Spin structure function of the virtual photon beyond the leading order in QCD

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Polarized photon structure can be studied in the future polarized e^+e^- colliding-beam experiments. We investigate the spin-dependent structure function of the virtual photon $g_1^\gamma(x,Q^2,P^2)$, in perturbative QCD for $\Lambda^2 \ll P^2 \ll Q^2$, where $-Q^2(-P^2)$ is the mass squared of the probe (target) photon. The analysis is performed to next-to-leading order in QCD. We particularly emphasize the renormalization scheme independence of the result. The nonleading corrections significantly modify the leading log result, in particular, at large x as well as at small x. We also discuss the nonvanishing first moment sum rule of g_1^γ, where $\mathcal{O}(\alpha_s)$ corrections are computed. [S0556-2821(99)08309-5]

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I. INTRODUCTION

In recent years there has been growing interest in the study of a polarized photon structure function. The information on the spin structure of the photon would be provided by the resolved photon process in polarized electron and proton collision in the polarized version of the DESY *ep* collider HERA [1,2]. More directly, the spin-dependent structure function of photon g_1^γ can be measured by the polarized e^+e^- collision in the future linear colliders (Fig. 1).

From the theoretical viewpoint, the first moment of a photon structure function g_1^γ has recently attracted attention in the literature [3–7] in connection with its relevance for the axial anomaly, which has also played an important role in the QCD analysis of the spin structure of the nucleon. Our aim here is to carry out the QCD computation of the photon's polarized structure function at the same level of the unpolarized case. Here we note that the two-loop splitting functions of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation or equivalently the two-loop anomalous dimensions have recently been calculated [8,9], and we can perform the next-to-leading order QCD analysis for the polarized photon structure function. Actually there has already been an analysis of spin-dependent structure function g_1^γ for the real photon target by Stratmann and Vogelsang [10].

In this paper we shall investigate the polarized virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$ to the next-leading order (NLO) in QCD, in the kinematical region:

$$\Lambda^2 \ll P^2 \ll Q^2, \tag{1.1}$$

where $-Q^2(-P^2)$ is the mass squared of the probe (target) photon, and Λ is the QCD scale parameter. We can base our arguments either on DGLAP-type Q^2 evolution equation for the parton distributions or on the framework of operator product expansion (OPE) and the renormalization group (RG) method. The unpolarized virtual photon structure functions $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$ were studied in the leading order (LO) [11] and in the NLO [12,13]. And the parton contents of virtual photon were studied in Refs. [14,15] and the target mass effect of unpolarized and polarized virtual photon structure in LO was discussed in Ref. [16].

The advantage to study the virtual photon target is that we can calculate the whole structure function entirely up to NLO by the perturbative method. On the other hand, for the real photon target [17,18] we can calculate the perturbative pieces, but not the nonperturbative contributions which may be estimated, for example, by vector-dominance model [19]. The perturbative pieces for the real photon target can be reproduced from the result for the virtual photon case.

In the next section we discuss the polarized photon structure functions. Next we present the two theoretical frameworks based on OPE (Sec. III) and on DGLAP parton model approach (Sec. IV). In Sec. V, the sum rule for the first moment of g_1^γ will be evaluated up to the order of α_s. The numerical analysis of g_1^γ will be given in Sec. VI. The final section is devoted to the conclusion and discussion.

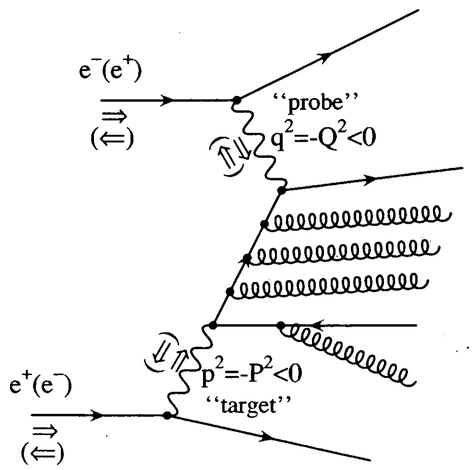

FIG. 1. Deep inelastic scattering on a polarized virtual photon in polarized e^+e^- collision, $e^+e^- \to e^+e^- +$ hadrons (quarks and gluons). The arrows indicate the polarizations of the e^+, e^- and virtual photons. The mass squared of the "probe" ("target") photon is $-Q^2(-P^2)(\Lambda^2 \ll P^2 \ll Q^2)$.

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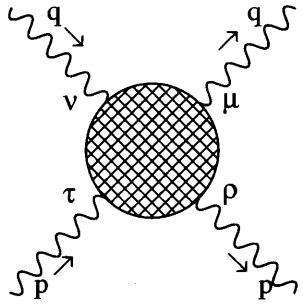

FIG. 2. Forward scattering of a virtual photon with momentum q and another virtual photon with momentum p. The Lorentz indices are denoted by μ,ν,ρ,τ.

II. POLARIZED PHOTON STRUCTURE FUNCTIONS

Let us consider the forward virtual photon scattering amplitude (Fig. 2),

$$T_{\mu\nu\rho\tau}(p,q)=i\int d^4xd^4yd^4z e^{iq\cdot x}e^{ip\cdot(y-z)} \times\langle 0|T(J_\mu(x)J_\nu(0)J_\rho(y)J_\tau(z))|0\rangle. \quad (2.1)$$

Its absorptive part is related to the structure tensor $W_{\mu\nu\rho\tau}(p,q)$ for the photon with mass squared $p^2=-P^2$ probed by the photon with $q^2=-Q^2$:

$$W_{\mu\nu\rho\tau}(p,q)=\frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu\rho\tau}(p,q). \quad (2.2)$$

The antisymmetric part, $W^A_{\mu\nu\rho\tau}$, which is antisymmetric under the interchange of μ and ν, can be decomposed as

$$W^A_{\mu\nu\rho\tau}=\epsilon_{\mu\nu\lambda\sigma}q^\lambda\epsilon_{\rho\tau}{}^{\sigma\beta}p_\beta\frac{1}{p\cdot q}g_1^\gamma+\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q\epsilon_{\rho\tau}{}^{\sigma\beta}p_\beta - \epsilon_{\rho\tau\alpha\beta}p^\beta p^\sigma q^\alpha)\frac{1}{(p\cdot q)^2}g_2^\gamma, \quad (2.3)$$

which gives two spin-dependent structure functions, $g_1^\gamma(x,Q^2,P^2)$ and $g_2^\gamma(x,Q^2,P^2)$. For a real photon, g_2^γ is identically zero, and there exists only one spin struture function, $g_1^\gamma(x,Q^2)$. On the other hand, for the off-shell or virtual photon ($P^2\neq 0$) target, we have two spin-dependent structure functions g_1^γ and g_2^γ. A more detailed argument on the structure functions is given in the Appendix D. The g_1^γ is related to the structure function W_4^γ, which was discussed some years ago in [20,21], such that $g_1^\gamma(x,Q^2)\equiv 2W_4^\gamma(x,Q^2)$. Here we note that the LO QCD correction was first studied by one of the authors in [22] and later in [23,3].

First we note that the same framework used in the analysis of the nucleon spin structure functions can be applied in our case. We can either base our argument on the OPE supplementd by the renormalization group (RG) method, or on the DGLAP type parton evolution equations. It should be noted the next-to-leading order analysis is now possible since the two-loop anomalous dimensions of twist-2 operators R_n^i relevant for the spin-dependent structure function (or equivalently two-loop parton splitting functions) were calculated independently by two groups, by Mertig-van Neerven [8] and by Vogelsang [9].

III. THEORETICAL FRAMEWORK BASED ON OPE

In our previous paper [24], we based our argument on the QCD improved parton model approach. Here we start with theoretical framework based on the OPE and RG method. Applying OPE for the product of two electromagnetic currents we get

$$\begin{aligned} i\int d^4x e^{iq\cdot x}T(J_\mu(x)J_\nu(0))^A &= -i\varepsilon_{\mu\nu\lambda\sigma}q^\lambda\sum_{n=odd}\left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_{n-1}} \\ &\quad\times\sum_i C^i_{1,n}R_{1,i}^{\sigma\mu_1\cdots\mu_{n-1}} - i(\varepsilon_{\mu\rho\lambda\sigma}q_\nu q^\rho \\ &\quad-\varepsilon_{\nu\rho\lambda\sigma}q_\mu q^\rho - q^2\varepsilon_{\mu\nu\lambda\sigma}) \\ &\quad\times\sum_{n=odd}\left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_{n-2}} \\ &\quad\times\sum_i C^i_{2,n}R_{2,i}^{\lambda\sigma\mu_1\cdots\mu_{n-2}}. \end{aligned} \quad (3.1)$$

For polarized deep inelastic scattering, the twist-2 and twist-3 operators: R_1^n, R_2^n contribute to the structure functions in the scaling limit. For g_1^γ only twist-2 operators are relevant. Now we can write down the moment sum rule for g_1^γ:

$$\begin{aligned}\int_0^1 dx x^{n-1}g_1^\gamma(x,Q^2,P^2) &= \sum_{i=\psi,G,NS,\gamma} C_n^i(Q^2/\mu^2,\bar g(\mu^2),\alpha) \\ &\quad\times\langle\gamma(p)|R_n^i(\mu^2)|\gamma(p)\rangle, \end{aligned}\quad (3.2)$$

where R_n^i and C_n^i are the twist-2 operators and their coefficient functions (hereafter we suppress the index 1 for twist-2 operators), with μ being the renormalization point and $\alpha=e^2/4\pi$, the QED coupling constant. ψ, G, NS and γ stand for singlet quark, gluon, nonsinglet quark and photon, respectively. The relevant twist-2 operators $R_n^i[i=\psi(S),G,NS,\gamma]$ are given by [22]

$$R_\psi^{\sigma\mu_1\cdots\mu_{n-1}}=i^{n-1}\bar\psi\gamma^{\{\sigma}D^{\mu_1}\cdots D^{\mu_{n-1}\}}\gamma_5 1\psi - \text{trace terms}, \quad (3.3)$$

$$R_G^{\sigma\mu_1\cdots\mu_{n-1}}=\frac{1}{4}i^{n-1}\epsilon^{\{\sigma}{}_{\alpha\beta\gamma}G^{\alpha\mu_1}D^{\mu_2}\cdots D^{\mu_{n-1}\}}G^{\beta\gamma} - \text{trace terms}, \quad (3.4)$$

$$R_{NS}^{\sigma\mu_1\cdots\mu_{n-1}}=i^{n-1}\bar{\psi}\gamma^{\{\sigma}D^{\mu_1}\cdots D^{\mu_{n-1}\}}\gamma_5(Q_{ch}^2-1)\psi$$
$$-\text{trace terms}, \tag{3.5}$$

$$R_{\gamma}^{\sigma\mu_1\cdots\mu_{n-1}}=\frac{1}{4}i^{n-1}\epsilon^{\{\sigma}{}_{\alpha\beta\gamma}F^{\alpha\mu_1}\partial^{\mu_2}\cdots\partial^{\mu_{n-1}\}}F^{\beta\gamma}$$
$$-\text{trace terms}, \tag{3.6}$$

where $\{\}$ means complete symmetrization over the Lorentz indices $\sigma\mu_1\cdots\mu_{n-1}$, D^{μ} denotes covariant derivative, 1 is an $n_f\times n_f$ unit matrix and Q_{ch}^2 is the square of the $n_f\times n_f$ quark-charge matrix, with n_f being the number of flavors. Here we note that the essential feature in the above equation is the appearance of photon operators R_n^{γ} in addition to the hadronic operators.

