Variety in Japan (1980−2000)

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1. Introduction

Since the Meiji period, the composition of Japan's foreign trade has had many changes. Along with the development of the economy, more and more products are available for export and, at the same time, more varieties of goods are imported for domestic consumption and production. However, since the mid-1980s, specialization and the expansion of foreign direct investment became trends in the Japanese economy, which might reduce the range of exported products. The 1980s also witnessed solid developments of Japan's economy, which might have the opposite effect on variety. From 1993, Japan's economy entered a period of economic stagnation. We expect that varieties of imports and exports might decrease in this period. Also the conclusion of many bilateral trade agreements in these periods might affect Japan's trade composition as well as import and export varieties. Therefore, it is interesting to access the real changes of variety in Japan during these two periods: the 1980s with an economic boom and the 1990s with an economic slowdown. In this paper we present calculations of variety of Japan's import and export for 21 industries over 21 years with a method developed by Feenstra (1994).

Many papers on economic growth theory have confirmed the importance of product variety (Romer, 1990; and Grossman and Helpman, 1991) and a number of empirical studies have tested this theory. These studies analyze the relationship between an increase in variety and total factor productivity (TFP) as in Feenstra et al. (1999a), between an increase in variety and Gross Domestic Product (GDP) in Funke and Ruhwedel (2005), between variety and business groups in Feenstra et al. (1999b) or between an increase in variety and welfare gains as in Broda and Weinstein (2006). A careful study of Japan's product variety, therefore, will be significant for other empirical studies of economic growth theory.

Firstly, we have to consider Japan's trade composition as well as trade policy to have the overall picture of Japanese foreign trade, which are the main factors affecting the variety of imported and exported goods.

Japan's trade composition

In the early Meiji period, Japan's major exports were coal and silk, and its major imports were manufactured goods, wool and cotton. From the late nineteenth century until the Pacific War, Japan's major export was textiles and its major imports were primary products including fuel. From 1960, Japan's major exports changed to automobiles, electronics goods and other manufactured goods. From 1945, its major imports have been petroleum and other natural resources.

The changes of Japan's trade composition are in accordance with the principle of comparative advantage (Saxonhouse, 1982). In the Meiji period, Japan had comparative advantages in producing coal and silk, which had a lower autarky price than in other countries. At the same time, it imported manufactured goods, wool and cotton, which

had higher autarky prices (Huber, 1971). As Japan industrialized and accumulated physical and human capital, its trade composition has changed. Japan has gained expanded production possibilities in producing automobiles, electronic goods and other manufactured goods and therefore it became the exporter of those goods and the importers of natural resources.

From 1980 to 2000, the basic trade composition of Japan remains, but with the conclusion of many bilateral trade agreements, we can expect changes in trading partners as well as in the trading pattern, which directly leads to changes in import and export varieties. In the next section, I would like to give an overview of Japan's trade policy during this period.

Japan's trade policy

Since the establishment of the General Agreement on Tariffs and Trade (GATT) in 1947, a number of multilateral trade policy agreements have cumulatively reduced the developed nations' ad valorem tariff rates from 40% in 1947 to 4% in 1994. Japanese trade policy also followed this liberalization trend. However, the elimination of protectionist trade policies of Japan is substantially due to foreign pressure (*gaiatsu*) (Lincoln, 2001).

Japan joined the GATT in 1955, however, it still maintained its system of foreign trade rationing and trade quotas in agricultural commodities. Only in 1963 were those quotas converted to tariffs and in 1964, foreign exchange restrictions on current transactions were abandoned. The liberalization of Japan's trade was largely due to the successive GATT rounds, especially the Tokyo Round concluded in 1979 and the Uruguay Round concluded in 1995. In 1988, Japan began lowering tariffs on many goods three or six years ahead of schedule. By 1992, all Uruguay Round concessions and more were made. By this year, Japan's average tariff rate for all imports was about 7% (GATT, 1994). Most of the manufactured products were duty free.

Although Japan is a full member of GATT, Japan has often been accused by the United States and Europe as having a closed market (Prestowitz, 1988). This criticism arises partly from the trade surplus of Japan with other countries, especially with the US (Flath, 2005). Before 1980, many US industries including textiles, steel, televisions and automobiles were seeking protection from Japan's imports.

The Japanese Government then agreed to voluntary export restraints of the above mentioned and other disputed commodities. The restraints of Japanese exports to US were then followed by similar restraints requested by the EU. Consequently, by the mid-1980s, a substantial part of Japan's exports to the US were subject to Japanese government restraints. However, the protectionist pressures seem to grow (Lincoln, 2001).

Since 1985, many bilateral negotiations between US and Japanese governments, which benefit American exporters to Japan, were obtained. The US industries which benefited included semiconductors, beef, oranges, wood products, insurance, telecommunications, auto parts and other items. Table 1 reports important Japan-US bilateral agreements on agriculture and manufactured products.

In the 1980s, although Japan lowered barriers to trade, Japan's trade surplus rose sharply. This was partially due to the liberalization of its foreign investment law beginning in 1980. Later, in 1992, after a long period of steady growth, Japan entered a period of economic stagnation. The GDP growth fell and in 1993, the GDP real growth was zero. This stagnation led to the decrease in nominal imports and thus leading to a higher trade surplus (Parsons, 2000).

