Investigation of thermodynamic performance of gas-loaded accumulator in hydraulic system

By

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Abstract

The objective of the thesis is to investigate dynamic performance of gas-loaded accumulators used in hydraulic systems. Gas-loaded accumulator is an important component in hydraulic circuits for energy conservation, shock/pulsation dampening, leakage compensation or thermal expansion. According to variety of accumulator working conditions including pre-charge pressures, charging or discharging speeds, isothermal or adiabatic operations et. al, the purpose of this study is to provide a reference for precisely sizing accumulator. The problem thesis of the study is “How much is thermal properties difference between ideal gas model and real gas model and how to derive isothermal and adiabatic equations of Soave-Redlich-Kwong gas model.”

In the first chapter of the thesis hydraulic accumulator design problems and research status are introduced. For sizing a gas-loaded accumulator, simultaneous equation model built by conservation of thermal energy equation and gas equation of state should be analyzed. The second chapter shows nomenclature used in this thesis.

One of important applications of gas-loaded accumulator in hydraulic drive system is energy regeneration. To show how the accumulator stores and reuses energy, a hydraulic system for supporting excavator boom cylinder is built and simulated by AMESIM software. Two types of energy regeneration systems assisting driving boom cylinder by regenerating gravitational potential energy are compared, influence of accumulator parameters is discussed.

As a key role in hydraulic system, real gas model considering molecular volume and intermolecular force is highlighted compared with ideal gas model. The well-known van der Waals equation is employed as the real gas model to compare with ideal gas model. The gas behavior shows different trends during compression and expansion by mathematical analysis and is confirmed in both isothermal and adiabatic operations.

Above part is followed by the chapter dealing with isothermal & adiabatic equation of Soave-Redlich-Kwong gas model. Based on former researchers’ study, the Soave-Redlich-Kwong equation of state is developed by modifying the van der Waals equation of state whose isothermal model and adiabatic model are relatively easy to be derived. As a complex mathematical expression, the Soave-Redlich-Kwong has not been analyzed.

Final chapter presents the conclusions of this study. The comparison of ideal gas model and van der Waals model shows that thermal properties of gas can be determined with the ideal gas model assumption if the operating conditions remain in low-pressure range (less than 10MPa, abs). More than 10% pressure deviation are produced giving a 20~32MPa, abs working pressure, however, the maximum pressure achievable by a commercial accumulator may not exceed about 35MPa, abs. The variation in volume that can be achieved in practice is less than 30% compared to that achievable by ideal gas model. The isothermal expression and adiabatic
expression of Soave-Redlich-Kwong was presented. The combination of Soave-Redlich-Kwong equation with Otis’s thermal time constant model was presented as well. Comparison results of adiabatic models (van der Waals equation and Soave-Redlich-Kwong equation) and thermal time-constant model that considered heat losses are concluded.

**Keywords**: Gas-loaded accumulator, Thermal properties, Equation of state, van der Waals model, Soave Redlich Kwong model, Isothermal change, Adiabatic change, Hydraulics
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Chapter 1

Introduction

This chapter gives a background introduction of this research. Focusing on hydraulic accumulator, its applications in engineering and the problem faced with recently are discussed. Section 1.1 introduced the definition of gas-loaded accumulator and its category. Section 1.2 showed the recent problem while applying accumulator in system designing. Section 1.3 introduced the history of mathematical modeling of gas in accumulator. Section 1.4 discussed the propose of this research and Section 1.5 gave the main structure of this thesis.

1.1 Background Information

The fast-economic development increases significant cost of energy during 21 centuries. The energy shortage comes with over-consumption of fossil resources, pessimistically, less than 100 years of fossil energy utilization is assumed. The unprecedented global increase in energy requirements have meant that the cost of traditional energy sources has been a rapid increase and that the dependence of economies development on an undistorted supply of such sources has become rigorous. Thus, newly energy resources are necessary for both society development and global environment. As engineering designer for hydraulic system, it is responsible to improve old systems and design novel systems. It has been estimated that a 300kW excavator will spent 2000t diesel, exhaust 6000t CO\textsubscript{2} in one lifecycle, a 5\% reduction of energy losses can save 100t diesel and reduce 300t CO\textsubscript{2} emission. Its batch production will have highlighted effect.

The final energy conservation result of hydraulic system mainly dependent on engineer designer, thus it should be considered before starting design the hydraulic system. Normally two ways are recommended.

1. Use effective hydraulic components. Effective pumps, valves, motors, frictionless sealing elements and lightweight constructions should be employed in hydraulic system.

2. Use effective hydraulic circuit.
   a. Flowrate is provided by hydraulic power based on requirements. Fixed hydraulic resource should be substituted by variable hydraulic resources to avoid exceeded flowrate. For instance, fixed pump is placed by common pressure pump is effective without system structural modify.
   b. Choose suitable parameters based on load and decrease operation pressure
distinguishes of all actuators.

c. Design suitable control strategy for specific actuator working process. Displacement control circuit is recommended rather than resistor circuit. By-pass throttling is recommended rather than in/out-let throttling and three-way pilot valve is recommended rather than two-way pilot valve.

(3) Energy regeneration. Appr eciable energy is produced while load braking or lowering and it is possible to regenerated and reutilized by hydraulic, electric or chemical reservoir.

In recent years, hydraulic hybrid power technology has been considered used in vehicle[1]–[3] or construction machinery. For instance, the gravitational/swing braking potential energy can be regenerated in hydraulic hybrid excavator. While conventional excavator swinging or boom cylinder lowering, the velocities are controlled by throttling valves, the entire potential energy is wasted in forms of heat loss or throttle loss. Novel excavators improved its efficiency of fuel consumption by adding energy regeneration system. Hydraulic transformer[4], independent metering valve[5], Effective circuit[6], hydraulic generator and motor[7], or hydraulic accumulator[8] are employed in various energy regeneration systems. This study is focusing on hydraulic gas-loaded accumulator. As a key hydraulic component, an accumulator is an energy storage device that accepts energy, save energy, and releases energy while be required in hydraulic circuit. It is usually a cylindrical or spherical vessel with a bladder[9], a diaphragm[10] or a piston[11] to separate hydraulic oil and charged gas and works by the correlation of compressible gas and incompressible fluid (hydraulic oil has a relatively low compressibility).

While other accumulator types exist, the gas-loaded accumulator is of the most common application. Theoretically, it is possible to store energy by ways of gravitational potential energy, kinetic energy or elastic potential energy. However, the high working pressure and relatively large power in modern hydraulic technology require the energy-storage device which can achieve energy storage/release in short time. Basically, the high-pressurized gas is the best option and the nitrogen is selected as accumulator gas to avoid burn.

1.2 Problem Statement

There are two key issues while using accumulator in hydraulic system. One is how to size the accumulator. As the accumulator is very heavy and has a large volume. It is reported that a 20t excavator produces $2.8 \times 10^5$J energy while boom cylinder lowering. At least a 50L, 150kg accumulator is needed for regenerating all energy. If the accumulator is not large enough, there is no enough space for hydraulic oil thus the performance specifications of the hydraulic system may not be met. On the other side, an oversized accumulator would increase unnecessary cost or too heavy for applications in hybrid
vehicle or aircraft landing-gear systems[12].

Another issue is to design suitable pre-charge pressure for different systems. During charging and discharging of accumulator, the gas pressure, volume and temperature are changing all the time, clear thermal properties analysis is significant. The pre-charge pressure of accumulator can be essential for the impact on system energy efficiency[13]. Accumulator study should include considering the gas model employed for the charged nitrogen gas, analyzing the thermodynamic properties between gas and ambient, and analyzing the accumulator structural parameters.

1.3 Research status of gas-loaded accumulator

As the hydraulic oil enters an accumulator, the pre-charged gas is compressed. The compression work represents a storage of available energy. While hydraulic oil being released from the vessel, the compressed gas is expanded, and the stored energy can be used for the hydraulic system. Gas pressure, volume and temperature are varying during all the process. Their correlation is mathematically described by equation of state named as gas model, which all research about thermodynamic performances in hydraulic gas-loaded accumulator should begin with.

The very first gas model is called ideal gas model, that it describes a theoretical gas composed of many randomly moving point particles whose only interactions are perfectly elastic collisions. Equation of state for ideal gas properly expresses thermal motion as expressed below[14]:

\[ PV = mRT. \] (1-1)

It assumes the molecular does not occupied volume space and intermolecular forces are neglected. In conditions of standard temperature and pressure, most real gases such as nitrogen oxygen, hydrogen qualitatively behave as an ideal gas. Thus, it is possible to be used to model the behavior of electrons in a metal[15] or applied in statistical mechanics[16].

However, the ideal gas model tends to fail to correctly predict gas thermodynamic properties at lower temperatures or higher pressures, as the volume that a real gas takes up and intermolecular attraction is not considered. The fixed equation of state is presented by van der Waals in 1873 as van der Waals gas model[17].

\[ \left( P + \frac{a_v}{V^2}\right)(V - mb_v) = mRT. \] (1-2)

\[ a_v = \frac{\gamma^2}{3}, b_v = \frac{9}{8}RT\gamma_v. \]

The constants \( a_v \) and \( b_v \) have positive values and their formula character the real gas
properties. This equation approaches the ideal gas model as the values of \(a_v\) and \(b_v\) approach zero. The constant \(a_v\) provides a correction for the intermolecular forces. Constant \(b_v\) is a correction for finite molecular size and its value is the size of one mole of the molecules.

In 1940, three researchers developed the equation of state names “Benedict–Webb–Rubin equation”[18] and the increased constants were modified by many other researchers[19–21]:

\[
P = \frac{mRT}{V} + m^3 \left( \frac{B_v - A_v}{V^2} \right) + m^2 \left( \frac{b_v - a_v}{V} \right) + \frac{m^3 c_0 \left( 1 + \frac{m^3 \gamma}{V^2} \right) e^{-\frac{\gamma}{T^2}}}{V^3 \gamma},
\]

where \(A_v, B_v, C_v, a_v, b_v, c_v, \alpha\) and \(\gamma\) are Benedict-Webb-Rubin parameters. In 1949 Otto Redlich and Joseph Neng Shun Kwong formulated a two-parameter, cubic equation of state that reflect accurately gas thermodynamics at temperatures above the critical temperature[22]. The named Redlich-Kwong equation is formulated as:

\[
P = \frac{mRT}{V - mb_{RK}} - \frac{a_{RK} m^2}{T^3 V (V + mb_{RK})},
\]

\[
a_{RK} = 0.42748 \frac{RT_{c}^{2.5}}{P_{c}}; b_{RK} = 0.08664 \frac{RT_{c}}{P_{c}}.
\]

Further modification of Redlich-Kwong equation was made by Soave in 1972[23]. The formulation is as follows:

\[
P = \frac{mRT}{V - mb} - \frac{a(T)m^2}{V (V + mb)},
\]

where

\[
a(T) = 0.42748 \frac{RT_{c}^{2.5}}{P_{c}} \left[ 1 + k \left( 1 - \left( \frac{T}{T_{c}} \right) \right)^{\frac{1}{2}} \right]^2
\]

\[
b = 0.08664 \frac{RT_{c}}{P_{c}}, \text{ and}
\]

\[
k = 0.480 + 1.57\omega - 0.176\omega^2.
\]

The Pitzer’s acentric factor \(\omega\) of nitrogen gas is assumed as 0.04. This equation of state
improved the accuracy when calculating the vapor pressures of pure substances, which are not influenced by any mixing rule.