For $-p^2=P^2\gg\Lambda^2$, we can calculate the photon matrix elements of the hadronic operators perturbatively. Choosing μ^2 to be close to P^2, we get, to the lowest order,

$$\langle\gamma(p)|R_n^i(\mu)|\gamma(p)\rangle=\frac{\alpha}{4\pi}\left(-\frac{1}{2}K_n^{0,i}\ln\frac{P^2}{\mu^2}+A_n^i\right),$$
$$i=\psi(S),G,NS, \tag{3.7}$$

where $K_n^{0,i}=(\boldsymbol{K}_n^0)^i$ are one-loop anomalous dimension matrix elements between the photon and hadronic operators. On the other hand, in the leading order of the QED coupling constant, α, we have for the photon operator R_n^{γ}:

$$\langle\gamma(p)|R_n^{\gamma}(\mu)|\gamma(p)\rangle=1. \tag{3.8}$$

It should be noted that the finite term A_n^i depends on the renormalization scheme for the operators R_n^i. Putting $\mu^2=-p^2=P^2$, we have

$$\langle\gamma(p)|R_n^i(\mu)|\gamma(p)\rangle|_{\mu^2=P^2}=\frac{\alpha}{4\pi}A_n^i, \tag{3.9}$$

and the nth moment with this choice $\mu^2=P^2$ in Eq. (3.2) becomes

$$\int_0^1 dx x^{n-1}g_1^{\gamma}(x,Q^2,P^2)$$
$$=\sum_{i,j=\psi,G,NS,\gamma}\langle\gamma(p)|R_n^i(\mu^2=P^2)|\gamma(p)\rangle$$
$$\times\left(T\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg\frac{\gamma_n(g)}{\beta(g)}\right]\right)_{ij}C_n^j(1,\bar{g},\alpha). \tag{3.10}$$

The evolution factor in the last equation is found to be [17]

$$T\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg\frac{\gamma_n(g)}{\beta(g)}\right]=\left(\begin{array}{c|c}\boldsymbol{M}_n & \boldsymbol{0}\\ \hline \boldsymbol{X}_n & \boldsymbol{1}\end{array}\right), \tag{3.11}$$

where

$$\boldsymbol{M}_n(Q^2/P^2,\bar{g}(P^2))=T\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg\frac{\hat{\gamma}_n(g)}{\beta(g)}\right],$$

$$\boldsymbol{X}_n(Q^2/P^2,\bar{g}(P^2),\alpha)=\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg\frac{\boldsymbol{K}_n(g,\alpha)}{\beta(g)}$$
$$\times T\exp\left[\int_{\bar{g}(Q^2)}^{g}dg'\frac{\hat{\gamma}_n(g')}{\beta(g')}\right] \tag{3.12}$$

with $\hat{\gamma}_n$ and $\boldsymbol{K}_n$ the hadronic anomalous dimension matrix and the off-diagonal element representing the mixing between the photon and hadron operators (see Appendix A). Thus we get

$$\int_0^1 dx x^{n-1}g_1^{\gamma}(x,Q^2,P^2)$$
$$=\frac{\alpha}{4\pi}\boldsymbol{A}_n\cdot\boldsymbol{M}_n(Q^2/P^2,\bar{g}(P^2))\boldsymbol{C}_n(1,\bar{g}(Q^2))$$
$$+\boldsymbol{X}_n(Q^2/P^2,\bar{g}(P^2),\alpha)\cdot\boldsymbol{C}_n(1,\bar{g}(Q^2))+C_n^{\gamma} \tag{3.13}$$

with

$$\boldsymbol{A}_n=(A_n^{\psi},A_n^G,A_n^{NS}). \tag{3.14}$$

The coefficient functions are given by (see Appendix C)

$$\boldsymbol{C}_n(1,\bar{g})=\begin{pmatrix}C_n^{\psi}(1,\bar{g})\\ C_n^{G}(1,\bar{g})\\ C_n^{NS}(1,\bar{g})\end{pmatrix}=\begin{pmatrix}\delta_{\psi}\left(1+\frac{\bar{g}^2}{16\pi^2}B_{\psi}^n\right)\\ \delta_{\psi}\frac{\bar{g}^2}{16\pi^2}B_G^n\\ \delta_{NS}\left(1+\frac{\bar{g}^2}{16\pi^2}B_{NS}^n\right)\end{pmatrix}$$

$$C_n^{\gamma}(1,\bar{g},\alpha)=\frac{\alpha}{4\pi}\delta_{\gamma}B_{\gamma}^n \tag{3.15}$$

with $\delta_{\psi}=\langle e^2\rangle=\Sigma_{i=1}^{n_f}e_i^2/n_f,\delta_{NS}=1,\delta_{\gamma}=3n_f\langle e^4\rangle=3\Sigma_{i=1}^{n_f}e_i^4$.

We then derive the following formula for the moments:

$$\int_0^1 dx x^{n-1} g_1^\gamma(x,Q^2,P^2)=\frac{\alpha}{4\pi}\frac{1}{2\beta_0}\Bigg[\sum_{i=+,-,NS}\widetilde{P}_i^n\frac{1}{1+\lambda_i^n/2\beta_0}\frac{4\pi}{\alpha_s(Q^2)}\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_i^n/2\beta_0+1}\right\}$$
$$+\sum_{i=+,-,NS}\mathcal{A}_i^n\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_i^n/2\beta_0}\right\}+\sum_{i=+,-,NS}\mathcal{B}_i^n\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_i^n/2\beta_0+1}\right\}+\mathcal{C}^n+\mathcal{O}(\alpha_s)\Bigg]. \tag{3.16}$$

Here $\alpha_s(Q^2)=\bar{g}^2(Q^2)/4\pi$ is the QCD running coupling constant. In Eq. (3.16), we have defined

$$\widetilde{P}_i^n=\boldsymbol{K}_n^0 P_i^n \boldsymbol{C}_n(1,0)(i=+,-,NS), \tag{3.17}$$

where P_i^n's are projection operators given in the Appendix A. The coefficients $\mathcal{A}_i^n$, $\mathcal{B}_i^n$ and $\mathcal{C}^n$ are computed from the NLO perturbative calculation, and are given by

$$\mathcal{A}_i^n=-\boldsymbol{K}_n^0\sum_j\frac{P_j^n\hat{\gamma}_n^{(1)}P_i^n}{2\beta_0+\lambda_j^n-\lambda_i^n}\boldsymbol{C}_n(1,0)\frac{1}{\lambda_i^n/2\beta_0}-\boldsymbol{K}_n^0\frac{\beta_1}{\beta_0}P_i^n\boldsymbol{C}_n(1,0)\frac{1-\lambda_i^n/2\beta_0}{\lambda_i^n/2\beta_0}+\boldsymbol{K}_n^1P_i^n\boldsymbol{C}_n(1,0)\frac{1}{\lambda_i^n/2\beta_0}-2\beta_0\boldsymbol{A}_nP_i^n\boldsymbol{C}_n(1,0), \tag{3.18}$$

$$\mathcal{B}_i^n=\boldsymbol{K}_n^0\sum_j\frac{P_i^n\hat{\gamma}_n^{(1)}P_j^n}{2\beta_0+\lambda_i^n-\lambda_j^n}\boldsymbol{C}_n(1,0)\frac{1}{1+\lambda_i^n/2\beta_0}+\boldsymbol{K}_n^0P_i^n\begin{pmatrix}\delta_\psi B_\psi^n\\ \delta_\psi B_G^n\\ \delta_{NS}B_{NS}^n\end{pmatrix}\frac{1}{1+\lambda_i^n/2\beta_0}-\boldsymbol{K}_n^0\frac{\beta_1}{\beta_0}P_i^n\boldsymbol{C}_n(1,0)\frac{\lambda_i^n/2\beta_0}{1+\lambda_i^n/2\beta_0}, \tag{3.19}$$

$$\mathcal{C}^n=2\beta_0(\delta_\gamma B_\gamma^n+\boldsymbol{A}_n\cdot\boldsymbol{C}_n(1,0)), \tag{3.20}$$

where $\lambda_i^n(i=+,-,NS)$ are eigenvalues of the 1-loop anomalous dimension matrix $\hat{\gamma}_n^{(0)}$ and are given in Appendix A. β_0 and the β_1 are the one- and two-loop β functions, and $\beta_0=11-2n_f/3$ and $\beta_1=102-38n_f/3$.

All the quantities necessary to evaluate $\widetilde{P}_i^n$, $\mathcal{A}_i^n$, $\mathcal{B}_i^n$, and $\mathcal{C}^n$ are now known and will be presented in Appendixes A–C. Two-loop results [8,9] have been calculated in the modified minimal subtraction ($\overline{\rm MS}$) scheme [25]. Actually the expressions of Eqs. (3.16) and (3.17)–(3.20) are the same in form as the ones obtained before by one of the authors and Walsh for the case of the virtual photon structure function F_2^γ [12]. The explicit expressions of the one-loop and two-loop anomalous dimensions [8,9] as well as one-loop coefficient functions [26–30,8,9] are given in Appendixes B and C.

Equation (3.16) is our main result of the present paper. The first term is the LO result, and the remaining terms are the NLO QCD corrections.

Now let us examine the renormalization scheme independence of the coefficients; $\mathcal{A}_i^n$, $\mathcal{B}_i^n$ and $\mathcal{C}^n$. As in the unpolarized case, $\mathcal{B}_i^n$ can be written in terms of a scheme-independent combination of 2-loop anomalous dimensions and 1-loop coefficient functions in the hadronic sector. Using the scheme-independent coefficients $R_{2,n}^i$ [31–33], we can write

$$\mathcal{B}_i^n=L_i^nR_{2,n}^i\ \ (i=+,-,NS), \tag{3.21}$$

where the explicit form for $R_{2,n}^i$ is given in Eqs. (9)–(12) of Ref. [31] (see also [12]) and

$$L_i^n=\widetilde{P}_i^n\frac{1}{1+\lambda_i^n/2\beta_0} \tag{3.22}$$

which is the coefficient of the leading-log term. The scheme independence of $\mathcal{B}_i^n$ follows from these two equations.

Regarding $\mathcal{C}^n$, we first consider the photon matrix elements of the renormalized quark and gluon operators. The finite matrix elements $\boldsymbol{A}_n$ and the tree-level coefficient functions $\boldsymbol{C}_n(1,0)$ are given by

$$\boldsymbol{A}_n=6(\langle e^2\rangle,0,\langle e^4\rangle-\langle e^2\rangle^2)\widetilde{A}_{nG}^\psi, \tag{3.23}$$

$$\boldsymbol{C}_n(1,0)=\begin{pmatrix}\langle e^2\rangle\\ 0\\ 1\end{pmatrix}. \tag{3.24}$$

Hence, we have

$$\boldsymbol{A}_n\cdot\boldsymbol{C}_n(1,0)=6\langle e^4\rangle\widetilde{A}_{nG}^\psi. \tag{3.25}$$

Noting that

$$B_\gamma^n = \frac{2}{n_f} B_G^n, \quad \delta_\gamma = 3 n_f \langle e^4 \rangle. \tag{3.26}$$

we find C^n equal to be

$$C^n = 12\beta_0 \langle e^4 \rangle (B_G^n + \tilde{A}_{nG}^\psi). \tag{3.27}$$

Since the combination $B_G^n + \tilde{A}_{nG}^\psi$ is scheme independent [25], so is C^n. In fact, in the $\overline{\text{MS}}$ scheme [8], the gluon matrix elements of quark operators ($i = \psi, NS$) read

$$\langle p,s | R_i^{\sigma\mu_1 \cdots \mu_{n-1}}(\mu^2) | p,s \rangle$$
$$= A_i(p^2, \mu^2, g)[\{s^\sigma p_1^\mu \cdots p^{\mu_{n-1}}\} - \text{traces}] \quad (i = \psi, NS),$$
$$A_\psi = \frac{g^2}{16\pi^2}\left(\frac{1}{2}\gamma_{\psi G}^{(0),n} \ln \frac{-p^2}{\mu^2} + \tilde{A}_{nG}^\psi\right), \tag{3.28}$$

where the finite matrix element $\tilde{A}_{nG}^\psi$ is given in the parton language as [8]

$$\tilde{A}_{nG}^\psi = \int_0^1 dx\, x^{n-1} a_{S,qg}^{(1)}(x), \tag{3.29}$$

where

$$a_{S,qg}^{(1)}(x) = T_f[(4-8x)\{\ln x + \ln(1-x)\}] \tag{3.30}$$

with $T_f = n_f/2$. Thus, we get

$$\tilde{A}_{nG}^\psi = 2n_f\left[\frac{n-1}{n(n+1)} S_1(n) + \frac{4}{(n+1)^2} - \frac{1}{n^2} - \frac{1}{n}\right]. \tag{3.31}$$

Therefore, from Eq. (3.27) we finally arrive at

$$C^n = 24\beta_0 f \langle e^4 \rangle \left[\frac{2}{n} - \frac{4}{n+1} - \frac{2}{n^2} + \frac{4}{(n+1)^2}\right] \tag{3.32}$$

which is consistent with the Box diagram calculation.

On the other hand, in the RG scheme adopted by Kodaira [28] which is the momentum subtraction scheme, we have, for $n \geq 3$,

$$\tilde{A}_{nG}^\psi = 0 \tag{3.33}$$

with the coefficient function given by

$$B_G^n = 2n_f\left[\frac{2}{n} - \frac{4}{n+1} - \frac{2}{n^2} + \frac{4}{(n+1)^2}\right]. \tag{3.34}$$

For $n=1$, because of the Adler-Bell-Jackiw anomaly

$$\tilde{A}_{n=1G}^\psi = -2n_f \tag{3.35}$$

and

$$B_G^{n=1} = 0, \tag{3.36}$$

by definition, since we have no gauge-invariant gluon operator for $n=1$. Combining this with the result for $\tilde{A}_{nG}^\psi$, we arrive at the same result for C^n.