The above overview of trade policy in Japan helps us to understand the import and export activities of Japan in the period 1980−2000 and thus, can help us to explain the changes in import and export varieties of Japan during this period.

Feenstra (1994) created a method to measure product variety. This method was used in many empirical studies to estimate the effects of variety on economic growth or other economic indices. In the following part of this paper, I will

Table 1 Major Bilateral Trade Agreements between Japan and the US (1980–1996)

	Year
Telecommunications	
NTT Procurement	1980
Telecomm.Equip. & Services- MOSS	1986
Telecomm.-Cellular &Third-Party Radio	1989
Telecomm.-Cellular & Third Party Radio	1994
Computers & High Technology	
Electronics – MOSS	1986
Medical/Pharmaceutical Products-MOSS	1986
Medical Technology Procurement	1994
Non-R&D Satellite Procurement	1990
Supercomputer Procurement	1990
Computer Products Procurement	1992
Semiconductors	1991
Semiconductors	1996
Transportation	
Auto and Auto Parts –MOSS	1987
Auto and Auto Parts	1992
Auto and Auto Parts	1995
Food and Agriculture	
Agriculture – Beef and Citrus	1988
Agriculture – Other Products	1988, 1992
Leather and Leather Footwear	1985
Wood Products – MOSS	1986
Wood Products	1990
Paper Products	1992
Building and Construction	
Flat Glass	1995

Source: ACCJ 1997

try to analyze and extend the product variety index in Feenstra (1994) to clearly separate import and export variety. The third part will describe the data used in this paper, which is Japan's trade data for 21 years, from 1980 to 2000. The result of the second part will then be applied in the fourth part of this paper to measure Japan's export and import varieties. Section 5 concludes the paper.

2. The measurement of import and export varieties

Generally, a product variety is defined as the brand produced by a firm, the output of a firm, the output of a country or the output within an industry in a country. In order to measure product variety, Feenstra (1994) and Feenstra and Markusen (1994) proposed a solution, in which new product varieties enter a constant-elasticity-of-substitution (CES)

aggregator function. The change of product variety between two periods is a function of the total expenditure and the expenditure on new varieties of the two periods.

In this paper, I extend Feenstra's (1994) variety index by incorporating import variety or export variety. The production function will take the form

$$y_t = f(x_t, z_t, I_t, J_t) = \left[\sum_{i \in I_t} a_i x_{it}^{(\sigma-1)/\sigma} + \sum_{i \in J_t} b_j z_{jt}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{1}$$

where $a_i > 0$ and $b_j > 0$ are parameters, $\sigma > 1$ is the elasticity of substitution, I_t denotes the set of domestic inputs in period t and J_t denotes the set of imported inputs in period t. The production of output y_t requires not only varieties of domestic inputs x_{it}, but also varieties of imported goods z_{jt}. This is different with the production function in Feenstra (1994) which does not distinguish domestic and imported inputs.

The firm will minimize its cost of production and come up with following CES unit-cost function (derived in the Appendix)

$$c(p_t, q_t, I_t, J_t) = \left[\sum_{i \in I_t} a_i^{\sigma} p_{it}^{1-\sigma} + \sum_{j \in J_t} b_i^{\sigma} q_{jt}^{1-\sigma} \right]^{1/(1-\sigma)} \tag{2}$$

where p is the price of domestic input i, and q is the price of imported input j. Now we consider two successive periods 0,1. In order to analyze the impacts of import varieties, I assume that the domestic input variety is unchanged over time, meaning $I_1 = I_0 = I$. The set of imported input is changing over time, but there are some inputs available in both periods $J = J_0 \cap J_1$. The cost ratio between the two periods can be measured by the price index developed by Sato (1976) and Vartia (1976)

$$\frac{c(p_1, q_1, I_1, J_1)}{c(p_0, q_0, I_0, J_0)} = \left(\frac{\lambda_1(J)}{\lambda_0(J)} \right)^{1/(\sigma-1)} \prod_{i \in I} \left(\frac{p_{i1}}{p_{i0}} \right)^{w_i(I)} \prod_{j \in J} \left(\frac{q_{j1}}{q_{j0}} \right)^{w_j(J)} \tag{3}$$

where the weights $w_i(I)$ and $w_i(J)$ are constructed from the expenditure shares $s_{\kappa t}(\mathrm{K}) \equiv q_{\kappa t} z_{\kappa t} / \sum_{\kappa \in \mathrm{K}} q_{\kappa t} z_{\kappa t}$ as:

$$w_{\kappa}(\mathrm{K}) \equiv \left(\frac{s_{\kappa 1}(\mathrm{K}) - s_{\kappa 0}(\mathrm{K})}{\ln s_{\kappa 1}(\mathrm{K}) - \ln s_{\kappa 0}(\mathrm{K})} \right) \Bigg/ \sum_{\kappa \in \mathrm{K}} \left(\frac{s_{\kappa 1}(\mathrm{K}) - s_{\kappa 0}(\mathrm{K})}{\ln s_{\kappa 1}(\mathrm{K}) - \ln s_{\kappa 0}(\mathrm{K})} \right) \tag{4}$$