The accumulator gas is modeled by gas models and combining with thermal energy balance, to be used in hydraulic system. The hydraulic control theory was quickly developed in 19 centuries. The military research results start to be applied in industry production. Meanwhile the basic theory of hydraulic accumulator was paid great attention such as preferences formulas and frequency calculation equation. At the end of 1970s the technology of energy-saving in vehicle promoted the accumulator application in energy conservation. Various accumulator configurations, types, functions were developed from 1980s. Until 1990s new research tools provided new ways and requirements for accumulator study as computational simulations, hardware and control theories. According to development of sorts of hydraulic systems, required performances are higher than past time, the basic accumulator theories which based on experimental data are not standard nor unitized. Thus, more detailed thermodynamics and processes analysis are necessary for future hydraulic system design.

### 1.4 Objectives

The objectives of this study include analysis of discrepancy of thermodynamic behavior of ideal gas model from van der Waals gas model, and the derivation of isothermal/adiabatic form of Soave-Redlich-Kwong equation of state, which is regarded as the most accurate gas model up to now.

### 1.5 Organization of the Thesis

Chapter 2 is the nomenclature of this thesis. Chapter 3 discusses the main application of accumulator in hydraulic system, especially in energy regeneration system. Chapter 4 discusses the thermal properties differences of ideal gas model and van der Waals gas model. Multiple parameters will be considered that how they influence accumulator performances in operating process. Chapter 5 will focus on the derivation of isothermal/adiabatic formula of Soave-Redlich-Kwong equation of state. And the main works are concluded in Chapter 6.
# Chapter 2

## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_v$</td>
<td>parameter in vdW model [Pa·m^6/mol^2]</td>
<td>$P_c$</td>
<td>critical pressure [MPa, abs]</td>
</tr>
<tr>
<td>$\alpha(T)$</td>
<td>parameter in SRK model [Pa·m^6/mol^2]</td>
<td>$P_s$</td>
<td>calculated pressure spot in pipeline [MPa, abs]</td>
</tr>
<tr>
<td>$A$</td>
<td>heat transfer area [m^2] $P_{d_{\text{max}}}$</td>
<td>$P_{d_{\text{min}}}$</td>
<td>maximum pulsation pressure [MPa, abs]</td>
</tr>
<tr>
<td>$A_A$</td>
<td>cylinder area at A [m^2]</td>
<td>$Q$</td>
<td>heat energy [J]</td>
</tr>
<tr>
<td>$A_B$</td>
<td>cylinder area at B [m^2]</td>
<td>$b$</td>
<td>parameter in vdW model [m^3/mol]</td>
</tr>
<tr>
<td>$b_v$</td>
<td>parameter in vdW model [m^3/mol]</td>
<td>$R$</td>
<td>gas constant [J/(mol·K)]</td>
</tr>
<tr>
<td>$b$</td>
<td>parameter in SRK model [m^3/mol]</td>
<td>$S$</td>
<td>entropy [J/K]</td>
</tr>
<tr>
<td>$C$</td>
<td>constant of SRK adiabatic expression [-]</td>
<td>$T_0$</td>
<td>initial gas temperature [K]</td>
</tr>
<tr>
<td>$C_v$</td>
<td>constant volume specific heat [J/(mol·K)]</td>
<td>$T$</td>
<td>absolute temperature [K]</td>
</tr>
<tr>
<td>$C_p$</td>
<td>constant pressure specific heat [J/(mol·K)]</td>
<td>$T_c$</td>
<td>critical temperature [K]</td>
</tr>
<tr>
<td>$d$</td>
<td>ratio exponent [-]</td>
<td>$T_w$</td>
<td>ambient temperature [K]</td>
</tr>
<tr>
<td>$f$</td>
<td>degree number of freedom of gas [-]</td>
<td>$\tau$</td>
<td>thermal time constant [s]</td>
</tr>
<tr>
<td>$F$</td>
<td>force on boom cylinder [N]</td>
<td>$U$</td>
<td>internal energy [J]</td>
</tr>
<tr>
<td>$G$</td>
<td>adiabatic exponent [-]</td>
<td>$\nu$</td>
<td>mole volume [m^3/mol]</td>
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<tr>
<td>$h$</td>
<td>heat transfer coefficient [W/(m^2·K)]</td>
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<td>critical volume [m^3/mol]</td>
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<tr>
<td>$H$</td>
<td>overall heat transfer coefficient [W/(m^2·K)]</td>
<td>$V_{\text{net}}$</td>
<td>net volume of accumulator [m^3]</td>
</tr>
<tr>
<td>$m$</td>
<td>amount of gas [mol]</td>
<td>$V$</td>
<td>gas volume [m^3]</td>
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<tr>
<td>$m_i$</td>
<td>amount of ideal gas [mol]</td>
<td>$V_0$</td>
<td>initial gas volume [m^3]</td>
</tr>
<tr>
<td>$m_v$</td>
<td>amount of van der Waals gas [mol]</td>
<td>$V_t$</td>
<td>final gas volume [m^3]</td>
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<td>$n$</td>
<td>polytropic exponent [-]</td>
<td>$\Delta V$</td>
<td>volume variation range [m^3]</td>
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<td>$V_{aq}$</td>
<td>bladder volume of accumulator [m^3]</td>
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<td>$P_{aq}$</td>
<td>accumulator pre-charged pressure [MPa, abs]</td>
<td>$W$</td>
<td>performed work in accumulator [J]</td>
</tr>
<tr>
<td>$P_A$</td>
<td>pressure at A [MPa, abs]</td>
<td>$\dot{x}$</td>
<td>velocity of boom cylinder [m/s]</td>
</tr>
<tr>
<td>$P_B$</td>
<td>pressure at B [MPa, abs]</td>
<td>$\alpha$</td>
<td>specific heat ratio [-]</td>
</tr>
<tr>
<td>$P_\delta$</td>
<td>pressure at B [MPa, abs]</td>
<td>$\omega$</td>
<td>acentric factor [-]</td>
</tr>
<tr>
<td>$P_\zeta$</td>
<td>ideal gas pressure [MPa, abs]</td>
<td>$\epsilon$</td>
<td>ratio in polytropic process [-]</td>
</tr>
<tr>
<td>$P_m$</td>
<td>average accumulator working pressure [MPa, abs]</td>
<td>$\delta_p$</td>
<td>allowable pressure pulsations factor [-]</td>
</tr>
<tr>
<td>$P_v$</td>
<td>van der Waals gas pressure [MPa, abs]</td>
<td>$\delta V_m$</td>
<td>pulsation volume of water pump [m^3]</td>
</tr>
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Chapter 3

Gas-loaded Accumulator in hydraulic System

Many construction machineries have a large flowrate change in different operations. Redundant power provided by pump, or gravitational/braking potential energy can be stored in accumulator and release it when needed[24]–[28]. In some hydraulic system, it is necessary to keep constant pressure for a while even after pump is stopped. The accumulator is possible to keep system as constant pressure by compensate system leakage[29]. Another application of accumulator is to absorb pulsation or moderate pressure shock in hydraulic systems which situations of pump, control valves, or hydraulic cylinders suddenly move or stop[30]. This chapter will explain how the accumulator functions the hydraulic system to reduce fuel consumption.

3.1 Regeneration of potential energy with a gas-loaded accumulator

In recent years, the gravitational potential energy regeneration by accumulator is studied by many researchers. It is important to reuse the potential energy to improve the power efficiency of the whole hydraulic system. For typical large hydraulic excavator, the boom weight is heavier than the load and its fast lowering is usually controlled by throttle valve and the gravitational potential energy converted into heat. The accumulator can store the energy and assist pump to drive the boom cylinder. The accumulator can be installed to save gravitational potential energy of boom cylinder in a hydraulic excavator[31]. Configuration of a hydraulic excavator includes travelling motor, swing motor, boom cylinder, arm cylinder and bucket cylinder. In the hydraulic drive system, the hydraulic oil is pumped by fixed displacement pump or variable displacement pump from tank, then pressurizes actuators through hydraulic line. Its pressure and flowrate are adjusted by multiple control valves. All these actuators are operated in sequence or synchronously in one typical working cycle. Considering the complex motion of actuators, the hydraulic performances are very important for power saving. For instance, the instantaneous acceleration of swing motor cases the pressure in motor increase momentarily, the relief valve near of inlet side of motor will open and large energy is lost. Braking energy of swing motor and gravitational potential energy of boom cylinder are usually transferred into heat by the throttle valve or relief valve. In some hydraulic systems the inconstant pressures of drive pumps are measured during excavation and loading work. As the duty cycle of a hydraulic excavator combinates traveling, leveling, truck loading and idling, the pump pressures are dramatically changing in just a single cycle. Every actuator undertakes different motion and has different pressure support by the pump. When only one pump is possible to drive all the cylinders, however, hydraulic systems with two or even more pumps have been designed. Even the number of pump relatively satisfies the multiple pressure requirement, it is inevitable on
losing energy in hydraulic circuits, and it is necessary to design specific energy regeneration system[32]. The hybrid power drive design provides a significant way to improve energy efficiency. An accumulator is possible to regenerate braking energy of swing motor, and gravitational potential energy of boom cylinder.

Using the measured pressures and cylinder velocities it is possible to calculate the gravitational potential energy. For an excavator which includes truck loading and the lifting motions, the swing kinetic energy and boom potential energy can be recovered by accumulator or other energy storage device[28]. Accurately, the amount of recoverable energy is different depending on operator, the gravitational potential energy from boom cylinder is mainly twice of the swing kinetic energy in truck loading period.

