学位論文及び審査結果の要旨

横浜国立大学

氏　　　　　　　名	唐 越之
学 位 の 種 類	博士（経済学）
学 位 記 番 号	国府博甲第 81 号
学位授与年月日	令和 5 年 3 月 23 日
学位授与の根拠	学位規則（昭和 28 年 4 月 1 日文部省令第 9 号）第 4 条第 1 項及び横浜国立大学学位規則第 5 条第 1 項
研究科(学府)・専攻名	国際社会科学府経済学専攻
学 位 論 文 題 目	Tests of Criticality for Branching Process with Immigration〔移民項のある分枝過程における臨界性検定〕
論 文 審 査 委 員	主査　横浜国立大学　永井 圭二　教授 横浜国立大学　藤生 源子　教授 横浜国立大学　古川 知志雄　准教授 京都大学　西山 慶彦　教授 京都工芸繊維大学　人見 光太郎　教授

論文の要旨

Branching processes serve as a mathematical model of a population in which each individual in generation n produces some random number of offspring in generation n+1. This study considers the case in which the offspring of each individual in each generation have the same fixed probability distribution that does not vary from generation to generation, or from individual to individual. Branching processes can be used to model not only population growth, but also cell kinetics, spreads of an infectious disease, etc.

A central problem in the theory of branching processes is whether the population in question is to explode or to extinct after some finite number of generations. It can be shown that starting with one individual in generation zero, the expected size of generation n equals to m^n where m is the expected number of offspring of each individual. If m < 1, the expected number of individuals goes rapidly to zero, which implies ultimate extinction with probability 1. This study uses a branching model with nonnegative immigration in each generation, so that the population does not become extinct.

Since the behaviors of branching processes differ depending on whether the offspring mean m is greater or less than 1. When m=1 (m<1 , m>1), the process is called a critical (resp. subcritical, supercritical) process. This study considers tests for the criticality of the offspring mean against the local alternatives including both super-critical and sub-critical hypotheses for branching processes with immigration. The effect of initial values, sample sizes on the test results and the effectiveness of the local alternatives are discussed.

This study is organized as follows. After the introduction of this study in Section 1, Section 2 describes the model and testing hypotheses, and gives the estimator of offspring mean as a test statistic and estimators of other parameters with preliminary asymptotic results.

Part I advocates the fixed-sample-size criticality tests (FCT). The FCT in this study is analyzed in the same method as the Dickey-Fuller unit root test in autoregressive models. First, the discrete-time branching process with immigration and close-to-one offspring mean is approximated to the continuous-time squared Bessel (BESQ) process with drift. Then the limiting test statistic is represented as the ratio of a BESQ and its integral, whose joint density can be derived from the theory of Bessel bridge in Pitman and Yor (1982). Rejection regions for the left-tailed and right-tailed criticality tests are determined and asymptotic powers are computed. The theoretical values of the asymptotic distribution of test statistic and the asymptotic power of the FCT are calculated using Mathematica and shown to be consistent with the simulation results. Numerical results show that for all m, the power of FCT increases as the sample size N increases. It is found that the dimension of the BESQ process determines the critical value of the FCT rejection region. The difficulty in the empirical analysis is that the dimension is usually unknown. This study provides a linear interpolation solution via using the estimator of the dimension. For the estimated dimension, the critical value is determined in the following way. First, critical values for sufficiently many dimensions are calculated, and that for the estimated dimension is determined by linear interpolation. Numerical results show that the linear interpolation method works well under the subcritical alternative hypothesis and under the supercritical alternative hypothesis, especially in the neighborhood of m=1. More specifically, the powers calculated by the linear interpolation fit with the theoretical results over the entire range of m under the subcritical alternative hypothesis with sample size N more than 100, or under the supercritical alternative hypothesis with sample size N more than 50. As for the circumstances when sample size N is less than 50 for the left-tailed test, or when sample size N is less than 30 for right-tailed tests, the simulated powers using the linear interpolation do not fit with the theoretical values when the local parameter drifts away from 0, but these powers still fit with the theoretical results well for m in (0.95,1.05), which is sufficient for the needs of criticality tests. Non-local alternative hypotheses should be used in such cases for m not in (0.95,1.05).

