Malapportionment in Space and Time:
Decompose It!

Version: January 7, 2021

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Earlier versions of this paper entitled "The State of Malapportionment in the World: One Person, One Vote?" and "The State of Malapportionment in the World" were delivered at the Annual Conference of the Australian Political Studies Association, Sydney, on September 28–October 1, 2014, at the Annual Meeting of the American Political Science Association, Philadelphia, on September 1–4, 2016, and at the 2017 Asian Political Methodology Meeting, Sydney, Australia, on January 9–11, 2017. Replication materials are available at Dataverse (https://dataverse.harvard.edu/dataverse/yutakamahara).

Declarations of interest: none

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Abstract (109)

Existing studies on legislative malapportionment often conceptualize and measure this phenomenon with little regard to intertemporal variations and the malapportionment-generating process (MGP). Our conceptualization leads us to introduce a measure called α-divergence that can identify the vote inequality derived from various stages of MGP. Using an originally created database that covers 440 elections in 112 countries, we decompose the overall degree of malapportionment into three stages: malapportionment that arises at the stages of interstate apportionment, intrastate districting, and allotment of seats to special districts. We also provide analyses that can decompose the demographic and political factors contributing to the *trends* of the overall degree of malapportionment for selected countries.

Keywords: Malapportionment, Election, Measurement, α-divergence, Database

1. Introduction

Malapportionment—the discrepancy between the share of legislative seats and the share of population or electorate within a given geographical unit—is anathema to representative democracy. It violates the norm of "one person, one vote" in the representative democratic process and creates a number of undesirable governance consequences. For instance, highly malapportioned countries tend to have less progressive tax schemes (Ardanaz and Scartascini 2013) and a skewed distribution of government subsidies favoring overrepresented areas (Horiuchi and Saito 2003). Authoritarian political systems are more likely to persist. Highly malapportioned systems usually make authoritarian incumbents advantageous, as in the case of Malaysia (Ostwald 2013; Washida 2018).

Despite the normative and practical importance of this issue, knowledge about malapportionment is still very limited. In particular, there is much to be learned about questions such as the following: To what extent does malapportionment exist across countries and time? Which stage of seat distribution mainly determines the degree of malapportionment, that is, interstate apportionment, intrastate districting, or seat allocation to special districts? What factors influence changes in the degree of malapportionment over time? The extant measures and databases of malapportionment are not well equipped to address these questions in a comprehensive manner. To provide answers to the questions above, this paper proposes a new *conceptualization*, *measurement method*, and *database*. In particular, the property of our suggested measure, *decomposability*, enables us to find factors for the *degree* and the *trends* of malapportionment.

Specifically, the extant research suffers from the following measurement issues.

First, almost all extant studies conceptualize malapportionment without accounting for the process of distributing seats. Generally, malapportionment is best understood in two stages: (1) at the apportionment stage, seats are initially distributed along administrative units and (2) at the districting stage, the distributed seats are assigned to delimited districts in each administrative unit. Moreover, given the inflow and outflow of people from each constituency, malapportionment inevitably worsens under the same seat distribution. Therefore, we propose a reconceptualization that accounts for the malapportionment-generating process (MGP) in terms of each stage for seat distribution as well as trend patterns. Our conceptualization leads us to realize that to understand malapportionment, we must consider the process by which malapportionment is generated at each readjustment and over time.

Second, practitioners frequently utilize the ratio of largest-to-smallest districts (hereafter *MaxMin*), while scholars almost always employ Samuels and Snyder's (2001) *MAL*. However, the former measure focuses on only the bipolar outliers among all districts (Samuels and Snyder 2001, p. 654), and there are inherently several problems with the latter, such as not fulfilling the Pigou–Dalton condition that every measure of inequality must meet (Sen 1973). *MAL* and *MaxMin* cannot properly capture MGP regarding the degree and trends of malapportionment.

Third, the existing databases cross-nationally compare the degree of malapportionment across the world but do not provide chronological comparison (e.g., Samuels and Snyder 2001; Ong et al. 2017). At present, we cannot understand the variation of malapportionment over time. Thus, employing conventional measures and databases precludes us from capturing the intertemporal variations of malapportionment and MGP and allows us to grasp only the *static* situation of malapportionment at a single point.

To fill the gap between conceptualization, measurement, and database, we introduce a new measure of malapportionment to decompose the factors of vote inequality at several stages where malapportionment is generated and the factors for change in inequality at these stages such as demographic trends and political processes.

Our proposed measure of malapportionment is α-divergence ($\alpha \to 0$), or Kullback–Leibler divergence, which is often used in information geometry (e.g., Amari 2009). Wada and Kamahara (2018) present this measure and calculate it using the same source as Samuels and Snyder (2001) and information for Italian and Taiwanese elections. We expand it in space and time. The two conventional measures, *MAL* and *MaxMin*, have defects, whereas α-divergence does not; thus, our measure is superior to theirs. It can decompose the overall degree of malapportionment into the contribution for each stage of seat allocation. Moreover, this paper demonstrates that using the change in the degree of malapportionment from one election to the next, we can detect the demographic and political contributing factors of malapportionment.

Furthermore, we provide the most extensive database currently available to capture the history of malapportionment across countries. Our database comprises data from 112 countries and 440 elections. Presently, Ong et al. (2017) develop the largest cross-national database (i.e., 160 countries), but they only provide a single data point (i.e., the specific election year) per country, whereas our database encompasses several election years per country. Our new conceptualization, measure, and database enable us to comprehend malapportionment by allowing us to compare the degree of malapportionment in space and time. Moreover, this paper also reminds us of the importance of measurement before conducting causal analysis because the well-defined measure can endogenously identify several factors for generating the value of the measure.

This paper is organized as follows. Section 2 formulates our conceptualization of malapportionment on the basis of MGP. Section 3 first discusses the three measures of malapportionment that we employ in our study. This section then describes the notable property of our measure, *decomposability*, and illustrates how one can decompose the sources of malapportionment. Section 4 applies our new measure to our original database, and Section 5 demonstrates the variation in malapportionment in time and space. Section 6 focuses on a decomposition analysis of the degree of and changes in malapportionment. This section illustrates how to decompose the shift in malapportionment over time. The concluding section summarizes our findings and suggests future research directions.

2. A Reconceptualization from the Malapportionment-generating Process

Malapportionment refers to "the discrepancy between the shares of legislative seats and the shares of population held by geographical units" (Samuels and Snyder 2001, p. 652). If a country adopts a nationwide single constituency, as in the case of legislative elections in Israel and most presidential elections, there is no malapportionment. In other words, malapportionment is an issue that needs to be addressed in most legislative elections as very few countries adopt a nationwide constituency. Although the measurement of malapportionment has been studied (e.g., Monroe 1994), the concept of malapportionment itself has not been fully discussed. In this section, therefore, we reconceptualize malapportionment through our simple mathematical formalization.

Let $\frac{N_j}{N}$ be the share of electorate or population and $\frac{n_j}{n}$ be the share of allocated seats in each geographical unit j, where N denotes the entire population or electorate $(N = \sum N_j)$, and n stands for entire seats $(n = \sum n_j)$. **P** represents the population quotient vector $(\frac{N_1}{N}, \frac{N_2}{N}, \ldots, \frac{N_k}{N})$ and **Q** represents the apportionment quotient vector $(\frac{n_1}{n}, \frac{n_2}{n}, \ldots, \frac{n_k}{n})$.[1] As referred at the beginning of this section, malapportionment is conceptualized as a discrepancy between **P** and **Q**. This discrepancy is measured using a function $f(\cdot)$. This general function includes several forms such as an absolute difference, or Manhattan distance *(MAL*, Samuels and Snyder 2001), Euclidean distance (Gallagher 1991), and *MaxMin*. In other words, malapportionment M can be expressed as follows:

$$M = f(\mathbf{P}, \mathbf{Q}). \tag{1}$$

Moreover, seat distribution is generally accompanied with a two-sequential process: the number of seats is initially determined for the administrative units according to the number of population or electorate (*apportionment*), and then by splitting the units into districts where the allocated seats within the units are distributed (*districting*). The process of malapportionment generated at any given election, MGP, should be able to be decomposed into apportionment and districting. Malapportionment can be caused at both stages; thus, the overall malapportionment M is decomposed into

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[1] In this paper, a bold letter stands for vector as **P** and **Q**.

the apportionment stage and the districting stage (M^A and M^D respectively):

$$M = M^A + M^D. \tag{2}$$

If a country employs a nationwide PR system, no malapportionment exists (i.e., $M = M^A = M^D = 0$). If electoral district conforms to administrative unit, there is no district within each unit; thus, the overall malapportionment is equal to apportionment stage one (i.e., $M^D = 0$ and $M = M^A \geq 0$). Conversely, if a country directly divides its entire territory into electoral districts without the apportionment-stage distribution of seats, then the overall malapportionment corresponds to districting stage one (i.e., $M^A = 0$ and $M = M^D \geq 0$). Identifying at which stage malapportionment is mainly generated is essential for decision makers, in order to make the value of a vote more equivalent.