3.1.1 Suction boost circuit of boom cylinder for hydraulic excavator

![Diagram of suction boost circuit](image)

(a) pump, (b) engine, (c) control valve, (d) boom cylinder, (e) load, (f) control valve, (g) accumulator, (h) hydraulic line, (i) check valve, (j) unload valve, (k) throttle valve

Figure 3.1 Accumulator in Suction-boost circuit

A simple example of accumulator application in oil-hydraulic circuit is a suction boost circuit, as shown in Figure 3.1. The accumulator is used to regenerate gravitational potential energy from the boom cylinder[33]. The metering pump (a) is driven by an engine (b). While lifting the boom cylinder (d) and the load (e), a proportional control valve (c) is switched to the direct position, the hydraulic oil supports the head side of the boom. While the load (e) is lowering by the boom cylinder (d), the control valve (c) moves to the center position, and another control
valve (f) moves to the right side to allow the hydraulic oil flows to accumulator (g) from boom cylinder (d). The gravitational potential energy of load (e) and boom cylinder (d) is stored in accumulator (g) by the pressurized gas. In the successive lifting operation, the control valve (f) is switched to the left side to open the hydraulic line (h). The accumulator (g) is connected to the suction port of the pump (a) and support it. Due to the pressurized flowrate from hydraulic line (h), the check valve (i) is closed. The control valve (c) returns to direct position to allow the flowrate through to the boom cylinder (d). The valve (j) is an unload valve.

While the boom lifting, the cylinder operates against gravity of device and load, to a certain level. While the boom lowering, the recoverable negative energy is produced. Neglecting the friction in the cylinder, the gravitational potential energy is defined as the integration of boom cylinder power due to gravity which can be expressed as Eq. (3-1).

\[ E_{\text{boom}} = \int F x dt = \int (P_A A_\text{boom} - P_B A_\text{boom}) \dot{x} dt \]

(3-1)

where the boom cylinder lowers from A to B as shown in Figure 3.1. To simulation this energy regeneration system with an accumulator, an AMESim model was built using parameters listed in Table 3.1, as shown in Figure 3.2. AMESim is a commercial simulation software for the modeling and analysis of multi-domain systems. It is part of systems engineering domain and falls into the mechatronic engineering field.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump displacement</td>
<td>45 cc/rev</td>
</tr>
<tr>
<td>Boom cylinder stroke length</td>
<td>1 m</td>
</tr>
<tr>
<td>Boom cylinder piston diameter</td>
<td>63 mm</td>
</tr>
<tr>
<td>Boom cylinder rod diameter</td>
<td>30 mm</td>
</tr>
<tr>
<td>Load mass</td>
<td>2500 kg</td>
</tr>
<tr>
<td>Accumulator volume</td>
<td>30 L</td>
</tr>
<tr>
<td>Accumulator pre-charge pressure</td>
<td>5 MPa</td>
</tr>
</tbody>
</table>
Chapter 3 Gas-loaded accumulator in energy regeneration system

Figure 3.2  Modeling of Suction-boost circuit

Figure 3.3  Volume change of gas in accumulator
Figure 3.4   Boom cylinder assisted by accumulator in suction-boost circuit

Figure 3.5   Boom negative power and accumulator power in Suction-boost circuit
The simulation gives properties of boom cylinder motions. The volume change of gas in accumulator has been shown in Figure 3.3. “A to B” is charging process and “C to D” is the discharging process. At the initial point A the accumulator has the maximum volume which is the accumulator full size. The minimum gas volume depends on pre-charge pressure and the load mass by the side of boom cylinder. Figure 3.4 shows boom cylinder displacement, cylinder velocity, and accumulator gas pressure. The actuation of boom cylinder including load is lifting and lowering periodically, the accumulator pressure is periodically increasing while boom lowering and decreasing while boom lifting. From the initial operation, the accumulator has a 5MPa, abs pre-charge pressure, then stores hydraulic oil from boom cylinder in head port. The gas pressure increases and stores energy. When the boom lifting in next duty task, the accumulator releases the stored energy to assist the main pump.

The recoverable negative power and regenerated power by accumulator can be calculated as shown in Figure 3.5. The positive power is provided from main pump in cylinder lifting direction. Then, the accumulator obtains gravitational negative power from cylinder in cylinder lowering direction. As a hydraulic accumulator can be integrated into a hydraulic hybrid system easily and seamlessly, it is recommended to be used as a key component for energy regeneration. However, the limited energy storage density is the only major defect in some hydraulic systems without enough installation space. Obviously, all the negative energy is impossible to be regenerated by accumulator completely, it depends on the hydraulic circuit and control strategy.

3.1.2 Real gas accumulator model in AMESim

![Different gas models in accumulator in Suction-boost circuit](image)

Figure 3.6 Different gas models in accumulator in Suction-boost circuit
Although many simulation results have been showing the energy regeneration performances by accumulator, it is necessary to consider the real situation that more virtual situations should consider, that is the simulating models have been simplified. Some hydraulic simulation software, including AMESim, provide real gas models to show more accurate simulation results for specific requirements. The software AMESim employs an Soave-Redlich-Kwong equation of state as the real gas model[34]. The simulation model of suction-boost circuit is built as shown in Figure 3.6. Not only the gas properties modelled by gas model reflect the accumulator thermodynamics, but the heat transfer between gas and ambient also should be considered. The very early research by Otis et. al. presented a concept of thermal time-constant, which describes heat transfer through rubber bag and vessel[35]. It is expressed by:

$$\tau = \frac{mC_p}{HA}$$

(3-2)

where

- $H$ is the overall heat transfer coefficient for the heat exchange of accumulator.
- $A$ the heat exchange area.

As shown in Figure 3.7, after a quick compression process, the gas pressure and temperature increase during A to B. Assuming the gas volume is constant, the gas pressure will drop down because of heat exchange through the rubber bag and the accumulator vessel. The time when gas pressure (or temperature) has a 63.2% drop starting from maximum pressure (Point B) is defines as the thermal time-constant. The time-constant for the heat transfer normally takes a few minutes. Basically it depends on the accumulator type and volume. This accumulator model predicts gas process in a more realistic manner and therefore it is useful when actual gas process is more significant.

Figure 3.7 Thermal time constant concept by Otis et.al[35]
The simulation is operated by AMESim. While the boom cylinder lifting, the accumulator keeps pre-charge pressure. Then boom cylinder lowers and the gravitational potential energy is saved in accumulator, that cases increasing of accumulator pressure. Before next lifting process of boom cylinder, the accumulator keeps constant pressure, however, a pressure decrease will happen due to the heat loss between accumulator gas and ambient. The gas pressure quickly decreases and be used for boom cylinder lifting. The gas temperature and pressure reach minimum value then slowly increase as the heat exchange happens. Figure 3.8 shows the gas pressures, modelled by different thermal time-constants, and the boundary pressures which are calculated in isothermal case and adiabatic case by ideal gas model. The accumulator in the hydraulic circuit were modelled with four different values of thermal time-constant, 10, 50, 100 second and ideal gas, which means there is no heat loss during all operations and the thermal time-constant can be assumed as infinite. The larger the thermal time-constant, the slower the gas pressure changes. The restored energies calculated by different thermal time-constant are distinguished, correspondingly. Figure 3.9 shows cylinder displacement properties simulated by different accumulator gas models. The 1m length cylinder can completely lower coupled with ideal-gas accumulator model. Gas pressure equals to hydraulic oil pressure at head side of boom cylinder. However, the cylinder cannot completely lower with real-gas accumulator model. The real gas pressure in accumulator rises faster than ideal gas during hydraulic oil flows in accumulator, and it becomes identical with hydraulic oil pressure at head side of boom cylinder before the oil in cylinder completely flows in accumulator. Here the pre-charged accumulator pressure is adjusted according to hydraulic system performance and the load. Thus, the difference of ideal gas model and real gas model in different pressure level should be investigated. It will be discussed in Chapter 4.

![Figure 3.8](image)

**Figure 3.8** Effect of gas thermal time-constant on gas properties
While using an accumulator to assist the regeneration system, it should be highlighted how will the accumulator influence the system performances. As in the suction-boost circuit, the accumulator flowrate is directly contacted to oil inlet side of pump. During accumulator operating, pressure pulsation and flowrate pulsation should be considered[36]. Thus, a main pump with higher performances is required in the hydraulic system, depending on the preferred performance analysis of accumulator.

3.1.3 Motor-assist circuit of boom cylinder for hydraulic excavator

As it might be difficult to directly assist pump by accumulator in some hydraulic circuit, other components coupled with accumulator is alternative. Figure 3.10 shows another energy regeneration system by accumulator (g) coupled with a variable hydraulic motor (h). The hydraulic motor (h) assists the main pump (a) with gears in parallel with an electric motor (b). An AMESim model for motor-assist circuit has been built, as shown in Figure 3.11. Main parameters in motor-assist circuit are as same as the suction-boost circuit. The gear ratio is 1:1. For hydraulic system designing, it is still difficult to deal with these parameters. How the parameters influence fuel consumption, hydraulic heat load, and the amount of usable power available on the hydraulic machine is critical for its overall performance[37]. Thus, more comprehensive mathematical models and experiment test are needed.
Figure 3.10 Accumulator in motor-assist circuit

Figure 3.11 Accumulator in motor-assist circuit
The simulation results are shown in Figure 3.12. While the boom cylinder lowering, the hydraulic oil flows into accumulator and the gas is compressed from A to B. While lifting the boom cylinder from C to D, the accumulator releases energy to drive the hydraulic motor. And the hydraulic motor is driven by hydraulic oil from accumulator. The main pump is driven by the shaft connected to hydraulic motor.

![Diagram of boom cylinder displacement and pressure properties in accumulator and hydraulic motor](image)

**Figure 3.12** Boom cylinder displacement and pressure properties in accumulator and hydraulic motor

**Figure 3.13** shows how the parameter affect the regenerated energy. While setting various displacement ranges of hydraulic motor, different energies have been regenerated by the circuit. In a hydraulic line with a hydraulic motor, its performances are influenced by many factors such as the parts precision, tolerances, leakage or friction. The volumetric efficiency depends on flow...
rate, which is driven by the accumulator. If the accumulator size changes, the volumetric efficiency of hydraulic motor is inevitably affected.

**Figure 3.13** Regenerated energy by hydraulic motor with different displacement ranges

**Figure 3.14** Power comparison of suction-boost circuit and motor-assist circuit

**Figure 3.14** shows the power comparison of engine power in main circuit and regenerated power by accumulator for suction-boost circuit and motor-assist circuit. The “MA-engine” line
represents engine power in motor-assist circuit that drives the boom cylinder. The lifting of boom cylinder is assisted by regenerated power by accumulator through a hydraulic motor, shown by the “MA-accumulator” line. The “SB-engine” line represents the engine power in suction-boost circuit that drives the boom cylinder. The lifting of boom cylinder is assisted by regenerated power by accumulator, shown by the “SB-accumulator” line. It can be seen the regenerated power by different regeneration systems are a little different.