The value of $\lambda_1(J)$ and $\lambda_0(J)$ are constructed as:

$$\lambda_t(J) = \left(\frac{\sum_{j \in J} q_{jt} z_{jt}}{\sum_{j \in J_t} q_{jt} z_{jt}} \right) = 1 - \left(\frac{\sum_{j \in J_t, j \notin J} q_{jt} z_{jt}}{\sum_{j \in J_t} q_{jt} z_{jt}} \right), \quad t = 0,1 \tag{5}$$

The term $\lambda_t(J)$ is the period t expenditure on the imported inputs in the set J, relative to that period's expenditure on the total imported inputs. It can also be understood to be 1 minus the period t expenditure on the new imported input, relative to the period t total expenditure on import. $\lambda_1(J)$ will be lower if there is a greater number of new imported inputs in period 1. The lower value of $\lambda_1(J)$ will lead to a lower value of the first ratio on the right hand side of equation (3), because $1/(s-1) > 0$. In conclusion, any new import variety in period 1 will reduce the unit cost of period 1 compared to that of period 0.

$\Delta VAR_{t-1,t}$ in this case will be the change in import variety of two years *t-1* and *t*, and to be defined as follows:

$$\Delta VAR_{t-1,t} = \ln\left(\frac{\lambda_{t-1}(J)}{\lambda_t(J)} \right) = \ln\left(\frac{\sum_{j \in J_t} q_{jt} z_{jt} \Big/ \sum_{j \in J} q_{jt} z_{jt}}{\sum_{j \in J_{t-1}} q_{jt-1} z_{jt-1} \Big/ \sum_{j \in J} q_{jt-1} z_{jt-1}} \right) \tag{6}$$

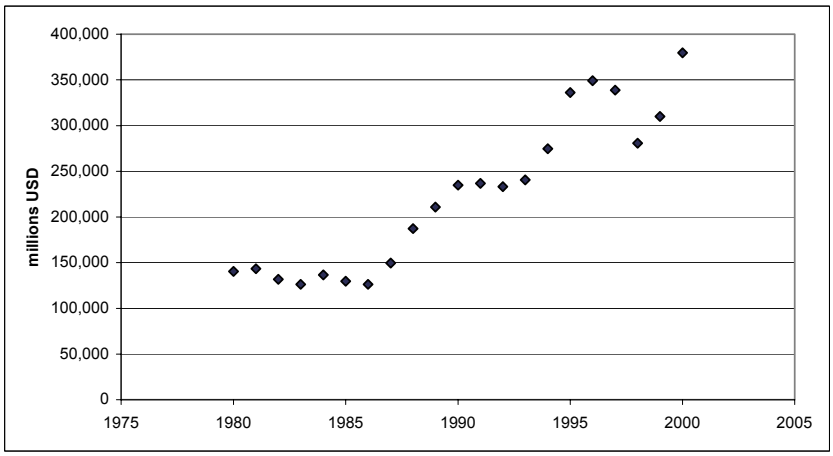
We can derive the same variety index as in (6) for export variety, with $\sigma < 0$. In the following part of the paper, I will use these indices to measure the changes in import and export varieties of Japan.

3. Data

I will use disaggregated trade data of Japan for the period 1980–2000 to construct the import and export variety indices. Figure 1 and figure 2 show the total of import and export volumes of Japan from 1980 to 2000. Import volumes were quite stable in the 1980s. However, during the 1990s, there were significant changes in the import volume of Japan with a sharp increase in 1993–1995 and a fall in 1997 and 1998. For exports, in the 1980s, the volumes steadily increased. In the early 1990s, despite stagnation, Japan's export volume still increased. However, there was some slowdown in exports in the late 1990s.

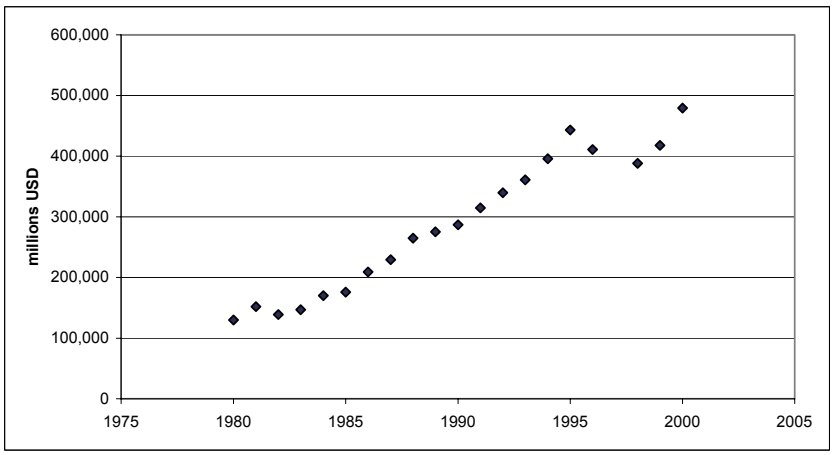
In order to construct variety indices and to maintain consistency in the classification of goods, I use the highly disaggregated trade data at the five-digit level of SITC revision 2 for Japan from 1980–2000. The classification distinguishes 1,473 commodities according to the Standard International Trade Classification (SITC Revision 2). Each commodity category will also differ if it is produced in a different country. In other words, the origin of the product plays an important role in defining the characteristics of the product. Therefore, I define a good to be a four or five digit SITC-2 category, and a variety is the import of a particular good from a particular country (as in Armington, 1969 and Broda and Weinstein, 2006).