The scheme-independence of the remaining coefficients $\mathcal{A}_i^n$ follows from the above arguments on $\mathcal{B}_i^n$ and C^n and the physically measurable moments given in Eq. (3.16).

IV. QCD IMPROVED PARTON MODEL APPROACH

We now turn to the analysis based on the QCD improved parton model [34] using the DGLAP parton evolution equations.

Let $q_\pm^i(x,Q^2,P^2)$, $G_\pm^\gamma(x,Q^2,P^2)$, $\Gamma_\pm^\gamma(x,Q^2,P^2)$ be quark with i-flavor, gluon, and photon distribution functions with $\pm$ helicities of the longitudinally polarized virtual photon with mass $-P^2$ [24]. Then the spin-dependent parton distributions are defined as

$$\Delta q^i \equiv q_+^i + \bar{q}_+^i - q_-^i - \bar{q}_-^i, \tag{4.1}$$

$$\Delta G^\gamma \equiv G_+^\gamma - G_-^\gamma, \quad \Delta\Gamma^\gamma \equiv \Gamma_+^\gamma - \Gamma_-^\gamma. \tag{4.2}$$

In the leading order of the electromagnetic coupling constant, $\alpha = e^2/4\pi$, $\Delta\Gamma^\gamma$ does not evolute with Q^2 and is set to be $\Delta\Gamma^\gamma(x,Q^2,P^2) = \delta(1-x)$. The quark and gluon distributions Δq^i and ΔG^γ satisfy the following evolution equations:

$$\begin{aligned}\frac{d\Delta q^i(x,Q^2,P^2)}{d\ln Q^2} = &\int_x^1 \frac{dy}{y}\left\{\sum_j \Delta \tilde{P}_{q^i q^j}\left(\frac{x}{y},Q^2\right)\Delta q^j(y,Q^2,P^2)\right.\\ &\left.+\Delta\tilde{P}_{qG}\left(\frac{x}{y},Q^2\right)\Delta G^\gamma(y,Q^2,P^2)\right\}\\ &+\Delta\tilde{P}_{q^i\gamma}(x,Q^2,P^2),\end{aligned} \tag{4.3}$$

$$\begin{aligned}\frac{d\Delta G^\gamma(x,Q^2,P^2)}{d\ln Q^2} = &\int_x^1 \frac{dy}{y}\left\{\Delta \tilde{P}_{Gq}\left(\frac{x}{y},Q^2\right)\sum_i \Delta q^i(y,Q^2,P^2)\right.\\ &\left.+\Delta\tilde{P}_{GG}\left(\frac{x}{y},Q^2\right)\Delta G^\gamma(y,Q^2,P^2)\right\}\\ &+\Delta\tilde{P}_{G\gamma}(x,Q^2,P^2),\end{aligned} \tag{4.4}$$

where $\Delta\tilde{P}_{AB}$ is a polarized splitting function of B-parton to A-parton, defined as $\Delta\tilde{P}_{AB} \equiv P_{A_+B_+} - P_{A_-B_+} (= P_{A_-B_-} - P_{A_+B_-}$, due to parity conservation in QCD and QED).

For later convenience we use, instead of Δq^i, the flavor singlet and nonsinglet combinations defined as follows:

$$\Delta q_S^\gamma \equiv \sum_i \Delta q^i, \tag{4.5}$$

$$\Delta q_{NS}^i \equiv \Delta q^i - \frac{\Delta q_S^\gamma}{n_f}, \tag{4.6}$$

so that $\Sigma_i \Delta q^i_{NS}=0$ and n_f is the number of relevant active quark flavors. The quark-quark splitting function $\Delta\widetilde{P}_{q^iq^j}$ is made up of two pieces, the one representing the case that j-quark splits into i-quark without through gluon, and the other one through gluon, and may be expressed as

$$\Delta\widetilde{P}_{q^iq^j}=\delta_{ij}\Delta\widetilde{P}_{qq}+\frac{1}{n_f}\Delta\widetilde{P}^S_{qq}, \tag{4.7}$$

where the second term is representing the splitting through gluon, and $\Delta\widetilde{P}_{qq}$ and $\Delta\widetilde{P}^S_{qq}$ are both independent of quark flavor, i and j. It is noted that by construction $\Delta\widetilde{P}^S_{qq}$ is relevant for the evolution of flavor-siglet Δq^γ_S and first appears in the order of α_s^2.

In the QCD improved parton model, which is based on the factorization theorem, the polarized virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$ is expressed as

$$g_1^\gamma(x,Q^2,P^2)=\int_x^1\frac{dy}{y}\left\{\sum_i \Delta q^i(y,Q^2,P^2)C^i\left(\frac{x}{y},Q^2\right)+\Delta G^\gamma(y,Q^2,P^2)C^G\left(\frac{x}{y},Q^2\right)\right\}+C^\gamma(x,Q^2), \tag{4.8}$$

where C^i, C^G, and C^γ are the coefficient functions of i-quark, gluon, and photon, respectively, and are independent of target photon mass P^2. Up to one-loop level they are given by

$$C^i(z,Q^2)=e_i^2\left\{\delta(1-z)+\frac{\alpha_s(Q^2)}{4\pi}B_q(z)\right\}, \tag{4.9}$$

$$C^G(z,Q^2)=\langle e^2\rangle\left\{0+\frac{\alpha_s(Q^2)}{4\pi}B_G(z)\right\}, \tag{4.10}$$

$$C^\gamma(z,Q^2)=\frac{\alpha}{4\pi}3n_f\langle e^4\rangle B_\gamma(z), \tag{4.11}$$

where $\langle e^2\rangle=\Sigma_i e_i^2/n_f$ and $\langle e^4\rangle=\Sigma_i e_i^4/n_f$. It is noted that $B_q(z)$ in Eq. (4.9) is independent of the quark flavor i. Since $\Sigma_i \Delta q^i C^i$ is rewritten as

$$\begin{aligned}\sum_i \Delta q^i C^i&=\sum_i\left\{\Delta q^i_{NS}+\frac{\Delta q^\gamma_S}{n_f}\right\}C^i\\
&=\Delta q^\gamma_S(y,Q^2,P^2)\langle e^2\rangle\left\{\delta\left(1-\frac{x}{y}\right)+\frac{\alpha_s(Q^2)}{4\pi}B_q\left(\frac{x}{y}\right)\right\}\\
&\quad+\sum_i e_i^2\Delta q^i_{NS}(y,Q^2,P^2)\left\{\delta\left(1-\frac{x}{y}\right)+\frac{\alpha_s(Q^2)}{4\pi}B_q\left(\frac{x}{y}\right)\right\},\end{aligned} \tag{4.12}$$

we obtain

$$\begin{aligned}g_1^\gamma(x,Q^2,P^2)=\int_x^1\frac{dy}{y}\bigg\{&\Delta q^\gamma_S(y,Q^2,P^2)C^S\left(\frac{x}{y},Q^2\right)\\
&+\Delta G^\gamma(y,Q^2,P^2)C^G\left(\frac{x}{y},Q^2\right)\\
&+\Delta q^\gamma_{NS}(y,Q^2,P^2)C^{NS}\left(\frac{x}{y},Q^2\right)\bigg\}+C^\gamma(x,Q^2),\end{aligned} \tag{4.13}$$

where we have defined

$$C^S(z,Q^2)\equiv\langle e^2\rangle\left\{\delta(1-z)+\frac{\alpha_s(Q^2)}{4\pi}B_S(z)\right\}, \tag{4.14}$$

$$C^{NS}(z,Q^2)\equiv\delta(1-z)+\frac{\alpha_s(Q^2)}{4\pi}B_{NS}(z), \tag{4.15}$$

$$\Delta q^\gamma_{NS}\equiv\sum_i e_i^2\Delta q^i_{NS}, \tag{4.16}$$

and $B_S(z)=B_{NS}(z)=B_q(z)$. From Eqs. (4.3)–(4.7) and (4.16), the evolution equations for Δq^γ_S, ΔG^γ, and Δq^γ_{NS} are now given by

$$\begin{aligned}\frac{d\Delta q^\gamma_S(x,Q^2,P^2)}{d\ln Q^2}=\int_x^1\frac{dy}{y}\bigg\{&\left[\Delta\widetilde{P}_{qq}\left(\frac{x}{y},Q^2\right)+\Delta\widetilde{P}^S_{qq}\left(\frac{x}{y},Q^2\right)\right]\Delta q^\gamma_S(y,Q^2,P^2)\\
&+n_f\Delta\widetilde{P}_{qG}\left(\frac{x}{y},Q^2\right)\Delta G^\gamma(y,Q^2,P^2)\bigg\}\\
&+\sum_i\Delta\widetilde{P}_{q^i\gamma}(x,Q^2),\end{aligned} \tag{4.17}$$

$$\begin{aligned}\frac{d\Delta G^\gamma(x,Q^2,P^2)}{d\ln Q^2}=\int_x^1\frac{dy}{y}\bigg\{&\Delta\widetilde{P}_{Gq}\left(\frac{x}{y},Q^2\right)\Delta q^\gamma_S(y,Q^2,P^2)\\
&+\Delta\widetilde{P}_{GG}\left(\frac{x}{y},Q^2\right)\Delta G^\gamma(y,Q^2,P^2)\bigg\}\\
&+\Delta\widetilde{P}_{G\gamma}(x,Q^2),\end{aligned} \tag{4.18}$$

$$\frac{d\Delta q^{\gamma}_{NS}(x,Q^2,P^2)}{d\ln Q^2}=\int_x^1\frac{dy}{y}\Delta\tilde{P}_{qq}\left(\frac{x}{y},Q^2\right)\Delta q^{\gamma}_{NS}(y,Q^2,P^2)+\sum_i e_i^2\left\{\Delta\tilde{P}_{q^i\gamma}(x,Q^2)-\frac{1}{n_f}\sum_j\Delta\tilde{P}_{q^j\gamma}(x,Q^2)\right\}. \quad (4.19)$$

Introducing a row vector $\Delta\boldsymbol{q}^{\gamma}=(\Delta q^{\gamma}_S,\Delta G^{\gamma},\Delta q^{\gamma}_{NS})$, the above evolution equations, Eqs. (4.17)–(4.19) are expressed in a compact matrix form

$$\frac{d\Delta\boldsymbol{q}^{\gamma}(x,Q^2,P^2)}{d\ln Q^2}=\Delta\boldsymbol{k}(x,Q^2)+\int_x^1\frac{dy}{y}\Delta\boldsymbol{q}^{\gamma}(y,Q^2,P^2)\Delta P\left(\frac{x}{y},Q^2\right), \quad (4.20)$$

where the elements of a row vector $\Delta\boldsymbol{k}=(\Delta K_S,\Delta K_G,\Delta K_{NS})$ are

$$\Delta K_S\equiv\sum_i\Delta\tilde{P}_{q^i\gamma},\quad \Delta K_G\equiv\Delta\tilde{P}_{G\gamma},$$

$$\Delta K_{NS}\equiv\sum_i e_i^2\left\{\Delta\tilde{P}_{q^i\gamma}-\frac{1}{n_f}\sum_j\Delta\tilde{P}_{q^j\gamma}\right\}. \quad (4.21)$$

Since $\Delta\tilde{P}_{q^i\gamma}$ is proportional to e_i^2, it is easily seen that ΔK_S and ΔK_{NS} have factors $n_f\langle e^2\rangle$ and $n_f(\langle e^4\rangle-\langle e^2\rangle^2)$, respectively. The 3×3 matrix $\Delta P(z,Q^2)$ is written as

$$\Delta P(z,Q^2)=\begin{pmatrix}\Delta P^S_{qq}(z,Q^2) & \Delta P_{Gq}(z,Q^2) & 0\\ \Delta P_{qG}(z,Q^2) & \Delta P_{GG}(z,Q^2) & 0\\ 0 & 0 & \Delta P^{NS}_{qq}(z,Q^2)\end{pmatrix}, \quad (4.22)$$

where

$$\Delta P^S_{qq}\equiv\Delta\tilde{P}_{qq}+\Delta\tilde{P}^S_{qq},\quad \Delta P_{qG}\equiv n_f\Delta\tilde{P}_{qG},$$

$$\Delta P_{Gq}\equiv\Delta\tilde{P}_{Gq},\quad \Delta P_{GG}\equiv\Delta\tilde{P}_{GG},\quad \Delta P^{NS}_{qq}\equiv\Delta\tilde{P}_{qq}. \quad (4.23)$$

Once we get the information on the coefficient functions in Eqs. (4.9)–(4.11) and parton splitting functions in Eqs. (4.21)–(4.23), we can predict the behavior of $g_1^{\gamma}(x,Q^2,P^2)$ in QCD. The NLO analysis is now possible since the spin-dependent one-loop coefficient functions and two-loop parton splitting functions are available [8,9]. There are two methods to obtain $g_1^{\gamma}(x,Q^2,P^2)$ in NLO. In one method, we use the parton splitting functions up to two-loop level and we solve numerically $\Delta\boldsymbol{q}^{\gamma}(x,Q^2,P^2)$ in Eq. (4.20) by iteration, starting from the initial quark and gluon distributions of the virtual photon at $Q^2=P^2$. The interesting point of studying the virtual photon with mass $-P^2$ is that when $P^2\gg\Lambda^2$, the initial parton distributions of the photon are completely known up to the one-loop level in QCD. Then inserting the solved $\Delta\boldsymbol{q}^{\gamma}(x,Q^2,P^2)$ into Eq. (4.13), and together with the known one-loop coefficient functions we can predict $g_1^{\gamma}(x,Q^2,P^2)$ in NLO.