Figure 3.15 shows the energy regeneration performances of accumulator in different hydraulic circuits. “Gas pressure-MA” line represents gas pressure in accumulator in motor-assist circuit during charging and discharging operations. “Energy-MA” line represents the regenerated energy in motor-assist circuit during the same period. “Gas pressure-SB” line represents gas pressure in accumulator in suction-boost circuit during charging and discharging operations. “Energy-SB” line represents the regenerated energy in suction-boost circuit during the same period. The “Gas pressure-MA” takes a little longer time than “Gas pressure-SB” in discharging process. The final regenerated energies in both circuits are also a little different. Thus, not only the accumulator parameters themselves affect energy regeneration, but also other hydraulic components determine the overall regeneration performances. Conversely, the accumulator parameters (size, pre-charge pressure, charge/discharge speed et.al) will influence whether other hydraulic components work effectively. The accumulator is an important component in hydraulic hybrid excavator. Numerical and experimental analysis have shown the advantage for machinery efficiency[38].
3.2 Some applications of accumulator in other fields

3.2.1 Accumulator for pulsation absorption

High-pressure water-jet propulsion system often utilizes accumulators to absorb flowrate pulsation and pressure pulsation when the high-velocity water jet from a nozzle is used for drilling, cutting or deburring et al.[40]–[42]. Generally, with an angle of more than 25 degree of the swash plate the axial piston water pump is employed. Thus, large flow pulsation is produced as the large displacement of the pump. While the flowrate comes through orifices, structural pipes, some pressure pulsations happens. The accumulator can assist for pulsation absorption based on water pump structure and its working characteristics, as shown in Figure 3.16. Transient flowrate pumped by water pump and flowrate passing by through nozzle have the force balance in the pipeline includes accumulator. As the specific working pressure range in high-velocity water-jet propulsion system, the accumulator pre-charge pressure, volume, and accumulator number will influence the overall pulsation absorption effect. According to working pressure, the accumulator bladder volume can be calculated by[39]

\[
V_{aj} = \frac{\delta V_m}{1 - \left(2 - \frac{\delta_p}{2 + \delta_p}\right)^{\frac{1}{\alpha}}}.
\]

\[
\delta_p = \frac{2(P_{\delta_{\text{max}}} - P_{\delta_{\text{min}}})}{(P_{\delta_{\text{max}}} + P_{\delta_{\text{min}}})^2}.
\]  

(3-3)
where parameters $[\delta_P]$ and $\delta V_m$ are determined by water pump and pipeline dynamic performances. For effectively absorbing flow pulsation, not only the accumulator position is important[43]–[45], but the number of accumulator also play a role. To select suitable parameter for the second accumulator if the first accumulator was determined, the following equation can be employed[39]:

$$\frac{P_{aq1}}{P_{s1}} V_{aq1} + \frac{P_{aq2}}{P_{s2}} V_{aq2} = V_{ar}$$  (3-4) 

However, only ideal gas properties were considered in above equation. The final parameter values would be different if the real gas properties are considered.

### 3.2.2 Accumulator for thermal change compensator

A steam accumulator can be used to store available energy in concentrating solar power system. Energy storage systems are regarded as a significant factor as the increasing shares of electrical energy produced by renewable energy sources[46]. The surplus steam is fed into liquid volume in charging process, then the pressure and temperature of the liquid volume will be increased by the condensing steam. During discharging, the saturated steam in taken out from the steam accumulator, and the pressure and temperature of the steam decreases. The steam temperature is acted as a gauge of energy storage content. The correlation between steam pressure, volume and temperature is important for measuring the volume-specific storage capacity of the accumulator.
3.3 Importance of analyzing gas thermodynamics in accumulator

Accumulator applications in various hydraulic systems have been introduced in this Chapter. The circuits discussed are examples to illustrate it is significant to study accumulator properties in hydraulic circuit. The capacity of an accumulator and its influences on energy regeneration phase are the key problems for the configuration design of hydraulic hybrid power systems. In specific application, the real gas compression properties are distinguished from ideal gas as the heat transfer and compressibility effects cannot be negligible.

3.4 Summary

This chapter shows how an accumulator is used for hydraulic energy regeneration system. Two different hydraulic circuits for driving boom cylinder of hydraulic excavator are simulated by AMESim. The thermodynamics of the accumulator and the performance of hydraulic circuit are analyzed. The parameter of thermal time constant of real gas model is also discussed. The results show the different of real gas model and ideal gas model should be considered. Accumulator applications in other fields are introduced as well.
Chapter 4

Thermal properties difference of ideal gas model and van der Waals model

Chapter 3 has shown how the gas models of accumulator influences the hydraulic system. It is significant to figure out how much difference of ideal gas and real gas model, and how the initial conditions impact the gas properties in gas state variation. The van der Waals gas model is employed as the real gas model, as its well-known mathematical expression can be used to calculate the isothermal change and adiabatic change for comparing with ideal gas model. Section 4.1 introduced the definition of accumulator gas. Section 4.2 analyzed the thermal energy balance in accumulator, that how the accumulator restores energy and output work to hydraulic system. Section 4.3 introduced the mathematical modeling of accumulator by ideal gas equation of state and van der Waals equation of state. Section 4.4 discussed how the parameters influence the accumulator thermodynamics. Based on the understanding of the actual variation of accumulator gas, the mathematical modeling method to solve engineering problem is concluded.

4.1 Supercritical fluid

The gas-loaded accumulator has pre-charge gas, usually nitrogen, and is compressed in accumulation process and expanded in energy release process. In high pressure condition, the nitrogen cannot be regarded as in ideal state any more, supercritical fluid is recommended for analyzing. Furthermore, the hypothesis of ideal gas behavior should also be reconsidered by real gas behavior. A substance basically has states of solid, liquid and gas, three kinds of state, as shown in Figure 4.1. The thermodynamics of a substance are determined by pressure and temperature. A substance can be changed to gaseous phase at a specific temperature point which is called a boiling point. If the pressure increases, the boiling point also increases. However, the increase of the boiling point is limited to a specific maximum point, which is called the super critical point. When the substance is subjected to a temperature and a pressure higher than the critical point value, it is named as supercritical fluid, which means it cannot be a liquid state while pressure continually increases, or a gas state while temperature continually increases. In the supercritical region, the substance exhibits specific properties and has an intermediate behavior between that of a gas and a liquid. A supercritical fluid possesses gas-like viscosities, liquid-like densities and diffusities intermediate to that of a gas and a liquid. Since 1822 when Baron Charles Cagniard de la Tour discovered the critical point of a substance from his experiments, much effort has been devoted to investigating the supercritical fluid and being used in many significant applications. However, it is still open problems of the
characteristics of various properties of the supercritical fluid and their theoretical analysis.

Figure 4.1  Phase diagram and supercritical fluid

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Temperature</td>
<td>126</td>
<td>K</td>
</tr>
<tr>
<td>Critical Pressure</td>
<td>3.4</td>
<td>MPa, abs.</td>
</tr>
<tr>
<td>Critical Density</td>
<td>0.31</td>
<td>g/ml</td>
</tr>
<tr>
<td>Practical Liquid Temperature</td>
<td>77</td>
<td>K</td>
</tr>
<tr>
<td>Practical Liquid Density</td>
<td>0.81</td>
<td>g/ml</td>
</tr>
</tbody>
</table>

Table 4.1  Supercritical fluid properties of Nitrogen

For a hydraulic system, the pressure of nitrogen gas-loaded in accumulator is assumed to be around MPa level, and the temperature is at normal temperature (about 300K), thus its supercritical fluid properties exactly matches its application. Table 4.1 shows the supercritical fluid properties of nitrogen. As the equation of state of nitrogen gas under practical conditions in hydraulic systems, the differences of ideal gas model and real gas model will be discussed in this chapter.
4.2 Thermal energy balance in accumulator

An accumulator operates by charging or discharging hydraulic oil in the hydraulic circuit, the energy in accumulator consequently increases or decreases while the gas pressure/temperature increasing or decreasing. The accumulator can be regarded as a thermodynamic system. The energy balance properties follow the first law of thermodynamics. It has a constant initial total energy, which can be transformed from one form to another, but can be neither created nor destroyed. Mathematically, the first law of thermodynamics is expressed by the following equation:

\[ dU = dQ - dW. \] (4-1)

Nitrogen gas in accumulator has internal energy of \( U \) as shown in Figure 4.2. The nitrogen gas receives heat energy \( Q \) from ambient and performs work \( W \) to outside. The increment of the energy \( U \) is equal to the difference between the heat accumulated by the system and the work done by it. The performed work \( W \) is positive while hydraulic oil charging into the accumulator by increasing the gas pressure. At the same time the gas temperature increases, the heat energy \( Q \) also increase. The total internal energy \( U \) is increasing in this process. However, heat exchange will happen if the temperature in accumulator is different from ambient temperature. The heat energy \( Q \) will decrease while the gas temperature higher than ambient temperature. In discharging process, the accumulator performs work to hydraulic system that the work \( W \) will be negative and the temperature has a decrease as the gas pressure decreasing. At the same time, if the gas temperature is lower than ambient temperature, heat exchange will happen from ambient to accumulator that cases an increase of heat energy \( Q \).

![Figure 4.2  Energy balance of a gas-loaded accumulator](image-url)
The total energy variation tendency depends on change of the thermal system. For the gas-loaded accumulator, namely the gas is assumed as isothermal, polytropic or adiabatic process for ideal gas model, and as isothermal or adiabatic process for real gas model. An isothermal process is a change of a thermal system that the temperature always keeps constant. It is applied when the thermal system is in contact with an outside thermal ambient, and the heat change in the thermal system will occur slowly enough to allow the thermal system continually adjusts the gas temperature in accumulator as equal as ambient temperature through heat exchange. Isothermal process states that the internal $U$ in accumulator of a fixed amount of the nitrogen gas depends only on its temperature if it is assumed as ideal. In the isothermal compression of the accumulator there is work $W$ done by hydraulic oil on the accumulator gas to decrease the gas volume and increase the gas pressure. The performed work $W$ will increase the internal energy $U$ and will results to increase the temperature. To keep the temperature as constant, energy must leave the accumulator as thermal form and transfers to outside. For the ideal gas model, the amount of energy transferring to ambient $Q$ is equal to the work done on the accumulator gas $dW$, as the internal energy $U$ does not change. In some hydraulic systems that using a group of accumulators as an energy storage device, it is “isothermal approximation”, which means there is large heat exchange area of accumulator as numbers of accumulators are employed and the size is large enough. The energy storing process also takes a long time that heat exchange has enough time to occur. Thus, the isothermal process is one of the important theories for accumulator study to model the gas states.

