

1 A theory of loosening earth pressure above a shallow tunnel in unsaturated ground

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3 **ABSTRACT**

4 A theory is proposed to evaluate the loosening earth pressure (vertical earth pressure after
5 excavation) acting on a shallow tunnel in unsaturated ground with an arbitrary groundwater
6 level. The theory is developed based on the limit equilibrium theory, combining soil–water
7 characteristic curves, Mohr-Coulomb failure criteria and effective stress for unsaturated soils.
8 The proposed theory is applied to predict the vertical distribution of loosening earth pressure
9 in unsaturated ground, which shows a significant difference from that in saturated ground. In
10 unsaturated ground, suction contributes to the increase in effective loosening earth pressure
11 and shear resistance. The remarkable effects of groundwater depth, soil type, and scale of
12 overburden height and trapdoor width on loosening earth pressure are also revealed. Based on
13 the soil–water characteristic curve, the degree of saturation decreases, which causes wet density
14 to decrease and the total and effective loosening earth pressures to have contrary tendencies.
15 Moreover, effective loosening earth pressures vary with soil type as the degree of saturation
16 varies. The total loosening earth pressures are, however, very similar regardless of soil type,
17 because wet density and shear resistance have similar tendencies. The proposed theory
18 provides a valid model for loosening earth pressure in unsaturated ground that will be useful
19 for shallow tunnel excavations.

20

21 **Keywords:** modelling; loosening earth pressure; shallow tunnel; unsaturated ground; limit
22 equilibrium theory

23

24 **Notation**

25	D	trapdoor width
26	e	void ratio
27	H	overburden height
28	H_w	groundwater level
29	g	gravity
30	K	coefficient of earth pressure
31	s	suction
32	S_{max}, S_{min}	maximum and minimum degrees of saturation
33	S_r	degree of saturation
34	u_a, u_w	pore air and pore water pressure
35	z	vertical coordinate
36	α, m, n	material parameters for van Genuchten's soil–water characteristic curve
37	$\rho_d, \rho_t, \rho_{sat}$	dry, wet and saturation density
38	ρ_s	solid density
39	ρ_w	the density of water
40	σ_z	total vertical stress
41	σ_z^{net}	net vertical stress
42	σ_x', σ_z'	Bishop's horizontal and vertical effective stresses
43	τ	shear stress
44	ϕ	internal friction angle
45	χ	effective stress parameter

46

47

48 **Introduction**

49 The stability of a shallow tunnel may be affected by variations in pressure that result from a
50 rise in the groundwater level. However, the ground above a tunnel is usually assumed to be in
51 a fully-dried or fully-saturated state in practical designs so that classical theories¹ can be
52 applied to evaluate loosening earth pressures.^{2,3} Although numerical methods can be used to
53 investigate the detailed behavior of ground during tunneling,^{4,5} they require
54 many parameters and those based on the mechanics of unsaturated soils have yet to be applied
55 to tunneling problems. Hence, a rational method for evaluating loosening earth pressure on a
56 tunnel in unsaturated ground is needed.

57 Theories for unsaturated soils have been gradually established over the past few decades.⁶
58 The shear strength of unsaturated soils is usually evaluated using two independent stress state
59 variables such as net normal stress and suction.⁷⁻⁹ The effective stress tensor for unsaturated
60 soils proposed by Bishop (1959) has also been applied widely to model the stress-strain
61 behavior of unsaturated soils.¹⁰⁻¹¹ This tensor can uniquely describe the critical state stress ratio
62 regardless of the degree of saturation.¹² Furthermore, various soil-water characteristic curve
63 (SWCC) models have been proposed to describe the relationship between suction and degree
64 of saturation.¹³⁻¹⁵

65 The objective of this paper is to develop a simple theory to evaluate loosening earth pressure
66 in unsaturated ground based on the mechanics of unsaturated soils. Specifically, the aim is to
67 model vertical earth pressure in unsaturated ground above a shallow tunnel after excavation.
68 For this, a trapdoor problem is used,¹⁶⁻¹⁹ in which tunnel excavation is directly modeled by
69 stress release triggered by the lowering of a trapdoor (a part of the bottom boundary of the
70 ground). The earth pressure acting on the trapdoor corresponds to the vertical earth pressure
71 that acts on the crown of the tunnel after excavation. The theory is verified through the
72 comparison with a classical theory for loosening earth pressure in fully-dried or fully-saturated

73 ground.¹ Through a series of simulations using the proposed theory, differences in loosening
74 earth pressures in different soil types such as sand, loam, and silty clay are investigated. The
75 effects of groundwater level, the scale of overburden height, and trapdoor width on loosening
76 earth pressure are also evaluated.