All the trade data are collected from the United Nations' COMTRADE database. I have divided the industries into 21 sectors, including primary and secondary industries. Table 2 and table 3 show the comparison of simple count-based varieties of those sectors (using our definition of variety) and total varieties between 1980 and 2000. We can see a sharp increase in import varieties in this period, from a total of 23885 varieties in 1980 to 36684 varieties in 2000, implying an increase of more than 50%. In contrast, export variety by the simple count-based method decreased quite sharply, from 58403 varieties in 1980 to 43552 varieties in 2000, meaning a decrease of nearly 30%.



Source: UN's Comtrade database

Figure 1 Japan's imports (1980−2000)



Source: UN's Comtrade database

Figure 2 Japan's exports (1980−2000)

4. Import and export varieties of Japan

Table 2 and table 3 are only the simple count-based varieties, which provide us with a rough estimate of the changes in variety. In this section, I will use the variety index calculation as developed in previous section to provide more comprehensive results. The variety index calculation also includes the volume of the imported or exported goods ($p_{it}\, x_{it}$) thus giving the weights to each variety. To compare the changes of variety between two years t and $t\text{-}1$, I will calculate $\Delta VAR_{t\text{-}1,t}$ by using equation (6) and multiplying it by 100.

Table 2 Simple count-based variety in Japan's imports (1980–2000)

	Industry	1980	2000
1	Agriculture	1607	2292
2	Food and kindred products	1536	2330
3	Textile mill products	2363	3146
4	Apparel	2036	4015
5	Lumber and wood	648	891
6	Furniture and fixture	237	354
7	Paper and allied	499	742
8	Printing, publishing and allied	398	444
9	Chemicals	2977	4364
10	Petroleum and coal products	278	337
11	Leather	419	462
12	Stone, clay, glass	1047	1696
13	Primary metal	1427	1960
14	Fabricated metal	1174	1699
15	Machinery, non-elect	2780	4402
16	Electrical machinery	1382	2466
17	Motor vehicles	220	417
18	Transportation equipment and ordnance	147	213
19	Precision instruments	630	1617
20	Rubber and misc. plastics	534	859
21	Misc. manufacturing	1546	1978
	Total	**23885**	**36684**

Source: UN's Comtrade database, counts compiled by author

Figure 3 and figure 4 show the changes in import and export varieties for 21 sectors of Japan during period 1980–2000.

The index $\Delta VAR_{t\text{-}1,t}$ presents the percent change of variety between two years t and $t\text{-}1$. A positive value of the index shows an increase in variety and a negative value shows a decrease in variety. In figure 3, 11 industries show downward trends of import varieties, with many variety indices below zero. Those industries are food and kindred products, apparel, lumber and wood, furniture and fixture, paper and allied, printing, publishing and allied, leather, stone, clay, glass, primary metal, non-electrical machinery and precision instruments. As mentioned in the introduction, the period 1980–2000 witnessed the conclusion of many bilateral trade agreements between Japan and the US. We expect that these agreements, with the desire of the US to increase exports to Japan, would increase import variety of Japan during the period. However, the graph does not show an increase in the varieties of targeted industries like paper products, wood products, leather and electrical products. This can be explained more clearly in Greaney (2001), in which the author studies the impacts of the US-Japan Trade Agreements during 1980–1995 and concludes that the expansion of the US exports to Japan created by these agreements was very limited.

Table 3 Simple count-based variety in Japan's exports (1980−2000)

	Industry	1980	2000
1	Agriculture	756	689
2	Food and kindred products	958	923
3	Textile mill products	5915	3846
4	Apparel	2642	1839
5	Lumber and wood	606	338
6	Furniture and fixture	589	433
7	Paper and allied	1309	992
8	Printing, publishing and allied	876	662
9	Chemicals	7807	6424
10	Petroleum and coal products	427	272
11	Leather	179	105
12	Stone, clay, glass	1648	1284
13	Primary metal	4091	2861
14	Fabricated metal	4950	3419
15	Machinery, non-elect	9436	7844
16	Electrical machinery	5279	3818
17	Motor vehicles	478	353
18	Transportation equipment and ordnance	447	372
19	Precision instruments	4480	3074
20	Rubber and misc. plastics	1531	1374
21	Misc. manufacturing	3999	2630
	Total	**58403**	**43552**

Source: UN's Comtrade database, compiled by author

Table 3 shows the decrease of export variety by the simple count-based method. If we look at figure 4, we can find the same result: 9 among 21 industries show downward trend of export varieties, with a lot of variety indices below zero. Those industries are food and kindred products, furniture and fixture, printing, publishing and allied, chemicals, leather, primary metal, fabricated metal, non-electrical machinery and motor vehicles. Only two industries, which are electrical machinery and miscellaneous manufacturing, show an upward trend of export varieties from 1980 to 2000.