The other method, which is more common than the former, is by making use of the inverse Mellin transformation. From now on we follow the latter method. First we take the Mellin moments of Eq. (4.13),

$$\int_0^1 dx x^{n-1}g_1^{\gamma}(x,Q^2,P^2)=\Delta\boldsymbol{q}^{\gamma}(n,Q^2,P^2)\cdot\boldsymbol{C}(n,Q^2)+C^{\gamma}(n,Q^2), \quad (4.24)$$

where we have defined the moments of an arbitrary function $f(x)$ as

$$f(n)\equiv\int_0^1 dx x^{n-1}f(x). \quad (4.25)$$

Comparing Eq. (4.24) with Eqs. (3.10) and (3.15), we can easily see the correspondence between the quantities in the QCD improved parton model and those in the framework of OPE as follows:

$$[\Delta\boldsymbol{q}^{\gamma}(n,Q^2,P^2)]_i=\sum_{j=S,G,NS,\gamma}\langle\gamma(p)|R_n^j(\mu^2=P^2)|\gamma(p)\rangle\times\left(T\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg\frac{\gamma_n(g)}{\beta(g)}\right]\right)_{ji}\quad (i=S,G,NS), \quad (4.26)$$

$$\boldsymbol{C}(n,Q^2)=\boldsymbol{C}_n(1,\bar{g}), \quad (4.27)$$

$$C^{\gamma}(n,Q^2)=C_n^{\gamma}(1,\bar{g},\alpha). \quad (4.28)$$

Henceforce we omit the obvious n-dependence for simplicity. We expand the splitting functions $\Delta\boldsymbol{k}(Q^2)$ and $\Delta P(Q^2)$ in powers of the QCD and QED coupling constants as

$$\Delta\boldsymbol{k}(Q^2)=\frac{\alpha}{2\pi}\Delta\boldsymbol{k}^{(0)}+\frac{\alpha\alpha_s(Q^2)}{(2\pi)^2}\Delta\boldsymbol{k}^{(1)}+\cdots, \quad (4.29)$$

$$\Delta P(Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\Delta P^{(0)}+\left[\frac{\alpha_s(Q^2)}{2\pi}\right]^2\Delta P^{(1)}+\cdots, \quad (4.30)$$

and introduce t instead of Q^2 as the evolution variable [35],

$$t\equiv\frac{2}{\beta_0}\ln\frac{\alpha_s(P^2)}{\alpha_s(Q^2)}. \quad (4.31)$$

Then, taking the Mellin moments of the both sides in Eq. (4.20), we find that $\Delta q^{\gamma}(t)(=\Delta q^{\gamma}(n,Q^2,P^2))$ satisfies the following inhomogenious differential equation [36,37]:

$$\frac{d\Delta q^{\gamma}(t)}{dt}=\frac{\alpha}{2\pi}\left\{\frac{2\pi}{\alpha_s}\Delta k^{(0)}+\left[\Delta k^{(1)}-\frac{\beta_1}{2\beta_0}\Delta k^{(0)}\right]+\mathcal{O}(\alpha_s)\right\}$$
$$+\Delta q^{\gamma}(t)\left\{\Delta P^{(0)}+\frac{\alpha_s}{2\pi}\left[\Delta P^{(1)}-\frac{\beta_1}{2\beta_0}\Delta P^{(0)}\right]\right.$$
$$\left.+\mathcal{O}(\alpha_s^2)\right\}, \tag{4.32}$$

where we have used the fact that the QCD effective coupling constant $\alpha_s(Q^2)$ satisfies

$$\frac{d\alpha_s(Q^2)}{d\ln Q^2}=-\beta_0\frac{\alpha_s(Q^2)^2}{4\pi}-\beta_1\frac{\alpha_s(Q^2)^3}{(4\pi)^2}+\cdots \tag{4.33}$$

with $\beta_0=11-2n_f/3$ and $\beta_1=102-38n_f/3$. Note that the P^2 dependence of Δq^{γ} solely comes from the initial condition (or boundary condition) as we will see below.

We look for the solution $\Delta q^{\gamma}(t)$ in the following form:

$$\Delta q^{\gamma}(t)=\Delta q^{\gamma(0)}(t)+\Delta q^{\gamma(1)}(t), \tag{4.34}$$

where the first (second) term represents the solution in the LO (NLO). First we discuss about the initial conditions of Δq^{γ}.

In Sec. III, we have observed that for $-p^2=P^2\gg\Lambda^2$ the photon matrix elements of the hadronic operators $R_n^i[i=\psi(S),G,NS]$ can be calculated perturbatively. Choosing the square of the renormalization point μ^2 to be close to P^2, we obtain, to the lowest order

$$\langle\gamma(p)|R_n^i(\mu)|\gamma(p)\rangle=\frac{\alpha}{4\pi}\left(-\frac{1}{2}K_n^{0,i}\ln\frac{P^2}{\mu^2}+A_n^i\right),$$
$$i=\psi(S),G,NS. \tag{4.35}$$

The $K_n^{0,i}$-terms and A_n^i-terms represent the operator mixing between the hadronic operators and photon operators in the LO and NLO, respectively. The operator mixing implies that there exists quark distribution in the photon. When we renormalize the photon matrix elements of the hadronic operators at $\mu^2=P^2$, we obtain

$$\langle\gamma(p)|R_n^i(\mu)|\gamma(p)\rangle|_{\mu^2=P^2}=\frac{\alpha}{4\pi}A_n^i \tag{4.36}$$

which shows that, at $\mu^2=P^2$, quark distribution exists in the photon, not in the LO but in the NLO. Thus we have

$$\Delta q^{\gamma(0)}(0)=0,\quad \Delta q^{\gamma(1)}(0)=\frac{\alpha}{4\pi}A_n. \tag{4.37}$$

Explicit expressions of A_n in the $\overline{\mathrm{MS}}$ scheme are given in Sec. III.

With these initial conditions, we obtain for the solution $\Delta q^{\gamma}(t)$ of Eq. (4.32),

$$\Delta q^{\gamma(0)}(t)=\frac{4\pi}{\alpha_s(t)}\mathbf{a}\left\{1-\left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{1-2\Delta P^{(0)}/\beta_0}\right\}, \tag{4.38}$$

$$\Delta q^{\gamma(1)}(t)=-2\mathbf{a}\left\{\int_0^t d\tau e^{(\Delta P^{(0)}-\beta_0/2)\tau}\right.$$
$$\left.\times\left[\Delta P^{(1)}-\frac{\beta_1}{2\beta_0}\Delta P^{(0)}\right]e^{-\Delta P^{(0)}\tau}\right\}e^{\Delta P^{(0)}t}$$
$$+\mathbf{b}\left\{1-\left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{-2\Delta P^{(0)}/\beta_0}\right\}$$
$$+\Delta q^{\gamma(1)}(0)\left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{-2\Delta P^{(0)}/\beta_0}, \tag{4.39}$$

where

$$\mathbf{a}=\frac{\alpha}{2\pi\beta_0}\Delta k^{(0)}\frac{1}{1-\dfrac{2\Delta P^{(0)}}{\beta_0}}, \tag{4.40}$$

$$\mathbf{b}=\left\{\frac{\alpha}{2\pi}\left[\Delta k^{(1)}-\frac{\beta_1}{2\beta_0}\Delta k^{(0)}\right]\right.$$
$$\left.+2\mathbf{a}\left[\Delta P^{(1)}-\frac{\beta_1}{2\beta_0}\Delta P^{(0)}\right]\right\}\frac{-1}{\Delta P^{(0)}}. \tag{4.41}$$

It is noted that the parton distributions $\Delta q^{\gamma}(t)$ *do* depend on the initial conditions $\Delta q^{\gamma}(0)=(\alpha/4\pi)A_n$, but we have seen in Sec. III that the structure function $g_1^{\gamma}(x,Q^2,P^2)$ itself is independent of $\Delta q^{\gamma}(0)$ in NLO in QCD.

The moments of the splitting functions are related to the anomalous dimensions of operators as follows:

$$\Delta P^{(0)}=-\frac{1}{4}\hat{\gamma}_n^0,\quad \Delta P^{(1)}=-\frac{1}{8}\hat{\gamma}_n^{(1)}, \tag{4.42}$$

$$\Delta k^{(0)}=\frac{1}{4}K_n^0,\quad \Delta k^{(1)}=\frac{1}{8}K_n^1. \tag{4.43}$$

The evaluation of $\Delta q^{\gamma(0)}(t)$ and $\Delta q^{\gamma(1)}(t)$ in Eqs. (4.38) and (4.39) can be easily done by introducing the projection operators P_i^n such as

$$\Delta P^{(0)}=-\frac{1}{4}\hat{\gamma}_n^0=-\frac{1}{4}\sum_{i=+,-,NS}\lambda_i^n P_i^n,\quad i=+,-,NS, \tag{4.44}$$

$$P_i^nP_j^n=\begin{cases}0 & i\neq j,\\ P_i^n & i=j\end{cases},\quad \sum_i P_i^n=\mathbf{1}, \tag{4.45}$$

where λ_i^n are the eigenvalues of the matrix $\hat{\gamma}_n^0$. Then the solution $\Delta q^\gamma(t)$ of Eq. (4.32) in the NLO is written as

$$
\begin{aligned}
\Delta q^\gamma(t)\Big/\left[\frac{\alpha}{8\pi\beta_0}\right]=&\frac{4\pi}{\alpha_s(t)}K_n^0\sum_i P_i^n\frac{1}{1+\dfrac{\lambda_i^n}{2\beta_0}}\left\{1-\left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{1+\lambda_i^n/2\beta_0}\right\}\\
&+\left\{K_n^1\sum_i P_i^n\frac{1}{\lambda_i^n/2\beta_0}+\frac{\beta_1}{\beta_0}K_n^0\sum_i P_i^n\left(1-\frac{1}{\lambda_i^n/2\beta_0}\right)\right.\\
&\left.-K_n^0\sum_{j,i}\frac{P_j^n\hat{\gamma}_n^{(1)}P_i^n}{2\beta_0+\lambda_j^n-\lambda_i^n}\frac{1}{\lambda_i^n/2\beta_0}-2\beta_0A_n\sum_i P_i^n\right\}\left\{1-\left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{\lambda_i^n/2\beta_0}\right\}\\
&+\left\{K_n^0\sum_{i,j}\frac{P_i^n\hat{\gamma}_n^{(1)}P_j^n}{2\beta_0+\lambda_i^n-\lambda_j^n}\frac{1}{1+\dfrac{\lambda_i^n}{2\beta_0}}-\frac{\beta_1}{\beta_0}K_n^0\sum_i P_i^n\frac{\lambda_i^n/2\beta_0}{1+\dfrac{\lambda_i^n}{2\beta_0}}\right\}\left\{1-\left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{1+\lambda_i^n/2\beta_0}\right\}+2\beta_0A_n.
\end{aligned}
\tag{4.46}
$$

Now inserting the above solution of $\Delta q^\gamma(t)$ into Eq. (4.24) and together with the information on the coefficient functions in Eq. (3.15), we reproduce the same formula for the moments of $g_1^\gamma(x,Q^2,P^2)$ given in Eqs. (3.16)–(3.20), as for the case of the OPE approach in the NLO.