77

78 **Proposed Theory for loosening earth pressure in unsaturated ground**

79 This study uses a trapdoor problem in unsaturated ground as shown in Fig. 1. The z coordinate
80 is taken vertically downward with the ground surface as the origin and groundwater level is set
81 as H_w . A trapdoor of width D , located near the ground surface at a depth H , simulates stress
82 release during tunnel excavation by its downward movement. The ground is assumed to be
83 uniform, having a solid density of ρ_s and dry density of ρ_d (void ratio, $e = \frac{\rho_s}{\rho_d} - 1$).

84 *Failure mode, equilibrium, and boundary conditions*

85 Several studies have indicated that shear bands tend to develop vertically upward from both
86 ends of a trapdoor and reach the ground surface when the overburden height, H , is no more
87 than 2–3 times larger than the trapdoor width, D .^{20–23} As our model uses a shallow trapdoor
88 problem, vertical failure surfaces were assumed as shown in Fig. 1. For this failure mode, an
89 equation of equilibrium of vertical force acting on a small soil element above the trapdoor is
90 given as:

$$91 \quad -\sigma_z(z + dz)D - 2\tau dz + \sigma_z(z)D + \rho_t g D dz = 0, \quad (1)$$

92 where $\sigma_z(z)$ is the vertical earth pressure in the ground above the trapdoor at a depth of z ; τ is
93 shear stress on the side boundary of the soil element; ρ_t is the wet density of soil; and g is the
94 acceleration due to gravity. Applying Taylor's expansion, the equilibrium equation (1) reduces
95 to Equation (2).

96
$$\frac{d\sigma_z}{dz} = \rho_t g - \frac{2}{D} \tau \quad (2)$$

97 As we consider a traction-free condition at the ground surface, the boundary condition is given
 98 as $\sigma_z(0) = 0$ kPa.

99 *Pore pressure and soil suction*

100 Pore air pressure ($u_a(z)$) and pore water pressure ($u_w(z)$) are assumed to be under static
 101 pressure conditions. The value of u_a is usually assumed to be zero (atmospheric pressure) at
 102 any depth as the density of air is negligible^{8,24,25} and u_w is assumed to have a linear distribution
 103 in the vertical direction with a slope of $\rho_w g$ (ρ_w is the density of water).^{8,24,26} The value of u_w
 104 becomes zero at the groundwater surface, is negative above the groundwater level, and can be
 105 written as:

106
$$u_w = \rho_w g(z - H_w). \quad (3)$$

107 Soil suction, s , is given as the difference between pore air pressure, u_a , and pore water
 108 pressure, u_w .

109
$$s = (u_a - u_w) = \rho_w g(H_w - z) \quad (4)$$

110 A simple, linear suction profile is assumed herein. However, the proposed method can
 111 incorporate the effects of infiltration and evaporation further by introducing a nonlinear suction
 112 profile.²⁷

113

114 *Degree of saturation and wet density*

115 The degree of saturation, S_r , is usually given as a function of suction. Although any SWCC
 116 can be applied to the proposed theory, the classical equation proposed by van Genuchten¹⁴ is
 117 employed herein:

118
$$S_r = (S_{max} - S_{min})\{1 + (\alpha s)^n\}^{-m} + S_{min}, \quad (5)$$

119 where $\langle \ \rangle$ denotes Macaulay brackets; S_{max} and S_{min} are the maximum and minimum degrees
 120 of saturation, respectively; and α , m , and n are material parameters. From Equations (4) and
 121 (5), S_r is given as a function of depth, $S_r(z)$. It is noted that the proposed method can
 122 incorporate several factors affecting the SWCC, such as air-entry suction,¹⁵ hydraulic
 123 hysteresis,²⁸ or the effect of density,²⁹ by replacing Equation (5) with an advanced SWCC
 124 model.^{11,30}