If the initial value is not negligible, the figures indicate that the linear interpolation method is valid in either the right-tailed or left-tailed test not only for local parameters close to 1, but also for nonlocal parameters away from 1.

Part II explores Sequential criticality tests (SCT) using stopping times based on observed Fisher information. The time change method is also implemented to transform the BESQ process into a Bessel (BES) process, thereby the asymptotic properties of the sequential test statistic for the SCT are investigated. Especially, Part II succeeds in deriving the asymptotic joint distribution of the sequential test statistic and the stopping time via the time change of the joint density, derived in Part I, of the BESQ and its integral. The sequential test statistic is found to be represented as a time-changed Brownian motion with the local parameter as drift, indicating that the SCT is actually a Z-test and that the initial value has no effect on the rejection region or the power. On the other hand, the stopping time is terminated earlier as the initial value is larger.

If the stationary alternative hypothesis is true, the stopping time tends to be terminated later. Therefore, in the SCT an upper bound is set at the 99th percentile point of the distribution of the stopping time under the null hypothesis. One attempt in this study is to perform a combined sequential test that rejects the null hypothesis when the stopping time exceeds the 99th percentile point. This prevents the sample size of the sequential test from becoming too large. The operating characteristics of the combined tests can be computed from the joint density of the sequential test statistic and the stopping time.

Part III validates the effectiveness of the testing methods for the local hypotheses in the FCT in Part I and the SCT in Part II. Comparisons are made with the testing methods for non-local stationary hypotheses: for the FCT, comparisons of power are made, and for the SCT, comparisons of the joint moments of the stopping time and sequential test statistics are made. It is checked whether the simulation results conform to the theoretical values computed from the local model or from the stationary model. It is confirmed that the local models perform better than the stationary model in the neighborhood of m=1.

In the case of the FCT, as is well known, the test statistic is normally distributed under the non-local stationary alternatives when the initial value can be neglected. Thus, the powers of the non-local FCT are computed via Z-test and compared to those of the local FCTs. It turns out that the power performances of the FCT in Part I are always better than that computed by a Z-test under the stationary alternative hypotheses for all m, but the difference between these two tests decreases as m drifts away from 1.

For the SCT, the sequential test statistics are normally distributed under both the non-local stationary alternatives and the local alternatives, thus comparisons of the joint moments of the stopping time and sequential test statistic are made. The joint moments of the stopping time and sequential test statistic under the null hypothesis are computed via the joint Laplace transform. Girsanov transformation is used to obtain the joint Laplace transform under the alternative hypothesis. The moments calculated from the local model fit well with the simulation results in the neighborhood of m=1. In other words, the local model performs better than the stationary model at the neighborhood of m=1, especially when the level c of the observed Fisher information is small. More specifically, the local model performs better for $0.9 \le m \le 1.1$ when the level c is around 2500, or for $0.95 \le m \le 1.05$, when the level c is around 10000. At the same time, moments calculated by the stationary model fit with the simulation results for $m < 0.9$ when the level c is around 2500, or for $m \le 0.95$ when c is around 10000. These results imply that when the level c of the observed Fisher information is large enough, the stationary model could perform well even in the neighborhood of m=1. However, when the c is small, the range in which the stationary model performs well becomes narrow. This study helps to decide whether local or non-local stationary models should be applied in inference. Those decisions differ depending on the value of c.