An adiabatic process in accumulator is one that occurs without heat exchange between the accumulator vessel and its surroundings. In an adiabatic process, the energy cannot be transferred to its surroundings by heat exchange, that the only way is by performing work $W$. As to expound the first law of thermodynamics, the adiabatic process is a significant concept in thermodynamics to analyze gas change properties in accumulator. While charging and discharging operations by hydraulic oil in accumulator happen so rapidly that they may be conveniently described by “adiabatic approximation”, which means that there is no enough time for heat exchange to take place to or from the accumulator through the accumulator vessel.

When the accumulator gas is compressed, the gas temperature rises as the work done by hydraulic system, the increasing internal energy can only slowly be dissipated by radiation or conduction of heat, that the pressure will be higher than that in isothermal process if given same initial conditions. When the accumulator is expanded, the gas temperature decreases as the accumulator does work to hydraulic system, the gas pressure will be lower than that in isothermal process if given same initial conditions.

Polytropic process is a concept which can describe multiple compression and expansion processes including heat exchange, mathematically expressed by:

$$PV^n = \text{const}$$  \hspace{1cm} (4-2)
Chapter 4 Thermal properties difference of ideal gas model and van der Waals model

For the ideal gas law, a process is polytropic if and only if the ratio \( \epsilon \) of heat exchange \( Q \) to work done \( W \) at each infinitesimal step is kept constant as expressed below:

\[
\epsilon = \frac{dQ}{dW} = \text{const}
\]  

(4-3)

The polytropic index \( n \) in Eq. (4-2) has been studied by researchers. The prediction of the value used for nitrogen gas in accumulator is presented by experiment data as shown in Table 4.2, in which it is related to average working pressure and charging/discharging time. It is an approximation of heat exchange process. Instead of ideal gas model, real gas model should be employed for high accuracy. The experimental polytropic index should be replaced by detailed expression described the heat exchange process. The difference of ideal gas model and real gas model should be investigated including isothermal change and adiabatic change.

### Table 4.2 Polytropic exponent of nitrogen[47]

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<td>&lt;2</td>
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<td>1.65</td>
<td>1.55</td>
<td>1.50</td>
<td>1.40</td>
</tr>
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</table>

According to accumulator operating performances in hydraulic system, it is selectable of these three kinds of gas change statement includes isothermal process, adiabatic process or polytropic process (only for ideal gas). A clear difference of thermodynamics characteristics has been shown in Figure 4.3. The heat produced in isothermal process is transferred to surrounding to keep temperature constant. Adiabatic compression means that no heat loss in accumulator in which the gas pressure has a rapid increase. The polytropic usually is more closed to various practical processes. Its pressure property is between isothermal process and adiabatic process.
To mathematically understand how the thermodynamics of the gas is in accumulator, it will be discussed in this section from the first law of thermodynamics. From Eq. (4-1), the work done $W$ of gas by hydraulic system can be formed in gas pressure and the change amount of gas volume:

$$dW = PdV.$$  \hspace{1cm} (4-4)

The heat energy $Q$ can also be expressed using temperature $T$ and the entropy $S$. The entropy is a measure of the unavailable energy in accumulator that is also usually considered to be a measure of the thermal system's disorder, that is a property of the thermal system's state, and that changes directly with any reversible change in heat in the thermal system and inversely with the temperature of the system. It can be expressed by the following equation:

$$dS = \frac{dQ}{T}.$$  \hspace{1cm} (4-5)

Thus, the internal energy can be expressed by substituting Eq. (4-4) and Eq. (4-5) to Eq. (4-1) yields:

$$dU = TdS - PdV.$$  \hspace{1cm} (4-6)

Neither by the heat energy nor the work done, the internal energy is described by quantity of state in Eq. (4-6). To calculate the internal energy, the entropy should be figure out. The total differential of internal energy is:

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV.$$  \hspace{1cm} (4-7)
The constant volume specific heat \( C_v \) is defined based on the temperature dependency to internal energy as the following formula:

\[
C_v = \frac{1}{m} \left( \frac{\partial U}{\partial T} \right)_V.
\] (4-8)

To eliminate the internal energy \( dU \), according to the definition of \( C_v \) its total differential formula is expressed by:

\[
dU = mC_v dT + \left( \frac{\partial U}{\partial V} \right)_T dV.
\] (4-9)

In Eq. (4-9) the differential form of \( U \) with respect to volume is expressed by:

\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial S}{\partial V} \right)_T - P.
\] (4-10)

Above equation includes entropy which can be represented according to Maxwell equations[48]. The Maxwell equations formulated by James Clerk Maxwell describe the interrelation between electric and magnetic fields. Usually they are formulated as four equation which named as “Gauss’s law”, “Absence of magnetic monopoles”, “Faraday’s law” and “Biot-Savart Law”. From the Maxwell relationship, it is known that

\[
\left( \frac{\partial S}{\partial V} \right)_T = \left( \frac{\partial P}{\partial T} \right)_V.
\] (4-11)

The correlation of entropy, volume with temperature is formulated by the correlation of pressure, temperature with volume. Then the Eq. (4-10) can be expressed as:

\[
\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_V - P.
\] (4-12)

The internal energy \( dU \) in Eq. (4-9) can be expressed by:

\[
dU = mC_v dT + T \left( \frac{\partial P}{\partial T} \right)_V dV - P dV.
\] (4-13)

The partial differential form of \( P \) with respect to \( T \) is decided by the gas state in accumulator calculated by gas model, which describes the relation between gas pressure, gas temperature and gas volume.
Chapter 4  Thermal properties difference of ideal gas model and van der Waals model

4.3 Equation of state of gas model in accumulator

4.3.1 Isothermal, polytropic and adiabatic changes of ideal gas

In a gas-loaded accumulator, it is assumed that the hydraulic oil pressure charging in accumulator is equal to compressed gas pressure. The hydraulic oil pressure is determined by working operations of the actuator and hydraulic pipeline characteristics in hydraulic system. To build the correlation of accumulator property with the hydraulic system, the gas pressure and volume should be understood. In isothermal process, the temperature is constant that the gas pressure can be expressed as:

\[ P = \frac{mRT}{V} = \frac{P_0V_0}{V}. \]  

(4-14)

where \( P_0 \) is the initial gas pressure which is equal to pre-charge pressure of accumulator, and \( V_0 \) is the initial gas volume which is usually 90\% of accumulator size. Once the gas volume has been known by the hydraulic oil volume (if the compression characteristic of hydraulic oil is negligible or calculable), the gas pressure can be figure out by this equation. It is difficult to keep temperature constant in accumulator. Either performances of gas charge or discharge, or heat exchange between gas and ambient will influence that. However, in pneumatic storage technologies application, a group of compressed gas-loaded accumulators together with the addition of a pump/motor and a motor/generator can be assumed operates in almost isothermal process as the large heat exchange area of accumulator[49].

The adiabatic pressure equation of nitrogen gas assumed as ideal in accumulator can be expressed as:

\[ P = \frac{mRT}{V^d} = P_0\left(\frac{V_0}{V}\right)^d \]  

(4-15)

where the ratio exponent

\[ d = \frac{C_p}{C_v} = \frac{f + 2}{f} \]  

(4-16)

As the degree number of freedom \( f \) is 5 for diatomic nitrogen gas, the adiabatic rate exponent \( d \) is 1.4 here.

From Eq. (4-2), the polytropic pressure equation of nitrogen gas assumed as ideal in accumulator can be expressed as:
\[
\frac{P_m R T}{V^n} = P_0 \left( \frac{V_0}{V} \right)^n \quad (4-17)
\]

where the polytropic exponent \( n \) is 1.2 for nitrogen gas which assumed as ideal.

Above pressure expressions of isothermal process, adiabatic process and polytropic process of ideal gas assumption have been presented. They will be compared with van der Waals real gas model in this chapter.

### 4.3.2 Isothermal and adiabatic changes of van der Waals gas

The assumptions of ideal gas are derived from the kinetic theory of gases that the only interaction among molecules is elastic collisions, however, the intermolecular forces among molecules are negligible. In the meanwhile, size of the molecules is small relatives to the distances separating them in low pressure case and the gas consists of molecules is in ceaseless random motion. However practical experience has shown that the ideal gas model has limited accuracy at low pressures. Real gas characteristics should be considered and therefore it is needed to quantify the difference gained by using a real gas approach compared to the ideal gas model. The concept of van der Waals gas equation of state was presented by van der Waals, which is based on plausible reasons that real gases do not follow the ideal gas model.

![Real gas concept](image)

**Figure 4.4** Real gas concept

The intermolecular forces and molecular size concept are illustrated in **Figure 4.4**. The gas pressure to the reservoir wall will be decreased by the intermolecular forces between molecules. For the nitrogen gas in accumulator, all the molecules in accumulator have a relatively lower pressure compared with gas model that neglects intermolecular force. In addition, at low pressure the molecule volume can be neglected compared to reservoir volume. Assuming molecules have zero volume of ideal gas, the volume available to the molecules for motion is
always the same as the container volume. For considering real gas characteristics, the molecules of a real gas have small but measurable volumes. At low pressures, the gaseous molecules are relatively far apart from each other, but as the pressure of the gas increases with decreasing container volume, the intermolecular distances are shortened. As a result, the volume occupied by the molecules becomes important compared to volume of the container. Therefore, the total volume occupied by the gas is larger than the volume predicted by the ideal gas law. The van der Waals equation of state shown in Eq. (1-2) models the real gas thermodynamics including both intermolecular forces and molecular size. The factor $a_v$ represents intermolecular forces and factor $b_v$ represents the molecular size. Here the van der Waals gas model will be more completely introduced in isothermal process and adiabatic process.