125 Wet density, ρ_t , is given as a function of the degree of saturation, $S_r(z)$, by its definition,
 126 which is written as:

$$127 \quad \rho_t = \frac{\rho_s + eS_r(z)\rho_w}{1 + e}. \quad (6)$$

128 From Equations (4) to (6), S_r and ρ_t are uniquely given by depth z , respectively; ρ_t equals
 129 saturation density (ρ_{sat}) at the saturated state ($S_r = 1$) and equals dry density (ρ_d) at the dry
 130 state ($S_r = 0$).

131 *Effective stress and shear resistance*

132 The effective stress proposed by Bishop¹⁰ is usually used for unsaturated soils, as it can
 133 uniquely arrange the critical state stress ratio of unsaturated soils regardless of suction, s , as
 134 shown in Fig. 2. By using this, σ_z in Equation (2) is given as:

$$135 \quad \sigma'_z = \sigma_z - \{\chi u_w + (1 - \chi)u_a\}, \quad (7)$$

136 where χ is the effective stress parameter, for which S_r is usually applied.^{11,31-33} Borja³⁴ proved
 137 that χ is equal to S_r based on the principles of thermodynamics. Therefore, in the proposed
 138 theory, Equation (7) can be written as:

$$139 \quad \sigma'_z = \sigma_z - u_a + S_r s. \quad (8)$$

140 The Mohr-Coulomb failure criterion is assumed to be satisfied by Bishop's effective stress
 141 along vertical failure surfaces (Fig. 1).

142
$$\tau = \sigma'_x \tan \phi, \quad (9)$$

143 where σ'_x is Bishop's horizontal effective stress; and ϕ is the internal friction angle of soil.

144 Herein, the internal friction angle is a constant regardless of the degree of saturation.

145 *Earth pressure coefficient*

146 A coefficient of earth pressure, K , is considered for Bishop's effective stress in the ground
147 above the trapdoor similarly as the saturated ground:¹

148
$$K = \frac{\sigma'_x}{\sigma'_z}. \quad (10)$$

149 Terzaghi¹ empirically recommended K to be at unity. According to numerical simulation of
150 the trapdoor problem in fully-dried ground, K tends to be larger than the earth pressure
151 coefficient at rest and is near unity.³⁵

152 *Loosening earth pressure*

153 Substituting Equations (4) to (10) into Equation (2), we can write the ordinary differential
154 equation as:

155
$$\frac{d\sigma_z}{dz} = \frac{\rho_s + eS_r\rho_w}{1 + e}g - \frac{2}{D}K[\sigma_z - u_a + S_r(z)\rho_w g(H_w - z)]\tan\phi. \quad (11)$$

156 Then, the loosening earth pressure, $\sigma_z(z)$, is derived by solving Equation (11) under the
157 boundary condition that $\sigma_z(0) = 0$ kPa. For this, a simple, explicit numerical scheme with
158 incremental depth Δz was applied:

159
$$\sigma_z(z + \Delta z) - \sigma_z(z) = \frac{d\sigma_z}{dz} \Delta z. \quad (12)$$

160 **Evaluation of loosening earth pressure**

161 The proposed theory was first applied to evaluate loosening earth pressure (vertical earth
162 pressure after lowering the trapdoor) in loamy ground under fully-dried and fully-saturated