To compare the degree of malapportionment cross-nationally, malapportionment can be formulated as Eqs. (1) and (2) around the same year. However, a chronological perspective is also required to fully understand the degree of malapportionment because malapportionment is not stable over years. For instance, Snyder and Samuels (2004) hold this perspective and calculate their measure (Samuels and Snyder 2001) of the degree of malapportionment for 11 Latin American countries between 1870 and 2000. They assume that malapportionment is generated through demographic and political processes. The former process, such as population changes involving movement from rural to urban areas, produces "natural malapportionment," or what we call demographic-driven malapportionment and in the latter process politicians manipulates the principle of "one person, one vote" for their political aims such as an electoral triumph, resulting in “unnatural malapportionment” or “politically-engineered malapportionment” (Snyder and Samuels 2004, pp. 137–138).[2] Given the existence of continuing demographic-driven malapportionment, without reapportionment or redistricting, we can expect an increase in the degree of malapportionment. Thus, the periodical readjustment of seat allocation and districting is quite essential to adhere to the principle of “one person, one vote.” Furthermore, if politicians and/or electorates demanded to achieve this principle over the years, we could see a decreasing trend in malapportionment. MGP can also be decomposed into several patterns typically shown in most time-series data. Thus, we draw on the components of a time-series data to reconceptualize malapportionment and MGP:

$$M_t = f(\mathbf{P}_t, \mathbf{Q}_t)$$

$$M_{t-1} = f(\mathbf{P}_{t-1}, \mathbf{Q}_{t-1})$$

$$\Delta M_t = M_t - M_{t-1} = f(\mathbf{P}_t, \mathbf{Q}_t) - f(\mathbf{P}_{t-1}, \mathbf{Q}_{t-1}) = \phi(\mathbf{\Delta P}, \mathbf{\Delta Q}; \mathbf{P}_{t-1}, \mathbf{Q}_{t-1}), \quad (3)$$

where t is a given election and $\phi(\cdot)$ is the simplified function of $f(\cdot_t) - f(\cdot_{t-1})$. M_t is a function of $\mathbf{P}_t$ and $\mathbf{Q}_t$, and ΔM_t is a function of $\mathbf{\Delta P}$ and $\mathbf{\Delta Q}$ between elections t and $t-1$. The value of $\mathbf{\Delta P}$ is the result of population movement between and/or within administrative units, and the value of $\mathbf{\Delta Q}$ is the result of a political process. In the study, political process includes reapportionment and redistricting decisions made

[2] Originally, Snyder and Samuels (2004) use the term “politically engineered malapportionment” (p. 138). We added a hyphen in accordance with demographic-driven malapportionment.

by incumbent legislators or a boundary authority under the influence of politics. Moreover, politicians can opt for inaction, such as not responding to population changes, to retain their overrepresented seat. Mathematically, $\mathbf{\Delta P} = \mathbf{0}$ indicates a lack of population movement, whereas $\mathbf{\Delta Q} = \mathbf{0}$ implies the absence of a redistribution or redistricting decision. Thus, we can decompose a time-series malapportionment data into a demographic component and a political component.

Based on this mathematical formulation, MGP theoretically includes at least four patterns. First, given that there usually exists demographic-driven malapportionment or movement of the population ($\mathbf{\Delta P} \neq \mathbf{0}$) without reapportionment and redistricting over years ($\mathbf{\Delta Q} = \mathbf{0}$, i.e., *inaction*), we can see incremental trend patterns in malapportionment ($\Delta M > 0$). Second, demographic-driven malapportionment is adjusted by regular seat redistribution ($\mathbf{\Delta P} \neq \mathbf{0}$, $\mathbf{\Delta Q} \neq \mathbf{0}$ and $\mathbf{\Delta Q} \propto \mathbf{\Delta P}$). For instance, Canada and the United States reapportion seats and redraw electoral districts every decade. This can be regarded as a kind of a seasonal pattern in the time-series process. Third, demographic-driven malapportionment is adjusted by abrupt intervention. Countries such as the United Kingdom and Sweden set rules to change electoral boundaries as a result of changes in demographic and/or administrative units (for details, see Handley's (2008) Appendix A). This shows a radical change with *action* ($\mathbf{\Delta P} \neq \mathbf{0}$, $\mathbf{\Delta Q} \neq \mathbf{0}$).[3] Fourth, there is a possibility that the movement of the

[3] Action includes the positive and negative interventions of politics. The authority lessens malapportionment by regulating and readjusting the seat-distribution process as well as worsens it by controlling and manipulating the process for the incumbent's

population itself could cause decremental patterns. This can be regarded as a decreasing trend without explicit action ($\mathbf{\Delta P} \neq \mathbf{0}$, $\mathbf{\Delta Q} = \mathbf{0}$, and $\Delta M < 0$).

To acknowledge these theoretical patterns based on Eq. (3), we illustrate the empirical patterns of malapportionment in India, Canada, and Australia. India avoided readjusting electoral boundaries between 1977 and 2004, so we can anticipate an upward trend in the degree of malapportionment or demographic-driven malapportionment. In Canada, under the same act on electoral boundaries redrawing in place since 1984,[4] we can expect a non-stationary pattern in malapportionment with a trend followed in the previous election, and periodical redistricting or a reduction in malapportionment regulated by provincial boundary commissions. In 1983 and 1984, Australia introduced the Commonwealth Electoral Legislation Amendment Act and an independent boundary authority, called the Australian Electoral Commission, respectively (Nohlen et al. 2001).[5] These reforms could change MGP radically in Australia.

Figure 1 verifies our expectations. The upper panels show the expected patterns based on natural trends, regular patterns, and an abrupt structural regime change, while the lower panels are empirical patterns of malapportionment in Indian, Canadian, and

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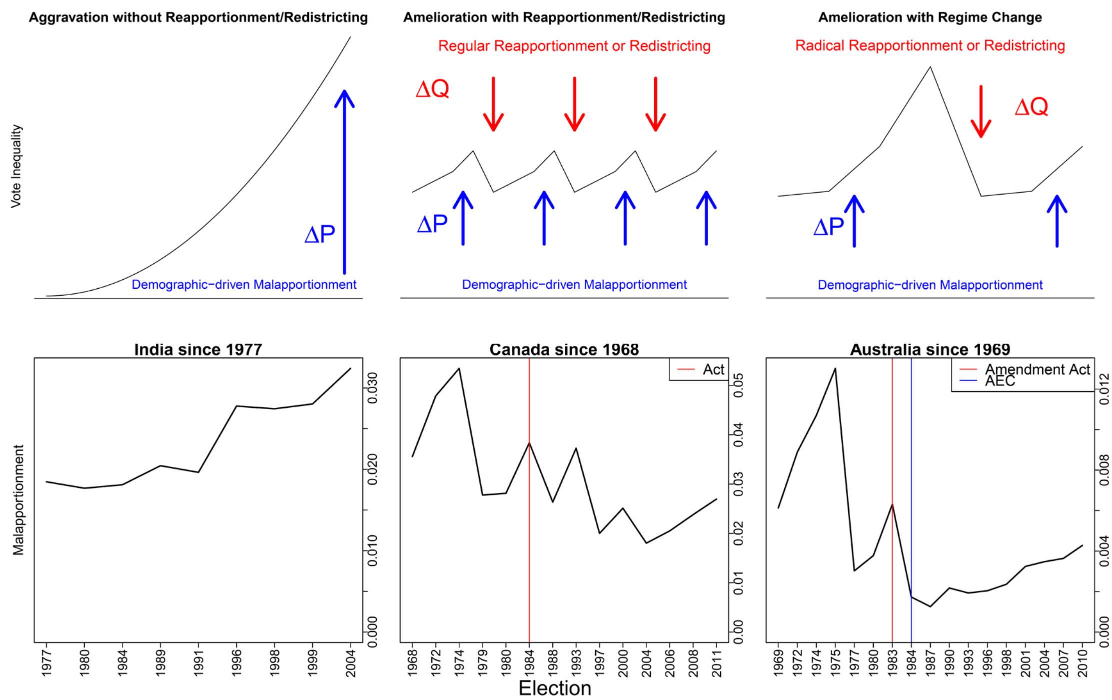
interests (e.g., Washida 2018).

[4] See ACE Project's Comparative Data. https://aceproject.org/epic-en/ (Accessed on September 16, 2020).

[5] See also the Commonwealth Electoral Legislation Amendment Act. https://www.legislation.gov.au/Details/C2004A02861 (Accessed on September 16, 2020) and the Australian Electoral Commission. https://www.aec.gov.au/ (Accessed on September 16, 2020).

Australian elections.[6] Figure 1 attests that the empirical examples show almost the same pattern as the expected patterns of malapportionment. This correspondence between the expectations and the empirical patterns proves the relevance of our reconceptualization of malapportionment based on MGP.

Figure 1 *Malapportionment-generating Processes in Expected Pattern and Empirical Data*



Scholars have ignored, intentionally or unintentionally, the process of malapportionment generated at several stages and over time when conceptualizing

[6] We calculate the degree of malapportionment using D^0 as proposed in Section 3.

malapportionment. One can, thereby, obtain only a fraction of information on, or the superficial state of, malapportionment using such a concept. In this section, we have reconceptualized malapportionment in a simple mathematical formula. Explicitly taking into account MGP, one can acquire a novel and deeper knowledge regarding the degree of malapportionment across countries and over years. In the two following sections, we discuss the problems of existing measures and databases and propose a new measure and database developed to satisfy our conceptualization of malapportionment.

3. Brief Discussion on Measuring Malapportionment

3.1 Introducing α-divergence ($\alpha \to 0$)

In this study, in order to gauge the degree of malapportionment, we compare three measures of malapportionment. The first measure is *MAL*, which is the most widely used measure among scholars. Samuels and Snyder (2001) introduce it as an index that employs the Loosemore–Hanby index (Loosemore and Hanby 1971):

$$MAL = \frac{1}{2}\sum\left|\frac{n_i}{n} - \frac{N_i}{N}\right|, \tag{4}$$

where i denotes a district, n_i the distributed seats in district i, and N_i the population or electorates in district i. This formula yields the aggregate value of the difference between the allocated seat share and the population share in each district.[7] If there is no

[7] In Eq. (1), $f(\mathbf{P},\mathbf{Q}) = \frac{1}{2}|\mathbf{Q} - \mathbf{P}|$.

difference, that is, if the value of *MAL* is zero, the country embodies the principle of "one person, one vote." If the value increases, it signifies that the legislature is composed of representatives selected from a more malapportioned electoral system. Samuels and Snyder (2001) apply this index to calculate malapportionment for Lower House and Upper House elections in 78 and 25 countries respectively. Their data set has been used by some scholars for analyzing the causes and the consequences of malapportionment (e.g., Ardanaz and Scartascini 2013; Horiuchi 2004).