V. SUM RULE FOR THE FIRST MOMENT OF $g_1^\gamma(x,Q^2,P^2)$

The polarized structure function g_1^γ of the real photon satisfies a remarkable sum rule [3–7]

$$\int_0^1 g_1^\gamma(x,Q^2)dx=0. \tag{5.1}$$

Now we can ask what happens to the first moment of the virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$. This can be studied by taking the $n\to 1$ limit of Eq. (3.16). Note that we have the following eigenvalues of the one-loop anomalous dimension matrix $\hat{\gamma}_{n=1}^0$:

$$\lambda_+^{n=1}=0,\ \lambda_-^{n=1}=-2\beta_0,\ \lambda_{NS}^{n=1}=0. \tag{5.2}$$

Physically speaking, the zero eigenvalues $\lambda_+^{n=1}=\lambda_{NS}^{n=1}=0$ correspond to the conservation of the axial-vector current at one-loop order, which has the counterpart for the unpolarized structure function F_2; the conservation of energy momentum tensor, $\lambda_-^{n=2}=0$. The other eigenvalue $\lambda_-^{n=1}=-2\beta_0$, which is negative, is rather an artifact of continuation of the anomalous dimension of the gluon operators to $n=1$, since there is no twist-2 gluon operator exists for $n=1$, in the RG scheme in which only gauge-invariant operators are allowed. But $n=1$ gluon operator exists in the so-called Adler-Bardeen scheme [38,39]. In fact, in the QCD improved parton model approach, there is no reason why the $n=1$ moment of the polarized gluon distribution should not be considered [40].

In the coefficients

$$\widetilde{P}_i^n\frac{1}{1+\lambda_i^n/2\beta_0},\quad \mathcal{A}_i^n,\quad \mathcal{B}_i^n\ (i=+,-,NS) \tag{5.3}$$

the special points (5.2) would develop the singularities at $n=1$, since in those coefficients there exist the factors

$$\frac{1}{\lambda_+^n},\ \frac{1}{\lambda_{NS}^n},\ \frac{1}{1+\lambda_-^n/2\beta_0}. \tag{5.4}$$

Now if we take the limit of n going to 1, we have

$$
\begin{gathered}
\widetilde{P}_i^n\frac{1}{1+\lambda_i^n/2\beta_0}\to 0\ (i=+,-,NS),\\
\mathcal{A}_+^n\to\text{finite},\ \mathcal{A}_-^n\to 0,\ \mathcal{A}_{NS}^n\to\text{finite},\\
\mathcal{B}_+^n\to 0,\ \mathcal{B}_-^n\to\text{finite},\ \mathcal{B}_{NS}^n\to 0.
\end{gathered}
\tag{5.5}
$$

However, $\mathcal{A}_+^n$, $\mathcal{A}_{NS}^n$, and $\mathcal{B}_-^n$ are multiplied by the following vanishing factors:

$$
\begin{gathered}
\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_+^n/2\beta_0}\right\},\ \left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_{NS}^n/2\beta_0}\right\},\\
\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\lambda_-^n/2\beta_0+1}\right\},
\end{gathered}
\tag{5.6}
$$

respectively, and thus the terms proportional to $\widetilde{P}_i^n$, $\mathcal{A}_i^n$, and $\mathcal{B}_i^n$ in Eq. (3.16) all vanish in the $n=1$ limit. Note that these vanishing factors are specific to the case of the virtual photon target, and that such factors do not appear when the target is real photon.

Thus, the first three terms in the 1st moment vanish irrespective of the RG scheme. So we get

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2)=\frac{\alpha}{4\pi}\frac{1}{2\beta_0}\mathcal{C}^{n=1}+\mathcal{O}(\alpha_s). \quad (5.7)$$

Now let us consider $\mathcal{C}^{n=1}$, which is given by

$$\mathcal{C}^{n=1}=12\beta_0\langle e^4\rangle(B_G^n+\widetilde{A}_{nG}^{\psi})|_{n=1}. \quad (5.8)$$

As we have seen, the combination $(B_G^n+\widetilde{A}_{nG}^{\psi})$ is renormalization-scheme independent [25]. The results in the $\overline{\text{MS}}$ scheme [8,9,29] are

$$B_G^{n=1}=0,\ \widetilde{A}_{n=1G}^{\psi}=-2n_f. \quad (5.9)$$

The same results have been obtained by Kodaira [28] in the framework of OPE and RG method. He set $B_G^{n=1}=0$, observing that there is no gauge-invariant twist-2 gluon operator for $n=1$ and obtained $\widetilde{A}_{n=1G}^{\psi}=-2n_f$ from the Adler-Bell-Jackiw anomaly. In the end, we have for the sum rule of the virtual photon structure function g_1^γ,

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2)=-\frac{3\alpha}{\pi}\sum_{i=1}^{n_f}e_i^4+\mathcal{O}(\alpha_s), \quad (5.10)$$

where

$$Q^2\gg P^2\gg m_i^2,\Lambda^2,\quad i=1,\ldots,n_f \quad (5.11)$$

with m_i the mass of ith flavor quark, and n_f the number of active flavors.

Now it should be pointed out that we can further pursue the QCD corrections of order α_s to the first moment of g_1^γ. In the above equation for the first moment, the leading order is $\mathcal{O}(1)$ not of order $1/\alpha_s(Q^2)$, which is the case for the general moments. So we now go to the order α_s QCD correction.

First we take the renormalization scheme of Kodaira [28]. We write down the first moment of $g_1^\gamma(x,Q^2,P^2)$:

$$\begin{aligned}\int_0^1 dx g_1^\gamma(x,Q^2,P^2)\\ &=C_1^{\psi}(Q^2/P^2,\bar{g}(P^2),\alpha)\langle\gamma(p)|R_1^{\psi}(\mu^2=P^2)|\gamma(p)\rangle\\ &\quad+C_1^{NS}(Q^2/P^2,\bar{g}(P^2),\alpha)\\ &\quad\times\langle\gamma(p)|R_1^{NS}(\mu^2=P^2)|\gamma(p)\rangle. \end{aligned}\quad (5.12)$$

Here it should be emphasized that because of the absence of the gauge-invariant $n=1$ gluon and photon operators, the mixing problem becomes much simpler. The coefficient functions can be given by

$$\begin{pmatrix}C_1^{\psi}(Q^2/P^2,\bar{g}(P^2),\alpha)\\ C_1^{NS}(Q^2/P^2,\bar{g}(P^2),\alpha)\end{pmatrix}=T\exp\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg'\frac{\hat{\gamma}_1(g')}{\beta(g')}\begin{pmatrix}C_1^{\psi}(1,\bar{g}(Q^2),\alpha)\\ C_1^{NS}(1,\bar{g}(Q^2),\alpha)\end{pmatrix}, \quad (5.13)$$

where $\hat{\gamma}_1(g)$ is a 2×2 diagonal matrix:

$$\hat{\gamma}_1(g)=\left(\begin{array}{c|c}\gamma_{\psi\psi}(g) & 0\\ \hline 0 & \gamma_{NS}(g)\end{array}\right). \quad (5.14)$$

Here anomalous dimensions are expanded in powers of the coupling constant:

$$\gamma(g)=\gamma^{(0)}\frac{g^2}{16\pi^2}+\gamma^{(1)}\left(\frac{g^2}{16\pi^2}\right)^2+\mathcal{O}(g^6), \quad (5.15)$$

where

$$\gamma_{\psi\psi}^{(0)}=\gamma_{NS}^{(0)}=0,$$

$$\gamma_{\psi\psi}^{(1)}=24C_FT_f=24\cdot\frac{N_c^2-1}{2N_c}\cdot\frac{n_f}{2}=16n_f,\quad \gamma_{NS}^{(1)}=0, \quad (5.16)$$

and the coefficient functions are

$$\begin{aligned}C_1^{\psi}(1,\bar{g}(Q^2),\alpha)&=\langle e^2\rangle\left(1-\frac{3}{4}C_F\frac{\alpha_s(Q^2)}{\pi}\right)\\ &=\langle e^2\rangle\left(1-\frac{\alpha_s(Q^2)}{\pi}\right),\\ C_1^{NS}(1,\bar{g}(Q^2),\alpha)&=1-\frac{3}{4}C_F\frac{\alpha_s(Q^2)}{\pi}=1-\frac{\alpha_s(Q^2)}{\pi},\end{aligned} \quad (5.17)$$

$$T\exp\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)}dg'\frac{\hat{\gamma}_1(g')}{\beta(g')}=\mathbf{1}-\frac{1}{16\pi^2}\frac{\hat{\gamma}^{(1)}}{2\beta_0}[\bar{g}^2(P^2)-\bar{g}^2(Q^2)]. \quad (5.18)$$

Here we have the finite matrix element of the quark operators between the virtual photon states:

$$\langle\gamma(p)|R_{n=1}^i(\mu^2=P^2)|\gamma(p)\rangle=\frac{\alpha}{4\pi}A_{n=1}^i, \quad (5.19)$$

where

$$A_n=6(\langle e^2\rangle,0,\langle e^4\rangle-\langle e^2\rangle^2)\widetilde{A}_{nG}^{\psi}. \quad (5.20)$$

Now we recall Kodaira's statement that the bare Green's function for the $n=1$ case does not receive divergent corrections, but the finite correction connected with Adler-Bell-Jackiw anomaly:

$$\tilde{A}^{\psi}_{n=1G}=-4T(R)=-2n_f. \tag{5.21}$$

Putting all these equations together, we finally obtain the $\mathcal{O}(\alpha_s)$ QCD correction:

$$\int_0^1 dx g_1^{\gamma}(x,Q^2,P^2)=-\frac{3\alpha}{\pi}\left[\sum_{i=1}^{n_f} e_i^4\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)\right.$$
$$\left.-\frac{2}{\beta_0}\left(\sum_{i=1}^{n_f} e_i^2\right)^2\left(\frac{\alpha_s(P^2)}{\pi}-\frac{\alpha_s(Q^2)}{\pi}\right)\right]$$
$$+\mathcal{O}(\alpha_s^2). \tag{5.22}$$

This result is perfectly in agreement with the one obtained by Narison, Shore and Veneziano in Ref. [5], apart from the overall sign for the definition of g_1^{γ}.

Now we show that we obtain the same result for the first moment of $g_1^{\gamma}(x,Q^2,P^2)$ in the $\overline{\rm MS}$ scheme. Although there exist no gauge-invariant twist-2 gluon- and photon-operators for $n=1$, the $\overline{\rm MS}$ calculation of the anomalous dimensions gives nonzero results for $\gamma_{GG}^{n=1}$ and $\gamma_{G\psi}^{n=1}$. Thus, the $\overline{\rm MS}$-scheme results rather correspond to the QCD parton model approach where the first moments of the gluon and photon distributions are as well defined as the other distributions.

Including the gluon and photon operators, let us start with Eq. (3.13) for the first moment of $g_1^{\gamma}(x,Q^2,P^2)$ in $\overline{\rm MS}$ scheme:

$$\int_0^1 dx g_1^{\gamma}(x,Q^2,P^2)$$
$$=\frac{\alpha}{4\pi}A_1\cdot M_1(Q^2/P^2,\bar{g}(P^2))\cdot C_1(1,\bar{g}(Q^2))$$
$$+X_1(Q^2/P^2,\bar{g}(P^2),\alpha)\cdot C_1(1,\bar{g}(Q^2))+C_1^{\gamma}. \tag{5.23}$$

We expand A_1 in powers of $\bar{g}^2(P^2)$ as

$$A_1=A_1^{(0)}+\frac{\bar{g}^2(P^2)}{16\pi^2}A_1^{(1)}+\cdots, \tag{5.24}$$

where $A_1^{(0)}$ is given in Eq. (3.23). Using the $\overline{\rm MS}$ results for the $n=1$ anomalous dimensions, we obtain up to the order $\mathcal{O}(g^2)$

$$M_1=\begin{pmatrix} 1-\frac{1}{2\beta_0}\frac{1}{16\pi^2}[\bar{g}^2(P^2)-\bar{g}^2(Q^2)]\times 24C_FT_f, & \cdots, & 0\\ 0, & \cdots, & 0\\ 0, & \cdots, & 1\end{pmatrix}, \tag{5.25}$$

where the second column is irrelevant since $B_G^1=0$ in $\overline{\rm MS}$ scheme and thus $C_1^G(1,\bar{g})$ starts in $\mathcal{O}(\alpha_s^2)$. Now it is easy to see that the first term of Eq. (5.23), to be more specific, $[(\alpha/4\pi)A_1^{(0)}M_1C_1(1,\bar{g}(Q^2))]$ gives the same result as in Eq. (5.22).

Let us now consider the contributions of other terms. If $A_1^{\psi(1)}$ and $A_1^{NS(1)}$ in the second term in Eq. (5.24) remain nonzero, then they give the $\mathcal{O}(g^2)$ contribution. But $A_1^{\psi(1)}=A_1^{NS(1)}=0$ due to the nonrenormalization theorem [41] for the triangle anomaly, so its contribution is at most in $\mathcal{O}(\alpha_s^2)$. The contribution of the second term in Eq. (5.23) is also in $\mathcal{O}(\alpha_s^2)$, since $K_1^0=K_1^{(1)}=0$ and we expect

$$K_{\psi}^{(2),n=1}=K_{NS}^{(2),n=1}=0, \tag{5.26}$$

for the three-loop mixing anomalous dimensions which are implied from the fact that the three-loop $\gamma_{\psi G}^{(2),n=1}=0$ [40].