163 conditions to check the validity of the theory against the classical Terzaghi's theory. The
164 loosening earth pressure in an unsaturated, loamy ground was then evaluated using the
165 proposed theory. Afterward, the loosening earth pressures in three types of unsaturated ground
166 (sandy, loamy, and silty clay soils) were investigated. The simulations in this section were
167 performed under an overburden height, H , of 10.0 m and trapdoor width, D , of 10.0 m. Thus,
168 the overburden ratio (H/D) was 1.0, for which a shallow failure mechanism with vertical
169 failure surfaces was expected. For simulation of unsaturated ground, the groundwater level was
170 set to 5.0 m and the vertical distributions of pore pressure and suction in the different ground
171 types were the same, as shown in Fig. 3(a) and (b). The principal difference between the
172 different soils is their water holding capacity. In general, water holding capacity is controlled
173 primarily by the particle size distribution of the soil. Small particles, such as clay and silt, tend
174 to be richer in capillaries than large sand particles, as they tend to present small pore sizes and
175 induce a strong capillary action. Therefore, fine soils rich in capillaries retain a higher degree
176 of saturation. Parameters for the SWCCs of the ground types were determined following
177 Hodnett and Tomasella³⁶ (Table 1). The physical properties (such as density and friction angle)
178 were the same for the different soils (Table 2). The SWCCs are compared in Fig. 4, and the
179 vertical distributions of the degree of saturation, S_r , for sand, loam, and silty clay are shown in
180 Fig. 3(c).

181 *Loosening earth pressure in fully-dried or fully-saturated ground (comparison with Terzaghi's*
182 *theory)*

183 Loosening earth pressures in loamy ground under fully-dried and fully-saturated conditions
184 were evaluated by the proposed theory and Terzaghi's theory. To model the fully-dried
185 condition using the proposed theory for unsaturated soils, the groundwater level, H_w , was set
186 to be very deep (10^6 m) so that soil suction, s , was remarkably large and the degree of saturation,
187 S_r , was almost zero. For fully-saturated ground, H_w was set to 0.0 m.

188 Loosening earth pressure was evaluated by the proposed theory and Terzaghi's theory in the
189 fully-dried and fully-saturated conditions (Figs 5 and 6). The initial earth pressure distribution
190 was calculated as the overburden pressure, which represents the earth pressure at rest before
191 lowering of the trapdoor. The saturation density, $\rho_{sat} \left(= \rho_d - \frac{\rho_s - \rho_d}{\rho_s} \rho_w \right)$, is 1.90 g/cm³.
192 Figures 5 and 6, show that both the total and effective loosening earth pressures are lower than
193 the corresponding initial values. This suggests that the earth pressure becomes reduced by
194 vertical-upward shear stress along failure surfaces acting on the ground above the trapdoor.
195 Both the total and effective loosening earth pressures calculated by the proposed theory were
196 identical with those calculated by Terzaghi's theory, which validates the consistency of the
197 proposed theory.

198 *Loosening earth pressure in the unsaturated ground*

199 The proposed theory was applied further to investigate loosening earth pressure in unsaturated
200 loamy ground. Total and effective loosening earth pressures were lower than the corresponding
201 initial earth pressures at all depths (Figure 7). In the unsaturated zone above the groundwater
202 level ($z < 5$ m), the effective loosening earth pressure was larger than the total loosening earth
203 pressure. This indicates that the proposed theory can consider the contribution of matric suction
204 to the increase in effective confining pressure and increase in frictional resistance (τ) in the
205 unsaturated zone. Therefore, the more significant arching effect in unsaturated ground, which
206 is known to occur, can be appropriately evaluated. On the other hand, the correlation between
207 total and effective loosening earth pressure magnitudes is reversed in the saturated zone below
208 the groundwater level.

209 *Loosening earth pressure in different types of unsaturated ground (sand, loam, and silty clay* 210 *soils)*

211 The loosening earth pressure in three types of unsaturated ground (sandy, loamy, and silty clay
212 soils) were investigated. The distributions of soil suction in the different ground types were
213 identical, as shown in Fig. 3(b). The same physical properties were assumed for the different
214 ground types as shown in Table 2. This is because the groundwater levels were identical
215 ($H_w = 5.0$ m). However, vertical distribution of the degree of saturation were different (Fig. 3(c))
216 as different sets of parameters (Table 1; Hodnett and Tomasella³⁸) were applied for the SWCCs.
217 Differences in the distributions of degree of saturation resulted in differences in the distribution
218 of wet density as shown in Fig. 3(d).