Additionally, the ratio of largest-to-smallest districts, which we call *MaxMin*, is a frequently used method among practitioners such as lawyers and journalists. For instance, the Supreme Court of Japan ruled that the current delimitation scheme is an "unconstitutional situation." This ruling was based on the ratio between the number of voters in the largest and smallest districts, which was 1:2.43 for the 2012 Lower House election and 1:2.13 for the 2014 Lower House election.[8] Journalists and citizens' groups frequently use this measurement method as it is intuitive and easy to calculate. However, Samuels and Snyder (2001) dismiss the method as "poor" (p. 654) because *MaxMin* calculates only outliers even when the rest of the districts have the same number of voters; thus, it fails to assess the complete picture.

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[8] Nikkei Shinbun. https://www.nikkei.com/article/DGXNZO62908690R21C13A1MM8000/ (Accessed on September 16, 2020) and https://www.nikkei.com/article/DGXLASDG25HAS_V21C15A1MM8000/ (Accessed on September 16, 2020).

Nevertheless, Samuels and Snyder's *MAL* also performs poorly in certain circumstances. As Monroe (1994) points out, one problem that we can intuitively apprehend is *insensitivity* to the different types of seat allocation.[9] We assume three cases where the distributions of the electorate for each state are the same, but the allocation of seats for each state varies across the cases. Although the measure of malapportionment must capture the situation of the three cases differently, *MAL* as well as *MaxMin* produces the same value across the different seat distributions to State B and State C for these cases (0.633 and 80.000 respectively in Table 1).

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[9] Monroe indicates the insensitivity problem using the Loosemore–Hanby index, which is mathematically identical to *MAL*.

Table 1 ***Insensitivity of MaxMin and MAL***

Case A

	Population (a)	Seats (b)	(a) / (b)	Population Share (c)	Seat Share (d)	| (c) - (d) |	$(c)\left(-\log\frac{(d)}{(c)}\right)$
Total	100000	6		1.000	1.000		
State A	1000	1	1000	0.010	0.167	0.157	-0.028
State B	4000	*3*	1333	0.040	0.500	0.460	-0.101
State C	15000	*1*	15000	0.150	0.167	0.017	-0.016
State D	80000	1	80000	0.800	0.167	0.633	1.255
Calculated Values		*MaxMin*	**80.000**		*MAL*	**0.633**	D^0 **1.110**

Case B

	Population (a)	Seats (b)	(a) / (b)	Population Share (c)	Seat Share (d)	| (c) - (d) |	$(c)\left(-\log\frac{(d)}{(c)}\right)$
Total	100000	6		1.000	1.000		
State A	1000	1	1000	0.010	0.167	0.157	-0.028
State B	4000	*2*	2000	0.040	0.333	0.293	-0.085
State C	15000	*2*	7500	0.150	0.333	0.183	-0.120
State D	80000	1	80000	0.800	0.167	0.633	1.255
Calculated Values		*MaxMin*	**80.000**		*MAL*	**0.633**	D^0 **1.022**

Case C

	Population (a)	Seats (b)	(a) / (b)	Population Share (c)	Seat Share (d)	| (c) - (d) |	$(c)\left(-\log\frac{(d)}{(c)}\right)$
Total	100000	6		1.000	1.000		
State A	1000	1	1000	0.010	0.167	0.157	-0.028
State B	4000	*1*	4000	0.040	0.167	0.127	-0.057
State C	15000	*3*	5000	0.150	0.500	0.350	-0.181
State D	80000	1	80000	0.800	0.167	0.633	1.255
Calculated Values		*MaxMin*	**80.000**		*MAL*	**0.633**	D^0 **0.989**

As the third measurement method of malapportionment, we introduce a measure that is based on divergence, which is frequently used in statistics and information geometry to judge how close a set of distributions are. To measure closeness from the population distribution **P** to the seat distribution **Q,** among several types of divergence, α-divergence ($\alpha \to 0$), notated as D^0 in this study, or Kullback–Leibler divergence is employed as the measure of malapportionment in Wada and

Kamahara (2018), because D^0 is impartial and unbiased compared to other parameterization of α. To calculate the degree of malapportionment, Theil and Schrage (1977) and Monroe (1994) suggest the use of the following identical mathematical formula of D^0. However, they do not explicitly argue a relationship between this measure and the geometric concept of divergence. D^0 is defined as follows:

$$D^0 = \sum \frac{N_i}{N}\left(-\log \frac{\frac{n_i}{n}}{\frac{N_i}{N}}\right). \qquad (5)$$

The smaller value of D^0 stands for the smaller divergence from **P** to **Q**, that is, a lesser degree of malapportionment.[10]

As shown in Table 1, *MAL* and *MaxMin* can be insensitive to different seat assignments for the same distribution of electorates, whereas D^0 is sensitive to these seat assignments or captures the degree of malapportionment generated from each assignment differently.[11] In this respect, we maintain that D^0 is superior to *MAL* and *MaxMin*.

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[10] Eq. (5) is a specification of Eq. (1). For other parameterizations of α, see Appendix A.

[11] The insensitivity problem can be interpreted as the unfulfillment of the Pigou–Dalton condition in economics (Sen 1973). D^0 satisfies this condition. For the definition of this condition and its mathematical proof, see Wada and Kamahara (2018).

3.2 Decomposability

Further, α-divergence has the additional excellent property of *decomposability*. Conventional measures, *MAL* and *MaxMin*, calculate only the overall degree of malapportionment in the country, whereas D^0 can decompose the factors related to such unfairness into apportionment and districting as follows (Wada and Kamahara 2018):

$$D^0 = \sum_{j=1}^{k}\sum_{i=1}^{k_j}\frac{N_{ji}}{N}\left(-\log\left(\frac{\frac{n_{ji}}{n}}{\frac{N_{ji}}{N}}\right)\right)$$

$$= \sum_{j=1}^{k}\frac{N_j}{N}\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}}\right)\right) + \sum_{j=1}^{k}\frac{N_j}{N}\sum_{i=1}^{kj}\frac{N_{ji}}{N_j}\left(-\log\left(\frac{\frac{n_{ji}}{n_j}}{\frac{N_{ji}}{N_j}}\right)\right)$$

$$= D^0_{apportionment} + \sum_{j=1}^{k} W_j^0\, D^0_{districting_j}, \qquad (6)$$

where j denotes a particular subnational jurisdiction or state, k_j the total number of the districts in state j, and k the total number of the states. The first term, $\sum_{j=1}^{k}\frac{N_j}{N}\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}}\right)\right)$, stands for the inequality across state js, and the second term, $\sum_{j=1}^{k}\frac{N_j}{N}\sum_{i=1}^{kj}\frac{N_{ji}}{N_j}\left(-\log\left(\frac{\frac{n_{ji}}{n_j}}{\frac{N_{ji}}{N_j}}\right)\right)$, stands for the weighted ($\frac{N_j}{N} \equiv W_j^0$) sum of the inequality across district is within state j. In other words, the overall degree of malapportionment D^0 can be decomposed into the apportionment- and the districting-stage degree of malapportionment, $D^0_{apportionment}$ and $\sum_{j=1}^{k} W_j^0\, D^0_{districting_j}$, respectively. This decomposition corresponds to Eq. (2).

Decomposing can be iterated any number of times. Thus, we can decompose not only apportionment and districting but also the effect of the existence of minority districts or overseas districts. Hereafter, these districts are called *special* districts or districts in *special areas*, and h denotes a special or general area. Here, we can use two kinds of weight sets for inequality across districts. Among the first set of weights, let the weight for either a special or general area h be $\frac{N_h}{N} \equiv W_h^0$ and the weight for state js within either a special or a general area h be $\frac{N_{hj}}{N_h} \equiv W_{hj}^0$. Let the second set of weight be $\frac{N_{hj}}{N} \equiv w_{hj}^0$.

$$D^0 = \sum_{h=1}^{k} \sum_{j=1}^{k_h} \sum_{i=1}^{k_{hj}} \frac{N_{hji}}{N} \left(-\log \left(\frac{\frac{n_{hji}}{n}}{\frac{N_{hji}}{N}} \right) \right)$$

$$= \sum_{h=1}^{k} \frac{N_h}{N} \left(-\log \left(\frac{\frac{n_h}{n}}{\frac{N_h}{N}} \right) \right) + \sum_{h=1}^{k} \frac{N_h}{N} \sum_{j=1}^{k_h} \frac{N_{hj}}{N_h} \left(-\log \left(\frac{\frac{n_{hj}}{n_h}}{\frac{N_{hj}}{N_h}} \right) \right)$$

$$+ \sum_{h=1}^{k} \frac{N_h}{N} \sum_{j=1}^{k_h} \frac{N_{hj}}{N_h} \sum_{i=1}^{k_{hj}} \frac{N_{hji}}{N_{hj}} \left(-\log \left(\frac{\frac{n_{hji}}{n_{hj}}}{\frac{N_{hjji}}{N_{hj}}} \right) \right)$$

$$= D_{special}^0 + \sum_{h=1}^{k} W_h^0 \, D_{appportionment_h}^0 + \sum_{h=1}^{k} W_h^0 \sum_{j=1}^{k_h} W_{hj}^0 D_{districting_{hj}}^0$$

$$= D_{special}^0 + \sum_{h=1}^{k} W_h^0 \, D_{appportionment_h}^0 + \sum_{h=1}^{k} \sum_{j=1}^{k_h} w_{hj}^0 D_{districting_{hj}}^0 \quad (7)$$

This decomposability is a notable property for considering MGP and demonstrating which factors significantly influence malapportionment: districting stage, apportionment stage, or special districts.

4. A New Database: Malapportionment in Space and Time

In addition to the mathematical problem of *MAL* as a measurement of malapportionment, Samuels and Snyder's *MAL*-based data set has several limitations. First, the scope of coverage is limited to 78 countries, whereas our database covers 112 countries.[12] Second, the time period covered is outdated. Their coverage is from 1986 to 1999, the most recent coverage being Israel's general election in 1999. If one wants to know the degree of the world's malapportionment in the current decade, an updated study is required. Third, their data set covers only one election per country. This provides a "snapshot" of malapportionment in a given country at a given election. In other words, some scholars have used this data for time-series cross-sectional (TSCS) analyses partially because of less data availability, implicitly assuming that the degree of malapportionment is more or less time-invariant (e.g., Ardanaz and Scartascini 2013). We maintain that this is a problematic use of the malapportionment index because the degree of malapportionment may abruptly fluctuate in some countries, or steadily change in other countries, as discussed in Section 2 and shown in Figure 1; thus, virtually treating malapportionment as a constant factor over time is likely to yield biased results.[13]

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[12] There exists a database with more coverage of countries (see Ong et al. 2017).