Finally we expand the third term of Eq. (5.23), C_1^{γ}, as

$$C_1^{\gamma}(1,\bar{g},\alpha)=\frac{\alpha}{4\pi}\delta_{\gamma}\left[B_{\gamma}^{(0),n=1}+\frac{\bar{g}^2}{16\pi^2}B_{\gamma}^{(1),n=1}+\cdots\right], \tag{5.27}$$

where $B_{\gamma}^{(0),n=1}=B_{\gamma}^{n=1}$ in Eq. (3.15). We already know that $B_{\gamma}^{(0),n=1}=0$ in $\overline{\rm MS}$ scheme. On the other hand, the two-loop ($\mathcal{O}(\alpha_s^2)$) coefficient function for the polarized gluon has been caluculated in the $\overline{\rm MS}$ scheme by Zijlstra and van Neerven [30]. It is made up of two terms, one proportional to factor $C_FT_fn_f$ and the other proportional to factor $C_AT_fn_f$. The first moments of both terms turn out to vanish. It can be shown that the two-loop ($\mathcal{O}(\alpha\alpha_s)$) coefficient function for the polarized photon, $B_{\gamma}^{(1)}$, is obtained from the two-loop gluon coefficient function, by picking up the term with the $C_FT_fn_f$ factor and by modifying the group factors. Thus we conclude that the first moment of $B_{\gamma}^{(1)}$ is zero. In the end, C_1^{γ} does not give $\mathcal{O}(1)$ nor $\mathcal{O}(\alpha_s)$ contributions to the first moment of $g_1^{\gamma}(x,Q^2,P^2)$. This means that we arrive at the same result for the 1st moment of g_1^{γ} given in Eq. (5.22) in the $\overline{\rm MS}$ scheme.

VI. NUMERICAL ANALYSIS

We now perform the inverse Mellin transform of Eq. (3.16) to get g_1^γ as a function of x. The n-th moment is denoted as

$$M(n,Q^2,P^2)=\int_0^1 x^{n-1} g_1^\gamma(x,Q^2,P^2)dx. \qquad (6.1)$$

Then by inverting the moments (6.1) we get

$$g_1^\gamma(x,Q^2,P^2)=\frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} M(n,Q^2,P^2)x^{-n}dn. \qquad (6.2)$$

In order to have better convergence of the numerical intergration, we change the contour in the complex n-plane from the vertical line connecting $C-i\infty$ with $C+i\infty$ (C is an appropriate positive constant), introducing a small positive constant ε, to

$$n=C-\varepsilon|y|+iy, -\infty<y<\infty. \qquad (6.3)$$

Hence we have

$$\begin{aligned} g_1^\gamma(x,Q^2,P^2)&=\frac{1}{2\pi i}\int_0^\infty M(C-\varepsilon y+iy,Q^2,P^2) \\ &\quad\times e^{-(C-\varepsilon y+iy)\log(x)}(i-\varepsilon)dy \\ &\quad+\frac{1}{2\pi i}\int_{-\infty}^0 M(C+\varepsilon y+iy,Q^2,P^2) \\ &\quad\times e^{-(C+\varepsilon y+iy)\log(x)}(i+\varepsilon)dy \\ &=\frac{1}{\pi}\int_0^\infty [\mathrm{Re}\{M(z,Q^2,P^2)e^{-z\log(x)}\} \\ &\quad-\varepsilon\,\mathrm{Im}\{M(z,Q^2,P^2)e^{-z\log(x)}\}]dy \end{aligned}$$

$$z=C-\varepsilon y+iy. \qquad (6.4)$$

In Fig. 3 we have plotted, as an illustration, the result for $n_f=3$, $Q^2=30\ \mathrm{GeV}^2$ and $P^2=1\ \mathrm{GeV}^2$ for the QCD scale parameter $\Lambda=0.2$ GeV. The vertical axis corresponds to

$$g_1^\gamma(x,Q^2,P^2)\Big/\frac{3\alpha}{\pi}n_f\langle e^4\rangle\ln\frac{Q^2}{P^2}. \qquad (6.5)$$

Here we have shown four cases; the Box (tree) diagram contribution,

$$g_1^{\gamma(\mathrm{Box})}(x,Q^2,P^2)=(2x-1)\frac{3\alpha}{\pi}n_f\langle e^4\rangle\ln\frac{Q^2}{P^2}, \qquad (6.6)$$

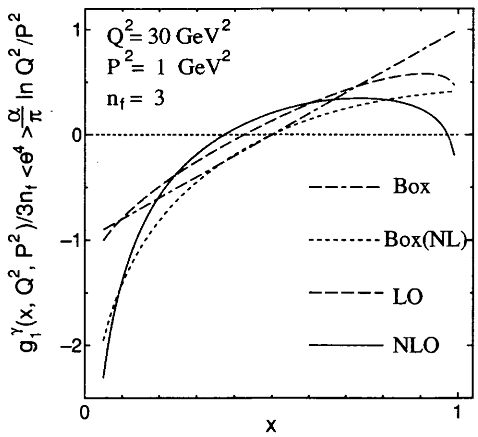

FIG. 3. Polarized virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$ to the next-to-leading order (NLO) in units of $(3\alpha n_f\langle e^4\rangle/\pi)\ln(Q^2/P^2)$ for $Q^2=30\ \mathrm{GeV}^2$, and $P^2=1\,\mathrm{GeV}^2$ and the QCD scale parameter $\Lambda=0.2$ GeV with $n_f=3$ (solid line). We also plot the leading order (LO) result (long-dashed line), the Box (tree) diagram (dash-dotted line) and the Box including non-leading contribution, Box (NL) (short-dashed line).

the Box diagram contribution including nonleading correction ignoring quark mass

$$g_1^{\gamma(\mathrm{Box,non\text{-}leading})}(x,Q^2,P^2)$$
$$=\frac{3\alpha}{\pi}n_f\langle e^4\rangle\left[(2x-1)\ln\frac{Q^2}{P^2}-2(2x-1)(\ln x+1)\right], \qquad (6.7)$$

the leading-order (LO) QCD correction and the next-to-leading order (NLO) QCD correction. We observe that the NLO QCD correction is significant at large x as well as at low x. We have also studied other examples with different Q^2 and P^2. In Fig. 4 we have plotted the case for $Q^2=100\,\mathrm{GeV}^2$ with $P^2=1\ \mathrm{GeV}^2$. Another case for $Q^2=30\ \mathrm{GeV}^2$ with $P^2=3\ \mathrm{GeV}^2$ is shown in Fig. 5. We have not seen any sizable change for the normalized structure function (6.5) for these different values of Q^2 and P^2. We examined the $n_f=4$ case as well. It is observed that the normalized structure function is insensitive to the number of

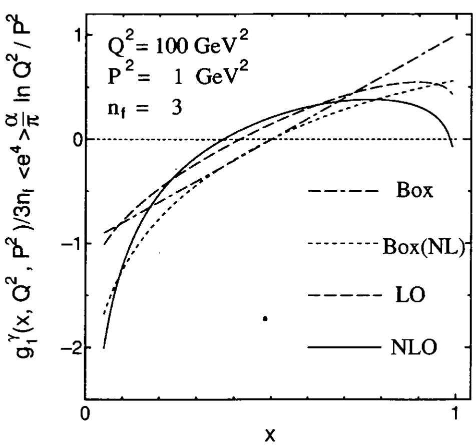

FIG. 4. Virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$ for $Q^2=100\ \mathrm{GeV}^2$, and $P^2=1\ \mathrm{GeV}^2$ with $\Lambda=0.2$ GeV, $n_f=3$.

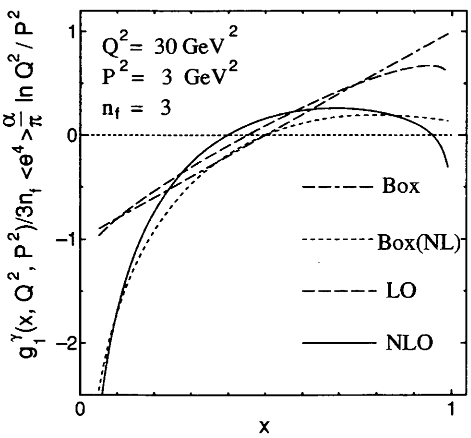

FIG. 5. Virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$ for $Q^2 = 30\ \mathrm{GeV}^2$, and $P^2 = 3\ \mathrm{GeV}^2$ with $\Lambda = 0.2$ GeV, $n_f = 3$.

active flavors. Here we have not directly taken into account the heavy quark mass dependence, but rather confined ourselves to the above kinematical region. It turns out from the numerical analyis as well as from theoretical arguments that, as P^2 increases, the NLO QCD result approaches the Box contribution including the nonleading correction, as in the unpolarized structure function [11,12].

Now let us consider the real photon case $P^2 = 0$. The structure function can be decomposed as

$$g_1^\gamma(x,Q^2) = g_1^\gamma(x,Q^2)|_{\text{pert.}} + g_1^\gamma(x,Q^2)|_{\text{non-pert.}}. \tag{6.8}$$

The second term can only be computed by some nonperturbative method like lattice QCD, or estimated by vector meson dominance (VMD) model. The first term, the pointlike piece, can be calculated in a perturbative method. Actually, it can formally be recovered in our analysis by setting $P^2 = \Lambda^2$ in Eq. (3.16). In Fig. 6, we have plotted the pointlike piece of g_1^γ of the real photon. The LO QCD result coincides with the previous result obtained by Sasaki in [22]. The NLO result is qualitatively consistent with the analysis by Stratmann and Vogelsang [10]. Finally, a comment on the $n=1$

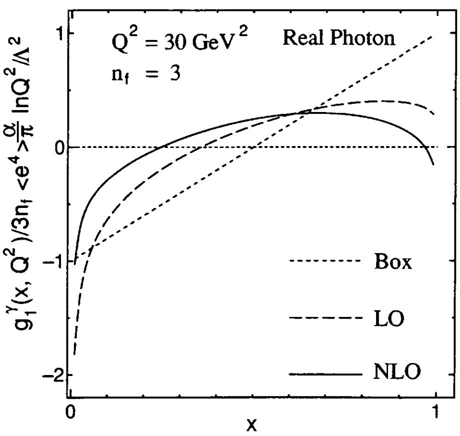

FIG. 6. Pointlike piece of the real photon structure function $g_1^\gamma(x,Q^2)$ in NLO for $Q^2 = 30\ \mathrm{GeV}^2$ with $\Lambda = 0.2$ GeV, $n_f = 3$ (solid line). Also plotted are the LO result (long-dashed line) and the Box (tree) diagram contribution (short-dashed line).

limit of the real photon structure function is in order. In the case of the unpolarized structure function F_2^γ we have a singularity of $\mathcal{A}_-^n$ at $n=2$ due to the vanishing of λ_-^n at $n=2$ which leads to the negative structure function [18]. As discussed in Refs. [42–44] we have to introduce some regularization prescription to recover positive structure function. For the polarized case we do not have such complication at $n=1$ as we have seen in Sec. V.

VII. CONCLUSION

Here in the present paper, we have investigated virtual photon's spin structure function $g_1^\gamma(x,Q^2,P^2)$ for the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$, in the next-to-leading order in QCD. We presented our arguments both in the framework of OPE supplemented by RG method and in the DGLAP equation approach. The results are shown to be independent of the renormalization scheme.

The first moment of g_1^γ for the virtual photon is nonvanishing in contrast to the vanishing first moment for the real photon case. We can go a step further to the order α_s which has turned out to reproduce the previous result of Narison, Shore, and Veneziano [5], and the result is RG scheme-independent.

The numerial evaluation of g_1^γ by the inverse Mellin transform was performed. The result shows that the NLO QCD corrections are significant at large x and also at small x. The numerical analysis can also be applied to the pointlike component of the real photon structure function. The result is qualitatively consistent with the previous analysis.

Although we have neglected in our kinematical region, we should also consider the power corrections of the form $(P^2/Q^2)^k (k=1,2,\dots)$, which are arising from the target mass effects as well as from higher-twist effects.

In the present paper we only presented the result for the polarized photon structure function g_1^γ itself. In the course of the parton model analysis, we also obtain the polarized parton distributions [2,45] of the longitudinally polarized photon, for the case of virtual photon, which will be discussed elsewhere.

As a future subject, it would be intriguing to study another spin structure function g_2^γ which only exists for off-shell photon. In the OPE language, the twist-2 as well as twist-3 operators contribute to the QCD effects for g_2^γ, which are now under investigation.