219 Figure 8 shows the distributions of the loosening earth pressure for the three ground types.
220 The effective earth pressure in clay and loam ground is much higher than that in the sandy
221 ground. This suggests that shear resistance of the soil along the sliding surfaces, which is
222 usually referred to as “arching effect”, becomes more substantial in the clayey and loamy
223 ground compared with the sandy ground. This can explain why the excavation of a tunnel in
224 sandy ground is relatively more challenging than doing so in clayey and loamy ground.
225 However, the differences between the total loosening earth pressures of the ground types is
226 small, although the total loosening earth pressure in the sandy ground is slightly higher than
227 that in the other types of soils.

228 **Effect of the groundwater level on loosening earth pressure**

229 A series of simulations was carried out using the proposed theory utilizing different
230 groundwater levels to explore its effect on loosening earth pressure. The trapdoor width, D ,
231 was fixed to 10.0 m and the parameters for loam given in Table 1 were used throughout this
232 section.

233 The overburden height, H , was set to $1.0D$ and the groundwater level, H_w was set to
234 different depths (0.0 m, 2.5 m, 5.0 m, 7.5 m, and 10.0 m) for each simulation. Figure 9 shows

235 the vertical distributions of the degree of saturation, S_r , the product of suction and degree of
236 saturation, sS_r , and total and effective loosening earth pressures. As the effective stress, σ'_z ,
237 in Equation (8) can be decomposed into net stress, $\sigma_z^{net}(= \sigma_z - u_a)$, sS_r represents the
238 contribution of suction on the increase in effective confining pressure. In the case of a deeper
239 H_w , the degree of saturation was smaller at any depth in the unsaturated zone (Fig. 9(a)) and
240 the wet density was also lower. This made overburden pressure lower in the deeper H_w case.
241 Meanwhile, sS_r is larger (Fig. 9(b)) in the deeper H_w case, which made effective earth pressure
242 higher (Fig. 9(d)) and total earth pressure lower (Fig. 9(c)). According to Equation (9), the
243 shear resistance along the vertical slip surfaces, which works to reduce the vertical earth
244 pressure on the trapdoor, also becomes more substantial in the case of a deeper H_w . Owing to
245 the effects of a lower overburden pressure and higher shear resistance, the total loosening
246 pressure becomes significantly smaller in the deeper H_w case.

247 The effect of the groundwater level on the loosening earth pressure acting on the trapdoor
248 was further investigated under different overburden heights, $H(\leq 30.0 \text{ m})$, and groundwater
249 levels, $H_w(= 0.0, 5.0, 10.0, 15.0, \text{ and } 20.0 \text{ m})$. Figures 10(a) and 10(b) show the total and
250 effective loosening pressures normalized by total and effective overburden pressure at the
251 depth, H , of $1.0D$, respectively (where ρ_{sat} and ρ_{sub} are the saturation and submerged
252 densities, respectively).

253 The total loosening earth pressure became higher both with the increase in overburden height,
254 H , and decrease in the groundwater level, H_w . Therefore, groundwater level and overburden
255 height had a significant effect on the total loosening pressure. On the other hand, the mean
256 effective stress varied depending on whether the groundwater level is shallower or deeper than
257 the overburden height. The effective earth pressure increased monotonically with the
258 overburden height in the unsaturated zone where the overburden height, H , was less than the

259 groundwater level H_w . In the case where the trapdoor is located below the groundwater level
260 ($H > H_w$), the effective loosening earth pressure would still increase with the increase in the
261 overburden when $H_w = 5.0$ m, but it would decrease with the overburden when H_w is 10.0 m
262 or larger.

263 **Scale effects on loosening earth pressure**

264 The characteristics of the loosening earth pressures in unsaturated ground were further
265 investigated in terms of the scale effect. In this section, a series of simulations was carried out
266 using the parameters for loamy ground (Table 1) by varying the width of the trapdoor, D (i.e.,
267 5.0 m, 10.0 m, and 20.0 m). For each trapdoor width, the overburden height, H , was varied
268 from 0.0 to $3.0D$ with keeping the ratio of the groundwater level, H_w , to the trapdoor width, D ,
269 constant.