[13] For an exemplary study that addresses fluctuations in malapportionment, see Snyder

In light of these limitations, our database includes a larger number of countries and extends the time period to include the most recent decades. Our database of 112 countries includes multiple elections for certain countries, and the time covered is from 1925 to 2017. We believe it is important to provide malapportionment measures for multiple elections so that scholars can analyze the intertemporal variations in one country. Furthermore, while recognizing their drawbacks, we include the two popular measures in our database, because it is important to know to what extent these measures deviate from the overall picture. In fact, to our knowledge, no studies even use, include, and compare *MAL* and *MaxMin* for a large number of countries.

To address the cross-national or intertemporal difference in electoral systems, we implement the Samuels and Snyder's (2001) calculation method explained in Appendix B. In calculating the three measures, we use the number of electorates rather than the population size in each constituency.[14] Samuels and Snyder mainly employ

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and Samuels (2004). They calculate the malapportionment index for 11 Latin American countries from 1870 to 2000.

[14] In this study, we use the terms "electorates," "eligible voters," and "registered voters" interchangeably. Technically, however, these terms refer to different bases of calculation for "one *person*, one vote," which is dependent on the voter registration system employed by a given country. Thus, any cross-national variation of malapportionment may be partially derived from the different registration systems. Given that the registration system remains typically constant over time, we can compare the degree of malapportionment without regard for the voter registration, although our database does

population size, because, according to them, most countries' delimitation rule is based on a population census rather than the number of eligible voters. However, some countries employ the size of electorates for seat allocation (Handley 2008). We believe that it is more appropriate to use the number of electorates because countries differ with regard to qualification standards to be eligible to vote. In other words, countries with the same population size can have different numbers of electorates depending on qualification standards.[15]

Another issue in calculating the three measures is whether to include districts that are reserved for ethnic minorities in a given country and/or for citizens overseas. Further, the existence of districts distant from the mainland (e.g., French overseas territories) would pose a problem for malapportionment. We calculate the indexes of malapportionment by including these special districts and capture their impact on the overall degree of malapportionment by using D^0 because of its decomposability. The data sources are derived from official statistics, researcher's data sources, and personal websites.[16]

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not address this difference. We appreciate a reviewer's suggestion regarding this point.

[15] Our data set includes a variable, *electorate*, indicating whether we employ electorates or people when calculating the measures. For the United States, we use the voting-age population. For details, see our codebook.

[16] For details, see Table A2 in Appendix C.

5. Malapportionment at a Glance

Employing the methods discussed in Section 4, we calculate the degree of malapportionment for 440 elections in 112 countries held from 1925 to 2017. In this study, in the case of bicameral systems, only the lower chamber is calculated and analyzed. This section overviews the overall degree of malapportionment in time and space.

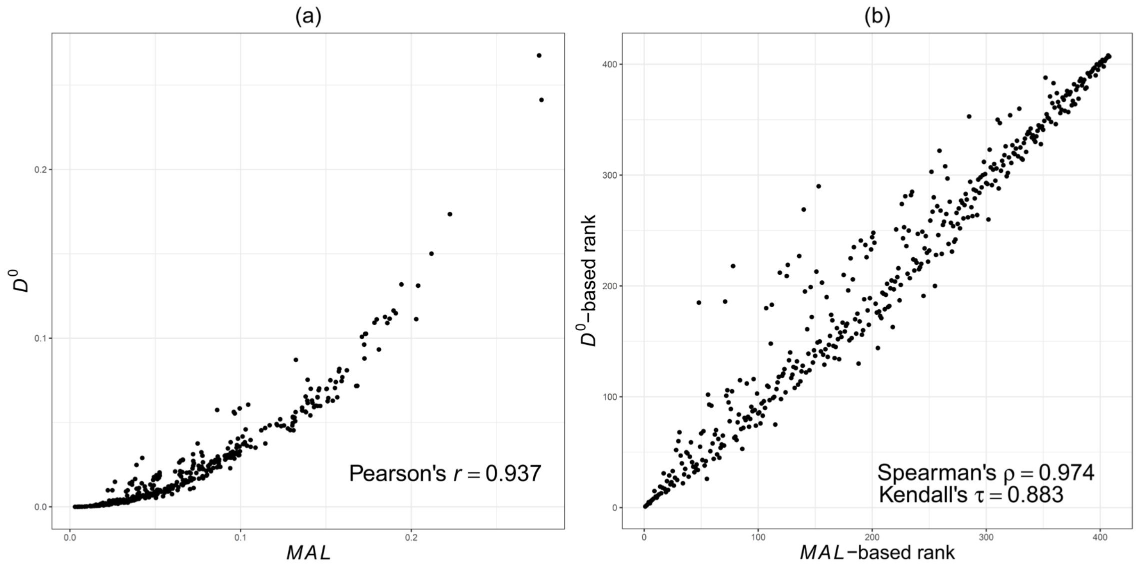
In our database, the largest number of elections of a given country is 34 for Australia, while our database includes a single election in 44 countries due to less resource availability. We compile time-series data for more than 60 countries and provide the largest database of malapportionment for 440 elections. Our database does not perfectly capture the historical trajectories of every election in all countries included in our database; however, this is the very first attempt to comprehend the degree of malapportionment from the perspective of MGP and TSCS.

Our correlation analysis shows that *MaxMin* on the one hand and *MAL* and D^0 on the other hand are not correlated, while *MAL* and D^0 strongly are (Table A4, Appendix E).[17] Figure 2 shows scatter plots between *MAL* and D^0 (Figure 2(a)) and between ranks of countries for the two measures (Figure 2(b)). Although this figure demonstrates that both Pearson's correlation and rank one are strong, Figure 2(b) shows that several countries are placed on the upper left over the diagonal lines, suggesting that these countries are located lower in the ranking of vote equality using D^0 than using *MAL*. This finding suggests that *MAL* and D^0 tend to produce similar results at the

[17] Descriptive statistics are displayed in Table A3, Appendix D.

aggregate level; thus, one may consider using *MAL* instead of D^0. This case may be true for when one is interested in gauging the overall degree of malapportionment. However, as Casper and Tufis (2003) demonstrate in their analyses of democracy, using highly correlated measures can produce different results. In this regard, we maintain that the use of D^0 is preferred particularly when conducting correlational or causal analyses.

Figure 2 *Correlation*



Note: When visualizing the relationship between the two measures and calculating their estimates, elections with a nationwide PR system were omitted because of the lack of malapportionment. Moreover, four out of six elections in the US were omitted because of the absence of trends. $\mathrm{N} = 408$.

Using D^0, we illustrate the geographic and temporal pattern of malapportionment around the world. Figure 3 indicates the degree of malapportionment in the most recent election for the countries in our database. A regional breakdown shows that there is lack of information in Africa and the most malapportioned region is Latin America. Figure 4 shows the intertemporal variation of D^0 for the countries for which we have data on multiple elections.[18] It reveals that some countries have a higher fluctuation in the degree of malapportionment (e.g., Canada and Japan), whereas others have relatively stable D^0 (e.g., Denmark and Germany).[19] The point we emphasize here is that treating malapportionment as a time-invariant factor in TSCS analysis should be avoided, in contrast to other scholars (e.g., Ardanaz and Scartascini 2013).

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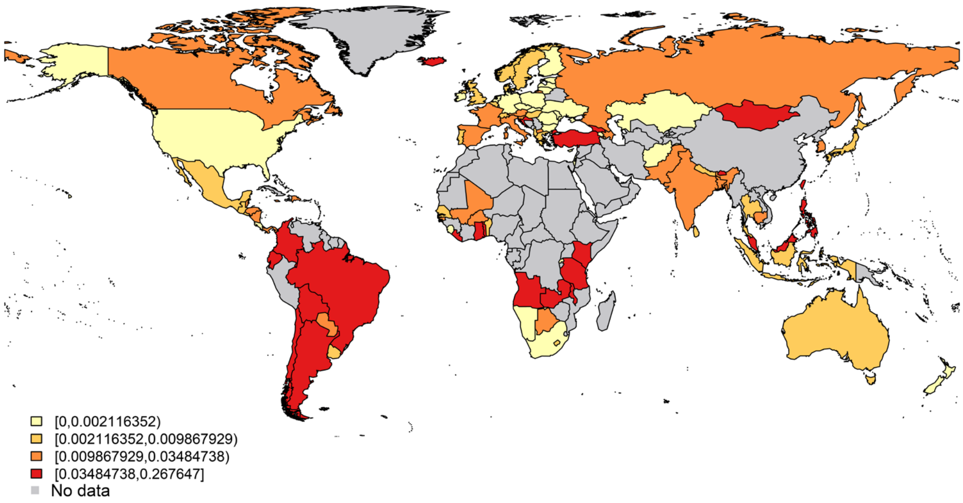
[18] We omitted countries for which we have only a single data point. Also, countries that employ a PR system with a single nationwide constituency are excluded because their degree of malapportionment is constantly zero. In the case of the Netherlands, there are 19 electoral districts. However, "the 19 districts do not function as true electoral districts, and the entire country effectively becomes a single 150-member electoral district" (Nohlen and Stöver 2010, p. 1389). Thus, in our database, the electoral system for the lower house of the Netherlands is classified as PR with a single nationwide constituency.