As the temperature is constant in isothermal process, the isothermal van der Waals gas model can be expressed as:

$$P = \left( P_0 + \frac{a_v m^2}{V_0^3} \right) \left( V_0 - mb_v \right) \frac{a_v m^2}{V^2}.$$  \hfill (4-18)

For the adiabatic process, the van der Waals equation of state is substituted into the Eq. (4-13):

$$dU = mC_v dT + \frac{a_v m^2}{V^2} dV.$$  \hfill (4-19)

Combining the first law of thermodynamics in Eq. (4-6) with above internal energy expression, the total differential form of entropy can be obtained:

$$dS = \frac{mC_v}{T} dT + \left( P + \frac{a_v m^2}{V^2} \right) dV.$$  \hfill (4-20)

Substituting the van der Waals equation of state in Eq. (1-2) into above equation:

$$dS = \frac{mC_v}{T} dT + \frac{mR}{V - mb_v} dV.$$  \hfill (4-21)

Integrate both sides of above equation yields

$$S = mC_v \ln \left( \frac{T (V - mb_v)^{\frac{g}{mR}}} {V^g} \right) + S_0.$$  \hfill (4-22)

As the gas temperature can be expressed by van der Waals equation of state:

$$T = \frac{\left( P + \frac{a_v m^2}{V^2} \right) (V - mb_v)} {mR}.$$  \hfill (4-23)
The Eq. (4-22) can be rewritten substituting above formula as:

\[
S = mC_v \ln \left[ \frac{1}{mR} \left( P + \frac{a_m m^2}{V^2} \right) \left( V - mb \right)^{\frac{R}{C_v} - 1} \right] + S_0. 
\] (4-24)

As the change of heat energy \( dQ = 0 \) in adiabatic process, it is known that the change of entropy \( dS = 0 \) according to Eq. (4-5), which means the entropy \( S \) is constant. Therefore, the logarithm part of above equation is constant. The following formula can be obtained:

\[
\left( P + \frac{a_m m^2}{V^2} \right) \left( V - mb \right)^{G} = \text{const}.
\] (4-25)

where the index \( G \) is defined as:

\[
G = \frac{R}{C_v} + 1.
\] (4-26)

According to the Eq. (4-25) and Eq. (4-26), the gas pressure in adiabatic process at any time can be calculated if the initial condition is known:

\[
P = \left( P_0 + \frac{a_m m^2}{V_0^2} \right) \left( \frac{V_0 - mb}{V - mb} \right)^{G}.
\] (4-27)

By these equations, it is possible to calculate the difference the ideal gas model and real gas model which described by van der Waals equation, in processes of isothermal and adiabatic.

### 4.4 Impact of thermal parameters on gas performance in accumulator [50]

#### 4.4.1 Compression and expansion

This section will compare the ideal gas and van der Waals gas in accumulator by the mathematical expressions derived in above sections. A gas-loaded accumulator uses a bladder or piston to separate gas and hydraulic fluid. The accumulator gas is compressed or expanded when charging or discharging by hydraulic oil. Basic calculation results by gas equation of state are shown in Figure 4.5. The pressure properties of van der Waals (vdW) gas model and ideal gas model corresponding to identical gas volume vary with different trends. Gas pressure varies begin with the point \((V_0, P_0)\) that real gas pressure is equal with ideal gas pressure, which can be regarded as initial point. To the left side, as gas charging process, the real gas pressure is higher than ideal gas pressure that the gas volume decreases, however, to the right side, as
gas discharging process, the trend is opposite that gas volume increases.

\[ P_i = \frac{m_i RT}{V}, \]  

(4-28)

and gas pressure of van der Waals model is:

\[ P_v = \frac{m_v RT}{V - m_v b_v} - \frac{am_v^2}{V^2}. \]  

(4-29)

It should be highlighted that the original gas amount \( m_v \) for van der Waals gas and \( m_i \) for ideal gas are different if the initial gas pressure, gas volume and gas temperature are assumed to be equal. Setting the identical initial pressure:

\[ P_{v0} = P_{i0} = P_0 \]  

(4-30)

The relationship of \( m_v \) and \( m_i \) in these two models can be calculated:
Chapter 4 Thermal properties difference of ideal gas model and van der Waals model

\[ m_i = \frac{V_0 m_i}{V_0 - m_i b_i} - \frac{a m_i^2}{V_0 R T_0} \]  \hspace{1cm} (4-31)

The magnitude relationship of gas amount \( m_p \) and \( m_i \) will keep constant during accumulator charging and discharging as the gas in accumulator keeps in sealed volume. Combing Eq. (4-28), Eq. (4-29) and Eq. (4-31) yields:

\[ \frac{P_v}{P_i} = \frac{V}{V_0 - m_i b_i} \frac{a m_i}{RT V} \frac{RT_0 V_0}{V_0 - m_i b_i} \]  \hspace{1cm} (4-32)

It is possible to mathematically compare the ideal gas pressure and van der Waals gas pressure by above equation. However, due to two variables \( V \) and \( T \) in the right side of this expression, it is difficult to calculate the specific value. While during accumulator operating, the gas compression or expansion happens between two extreme cases isothermal and adiabatic processes, the specific value can be predicted by these extreme cases. In isothermal process, the temperature is constant as:

\[ T = T_0. \]  \hspace{1cm} (4-33)

It is calculable using a constant to instead of the variable in Eq. (4-32). A function of \( h(x) \) is created in which the variable \( T \) in Eq. (4-32) is substituted by constant \( T_0 \) and the \( x \) represents the only variable \( V \):

\[ h(x) = \frac{x}{x - m_i b_i} \frac{a m_i}{RT_0 V_0} - \frac{a m_i}{RT V}. \]  \hspace{1cm} (4-34)

Compared with initial temperature \( T_0 \), the gas temperature \( T \) in adiabatic process rises during charging as the work done by hydraulic oil cannot be transferred to its surroundings by heat exchange and drops during discharging as the work to hydraulic oil cannot be compensated by thermal energy from surroundings. Give \( x \) in Eq. (4-34) the value \( V \) in Eq. (4-32), the specific value of \( P_v/P_i \) can be predicated by the comparison of \( T \) with \( T_0 \). If \( T \) equals to \( T_0 \), \( P_v/P_i \) is equal to \( h(V) \). While giving a value of \( T \) larger than \( T_0 \), a larger value of \( P_v/P_i \) will be obtained, thus it is larger than \( h(V) \). Therefore, the value of Eq. (4-32) can be compared with function \( h(x) \) by this relation. To plot the function \( h(x) \), parameter values in Eq. (4-34) are set according to accumulator operating conditions and accumulator gas properties. The initial conditions van der Waals parameters, and gas constant reflects nitrogen thermodynamics are
listed in Table 4.3. The molecular number of van der Waals gas $m_v$ can be calculated by which in ideal gas model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>van der Waals parameter $a_v$</td>
<td>0.1062</td>
</tr>
<tr>
<td>van der Waals parameter $b_v$</td>
<td>$2.7009 \times 10^{-5}$</td>
</tr>
<tr>
<td>Amount of van der Waals gas $m_v$</td>
<td>40.5142 mol</td>
</tr>
<tr>
<td>Nitrogen gas constant $R$</td>
<td>8.3145 J/(mol·K)</td>
</tr>
<tr>
<td>Initial gas temperature $T_0$</td>
<td>303 K</td>
</tr>
<tr>
<td>Initial gas volume $V_0$</td>
<td>$2.7 \times 10^{-3}$ m$^3$</td>
</tr>
</tbody>
</table>

The plot of $h(x)$ is shown in Figure 4.6. Abscissa represents $x$ that varies in range of gas volume $V$ during compression and expansion. The initial volume is $V_0$ in operating process which demonstrates that the $h(x)$ is larger than 1 for $0 < x < V_0$. This range can be regarded as accumulator compression process that the gas is contradictorily compressed as the occupied molecule volume is extruded. Compare with ideal gas without considering the molecule
volume, external pressure is needed in compression process. While \( x \) varies in range of \( x > V_0 \), the value of \( h(x) \) is smaller than 1 which can be supposed as accumulator discharging process that the intermolecular force among molecules has an obvious influence on gas properties. It decreases gas pressure to reservoir wall. For a long completed discharging process, the low pressure can be supposed as pre-charged pressure as accumulator has a fixed size installed in hydraulic system. Thus, the pressure difference of ideal gas and van der Waals gas is negligible, which is confirmed by the value of \( h(x) \) whose value trends to 1 while the \( x \) increasing. The value of \( P_v/P_i \) can be explicit by the relation depicted in Table 4.4. Here according to gas thermodynamic properties and accumulator operating processes, mathematically, the trends that higher van der Waals gas pressure during charging process and lower van der Waals gas pressure during discharging compared with ideal gas model has been explained.

<table>
<thead>
<tr>
<th>Charging</th>
<th>( 0 &lt; V &lt; V_0 )</th>
<th>( T &gt; T_0 )</th>
<th>( P_v/P_i &gt; h(V) &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharging</td>
<td>( V &gt; V_0 )</td>
<td>( T &lt; T_0 )</td>
<td>( P_v/P_i &lt; h(V) &lt; 1 )</td>
</tr>
</tbody>
</table>

The accumulator works between isothermal process and adiabatic process. During accumulator charging process, the hydraulic oil flows into accumulator, the accumulator gas is compressed by hydraulic oil and its volume reduces and thereby increase the gas pressure. The physical state of gas undergoes change depends on the accumulator work process, whether it is isothermal or adiabatic. Respectively, the gas thermodynamics in accumulator in isothermal and adiabatic operations calculated by ideal gas model and van der Waals gas model are analyzed. The initial gas pressure is set as 13MPa, abs for compression process and gas volume is assumed to decreases to 2.7L from 5L. The equations introduced in Section 1.3 and this chapter are employed. The calculation results of isothermal and adiabatic compression are shown in Figure 4.7. The blue dot-and-dash line and green solid line are isothermal compression processes by ideal gas model and van der Waals model, respectively. The orange dash line and red dot line are adiabatic compression processes by ideal gas model and van der Waals model, respectively. The van der Waals gas has a relatively higher pressure than ideal gas, especially the adiabatic process. The pressure difference is large enough that it has a significant influence on accumulator performance which serving in hydraulic system. In isothermal process, the compressed gas in accumulator has an increasing temperature, then transfer to surrounding by heat exchange. However, While the accumulator operation process trends towards adiabatic from isothermal, the pressure difference becomes larger. While the maximum pressure reached by ideal gas model is 31MPa, abs, that reached by van der Waals
gas model is 38MPa. Thus, the actual gas properties should be considered for accumulator design.

![Figure 4.7 Isothermal and adiabatic compression of gas-loaded accumulator](image1)

![Figure 4.8 Isothermal and adiabatic expansion of gas-loaded accumulator](image2)

Then gas expansion process has also been calculated, and the results are shown in Figure 4.8. The accumulator is expanded from 2.7L of gas volume to 5.0L. The initial gas pressure is assumed as 33MPa, abs. The blue dot-and-dash line and green solid line are isothermal
expansions of ideal gas model and van der Waals gas model, respectively. The orange dash line and red dot line are adiabatic expansions of ideal gas model and van der Waals gas model, respectively. The van der Waals gas has a relatively lower pressure than ideal gas, especially the adiabatic process. The pressure difference is large enough that it has substantial influence on energy storage capacity. In isothermal process, the expanded gas in accumulator has a decreasing temperature, then ambient thermal energy transfer to accumulator by heat exchange. If the accumulator operation process trends towards adiabatic from isothermal, the pressure difference becomes larger. While the minimum pressure reached by ideal gas model is 14MPa, abs, that reached by van der Waals gas model is 10MPa. Both isothermal and adiabatic operations are in accord with previous mathematical analysis.