270 *Scale effect in fully-saturated ground ($H_w/D = 0$)*

271 The scale effect on the total and effective loosening earth pressures in the fully-saturated
272 ground ($H_w = 0.0$ m) was investigated herein. Figure 11 shows the total and effective
273 loosening earth pressures normalized by the corresponding overburden pressures for the
274 overburden height of $1.0D$. The normalized total and effective loosening earth pressures were
275 identical, regardless of the width of the trapdoor, D . Therefore, trapdoor scale can be ignored
276 in fully-saturated ground, which has been demonstrated by previous studies.²¹

277 *Scale effect in unsaturated ground ($H_w/D > 0$)*

278 The scale effect on the total and effective loosening earth pressures in the partially-saturated
279 ground for three kinds of groundwater level ratio, H_w/D , were investigated. The total and
280 effective loosening earth pressures were normalized by their initial values of saturated ground
281 for the overburden depth of $1.0D$.

282 Figure 12 shows the vertical distribution of the normalized total and effective loosening earth
283 pressures, degree of saturation, and wet density for the case where the groundwater level, H_w ,
284 was $1.5D$. The normalized total and effective pressures in the unsaturated zone for the different
285 trapdoor widths were different, which implies trapdoor scale affects the normalized loosening
286 earth pressure in the unsaturated ground. For the case of the larger trapdoor width, the
287 normalized effective loosening earth pressure in the unsaturated zone tended to be lower and
288 the normalized total loosening earth pressure tended to be higher. These differences in the
289 distributions of the normalized loosening pressures are related to the differences in the
290 distributions of saturation degree and wet density.

291 Figures 13 and 14 show the distributions of the normalized loosening pressures, degree of
292 saturation, and wet density for the cases where the groundwater level, H_w , was $0.5D$ and $2.0D$,
293 respectively. The scale effect was more significant in the higher H_w case. This is because, in
294 the higher H_w case, the unsaturated zone was thicker, suction tended to be stronger, and the
295 shear resistance along the slip surfaces was larger. These results suggest that the scale effect
296 on the normalized total and effective loosening earth pressure exists in unsaturated grounds,
297 particularly for the deeper groundwater levels. This should be considered in the excavation of
298 tunnels in unsaturated ground.

299 However, by scaling down the α parameter of the SWCC with the increase in the trapdoor
300 width, the distributions of the normalized total and effective loosening earth pressures in the
301 partially-saturated ground became identical regardless of the trapdoor width. For the case of
302 the overburden height of $3D$ and groundwater level of $2D$, the loosening earth pressures were
303 calculated for three different trapdoor widths (5 m, 10 m, and 20 m), with the scaled parameter
304 α given in Table 3. The distributions of the degree of saturation, wet density, and the
305 normalized total and effective loosening earth pressures became identical regardless of the
306 overburden height ratio, as shown in Figure 15.

307 **Conclusions**

308 A simple theory for evaluating loosening earth pressure acting on a shallow trapdoor in
309 unsaturated ground is proposed in this paper. The proposed theory was developed based on the
310 limit equilibrium method by combining Bishop's effective stress for unsaturated soils, a classic
311 SWCC curve, and Mohr-Coulomb failure criteria. The proposed theory predicted valid
312 loosening earth pressures in both fully-saturated and fully-dried grounds compared to pressures
313 calculated by Terzaghi's theory.

314 Differences in effective loosening earth pressure were demonstrated for different types of
315 unsaturated ground (sandy, loamy, and silty clay grounds). In unsaturated ground with high
316 water retention (such as clay soils), higher effective confining pressure and higher shear
317 resistance along the slip surfaces can be expected.

318 Depth of the groundwater level was shown to have a significant effect on total and effective
319 loosening earth pressures. In unsaturated ground, the total loosening earth pressure was lower
320 than that in the saturated ground, but the effective loosening earth pressure tended to be higher.
321 Therefore, the mechanical stability of a shallow tunnel in unsaturated ground will vary with
322 groundwater level fluctuations.