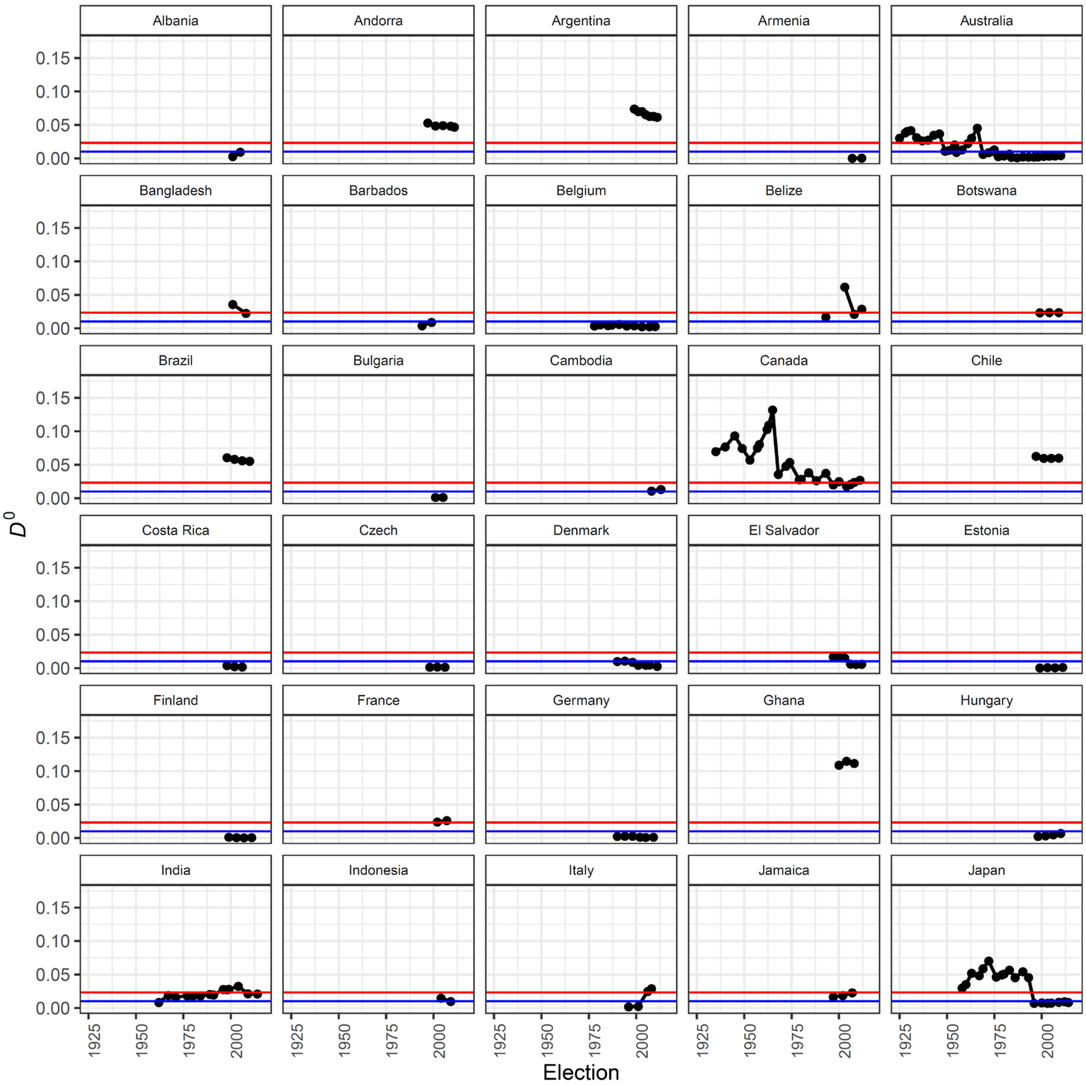
[19] Figure A1 in our Appendix F compares an intertemporal change in D^0 and that in *MaxMin*. Although many countries have almost the same pattern between the two measures, the patterns of some countries diverge such as France and India.

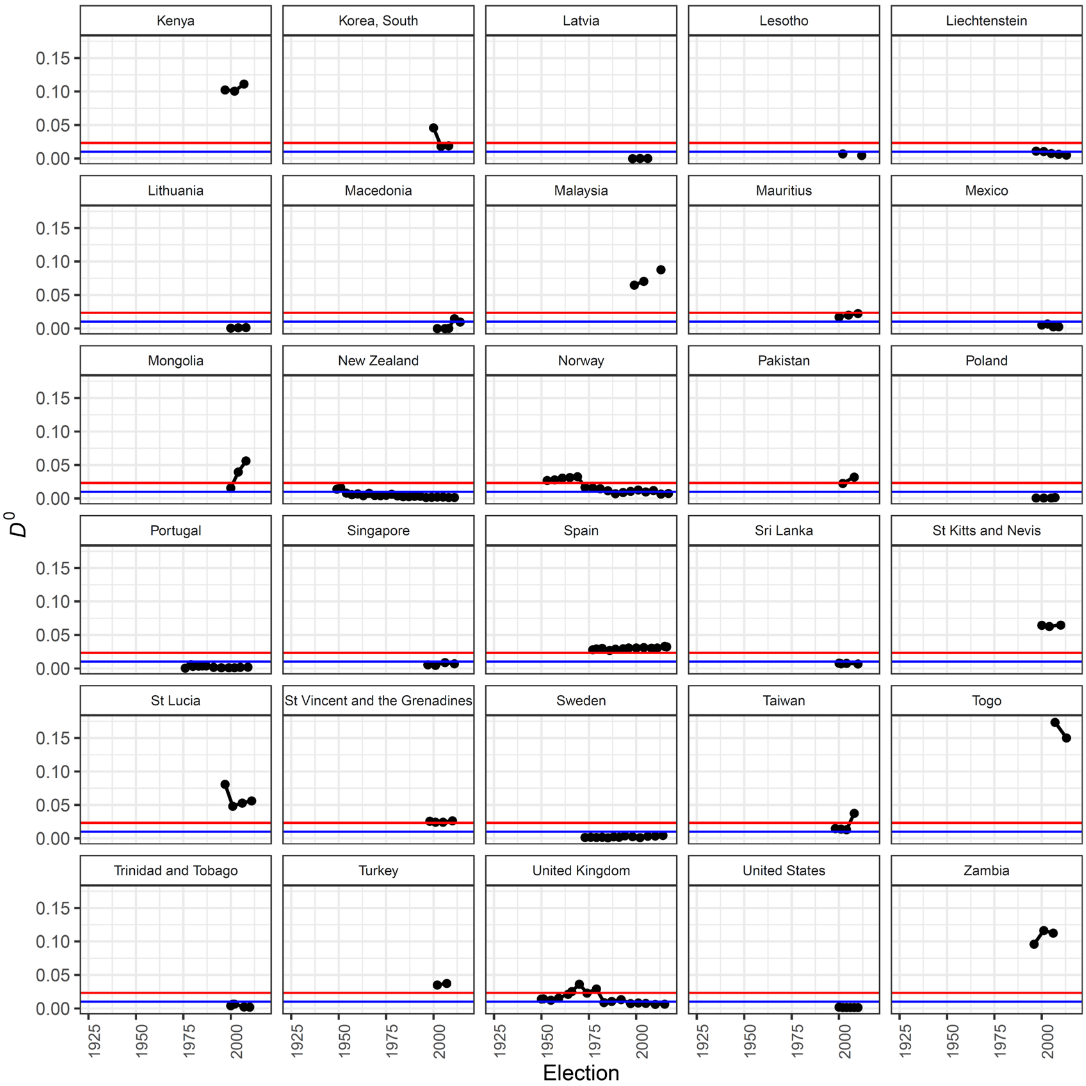
Figure 3 *Malapportionment in the World*



Note: D^0 is used as malapportionment. Each value of D^0 is derived from the most recent election for each country in this data set.

Figure 4 *Historical Change in Malapportionment (D^0) in Each Country by Using Time-series Cross-sectional Data*





Note: Red lines stand for the mean value of D^0 and blue lines for its median for 440 elections. Cases are shown in this panel if we observe D^0 in more than one year. Thus, we omit single-year observations. Moreover, we do not report on countries only with nationwide PR systems because of no time-series variance of D^0. Omitted cases are enumerated in Note of Figure A1, Appendix F.