Another adiabatic compression calculation case is built that the accumulator size is set as more than 30L, which is normal size using in hydraulic system. To compare the pressure and volume variation trends, the accumulator compression is operated by a 30L to 20L volume change by ideal gas model and van der Waals gas model, and a 40L to 30L volume change by van der Waals gas model. The initial gas pressure is 20MPa, abs. The calculation result is shown in Figure 4.9. This confirmed designed accumulator has a size that is not suitable to store hydraulic energy as the reduced storage capacity is produced in the real case (van der Waal gas model). In fact, since the achievable maximum pressure by a commercial gas-loaded accumulator may not exceed about 35 MPa, abs[51], the achievable volume variation in practice is less than 25% compared to that assumed by ideal gas model. For instance, an 32L bladder accumulator which has a 30L maximum gas volume is designed by ideal gas model for
receiving 10L hydraulic oil for energy saving from actuator in hydraulic system. For compression operation, the largest gas pressure can keep by 35 MPa, abs. However, considering the actual gas properties, this design may not be achieved by a commercial bladder accumulator charged by nitrogen gas. To match the hydraulic system requirement and ensure maximum energy saving capability, the same maximum gas pressure and minimum gas pressure obtained with the ideal gas model are needed. It means a 45L bladder accumulator which has a 40L maximum gas volume is necessary for the hydraulic system.

### 4.4.2 Impact of parameters in van der Waals model on gas performance in accumulator

Above discussion has shown that the actual gas properties are different from ideal gas. It was confirmed by van der Waals equation of state. The design of accumulator in high pressure level is strongly influenced by the gas behavior. As the van der Waals equation of state describes actual gas behaviors including intermolecular forces and molecular size. To highlight their effects on gas thermodynamics, it is necessary to analyze the influence of intermolecular force and molecular size, respectively. From Eq. (1-2), it can be rewritten as follows if only intermolecular force is considered:

\[
P + \frac{a_m}{V^2} = \frac{mRT}{V}.
\]  

\[a_m = \frac{\alpha}{3}.
\]  

(4-35)

and it can be rewritten as follows if only molecular size is considered:

\[
P(V - mb_v) = \frac{mRT}{V}.
\]  

\[b_v = \frac{9}{8}RTv_v.
\]  

(4-36)

Based on above two equations, the gas variation in pressure is calculated and compared to the calculation of original van der Waals equation of state. The calculation cases include adiabatic compression and adiabatic expansion. In adiabatic compression, the initial gas pressure is set as 13MPa, abs. The gas is compressed from 5L to 2.7L in a mini-accumulator. In adiabatic expansion, the initial gas pressure is set as 30 MPa, abs. The gas is expanded from 2.7L to 5L. The calculation result of adiabatic compression is shown in **Figure 4.10**. The black solid line is calculated data by van der Waals equation of state, the blue dash line is calculated data by Eq. (4-35) only intermolecular force is considered and the red dot line is calculated data by Eq. (4-36) that only molecular volume is considered. Results indicate that compared to intermolecular force, molecular size takes a significant role influencing gas properties in accumulator.
Chapter 4 Thermal properties difference of ideal gas model and van der Waals model

Figure 4.10 van der Waals properties in adiabatic compression process

Figure 4.11 van der Waals properties in adiabatic expansion process

The calculation result of adiabatic expansion is shown in Figure 4.11. It gives similar variation trends by these three cases as adiabatic compression process. The black solid line is calculated data by van der Waals equation of state, the blue dash line is calculated data by Eq. (4-35) only intermolecular force is considered and the red dot line is calculated data by Eq. (4-36) that only molecular volume is considered. The reason why it is not suitable to design accumulator by ideal gas model is because the gas pressure usually keeps in high level, which means the
compressed gas molecules having short distance, the molecular size should not be negligible. The other factor, intermolecular force, is evaluated by temperature. At the micro level, the microparticles have random motion that the active degree is described by temperature. The molecular movement is static when the temperature is absolute zero, and it becomes active when the temperature reach to 300K. The motion of accumulator gas is increased by raising the temperature, thus the molecules are further apart and have little interaction with one another. Thus, rather than the intermolecular force in van der Waals model, the molecular size is significant for the gas properties.

4.4.3 Impact of initial pressure on gas performance in accumulator

Proper design of an accumulator can be achieved by real gas model which is more accurate to evaluate the actual gas behaviors. As the accumulator working in low operating pressure, actually the real gas properties are inconspicuous. It is reasonable to model the accumulator gas in ideal. The obvious pressure difference in high-level initial pressure and low-level pressure is shown in Figure 4.12. The diabatic expansion is operated by different initial pressure but same volume variation range (2.7L to 5L). The red solid line is calculation result in high-level initial pressure (34MPa, abs) by van der Waals gas model, the orange dash line is that by ideal gas model. The blue dot line is calculation result in low-level initial pressure (5MPa, abs) by van der Waals gas model, the green triangle is that by ideal gas model. At low-pressure level condition, the ideal gas model achieves a good approximation of real gas. However, the pressure difference becomes large while the accumulator works from low-
pressure range to high-pressure range.

Figure 4.13  Force balance of actuator with accumulator

For the application of accumulator in hydraulic system, as shown in Figure 4.13. The hydraulic oil flows into accumulator and compress the gas that increases the gas pressure. The force balance can be expressed by (assuming the pressures of oil side and gas side in accumulator are same):

\[ PA = F. \]  \hspace{1cm} (4-37)

The gas pressure \( P \) is determined by the Force \( F \) from actuator side. Meanwhile, the actuator cylinder position is determined by the gas pressure \( P \). If the designed accumulator by ideal gas model is employed in this hydraulic system, the actual compressed gas pressure will be higher than predicted by ideal gas model, which means the \( PA \) in accumulator becomes higher than \( F \) at cylinder side before the head rod of cylinder completely lowered. Thus, the designed accumulator is not favorable for matching the hydraulic performances.

To quantitatively evaluate the pressure difference between ideal gas model and van der Waals gas model, the error range is calculated in series of different initial pressure. Mathematically, the pressure difference is expressed by the following formula:

\[ \delta P = P_i - P. \]  \hspace{1cm} (4-38)

Calculating by series of initial gas pressures, the results of expansion process of accumulator in both isothermal and adiabatic operations are shown in Figure 4.14. The initial pressure is set as 4, 10, 20, 30, 40, 50 (MPa, abs) and the accumulator is expanded from 5L to 10L. While the initial pressure is very low as 4MPa, abs, the pressure differences between ideal gas model
and van der Waals gas model in both isothermal and adiabatic operations are closed to zero. At such work condition, it is possible to design an accumulator by ideal gas model which means the rathe simple equation of state will also make sense for engineers. While the initial pressure is 10MPa, abs, the pressure differences are still inapparent for some applications in hydraulic systems. However, the adiabatic process has more than 1MPa, abs pressure difference at 20MPa, abs initial pressure case. It is only reasonable to be calculated by gas model that considering real gas properties. While the initial pressure exceeds 30 MPa, abs, the pressure difference is large enough that only real gas model is reasonable for designing, because the accumulator is possible to be damaged if the ideal gas model is employed. Varies of applications have different requirements of accuracy level for designing. Other factors as accumulator working situation and accumulator lifetime are necessary to be considered.

(a) Expansion, $P_0=4\text{MPa, abs}$.

(b) Expansion, $P_0=10\text{MPa, abs}$. 
Chapter 4 Thermal properties difference of ideal gas model and van der Waals model

(c) Expansion, P₀=20MPa, abs.

(d) Expansion, P₀=30MPa, abs.

(e) Expansion, P₀=40MPa, abs.
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(f) Expansion, $P_0=50$ MPa, abs.

**Figure 4.14** Pressure difference of gas models at different initial pressures in expansion process

Generally, the accumulator is classified as low-pressure accumulator, middle-pressure accumulator and high-pressure accumulator. However, the pressure standards of the classification are determined by engineers or manufacturers. Thus, the allowable pressure errors are different. As sometimes the complexed real gas models (as shown in Section 1.3 in Chapter 1), here provides an example of method to choose equation of state of ideal gas model and real gas model. Once the average working pressure is settled, the allowable pressure error should be settled as well based on the hydraulic system performances. For instance, the pressure errors of a series of hydraulic systems having accumulator meet the requirements as shown in Table 4.5, the pressure point that distinguish real gas properties with ideal gas assumption can be figured out.

<table>
<thead>
<tr>
<th>$P_m$ (MPa, abs)</th>
<th>$P_m&lt;4$</th>
<th>$4&lt;P_m&lt;20$</th>
<th>$20&lt;P_m&lt;33$</th>
<th>$P_m&gt;33$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$REQ$</td>
<td>±20%</td>
<td>±8%</td>
<td>±5%</td>
<td>±2.5%</td>
</tr>
</tbody>
</table>

If the average working pressure is between 4MPa, abs and 20MPa, abs, the real gas model should be used while the pressure difference ($DEL$) of real gas model and ideal gas model exceeds ±8% of average working pressure $P_m$ of the hydraulic system, as expressed by the following correlation:
\[ \text{DEL} = P_v - P_i < P_m \times \text{REQ}. \] (4-38)

which means the real gas thermodynamics has the possibility to affect the system performances. The value of pressure error limitation (\(\text{REQ}\)) is the key factor that should be referred the hydraulic system characterizes and engineer experiences. The pressure errors are shown in Figure 4.15. The dot line is the pressure error limitation (\(\text{REQ}\)) that determined by the hydraulic performances. The red solid line and blue solid line are the pressure differences of ideal gas model and van der Waals gas model in isothermal process (\(\text{DEL-Isothermal}\)) and adiabatic process (\(\text{DEL-Adiabatic}\)), respectively. The \(PV\) diagram calculated by ideal gas model in polytropic process is also provided as reference pressure. While the average working pressure \(P_m\) under 4MPa, abs, the pressure difference \(\text{DEL}\) in both charging and discharging processes are not exceed the pressure error limitation (\(\text{REQ}\)). Thus, the ideal gas model can be used for accumulator designing in hydraulic circuit, as shown in Table 4.6. While the average working pressure is between 4MPa, abs and 20MPa, abs, the adiabatic charge process should employ real gas model above 5MPa, abs of average working pressure, and the isothermal charging process should employ real gas model above 15MPa, abs of average working pressure. The calculation results also show that the discharge process in both isothermal and adiabatic can use ideal gas model for design under 20MPa, abs. While the 5\% of pressure error limitation is settled for the average working pressure range from 20MPa, abs to 33MPa, the real gas model is necessary for adiabatic charging if \(P_m\) above 20MPa, abs and for adiabatic discharging if \(P_m\) above 23MPa, abs, and for isothermal charging if \(P_m\) above 20MPa, abs as well. It is still possible to use ideal gas model for isothermal discharge process if \(P_m\) under 33MPa, abs. While the \(P_m\) is above 33MPa, abs, all the gas compression process and expansion process should employ the real gas model.