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APPENDIX A: NOTATION FOR ANOMALOUS DIMENSIONS

To the lowest order in α, the anomalous dimension matrix has the form

$$\gamma_n(g,\alpha)=\begin{pmatrix} \hat{\gamma}_n(g^2) & 0 \\ K_n(g^2,\alpha) & 0 \end{pmatrix}, \tag{A1}$$

where $\hat{\gamma}_n(g^2)$ is the usual 3×3 anomalous dimension matrix in the hadronic sector

$$\hat{\gamma}_n(g)=\begin{pmatrix} \gamma^n_{\psi\psi}(g) & \gamma^n_{G\psi}(g) & 0 \\ \gamma^n_{\psi G}(g) & \gamma^n_{GG}(g) & 0 \\ 0 & 0 & \gamma^n_{NS}(g) \end{pmatrix}, \tag{A2}$$

and $K_n(g,\alpha)$ is the three-component row vector

$$K_n(g,\alpha)=(K^n_\psi(g,\alpha),K^n_G(g,\alpha),K^n_{NS}(g,\alpha)) \tag{A3}$$

representing the mixing between photon and three hadronic operators. The anomalous dimensions are expanded as

$$\hat{\gamma}_n(g)=\frac{g^2}{16\pi^2}\hat{\gamma}^0_n+\frac{g^4}{(16\pi^2)^2}\hat{\gamma}^{(1)}_n+O(g^6), \tag{A4}$$

$$K_n(g,\alpha)=-\frac{e^2}{16\pi^2}K^0_n-\frac{e^2g^2}{(16\pi^2)^2}K^{(1)}_n+O(e^2g^4). \tag{A5}$$

The one-loop anomalous dimension matrix $\hat{\gamma}^0_n$ can be expressed in terms of its eigenvalues $\lambda^n_i(i=+,-,NS)$ as

$$\hat{\gamma}^0_n=\sum_{i=+,-,NS}\lambda^n_i P^n_i, \tag{A6}$$

where

$$\lambda^n_\pm=\frac{1}{2}\{\gamma^{0,n}_{\psi\psi}+\gamma^{0,n}_{GG}\pm[(\gamma^{0,n}_{\psi\psi}-\gamma^{0,n}_{GG})^2+4\gamma^{0,n}_{\psi G}\gamma^{0,n}_{G\psi}]^{1/2}\}, \tag{A7}$$

$$\lambda^n_{NS}=\gamma^{0,n}_{NS}, \tag{A8}$$

and P^n_i are the corresponding projection operators,

$$P^n_\pm=\frac{1}{\lambda^n_\pm-\lambda^n_\mp}\begin{pmatrix} \gamma^{0,n}_{\psi\psi}-\lambda^n_\mp & \gamma^{0,n}_{G\psi} & 0 \\ \gamma^{0,n}_{\psi G} & \gamma^{0,n}_{GG}-\lambda^n_\mp & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{A9}$$

$$P^n_{NS}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A10}$$

APPENDIX B: EXPLICIT EXPRESSIONS FOR ANOMALOUS DIMENSIONS

1. One-loop order

$$\gamma^{0,n}_{\psi\psi}=\gamma^{0,n}_{NS}=2C_F\left[-3-\frac{2}{n(n+1)}+4S_1(n)\right], \tag{B1}$$

$$\gamma^{0,n}_{\psi G}=-8T_f\frac{n-1}{n(n+1)}, \tag{B2}$$

$$\gamma^{0,n}_{G\psi}=-4C_F\frac{n+2}{n(n+1)}, \tag{B3}$$

$$\gamma^{0,n}_{GG}=2C_A\left[-\frac{11}{3}-\frac{8}{n(n+1)}+4S_1(n)\right]+\frac{8}{3}T_f, \tag{B4}$$

where

$$S_1(n)=\sum_{j=1}^{n}\frac{1}{j} \tag{B5}$$

and

$$C_A=3,\quad C_F=\frac{4}{3},\quad T_f=T_Rn_f=n_f/2 \tag{B6}$$

with n_f being the number of flavors

$$K^0_n=(K^{0,n}_\psi,0,K^{0,n}_{NS}), \tag{B7}$$

$$K^{0,n}_\psi=24n_f\langle e^2\rangle\frac{n-1}{n(n+1)}, \tag{B8}$$

$$K^{0,n}_{NS}=24n_f(\langle e^4\rangle-\langle e^2\rangle^2)\frac{n-1}{n(n+1)}. \tag{B9}$$

2. Two-loop order [8,9]

a. Non-singlet sector

$$\gamma^{(1),n}_{NS}=8C_F^2A^n_{NS}+8C_AC_FB^n_{NS}+8C_FT_fD^n_{NS}, \tag{B10}$$

with

$$A_{NS}^n=-\frac{3}{8}+\frac{5}{n}-\frac{5}{n+1}-\frac{3}{n^2}-\frac{2}{(n+1)^2}+\frac{1}{n^3}-\frac{3}{(n+1)^3}+(-1)^n\left\{-\frac{4}{n}+\frac{4}{n+1}+\frac{2}{n^2}+\frac{2}{(n+1)^2}-\frac{2}{n^3}+\frac{2}{(n+1)^3}\right\}+S_1(n)\left(\frac{2}{n^2}-\frac{2}{(n+1)^2}\right)+S_2(n)\left(3-\frac{2}{n}+\frac{2}{n+1}+4S_1(n)\right)+S_2'\left(\frac{n}{2}\right)\left(\frac{2}{n}-\frac{2}{n+1}-4S_1(n)\right)-S_3'\left(\frac{n}{2}\right)+8\widetilde{S}(n), \tag{B11}$$

$$B_{NS}^n=-\frac{17}{24}-\frac{187}{18}\frac{1}{n}+\frac{187}{18}\frac{1}{n+1}+\frac{17}{6}\frac{1}{n^2}-\frac{5}{6}\frac{1}{(n+1)^2}-\frac{1}{n^3}+\frac{1}{(n+1)^3}+(-1)^n\left\{\frac{2}{n}-\frac{2}{n+1}-\frac{1}{n^2}-\frac{1}{(n+1)^2}+\frac{1}{n^3}-\frac{1}{(n+1)^3}\right\}+\frac{67}{9}S_1(n)+S_2(n)\left(-\frac{11}{3}+\frac{2}{n}-\frac{2}{n+1}-4S_1(n)\right)+S_2'\left(\frac{n}{2}\right)\left(-\frac{1}{n}+\frac{1}{n+1}+2S_1(n)\right)+\frac{1}{2}S_3'\left(\frac{n}{2}\right)-4\widetilde{S}(n), \tag{B12}$$

$$D_{NS}^n=\frac{1}{6}+\frac{22}{9}\frac{1}{n}-\frac{22}{9}\frac{1}{n+1}-\frac{2}{3}\frac{1}{n^2}+\frac{2}{3}\frac{1}{(n+1)^2}-\frac{20}{9}S_1(n)+\frac{4}{3}S_2(n), \tag{B13}$$

where

$$S_2(n)=\sum_{j=1}^{n}\frac{1}{j^2},\quad S_3(n)=\sum_{j=1}^{n}\frac{1}{j^3},\quad \widetilde{S}(n)=\sum_{j=1}^{n}\frac{(-1)^j}{j^2}S_1(j), \tag{B14}$$

and

$$S_2'\left(\frac{n}{2}\right)=\frac{1+(-1)^n}{2}S_2\left(\frac{n}{2}\right)+\frac{1-(-1)^n}{2}S_2\left(\frac{n-1}{2}\right), \tag{B15}$$

$$S_3'\left(\frac{n}{2}\right)=\frac{1+(-1)^n}{2}S_3\left(\frac{n}{2}\right)+\frac{1-(-1)^n}{2}S_3\left(\frac{n-1}{2}\right). \tag{B16}$$

b. Singlet sector

(1) $\gamma_{\psi\psi}$:

$$\gamma_{\psi\psi}^{(1),n}=\gamma_{NS}^{(1),n}+\gamma_{PS,\psi\psi}^{(1),n} \tag{B17}$$

with

$$\gamma_{PS,\psi\psi}^{(1),n}=8C_FT_f\left\{-\frac{2}{n}+\frac{2}{n+1}-\frac{2}{n^2}+\frac{6}{(n+1)^2}+\frac{4}{n^3}+\frac{4}{(n+1)^3}\right\}. \tag{B18}$$

(2) $\gamma_{\psi G}$:

$$\gamma_{\psi G}^{(1),n}=8C_FT_fD_{\psi G}+8C_AT_fE_{\psi G}, \tag{B19}$$

with

$$D_{\psi G}^n=S_1^2(n)\left(\frac{2}{n}-\frac{4}{n+1}\right)-S_2(n)\left(\frac{2}{n}-\frac{4}{n+1}\right)+S_1(n)\left(\frac{8}{n}-\frac{8}{n+1}-\frac{4}{n^2}\right)+\frac{22}{n}-\frac{27}{n+1}-\frac{9}{n^2}-\frac{8}{(n+1)^2}+\frac{2}{n^3}+\frac{4}{(n+1)^3}, \tag{B20}$$

$$E_{\psi G}^n=-\frac{24}{n}+\frac{22}{n+1}+\frac{2}{n^2}+\frac{24}{(n+1)^2}+\frac{4}{n^3}+\frac{24}{(n+1)^3}-S_1(n)\left(\frac{8}{n}-\frac{8}{n+1}-\frac{8}{(n+1)^2}\right)-S_1^2(n)\left(\frac{2}{n}-\frac{4}{n+1}\right)+S_2(n)\left(\frac{2}{n}-\frac{4}{n+1}\right)-S_2'\left(\frac{n}{2}\right)\left(\frac{2}{n}-\frac{4}{n+1}\right)+[1+(-1)^n]\left(\frac{2}{n}-\frac{4}{n+1}\right)\left(-2S_2(n)+S_2'\left(\frac{n}{2}\right)+\zeta(2)\right). \tag{B21}$$

(3) $\gamma_{G\psi}$:

$$\gamma_{G\psi}^{(1),n}=8C_F^2A_{G\psi}^n+8C_AC_FB_{G\psi}^n+8C_FT_fD_{G\psi}^n, \quad \text{(B22)}$$

with

$$A_{G\psi}^n=\frac{17}{2}\frac{1}{n}-\frac{4}{n+1}-\frac{2}{n^2}-\frac{1}{2}\frac{1}{(n+1)^2}-\frac{2}{n^3}-\frac{1}{(n+1)^3}-S_1(n)\left(\frac{2}{n}+\frac{1}{n+1}+\frac{2}{(n+1)^2}\right)+S_1^2(n)\left(\frac{2}{n}-\frac{1}{n+1}\right)+S_2(n)\left(\frac{2}{n}-\frac{1}{n+1}\right), \quad \text{(B23)}$$

$$B_{G\psi}^n=-\frac{41}{9}\frac{1}{n}-\frac{35}{9}\frac{1}{n+1}+\frac{4}{n^2}-\frac{38}{3}\frac{1}{(n+1)^2}-\frac{4}{n^3}-\frac{6}{(n+1)^3}+S_1(n)\left(\frac{10}{3}\frac{1}{n}+\frac{1}{3}\frac{1}{n+1}+\frac{4}{n^2}\right)-S_1^2(n)\left(\frac{2}{n}-\frac{1}{n+1}\right)-S_2(n)\left(\frac{2}{n}-\frac{1}{n+1}\right)+S_2'\left(\frac{n}{2}\right)\left(\frac{2}{n}-\frac{1}{n+1}\right)+[1+(-1)^n]\times\left(\frac{2}{n}-\frac{1}{n+1}\right)\left(2S_2(n)-S_2'\left(\frac{n}{2}\right)-\zeta(2)\right), \quad \text{(B24)}$$

$$D_{G\psi}^n=\frac{16}{9}\frac{1}{n}+\frac{4}{9}\frac{1}{n+1}+\frac{4}{3}\frac{1}{(n+1)^2}-S_1(n)\left(\frac{8}{3}\frac{1}{n}-\frac{4}{3}\frac{1}{n+1}\right). \quad \text{(B25)}$$