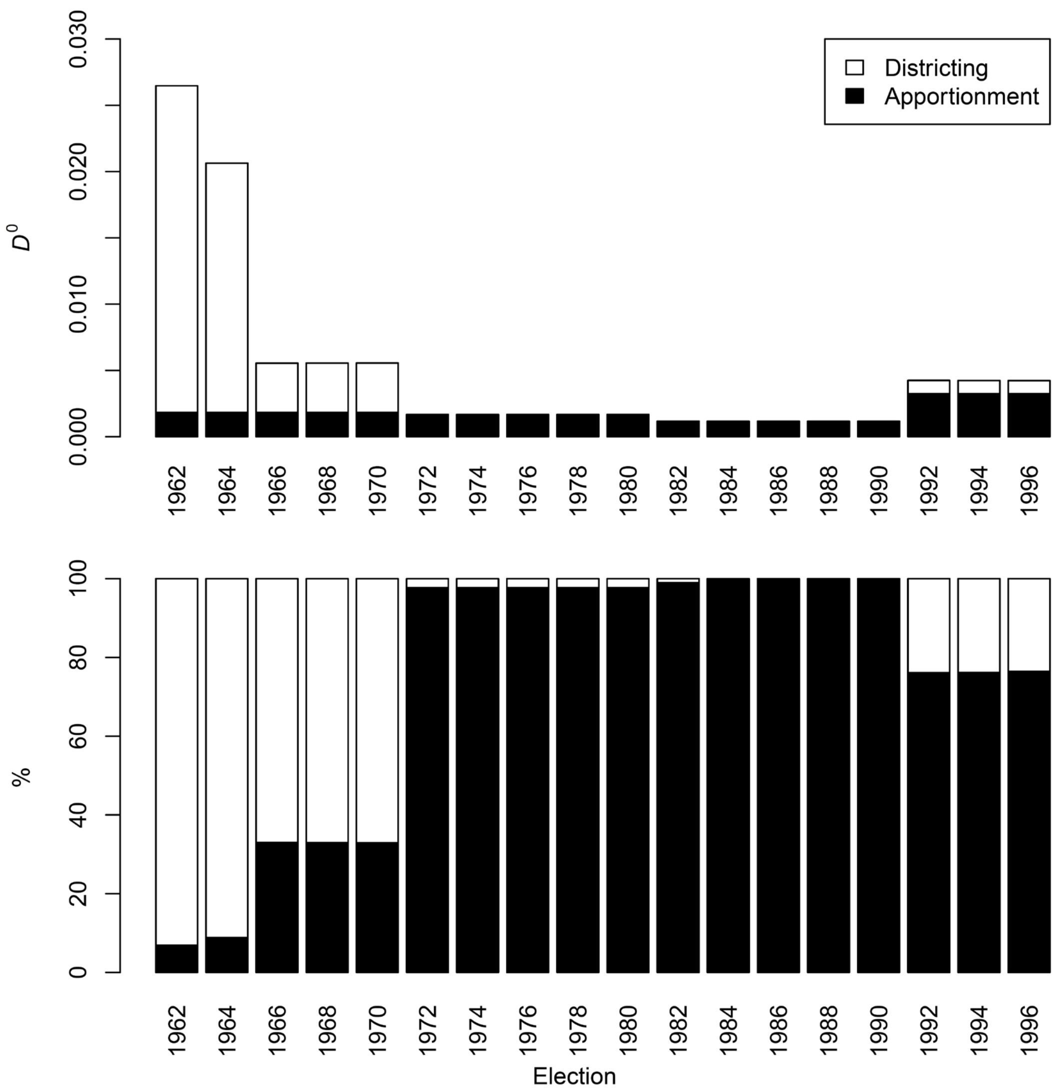
6. Decomposition Analyses

6.1 Decomposing the degree of malapportionment

To understand the whole picture of malapportionment, MGP should never be ignored.

Eq. (2) helps us realize that malapportionment can be the total of several-stage malapportionments. Therefore, we require not only a database with more extensive cross-national and intertemporal coverage but also a measure with decomposability. As discussed above, D^0 is advantageous as it can decompose the sources of national-level malapportionment into various contributing factors, most notably those stemming from apportionment, districting, and special districts such as overseas territories or reserved seats for minorities.

Figure 5 *Decomposing Apportionment and Districting in the United States*

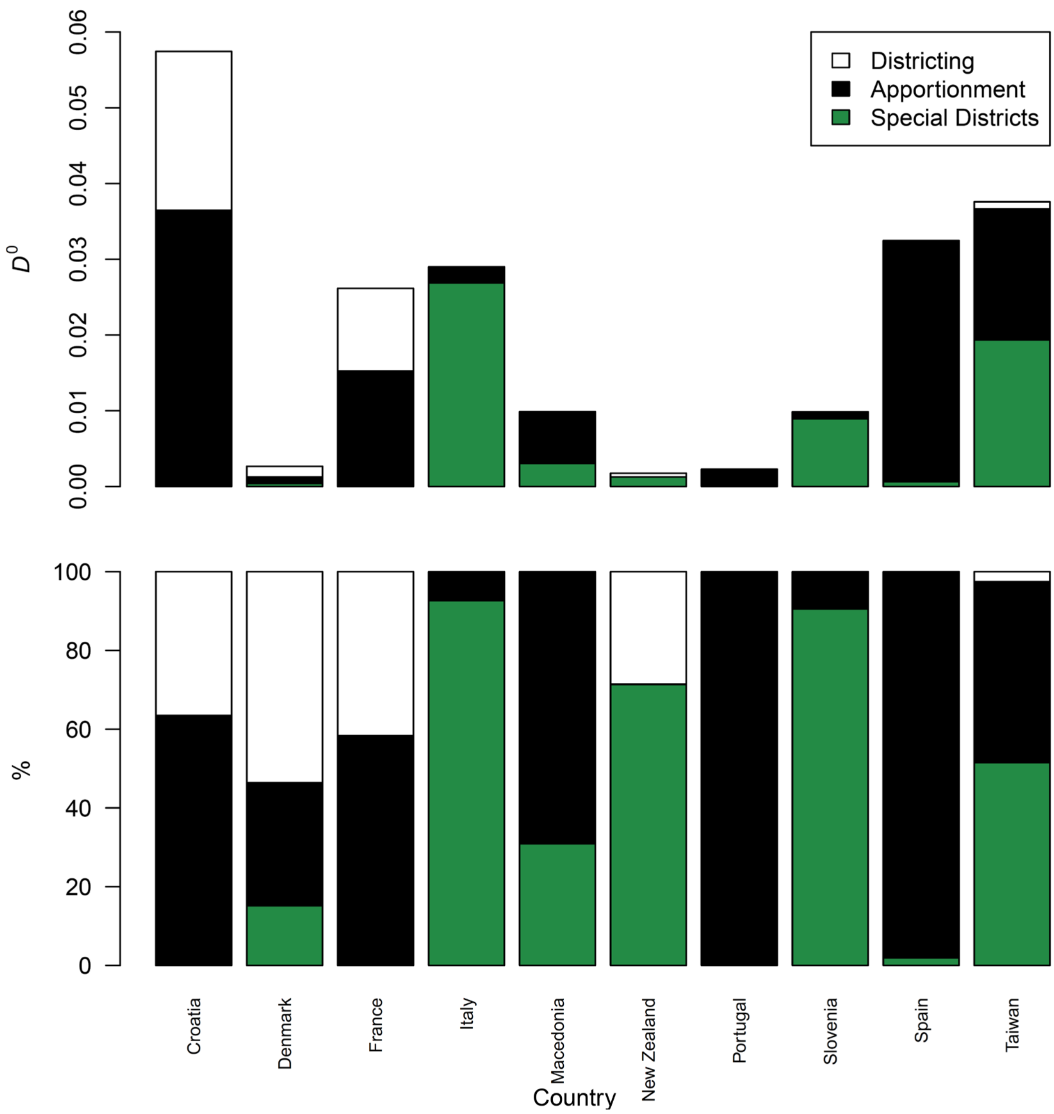


Notes: We cannot calculate the degree of malapportionment at the districting stage between the 1942 and 1960 elections because Adler's data set has several errors. Moreover, this data set also includes incorrect information regarding New Mexico and Hawaii. Thus, we omitted elections between 1942 and 1960, rectified the information, and estimated the number of electorates following Martis (1982).
Source: Compiled by the authors based on the data provided by Scott Adler's web page (Adler N.d.).

Figure 5 illustrates the sources of malapportionment by distinguishing apportionment and districting in the case of the US House of Representatives between the 88th Congress (1963–1964) and the 105th Congress (1997–1998). It reveals that the drastic reduction in the degree of malapportionment in the 1960s, often known as the "reapportionment revolution," was actually the "redistricting revolution." Upon the Constitutional mandate of equal apportionment (14th Amendment), each state received a relatively equal number of seats per population. The problem, however, was how district boundaries were drawn within the states. The legislature in each state was in charge of districting, but after the so-called reapportionment revolution, the state courts were given the discretionary authority to draw boundaries in the event that the state legislature failed to do a fair districting. This change of rule drastically reduced intrastate inequality in districting (Cox and Katz 2002).

Our database includes the decomposed components of D^0 for several countries. Among countries, some create special districts to better reflect the voice of citizens abroad, in overseas territories, and in certain ethnic communities. One might think that the existence of such districts tends to produce an unequally apportioned legislature. However, as Figure 6 shows, while in some cases it does (e.g., New Zealand and Slovenia), in other cases it does not (e.g., Croatia and France).

Figure 6 *Decomposing Apportionment, Districting, and Special Districts*



Note: Degree of malapportionment for each country in the latest election in our database. The loading rates of special districts on the degrees of malapportionment in Croatia, France, and Portugal are almost zero. Thus, this figure cannot capture them.

6.2. Decomposing trends in malapportionment

As discussed in Section 2, the change in malapportionment between two successive elections can be caused by population shifts and by either politically active or inactive manipulation for an incumbent's interests. Based on MGP, we can call the type of malapportionment generated by demographic movement "demographic-driven malapportionment" and the other type caused by political motivation "politically-engineered malapportionment." Although our database includes only the degree of malapportionment and its decomposed factors, this subsection provides a decomposition analysis of the trend of malapportionment if time-series information is available and the number of administrative units is fixed.

The degree of malapportionment using D^0 in any given election t can be defined as ${D^0}^t$. The difference between the degree of malapportionment in the current election and the previous one, ${D^0}^1 - {D^0}^0$, can be decomposed into pure demographic change $\left(\Delta\left(\frac{N_j}{N}\right)\right)$ and politics-related change of apportionment $\left(\Delta\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}}\right)\right)\right)$ or districting $\left(\Delta\left(D^0_{districting_j}\right)\right)$ as follows:[20]

[20] To use this equation, the number of states or prefectures, k, must be invariable. The mathematical extension, including the case of three-stage decomposition with special areas, is shown in Appendix G. Mookherjee and Shorrocks (1982) conduct a similar decomposition in the context of economic equality across generations.

$$\Delta D^0 = D^{0^1} - D^{0^0}$$

$$= \left(\sum_{j=1}^{k} \overline{\left(\frac{N_j}{N}\right)} \Delta \left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}} \right) \right) \right)$$

$$+ \left(\sum_{j=1}^{k} \overline{\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}} \right) \right)} \Delta \left(\frac{N_j}{N}\right) \right) + \left(\sum_{j=1}^{k} \overline{\left(D^0_{districting_j}\right)} \Delta \left(\frac{N_j}{N}\right) \right)$$

$$+ \left(\sum_{j=1}^{k} \overline{\left(\frac{N_j}{N}\right)} \Delta \left(D^0_{districting_j}\right) \right). \tag{8}$$

Each decomposed term includes a notation for summation (Σ), an intertemporal weight between two elections ($\overline{(\cdot)}$),[21] and the difference in either a demographic- or politics-driven change ($\Delta(\cdot)$). The first term can be interpreted as the weighted sum of the change in the disproportion of each state's apportionment.[22] If the authorities make

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[21] A weight $\overline{\left(\frac{N_j}{N}\right)}$ in the first and fourth terms pertains to the intertemporal average of the population share of the state between two elections. A weight $\overline{\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}} \right) \right)}$ in the second term denotes the intertemporal average of the value of each state's disproportionate apportionment between two elections. Lastly, a weight $\overline{\left(D^0_{districting_j}\right)}$ in the third term stands for the intertemporal average of the inequality in districting within each state between two elections. For details, see Appendix G.