323 The normalized loosening pressure was identical irrespective of scale in fully-saturated
324 ground, confirming that the scale effect is not significant in fully-saturated ground. In
325 unsaturated ground, however, the scale effect on the normalized loosening earth pressure was
326 significant, particularly in the case of deeper groundwater level. With a broader trapdoor, the
327 normalized effective loosening earth pressure in the unsaturated zone tended to be smaller, but
328 the normalized total loosening earth pressure tended to be more significant. However, by
329 scaling down the SWCC parameter α with the increase in the width of the trapdoor, unique
330 distributions for the normalized loosening earth pressures could be achieved and the scale
331 effect in the unsaturated ground disappeared.

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423 **Figure captions**

424 Fig. 1. Trapdoor problem in unsaturated soil.

425 Fig. 2. Relation between mean stress, p'' , and deviator stress, q , of Bishop's effective stress
426 (replotted from Sivakumar¹²).

427 Fig. 3. Vertical distribution of (a) pore pressure, u_a, u_w ; (b) suction, s ; (c) degree of saturation,
428 S_r ; and (d) wet density, ρ_t (overburden height, $H = 10.0$ m, width of trapdoor, $D = 10.0$ m, and
429 groundwater level, $H_w = 5.0$ m).

430 Fig. 4. Soil–water characteristic curves for sand, loam, and silty clay.

431 Fig. 5. Depth vs. loosening earth pressure in fully-dried ground.

432 Fig. 6. Depth vs. total and effective loosening earth pressures in the fully-saturated ground
433 (groundwater level, $H_w = 0.0$ m).

434 Fig. 7. Depth vs. total and effective loosening earth pressures in unsaturated ground
435 (groundwater level, $H_w = 5.0$ m).

436 Fig. 8. Vertical distribution of (a) total loosening earth pressure; and (b) effective loosening
437 earth pressure in sand, loam, and silty clay (overburden height, $H = 10.0$ m, and groundwater
438 level, $H_w = 5.0$ m).

439 Fig. 9. Vertical distribution of (a) degree of saturation; (b) product of degree of saturation and
440 suction; (c) total loosening earth pressure; and (d) effective loosening earth pressure in loamy
441 ground. (overburden height, $H = 10$ m, and groundwater level, $H_w = 0.0, 2.5, 5.0, 7.5, 10.0$ m).

442 Fig. 10. Overburden height vs. (a) total loosening earth pressure and (b) effective loosening
443 earth pressure in unsaturated ground at different groundwater levels (overburden height, $H =$
444 $0\text{--}30$ m, width of trapdoor, $D = 10$ m, and groundwater level, $H_w = 0, 5, 10, 15, 20$ m).

445 Fig. 11. Vertical distribution of (a) total loosening earth pressure; (b) effective loosening earth
446 pressure in loamy ground (overburden ratio, $H = 3.0D$, and groundwater level, $H_w = 0.0D$).

447 Fig. 12. Vertical distribution of (a) degree of saturation; (b) wet density; (c) total loosening
448 earth pressure; and (d) effective loosening earth pressure in loamy ground (overburden, $H =$
449 $3.0D$, and the groundwater level ratio, $H_w = 1.5D$).

450 Fig. 13. Vertical distribution of (a) degree of saturation; (b) wet density; (c) total loosening
451 earth pressure; and (d) effective loosening earth pressure in loamy ground (overburden ratio,
452 $H = 3.0D$, and the groundwater level ratio, $H_w = 0.5D$).

453 Fig. 14. Vertical distribution of (a) degree of saturation; (b) wet density; (c) total loosening
454 earth pressure; and (d) effective loosening earth pressure in loamy ground (overburden ratio,
455 $H = 3.0D$, and groundwater level ratio, $H_w = 2.0D$).

456 Fig. 15. Vertical distribution of (a) normalized degree of saturation; (b) normalized sS_r ; (c)
457 normalized effective loosening earth pressure; and (d) normalized total loosening earth
458 pressure in loamy ground (overburden ratio, $H = 0.0-3.0D$, and groundwater level ratio, $H_w =$
459 $2.0D$).

460