5. Conclusion

This paper provides a complement to Feenstra's variety index, with a focus on import and export varieties. Based on this calculation method, we measure Japan's export and import varieties over 21 years, from 1980 to 2000.

The result suggests that both export and import variety of Japan show downward trends in many industries. Specialization and the expansion of foreign direct investment from mid-1980s might have reduced the range of imported

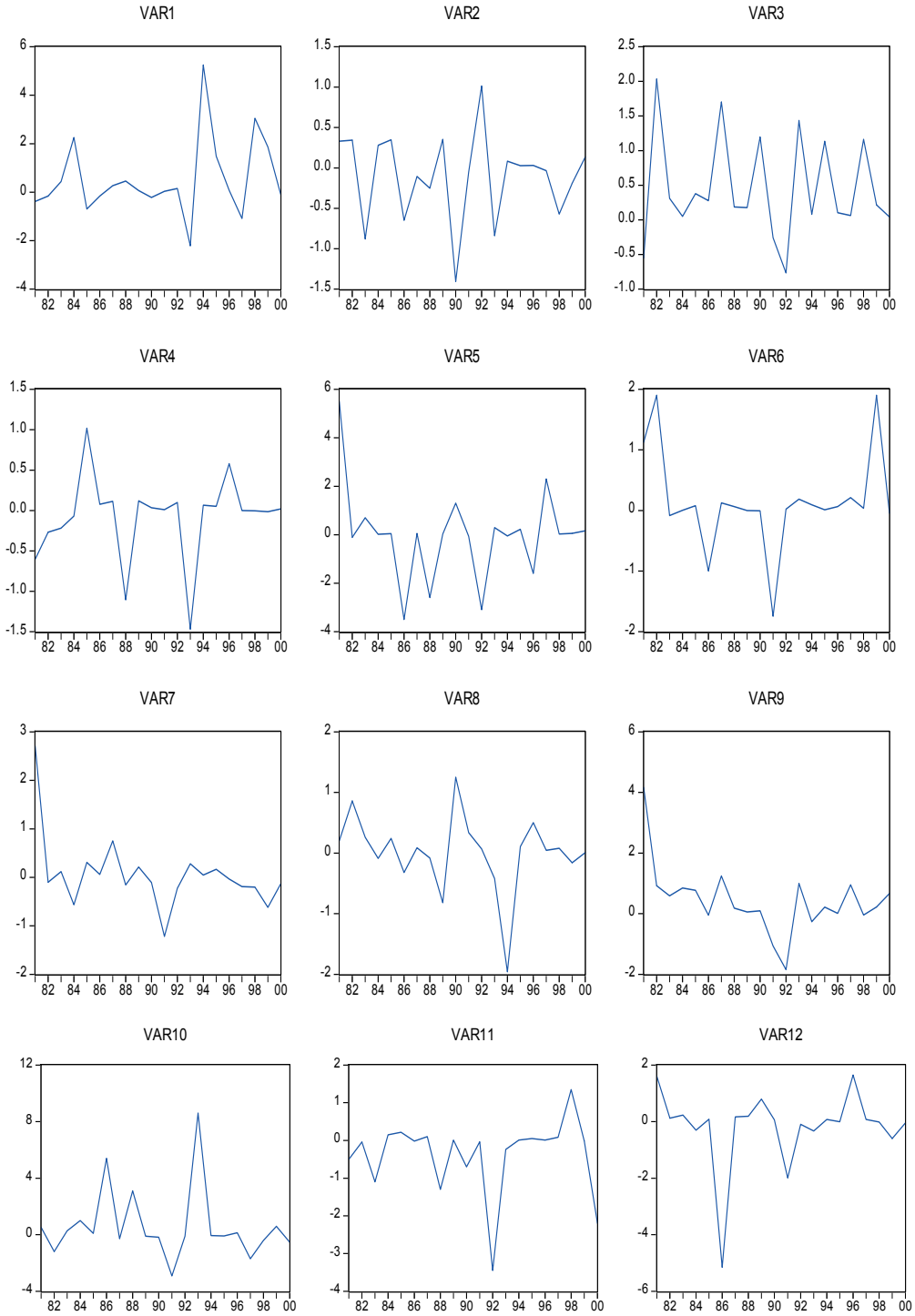
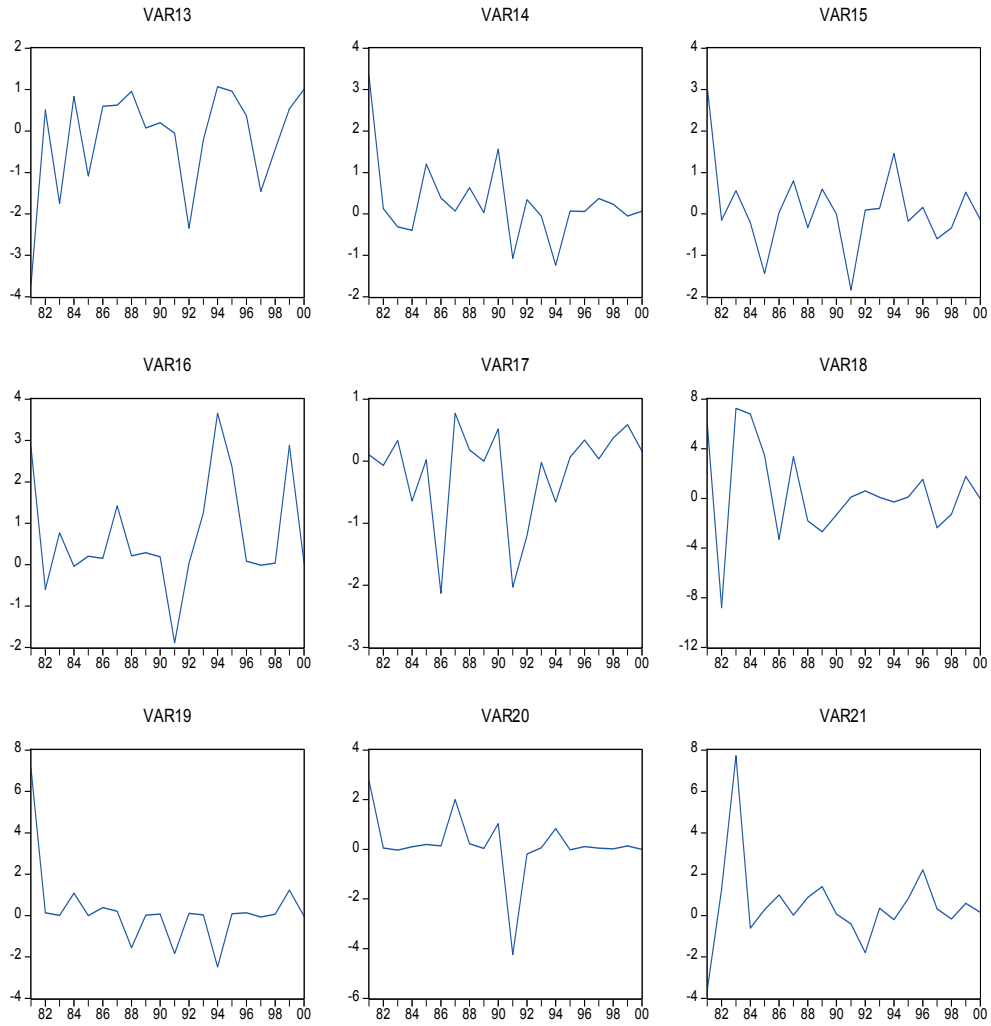