[22] The last component $\Delta\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}} \right) \right)$ can be interpreted as the difference ($\Delta(\cdot)$) in the

a reapportionment in such a manner as to distort the seat apportionment to each state, or do not make a suitable reapportionment that responds to population changes across the state between two elections, this term shows the aggravation of malapportionment, indicating the disproportionate apportionment regardless of the demographic changes. The second term is the weighted sum of the change in the state j's population share. Thus, for instance, if the population share of an undervalued state becomes large due to demographic shifts, this term indicates a worsening trend in malapportionment between two elections. The third term is another weighted sum of the change in the states' population share between two elections. If the population share of an unequally districted state becomes large, this term shows that malapportionment worsens. The last term is the weighted sum of the change in the inequality of the districting, or districting-stage malapportionment, within each state j. If the authorities do redistricting unequally or do not do a suitable redistricting that corresponds to the population change within the state, this term indicates that malapportionment becomes worse between two elections.

Simply put, we can decompose the change in the degrees of malapportionment between two elections into demographically caused and politically caused components:

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log deviation from the population to the seat quotients in state j ($-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}}\right) \equiv -\left(\log\left(\frac{n_j}{n}\right) - \log\left(\frac{N_j}{N}\right)\right) = \log\left(\frac{N_j}{N}\right) - \log\left(\frac{n_j}{n}\right)$). This deviation stands for the disproportionate seat apportionment ($\frac{n_j}{n}$) to the size of the electorate ($\frac{N_j}{N}$).

1. The first term

Change in vote inequality caused by change in disproportionate apportionment.

The first term is labeled *Political (apportionment)* in Figure 7.

2. The second term

Change in vote inequality caused by the state's population change weighted by the disproportionate *apportionment* of the seats between two elections.

The second term is labeled *Demographic (cross-state migration weighted by apportionment)* in Figure 7.

3. The third term

Change in vote inequality caused by the state's population change weighted by the inequality in each state's *districting*.

The third term is labeled *Demographic (cross-state migration weighted by districting)* in Figure 7.

4. The fourth term

Change in vote inequality caused by the change in the inequality in the districting within each state.

The fourth term is labeled *Political (districting)* in Figure 7.

The second and third terms in Eq. (8) demonstrate only the total influence of the *state* population ($\Delta\left(\frac{N_j}{N}\right)$). However, Eq. (8) cannot generally calculate the influence of the population changes between *districts*. Thus, MGP at the districting stage is still in a black box. Yet, if and only if the number of districts in each state, k_j, is invariable during the sample period, can we unpack this black box and identify the influence of the population change at the district level ($\Delta\left(\frac{N_{ji}}{N}\right)$). The decomposition for this special case appears in Appendix H. In other words, changes in malapportionment can be decomposed to demographic- and politics-driven components. Thus, we can investigate the political process of malapportionment by decomposing trends in malapportionment. Moreover, decomposing enables practitioners as well as researchers to propose more focused policies for the cause of malapportionment across times.

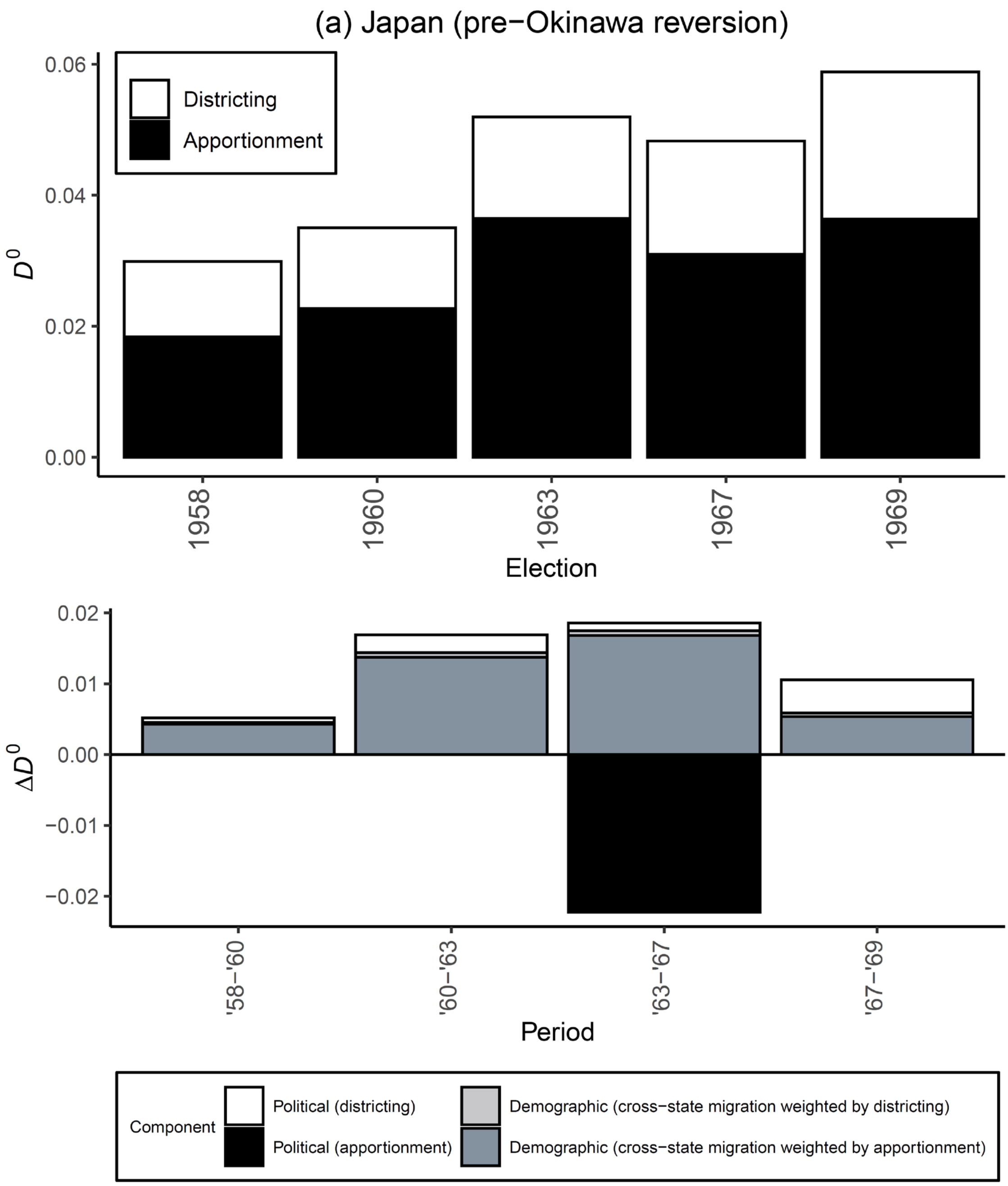
The decomposability of D^0 enables us simply to analyze what factors influence the *degree* of malapportionment for each election (i.e., special districts, apportionment, and districting), and the decomposability of ΔD^0 also allows us to identify what factors affect the *changes* in malapportionment (i.e., demographic-driven and politically-engineered malapportionment) without regression-type analysis.

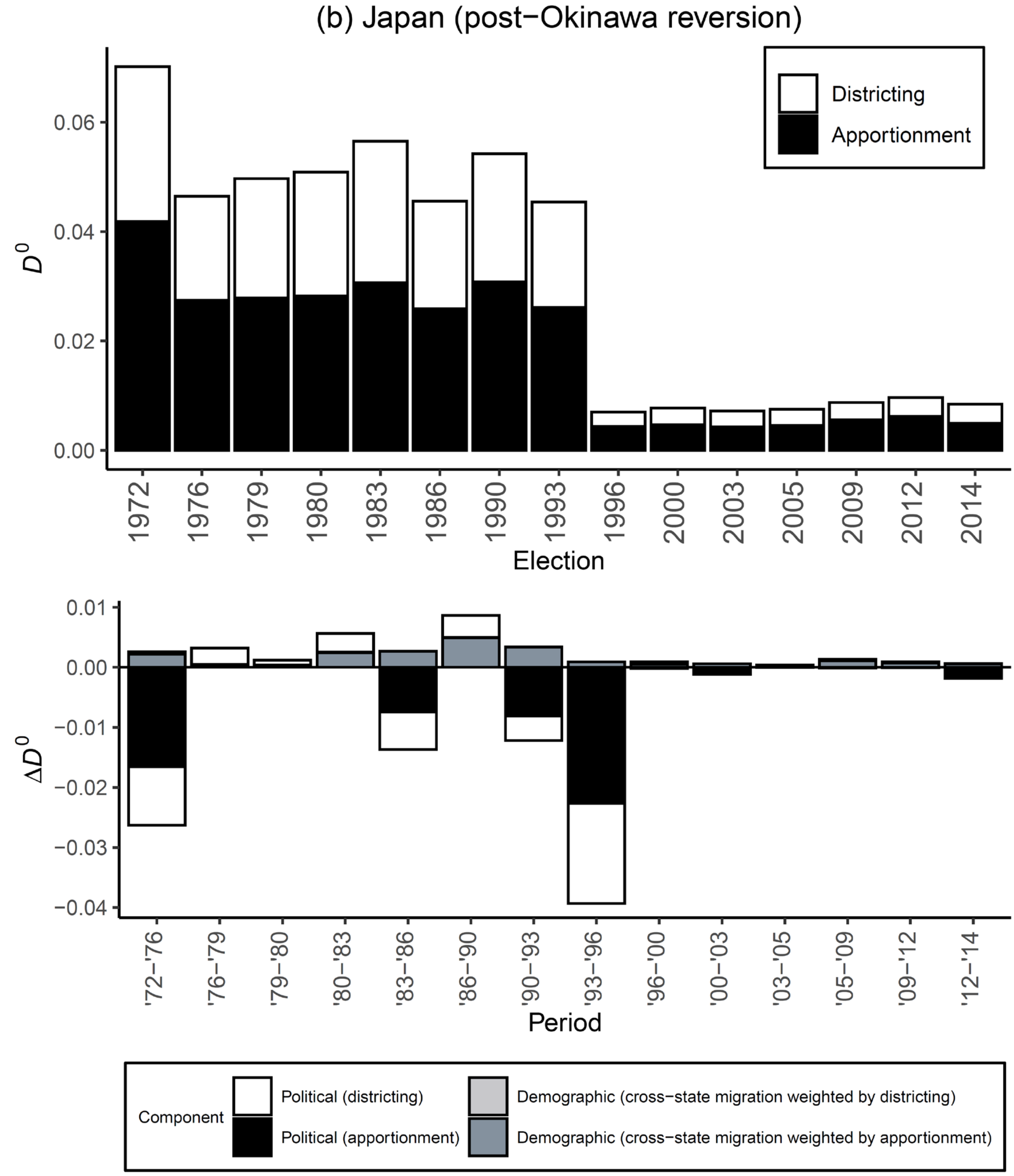
As Figure 4 indicates, the degree of malapportionment has fluctuated in several countries. Thus, using ΔD^0 we can infer a mechanism that causes these fluctuations. As illustrative cases, we select two periods of Japan (pre- and post-Okinawa reversion) and, in Appendix I, also analyze other three countries (Australia since 1949, the United Kingdom since 1950, and New Zealand since 1949). We can identify what factor mainly determines the degree of malapportionment using D^0 and ΔD^0 because both indexes inherently and endogenously have their own factors, as described earlier.