審査結果の要旨

本論文では、移民項のある分枝過程が局所パラメータで表される１に近い基本再生産数 m を有するとき、優臨界対立仮説(m>1)と劣臨界対立仮説(m<1)に対する臨界性検定について漸近理論を構築している。論文は３部構成となっており、第Ⅰ部では固定サンプルサイズの臨界性検定(fixed-sample-size criticality test; FCT)について、第Ⅱ部ではオンラインデータを想定した停止時刻を用いる逐次臨界性検定(sequential criticality test; SCT)について、それぞれ初期値の影響を考慮した漸近理論を構築している。第Ⅲ部では、FCT 理論から求められる検出力を定常理論から求められる検出力と比較し優位性を示し、また SCT については停止時刻と逐次検定統計量の同時ラプラス変換を導出し、それにより求められるモーメントの理論値をシミュレーションと比較し有効性を確認している。

第Ⅰ部で扱う FCT の理論では、Wei and Winnicki (1990,p1771)が提起した分枝過程の臨界性検定の問題、すなわち基本再生産数が１に近い分枝過程において、どの臨界性の状態にあるかを検定するという問題を解く。第Ⅱ部で扱う SCT の理論では、第Ⅰ部と同じ問題をオンラインデータ観測されたデータに対して Sriram, Basawa, and Huggins (1991) の逐次推定の方法を使って解決する。

第Ⅰ部では、基本再生産数 m の最尤推定量で表される FCT の検定統計量の累積分布関数を、Pitman and Yor (1982)の Bessel bridge の理論から逆ラプラス変換を実行することで導出する。ここでは、初期値がある場合とない場合についての検定統計量の棄却域と検出力を、子孫分散と移民平均の２つのパラメータに依存する形で計算している。また子孫分散と移民平均を推定した場合の FCT の棄却域を、線形補間の方法によって提案し、サンプルサイズが小さい場合は２つのパラメータが既知の場合に比べて極限理論値よりも少し検出力が落ちるが、大きい場合はよく一致することをシミュレーションにより確認している。

第Ⅱ部で扱う SCT の理論では、観測されるフィッシャー情報量に基づく停止時刻に対し基本再生産数の逐次最尤推定量を検定統計量とし、停止時刻と検定統計量の極限結合密度関数が求められている。また、停止時刻については帰無における周辺密度関数も求められている。さらに、劣臨界対立仮説の場合に、臨界仮説（m=1）における停止時刻の 99 パーセント点を上限に設定し、停止時間が上限に達した時点で帰無仮説を棄却することを SCT に組み込み、固定サンプルサイズと逐次的な方法を複合した検定法も提案している。

第Ⅲ部では、まず FCT について、劣臨界対立仮説に対し局所理論から求めた検出力を非局所強

定常理論から求めた検出力と比較し､前者が必ず後者より大きくなることを理論値から確かめている。SCT については、逐次検定統計量と停止時刻の結合密度関数は停止時刻の値が 0 に近いところで大きく振動する欠点があるので、検定統計量と停止時刻の結合ラプラス変換を求めている。これにより検定統計量と停止時刻の同時モーメントが計算できる。ここでは、シミュレーション結果にたいして､結合ラプラス変換から求めたモーメントの理論値と非局所強定常理論から求められるモーメントの理論値のどちらがフィットするかを調べ､m が 1 の近傍にある時は、結合ラプラス変換から求めたモーメントが有効であることを確かめている。

本研究に対する課題は以下のとおりである。第 I 部の FCT の検定統計量については基本再生産数の局所パラメータの最尤推定量を用いるより、t 検定統計量を用いる方が、子孫分散と移民平均の２つのパラメータに対し頑健な結果が得られると考えられる。すなわち、t 統計量の方が、数表の作成や線形補間による棄却域の臨界値の決定が容易であると考えられる。第Ⅱ部の SCT に関しては、提案している停止時刻に上限を設定する複合的検定方法について、動作特性に対する十分な考察が加えられていない。第Ⅲ部の非局所理論との比較は劣臨界対立仮説についてのみ行っている。非局所優臨界対立仮説の時の逐次についての逐次理論非局所理論との比較が行われていない。

本論文の研究は以上のような課題を残してはいるが､学術的水準の高さおよび内容の豊富さを鑑みると博士学位授与に十分に値すると判断できる。

以上の博士号請求論文の内容及びその評価に基づき、審査委員会は、国際社会科学府学位基準②を適用し、唐越之氏の研究が博士号学位に値するものと判断する。