Figure 3 Changes in Japan's import varieties for 21 industries (1980–2000)



Note: The numbers 1 to 21 stand for the names of the 21 industries as presented in table 2 and table 3

Figure 3 continued

and exported goods, as we expected. These negative effects on variety might be larger than the positive effects of more expenditure on R&D and the expansion of foreign markets during the period.

The variety indices calculated in this paper are the percent change of variety between two years. This paper provides import and export variety indices of Japan during period 1980−2000. As suggested by endogenous growth theory, the changes of variety may have effects on other economic indices or measures, such as Total Factor Productivity, Gross Domestic Product or welfare gains. The results of this paper, therefore, can be used for further empirical studies .

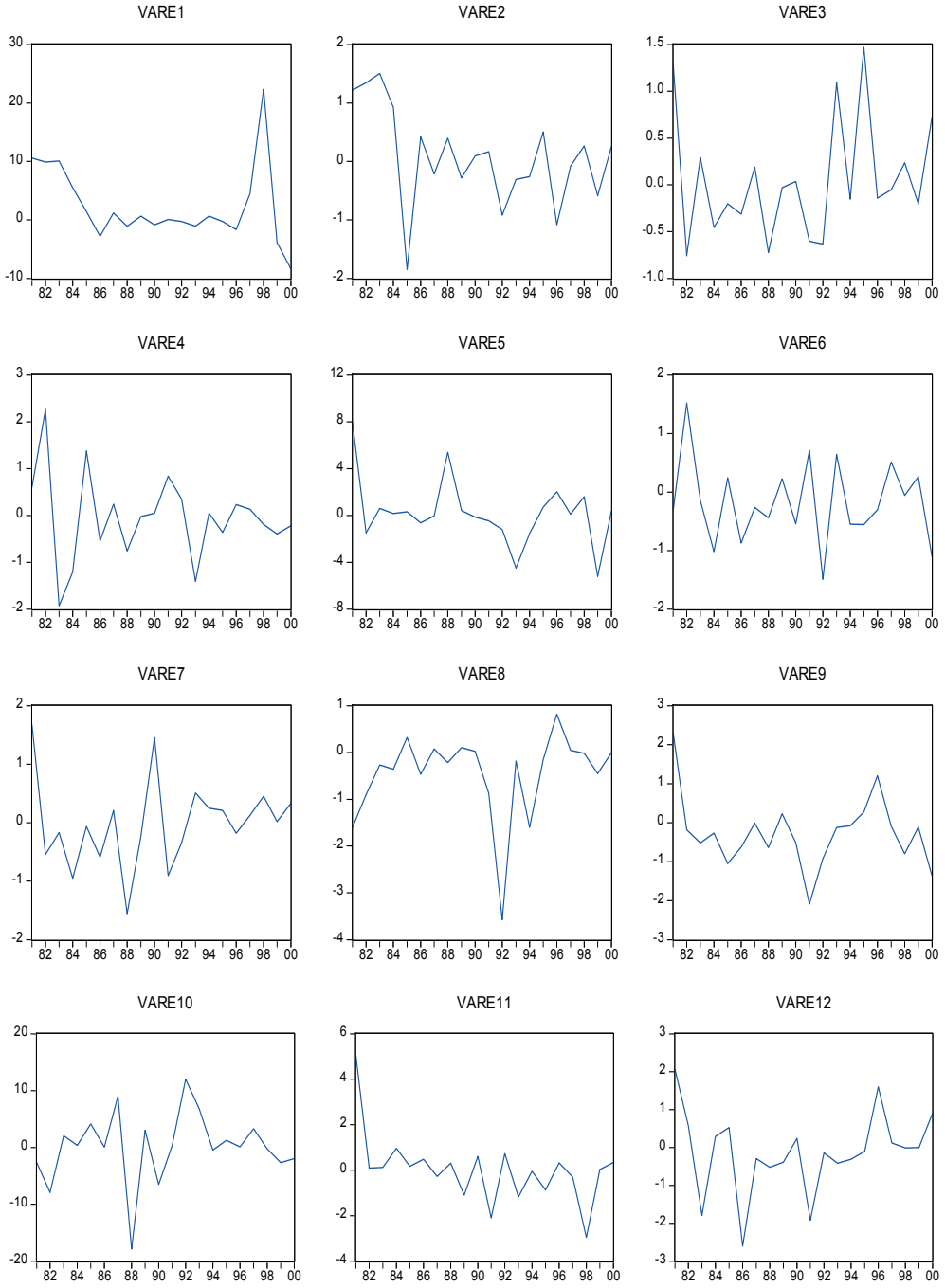
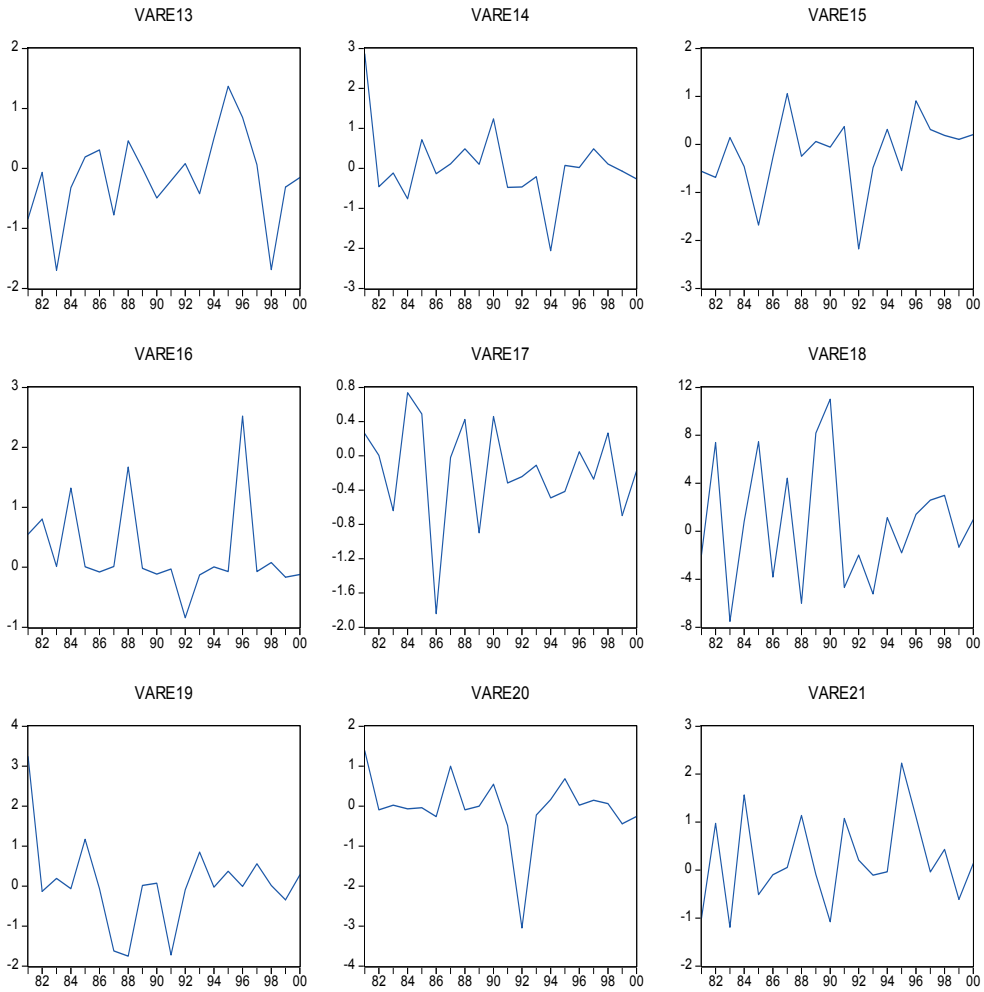


Figure 4 Changes in Japan's export varieties for 21 industries (1980–2000)



Note: The numbers 1 to 21 stand for the names of the 21 industries as presented in table 2 and table 3