6.3. Case studies

This subsection provides the inference regarding several factors of malapportionment by observing Japan and three countries. Using cross-national analysis, many previous studies have identified electoral systems, for instance, single-member districts (SMDs), as institutional factors causing malapportionment (Samuels and Snyder 2001; Horiuchi 2004; Ardanaz and Scartascini 2013). However, as described above, these studies lack a temporal perspective because countries do not change electoral systems frequently, while the degree of malapportionment does. Thus, our database and the decomposability of D^0 lead to more robust reasoning regarding the factors of malapportionment and its trends.

Figure 7 ***Decomposition Analyses of Degrees of and Changes in Malapportionment (Japan)***





Notes: Each term in Eq. (8) is labeled in Figure 7 as follows: *Political (districting)* $= \sum_{j=1}^{k}\overline{\left(\frac{N_j}{N}\right)}\,\Delta\left(D^0_{districting_j}\right)$, *Political (apportionment)* $= \sum_{j=1}^{k}\overline{\left(\frac{N_j}{N}\right)}\,\Delta\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}}\right)\right)$, *Demographic (cross-state migration weighted by districting)* $= \sum_{j=1}^{k}\overline{\left(D^0_{districting_j}\right)}\,\Delta\left(\frac{N_j}{N}\right)$, and *Demographic (cross-state migration weighted by apportionment)* $= \sum_{j=1}^{k}\overline{\left(-\log\left(\frac{\frac{n_j}{n}}{\frac{N_j}{N}}\right)\right)}\,\Delta\left(\frac{N_j}{N}\right)$. For details, see Appendix G.

Because the number of prefectures in Japan changed from 46 to 47 in 1972,[23] we must calculate separately the decomposed trends for the period 1958–1969 and the period 1972–2014 (Figures 7(a) and 7(b) respectively). The Japanese government reapportioned the Lower House seats in 1967, 1976, 1986, 1993, 2000, 2003, and 2014.[24] We can thus observe the impact of reapportionment, even though these reapportionments fell far short of the principle of fair representation. In 1996, the Japanese electoral system for the Lower House was changed from multi-member districts with single nontransferable voting (MMDs-SNTV) to SMDs with regional block PR. In contrast to the cross-national analyses of previous research (e.g., Horiuchi 2004), malapportionment at the districting stage became more equal after introducing SMDs (Figure 7(b)). As discussed in Section 3.1, Japan conceptualizes the value of a vote in terms of *MaxMin*, or the ratio of the most populous district to the least populous district, without considering the remaining systematic malapportionment in the apportionment stage other than the two extreme districts. In other words, when shaping districts within each prefecture, only the most advantageous (less populous) prefectures and the most disadvantageous (more populous) prefectures are split equally, to ameliorate the value of a vote based on the above-discussed conceptualization of the vote value. However, Japanese authorities do not care much about vote equality for the

[23] In 1972, Okinawa Prefecture was reverted to Japan from the United States.

[24] The 2000 reapportionment decreased only the upper-tier seats distributed by the system of PR. Thus, the degree of malapportionment (D^0) became worse (Figure 7(b)).

other remaining prefectures (Wada 2010).

The lower panels of Figures 7(a) and (b) show that the deteriorating change in malapportionment is caused mainly by demographic change between two elections (*Demographic (cross-state migration weighted by apportionment)*) and/or change in vote inequality caused by unequal districting within each prefecture (*Political (districting)*). This figure also indicates that improved change in malapportionment is caused mainly by less disproportionate apportionment (*Political (apportionment)*) and by more equal districting within each prefecture than in the previous election (*Political (districting)*; see ΔD^0 in the '72-'76, '83-'86, '90-'93, and '93-'96 periods, Figure 7(b)).[25] More interestingly, although malapportionment was improved due to the 1967 reapportionment, districting worsened it slightly. In other words, apportionment and districting had opposite effects between 1963 and 1967 (Figure 7(a)). Generally, Japan's deteriorating change is caused by population shift and political inactiveness. Although political interventions are not enough to achieve the equal value of a vote, reapportionment, though less effective, has historically improved the degree of malapportionment.

Moreover, Appendix I also illustrates the degree of and changes in

[25] Appendix H and Figures A3(a1) and (a2) in Appendix I for the calculable cases—where the number of districts in a prefecture remains unchanged—demonstrate that the main factors are demographic changes that occur not only in the apportionment (*Demographic (cross-state migration weighted by apportionment)*) but also in the districting stage (*Demographic (cross-district migration)*).

malapportionment for other three countries. In Australia, we can observe that fluctuation in malapportionment is mainly caused by the change in the vote inequality at the districting stage in each state (*Political (districting)*; see Figure A2(a)). In the UK, the lower panel of Figure A2(b) shows that, for instance, the drastic improvement in malapportionment from 1979 to 1983 is attributed to fair districting (*Political (districting)*) as well as the introduction of proportionate apportionment (*Political (apportionment)*). Last, in Figure A2(c), we identify politically-engineered malapportionment caused by disproportionate apportionment to the special district (*Political (special)*), whereas demographic change in the special district (*Demographic (special or general area)*) reduced the overall degree of malapportionment during the three periods ('93–'96, '96–'99, and '99–'02) after New Zealand introduced the mixed-member majoritarian system.

Our case studies have demonstrated that decomposing the degree of and the trends in malapportionment enables us to identify where malapportionment emanates from, such as apportionment, districting, or demographic-driven or politically-engineered malapportionment. This is a feature previously unknown by using the extant measures. Using only an algebraic identity without conducting statistical inference, our study recognizes the occurrence in MGP only by decomposing changes in malapportionment. Especially among the politics-related components in MGP, we can identify the opposite effects of reapportionment and redistricting during the 1963–1967 period in Japan. Our study proposes that D^0 is a reliable tool for analyzing unequal elections.

7. Conclusion

Most previous studies have conceptualized malapportionment as “as-if” time-invariant and without accounting for MGP, whereas our conceptualization explicitly formulates MGP, thus leading us to utilize a measure, D^0. This index can identify the factors of malapportionment. Using an originally created and most extensive database of malapportionment, we applied our measure to capture the historical trajectory of malapportionment across 112 countries. Comparing our measure with the two popular measures, we have shown that these extant measures and databases capture only the *static* situation of malapportionment. In contrast, our measure and database enable us to comprehend the whole picture of malapportionment because we can decompose the overall *degree* of malapportionment into that generated at the special district, apportionment, and districting stages. Moreover, for selected countries, we can also find demographic and political factors that change the degree of malapportionment by decomposing its *trends* without conducting causal analysis.

Several avenues for future research can be suggested. First, the study of malapportionment needs a more expansive database, particularly in the following three areas. Building upon the existing databases, we need to expand their coverage both longitudinally and cross-nationally. In addition, we need to build new databases for elections in the upper house of national assemblies and in local legislatures. These elections are neglected in the current databases. Enhancing this database coverage would facilitate the analyses of the causes and consequences of malapportionment.

Second, our decomposition analyses help researchers realize that selecting an appropriate measure is crucial before conducting analysis on the causes and consequences of a given political phenomenon of interest. If researchers employed independent variables as the institutional or political factors of phenomena, it would be appropriate to use measures that decompose these factors into the relevant and irrelevant components, as our measure decomposes the degree of and the trends in malapportionment into several contributing factors. Appropriate research designs as well as measures are required to estimate causal effects more properly.

Acknowledgments: We thank Brad Horton, Sachin Karmacharya, Tomoka Kami, Taehee Kim, Verena Mack, Kosuke Nakamura, Yuuki Oda, and Munkhjin Oyunbatar for the help in obtaining electoral data. We also appreciate the valuable comments from Hiromi Akiyama and the two anonymous reviewers. Moreover, we thank the participants of the 2014 Annual Meeting of the Australian Political Science Association, the 2016 Annual Meeting of the American Political Science Association, and the 2017 Asian Political Methodology Meeting.

Funding: This research was supported by the Japan Society for the Promotion of Science Grant-in-Aid for Scientific Research (KAKENHI grant numbers: 26285032, 17K03550, 17H00969, and 17H02483) and the Nomura Foundation research grant.

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