(4) γ_{GG}:

$$\gamma_{GG}^{(1),n}=8C_FT_fD_{GG}^n+8C_AT_fE_{GG}^n+8C_A^2F_{GG}^n, \quad \text{(B26)}$$

with

$$D_{GG}^n=1+\frac{10}{n}-\frac{10}{n+1}-\frac{10}{n^2}+\frac{2}{(n+1)^2}+\frac{4}{n^3}+\frac{4}{(n+1)^3}, \quad \text{(B27)}$$

$$E_{GG}^n=\frac{4}{3}+\frac{76}{9}\frac{1}{n}-\frac{76}{9}\frac{1}{n+1}-\frac{4}{3}\frac{1}{n^2}-\frac{4}{3}\frac{1}{(n+1)^2}-\frac{20}{9}S_1(n), \quad \text{(B28)}$$

$$F_{GG}^n=-\frac{8}{3}-\frac{97}{18}\frac{1}{n}+\frac{97}{18}\frac{1}{n+1}+\frac{29}{3}\frac{1}{n^2}-\frac{67}{3}\frac{1}{(n+1)^2}-\frac{8}{n^3}-\frac{24}{(n+1)^3}+S_1(n)\left(\frac{67}{9}+\frac{8}{n^2}-\frac{8}{(n+1)^2}\right)-\frac{1}{2}S_3'\left(\frac{n}{2}\right)+4\tilde{S}(n)-2S_2'\left(\frac{n}{2}\right)\left(S_1(n)-\frac{2}{n}+\frac{2}{n+1}\right)+[1+(-1)^n]\left[8S_2(n)\left(\frac{1}{n}-\frac{1}{n+1}\right)-2S_3(n)-4S_1(n)S_2(n)+2S_2'\left(\frac{n}{2}\right)\left(S_1(n)-\frac{2}{n}+\frac{2}{n+1}\right)+\frac{1}{2}S_3'\left(\frac{n}{2}\right)-4\tilde{S}(n)+\zeta(2)\left(2S_1(n)-\frac{4}{n}+\frac{4}{n+1}\right)-\zeta(3)\right]. \quad \text{(B29)}$$

c. $K_n^{(1)}$

$$K_n^1=\left(K_\psi^{1,n},\quad K_G^{1,n},\quad K_{NS}^{1,n}\right), \quad \text{(B30)}$$

with

$$K_\psi^{1,n}=-3n_f\langle e^2\rangle C_F8D_{\psi G}^n, \quad \text{(B31)}$$

$$K_G^{1,n}=-3n_f\langle e^2\rangle C_F8(D_{GG}^n-1), \quad \text{(B32)}$$

$$K_{NS}^{1,n}=-3n_f(\langle e^4\rangle-\langle e^2\rangle^2)C_F8D_{\psi G}^n. \quad \text{(B33)}$$

d. Anomalous dimensions at $n=1$ *($\overline{\mathrm{MS}}$ scheme)*

$$\gamma_{NS}^{0,n=1}=\gamma_{\psi\psi}^{0,n=1}=0, \quad \text{(B34)}$$

$$\gamma_{\psi G}^{0,n=1}=0, \quad \text{(B35)}$$

$$\gamma_{G\psi}^{0,n=1}=-6C_F, \tag{B36}$$

$$\gamma_{GG}^{0,n=1}=-\frac{22}{3}C_A+\frac{8}{3}T_f=-2\beta_0, \tag{B37}$$

$$K_{NS}^{0,n=1}=K_{\psi}^{0,n=1}=0, \tag{B38}$$

$$\gamma_{NS}^{(1),n=1}=0, \tag{B39}$$

$$\gamma_{\psi\psi}^{(1),n=1}=24C_FT_f, \tag{B40}$$

$$\gamma_{\psi G}^{(1),n=1}=0, \tag{B41}$$

$$\gamma_{G\psi}^{(1),n=1}=18C_F^2-\frac{142}{3}C_AC_F+\frac{8}{3}C_FT_f, \tag{B42}$$

$$\gamma_{GG}^{(1),n=1}=8C_FT_f+\frac{40}{3}C_AT_f-\frac{68}{3}C_A^2=-2\beta_1, \tag{B43}$$

$$K_{\psi}^{(1),n=1}=K_G^{(1),n=1}=K_{NS}^{(1),n=1}=0. \tag{B44}$$

APPENDIX C: COEFFICIENT FUNCTIONS

$C_n(C_n^\gamma)$ is the coefficient function of the hadronic (photon) operators [17]:

$$C_n(1,\bar{g}(Q^2))=\begin{pmatrix}\delta_\psi\left(1+\dfrac{\bar{g}^2(Q^2)}{16\pi^2}B_\psi^n\right)\\ \delta_\psi\dfrac{\bar{g}^2(Q^2)}{16\pi^2}B_G^n\\ \delta_{NS}\left(1+\dfrac{\bar{g}^2(Q^2)}{16\pi^2}B_{NS}^n\right)\end{pmatrix}, \tag{C1}$$

$$C_n(1,\bar{g}(Q^2),\alpha)=\frac{e^2}{16\pi^2}\delta_\gamma B_\gamma^n, \tag{C2}$$

and

$$B_\gamma^n=(2/n_f)B_G^n. \tag{C3}$$

1. $\overline{\text{MS}}$ scheme [8,9]

$$B_\psi^n=B_{NS}^n=C_F\left[\left(\frac{2}{n}+\frac{2}{n+1}+3\right)S_1(n-1)+4\sum_{j=1}^{n-1}\frac{1}{j}S_1(j)-4S_2(n-1)+\frac{6}{n}-9\right], \tag{C4}$$

$$B_G^n=4T_f\left[-\frac{n-1}{n(n+1)}(S_1(n)+1)-\frac{1}{n^2}+\frac{2}{n(n+1)}\right]. \tag{C5}$$

2. Momentum subtraction [27,28]

$$B_\psi^n=B_{NS}^n=C_F\left[-2-\frac{3}{n}+\frac{8}{n+1}+\frac{4}{n^2}-\frac{4}{(n+1)^2}+3S_1(n)-8S_2(n)\right], \tag{C6}$$

$$B_G^n=8T_f\left[\frac{1}{n}-\frac{2}{n+1}-\frac{1}{n^2}+\frac{2}{(n+1)^2}\right]\quad n\geqslant 3,$$

$$B_G^{n=1}=0. \tag{C7}$$

APPENDIX D: TENSOR DECOMPOSITION OF VIRTUAL PHOTON-PHOTON AMPLITUDE

After using parity invariance, time-reversal invariance, and gauge invariance, Brown and Muzinich [20] have shown that there are eight independent tensors, in other words, eight-invariant amplitudes for virtual photon-photon scattering. Those eight independent tensors, which are free from kinematic singularities and kinematic zeros, are given in Eqs. (A3)–(A10) of Ref. [20].

Using these tensors $(I_i)_{\mu\nu\rho\tau}$, the absorptive part of the forward virtual photon-photon scattering amplitude $W_{\mu\nu\rho\tau}$ is decomposed as

$$W_{\mu\nu\rho\tau}=\sum_{i=1}^{8}(I_i)_{\mu\nu\rho\tau}A_i(w,t_1,t_2), \tag{D1}$$

where the A_i are the invariant amplitudes and

$$w=p\cdot q,\quad t_1=q^2=-Q^2,\quad t_2=p^2=-P^2. \tag{D2}$$

In order to implement crossing symmetry under $q\to-q$ and $\mu\leftrightarrow\nu$, we form the even combinations, I_1, I_2+I_3, I_4, I_5, I_7+I_8, and the odd combinations, I_2-I_3, I_7-I_8, $I_6'=2I_6-3wI_7-wI_8+(t_1t_2/w)(I_2-I_3)$. It is noted that the combinations I_2-I_3 and I_7-I_8 are antisymmetric under the interchange of $\mu\leftrightarrow\nu$ and $\rho\leftrightarrow\tau$, while the rest of the combinations are symmetric. In terms of these crossing-even and

-odd combinations, the amplitude $W=\Sigma_{i=1}^{8}I_iA_i$ is rearranged as follows:

$$\sum_{i=1}^{8} I_iA_i=I_1A_1+\frac{1}{2}(I_2+I_3)(A_2+A_3)+I_4A_4+I_5A_5+\frac{1}{2}(I_7+I_8)(A_7+A_8+2wA_6)+\frac{1}{2}I_6'A_6+\frac{1}{2}(I_2-I_3)\left(A_2-A_3-\frac{t_1t_2}{w}A_6\right)+\frac{1}{2}(I_7-I_8)(A_7-A_8+wA_6). \quad \text{(D3)}$$

Now $g_1^\gamma=2W_4^\gamma$ is written in terms of invariant amplitudes as

$$g_1^\gamma \propto a_{1111}-a_{1-11-1} \quad \text{(D4)}$$

$$=w^2\left(A_2-A_3-\frac{t_1t_2}{w}A_6\right)-t_1t_2(A_7-A_8+wA_6) \quad \text{(D5)}$$

which is obtained from Eq. (A14) of Ref. [20]. Here a_{1111} (a_{1-11-1}) represents the s-channel helicity amplitude for $(+1)\gamma+(\pm1)\gamma\to(+1)\gamma+(\pm1)\gamma$. It is noted that $[A_2-A_3-(t_1t_2/w)A_6]$ is the invariant amplitude associated with (I_2-I_3), while $(A_7-A_8+wA_6)$ is associated with (I_7-I_8). In the limit $t_2=p^2=0$ or in the case that $w=p\cdot q, t_1=q^2\gg t_2=p^2$, the second term $t_1t_2(A_7-A_8+wA_6)$ does not contribute to g_1^γ.

In fact we observe that the tensor $(I_2-I_3)\equiv I_-$ is associated to g_1^γ while $(I_7-I_8)\equiv J_-$ is associated to g_2^γ. It can be shown that

$$\epsilon_{\mu\nu\lambda\sigma}q^\lambda\epsilon_{\rho\tau}{}^{\sigma\beta}p_\beta=\frac{1}{p\cdot q}I_-, \quad \text{(D6)}$$

and in the limit of $-q^2, p\cdot q\gg -p^2$

$$[\epsilon_{\mu\alpha\lambda\sigma}q_\nu q^\alpha-\epsilon_{\nu\alpha\lambda\sigma}q_\mu q^\alpha-\epsilon_{\mu\nu\lambda\sigma}]\epsilon_{\rho\tau}{}^{\sigma\beta}p_\beta p^\lambda=J_-. \quad \text{(D7)}$$

Now using an identity

$$g_{\mu\nu}\epsilon_{\alpha\beta\gamma\delta}=g_{\mu\alpha}\epsilon_{\nu\beta\gamma\delta}+g_{\mu\beta}\epsilon_{\alpha\nu\gamma\delta}+g_{\mu\gamma}\epsilon_{\alpha\beta\nu\delta}+g_{\mu\delta}\epsilon_{\alpha\beta\gamma\nu}, \quad \text{(D8)}$$

we get

$$\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q\,\epsilon_{\rho\tau}{}^{\sigma\beta}p_\beta-\epsilon_{\rho\tau\alpha\beta}p^\beta p^\sigma q^\alpha)=-[\epsilon_{\mu\alpha\lambda\sigma}q_\nu q^\alpha-\epsilon_{\nu\alpha\lambda\sigma}q_\mu q^\alpha-\epsilon_{\mu\nu\lambda\sigma}q^2]\epsilon_{\rho\tau}{}^{\alpha\beta}p_\beta p^\lambda. \quad \text{(D9)}$$

Hence we have from Eq. (2.3)

$$W_{\mu\nu\rho\tau}=\frac{1}{(p\cdot q)^2}[(I_-)_{\mu\nu\rho\tau}g_1^\gamma-(J_-)_{\mu\nu\rho\tau}g_2^\gamma]. \quad \text{(D10)}$$

Finally it is interesting to see the relation between the polarized photon structure functions g_1^γ and g_2^γ and polarized nucleon structure functions g_1 and g_2. By introducing the polarization vectors, $\epsilon^{*\rho}$ and ϵ^τ for the target photon just like those for the gluon target discussed by Gabrieli and Ridolfi [46], we have

$$iW_{\mu\nu}^A=\epsilon^{*\rho}W_{\mu\nu\rho\tau}\epsilon^\tau=W_{\mu\nu\rho\tau}\frac{1}{2}(\epsilon^{*\rho}\epsilon^\tau-\epsilon^{*\tau}\epsilon^\rho)=W_{\mu\nu\rho\tau}\left(-\frac{i}{2\sqrt{|p^2|}}\epsilon^{\rho\tau\gamma\delta}p_\gamma s_\delta\right), \quad \text{(D11)}$$

where s is the longitudinal spin vector for the target photon. After using the relation $p\cdot s=0$, we get

$$W_{\mu\nu}^A=\frac{\sqrt{|p^2|}}{p\cdot q}\left[\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma g_1^\gamma+\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q s^\sigma-q\cdot s p^\sigma)\frac{g_2^\gamma}{p\cdot q}\right] \quad \text{(D12)}$$

which, apart from the factor $\sqrt{|p^2|}$, has exactly the same form as Eq. (2.4) of Kodaira *et al.* [27] which defines the polarized nucleon structure functions g_1 and g_2, and also as Eq. (9) in Ref. [46].

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