Figure 4 continued

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Appendix

Unit-cost function derivation

In each period t, the firm maximizes its profit in producing y based on the production function in (1) as described in section 2:

$$y = f(x, z, I, J) = \left[\sum_{i \in I} a_i x_i^{(\sigma-1)/\sigma} + \sum_{i \in J} b_j z_j^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{A1}$$

The firm faces the following budget constraint:

$$B = \sum_{i \in I} p_i x_i + \sum_{j \in J} q_j z_j \tag{A2}$$

Then the firm will produce product y according to the production function (A1) with the budget constraint (A2). The maximization problem of the firm will be:

$$L=\left[\sum_{i\in I}a_i x_i^{(\sigma-1)/\sigma}+\sum_{i\in J}b_j z_j^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}+\lambda\left(B-\sum_{i\in I}p_i x_i-\sum_{j\in J}q_j z_j\right)$$

$$\frac{\partial L}{\partial x_i}=\left[\sum_{i\in I}a_i x_i^{(\sigma-1)/\sigma}+\sum_{i\in J}b_j z_j^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}a_i x_i^{-1/\sigma}-\lambda p_i=0$$

$$\frac{\partial L}{\partial z_j}=\left[\sum_{i\in I}a_i x_i^{(\sigma-1)/\sigma}+\sum_{i\in J}b_j z_j^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}b_j z_j^{-1/\sigma}-\lambda q_j=0$$

From the above maximization problem, we have:

$$\frac{x_1}{x_2}=\left(\frac{a_1p_2}{a_2p_1}\right)^{\sigma},\quad \frac{x_1}{x_3}=\left(\frac{a_1p_3}{a_3p_1}\right)^{\sigma},\ldots\quad \frac{x_1}{z_1}=\left(\frac{a_1q_1}{b_1p_1}\right)^{\sigma},\quad \frac{x_1}{z_2}=\left(\frac{a_1q_2}{b_2p_1}\right)^{\sigma},\ldots \tag{A3}$$

$$\Rightarrow\left(\frac{p_1}{a_1}\right)^{\sigma}x_1=\left(\frac{p_2}{a_2}\right)^{\sigma}x_2=\ldots=\left(\frac{q_1}{b_1}\right)^{\sigma}z_1=\left(\frac{q_2}{b_2}\right)^{\sigma}z_2=\ldots \tag{A4}$$

Substituting (A3), (A4) into the budget constraint in (A2), we obtain:

$$p_1\left(\frac{a_1p_2}{a_2p_1}\right)^{\sigma}x_2+p_2\left(\frac{a_2p_3}{a_3p_2}\right)^{\sigma}x_3+\ldots+q_1\left(\frac{b_1q_2}{b_2q_1}\right)^{\sigma}z_2+q_2\left(\frac{b_2q_3}{b_3q_2}\right)^{\sigma}z_3+\ldots=B$$

$$\Rightarrow p_1\left(\frac{a_1}{p_1}\right)^{\sigma}\left(\frac{p_1}{a_1}\right)^{\sigma}x_1+p_2\left(\frac{a_2}{p_2}\right)^{\sigma}\left(\frac{p_1}{a_1}\right)^{\sigma}x_1+\ldots$$
$$+q_1\left(\frac{b_1}{q_1}\right)^{\sigma}\left(\frac{p_1}{a_1}\right)^{\sigma}x_1+q_1\left(\frac{b_2}{q_2}\right)^{\sigma}\left(\frac{p_1}{a_1}\right)^{\sigma}x_1+\ldots=B$$

$$\Rightarrow\left(\frac{p_1}{a_1}\right)^{\sigma}x_1\left(\sum_{i\in I}a_i^{\sigma}p_i^{1-\sigma}+\sum_{j\in J}b_j^{\sigma}q_j^{1-\sigma}\right)=B$$

$$\Rightarrow x_1=\frac{B}{\left(\sum_{i\in I}a_i^{\sigma}p_i^{1-\sigma}+\sum_{j\in J}b_j^{\sigma}q_j^{1-\sigma}\right)}\left(\frac{a_1}{p_1}\right)^{\sigma}$$

Similarly for other values of x_i and z_i, we have the following expressions for x_i and z_i

$$x_i = \frac{B}{\left(\sum_{i\in I} a_i^{\sigma} p_i^{1-\sigma} + \sum_{j\in J} b_j^{\sigma} q_j^{1-\sigma}\right)} \left(\frac{a_i}{p_i}\right)^{\sigma}$$

$$z_j = \frac{B}{\left(\sum_{i\in I} a_i^{\sigma} p_i^{1-\sigma} + \sum_{j\in J} b_j^{\sigma} q_j^{1-\sigma}\right)} \left(\frac{b_j}{q_j}\right)^{\sigma}$$

With the above equations for x_i and z_i, the firm will come up with the following unit-cost function as in (2)

$$c(p_t, q_t, I_t, J_t) = \left[\sum_{i\in I_t} a_i^{\sigma} p_{it}^{1-\sigma} + \sum_{j\in J_t} b_i^{\sigma} q_{jt}^{1-\sigma}\right]^{1/(1-\sigma)} \tag{A5}$$

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