the independent operators and reached a simple form for the moments of $\bar{g}_2$ in the large N_C limit. However, arbitrariness in the choice of the operator bases should not enter into physical quantities [17]. A different choice of the operator bases leads to different forms for the anomalous dimension matrix and the coefficient functions. Recall that the constraint, Eq. (9), gives a relation among the tree-level coefficient functions and also a relation among the matrix elements of the operators. After diagonalizing the anomalous dimension matrix and using these relations, we can arrive at the same conclusion for the moments of $\bar{g}_2$ in the $N_C\to\infty$ limit. What we did in this paper is that we chose particular sets of bases from the beginning which include an operator that represents the whole contribution to $\bar{g}_2$ for large N_C.

Finally, the nucleon has other twist-3 distributions, namely, chiral-odd distributions $h_L(x,Q^2)$ and $e(x,Q^2)$ [22]. Just like the $\bar{g}_2$ case, the Q^2 evolutions of flavor nonsinglet $h_L(x,Q^2)$ and $e(x,Q^2)$ turn out to be quite complicated due to mixing with quark-gluon operators, the number of which increases with spin. However, it has been proved [23] that in the large N_C limit, these twist-3 distributions also obey a simple DGLAP equation. The proof holds true only when we work with massless quarks.

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