(a) Charging \(P_m = 4\text{MPa, abs.}\)
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(b) Discharging $P_m = 4\text{MPa, abs.}$

(c) Charging $P_m = 5\text{MPa, abs.}$

(d) Charging $P_m = 15\text{MPa, abs.}$
(e) Discharging $P_m = 20\text{MPa, abs.}$

(f) Charging $P_m = 20\text{MPa, abs.}$

(f) Discharging $P_m = 23\text{MPa, abs.}$
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(g) Discharging $P_m = 33\text{MPa, abs.}$

(h) Charging $P_m = 33\text{MPa, abs.}$

Figure 4.15 Pressure difference of gas models at different initial pressures in expansion process

<table>
<thead>
<tr>
<th>$P_m$ (MPa, abs)</th>
<th>$P_m &lt; 4$</th>
<th>$4 &lt; P_m \leq 20$</th>
<th>$20 &lt; P_m \leq 33$</th>
<th>$P_m &gt; 33$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 20%$</td>
<td>$\pm 8%$</td>
<td>$\pm 5%$</td>
<td>$\pm 2.5%$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6 Gas model selection reference

<table>
<thead>
<tr>
<th>$P_m$ (MPa, abs)</th>
<th>$P_m &lt; 4$</th>
<th>$4 &lt; P_m \leq 20$</th>
<th>$20 &lt; P_m \leq 33$</th>
<th>$P_m &gt; 33$</th>
</tr>
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<tbody>
<tr>
<td>$\pm 20%$</td>
<td>$\pm 8%$</td>
<td>$\pm 5%$</td>
<td>$\pm 2.5%$</td>
<td></td>
</tr>
</tbody>
</table>

Adiabatic

- Charge: ideal if $P_m > 5$
- Discharge: ideal if $P_m > 20$

Isothermal

- Charge: ideal if $P_m > 15$
- Discharge: ideal if $P_m > 23$

- Charge: ideal
- Discharge: ideal
It should be highlighted there is no common standards of pressure error limitation by now. It depends on hydraulic requirements usually the average working pressure. The pressure error calculation of gas compression or expansion processes has an initial pressure as one of the initial conditions. The correlation of initial pressure of accumulator and average working pressure of hydraulic system is also determined by engineer or accumulator manufacturer.

### 4.4.4 Impact of volume variation on gas performance in accumulator

![Figure 4.16](imageurl)  
**Figure 4.16**  Pressure properties in different accumulator volumes

One of the key problems in accumulator design for engineer is to balance the accumulator size and the energy density. Larger accumulator has larger energy storage capacity, however, the space for its weight and installation space are the limitation factors. How different of accumulator thermodynamics according to the volume is analyzed in this section. Accumulator with different volumes is compressed in adiabatic process. The initial pressure is settled as 20MPa, abs and the initial temperature is 303K. All the range of variation of accumulator volume is 10L. The initial volumes in series of calculation cases are settled as 40L, 30L, and 20L by ideal gas model and van der Waals gas model. The pressure properties in different accumulator volumes are shown in **Figure 4.16**. While the range of variation of accumulator volume is from 40L to 30L, the $PV$ diagram shows that the pressure calculated by van der Waal gas model is larger than that calculated by ideal gas model. The highest pressure calculated by ideal gas model reaches 30MPa, abs can be achieved by van der Waals gas model.
in 40L-32.5L volume range. However, while the range of variation of accumulator volume is from 30L to 20L, which means the accumulator size is smaller than previous case, the pressure trend by van der Waals gas model becomes larger compared that in 40L-30L volume range. It indicates that the pressure trends are different in various accumulator volumes although in same volume variation range. The pressure properties in 20L-10L volume range shows more obvious difference with other cases. The highest pressure in 40L-30L volume range reaches 35MPa, abs while that in 20L-10L volume reaches more than 90MPa, abs. Thus, the accumulator volume has outstanding influence on gas pressure. The accumulator volume sized by ideal gas model is not suitable if considering the real gas properties.

The temperature trends are shown in Figure 4.17. While the range of variation of accumulator volume is from 40L to 30L, the $TV$ diagram shows that the temperature calculated by van der Waals gas model is higher than that calculated by ideal gas model. The highest temperature calculated by ideal gas model reaches 340K while that reaches 355K by van der Waals gas model in same volume range. When the range of variation of accumulator volume is from 30L to 20L, which means the accumulator size is smaller than previous case, the temperature trend by van der Waals gas model becomes higher compared that in 40L-30L volume range. It indicates that the temperature trends are different in various accumulator volumes although in same volume variation range. The temperature properties in 20L-10L volume range shows more obvious difference with other cases. For the van der Waal gas model, the highest

**Figure 4.17** Temperature properties in different accumulator volumes
temperature in 40L-30L volume range reaches 355K while that in 20L-10L volume reaches more than 450K. The effect of gas temperature and duty cycle in hydraulic systems can easily be overlooked. The temperature is influenced by ambient air, hydraulic oil and rate of compression/expansion. Therefore, the accuracy of temperature prediction is significant for the accumulator applications. The accumulator type is the key role that affect the accumulator thermodynamics. For instance, the ambient of a piston-type accumulator may be more related to environment temperature. The ambient of a bladder-type accumulator may be more related to hydraulic oil temperature. Further temperature increase will happen as the hydraulic oil in accumulator membrane may not be well circulated within the pipeline. Nearly isothermal process or adiabatic process is defined depending on how rapid compression/expansion is operated, and how soon the next cycle starts.

![Diagram showing work done properties in different accumulator volumes](image)

**Figure 4.18** Work done properties in different accumulator volumes

The work done $W$ trends in compression process are shown in **Figure 4.18**. While the range of variation of accumulator volume is from 40L to 30L, the $WP$ diagram shows that the work done calculated by van der Waal gas model is higher than that calculated by ideal gas model. The highest work done calculated by ideal gas model reaches 100kJ while that reaches 150kJ by van der Waals gas model in same volume range. When the range of variation of accumulator volume is from 30L to 20L, which means the accumulator size is smaller than previous case, the work done trend by van der Waal gas model (250kJ) becomes higher compared that in 40L-30L volume range. It indicates that the work done trends are different in various accumulator volumes although in same volume variation range. The work done properties in
20L-10L volume range shows more obvious difference with other cases. The energy storage capacity is different in different accumulator sizes or evaluated by ideal gas model and van der Waals gas model. As a gas-loaded accumulator enables hydraulic circuits to cope with extremes of demand to respond rapidly, the calculated energy capacity error by ideal gas model should be highlighted, which means the real gas thermodynamical behaviors and realistic operating conditions should be considered when evaluating the kinetic energy recoverability.

### 4.4.5 Gas volume compression ratio

According to above advanced theoretical analysis of accumulator gas thermodynamic behaviors by ideal gas model and van der Waals gas model in series of conditions, the gas pressure, volume et al. variation trends during accumulator compression and expansion processes are compared. The calculation results show that, in the van der Waals gas model which considering intermolecular force and molecular volume, the molecular volume produces more evident difference from ideal gas model. In compression process, the gas compressibility significantly impacts the gas volumetric behaviors in accumulator and thus the volume design of accumulator. With a constant initial pressure, the ratio of gas compression in accumulator determines the final pressure, thus determines the pressure balance from gas side to oil side. A proper compression ratio prediction must consider the hydraulic flow volume and the oil pressure at outlet of accumulator in pipeline. To understand how the pressure error predicted by ideal gas model and van der Waals gas model varies coupled with increasing initial pressure, calculation cases should be compared with the conditions of different initial pressures and gas volume compression ratios. During accumulator compression process, the gas pressure increases and gas volume decreases. The compression ratio can be described as:

\[
CR = \frac{V_0}{V}.
\]  

(4-39)

For compression period, the initial gas volume \(V_0\) is maximum that the value of \(CR\) will be larger than 1.0. During expansion period, the initial gas volume \(V_0\) is minimum that the value of \(CR\) will be smaller than 1.0. The correlation of pressure error rate and the initial pressure is compared based on series of \(CR\) values. The graphical representation of pressure and pressure error rate is shown in Figure 4.19. The horizontal axis is the initial gas pressure \(P_0\) in compression and the vertical axis represents the error ratio of the pressure deviation of calculation results of ideal gas from van der Waals gas with respective to the initial pressure \(P_0\). The pressure deviation is predicted by calculating the pressure difference between ideal gas model and van der Waals gas model. As analyzed in above sections, the increased initial pressure will lead to an increase of pressure error ratio of ideal gas assumption from actual gas behaviors. Different pressure error rates have been shown in the diagram with different compression ratios. When the accumulator being used in hydraulic system, it should work in specific conditions as specific acceptability of pressure error is necessary for hydraulic system.
performance. It indicates that the accumulator gas volume compression ratio plays a significant role. With a high compression ratio $CR$, the pressure error will be high. For the case of $CR = 1.1$, the pressure error reaches 10 percent at 30MPa, abs which means an error of 3MPa, abs is predicted by ideal gas model compared with van der Waals gas model in accumulator. The pressure error rate becomes higher while the compression ratio is larger. As $CR = 3$ represented by rod solid line, the pressure error is higher than that of $CR = 1.1$ case with same initial pressure. The curves give detailed correlation of initial pressure and pressure error rate.

![Figure 4.19](image)

**Figure 4.19** Relation of pressure error ratio in series of different volume compression ratios

The effect of gas compressibility is sure that in high-pressure-level accumulator, the accumulator design is strongly influenced by the actual behaviors of gas and it is therefore obvious that the ideal gas model is not adequate for sizing the capacity of energy storage. Actuator mobility characteristic and control strategies in hydraulic system decide how the characteristics of the gas-loaded accumulator. Energy storage device consisted of series of accumulators used in a liquid-piston power generation system often operates in nearly isothermal and the average pressure variation range is between 2MPa, abs and 10MPa, abs. The pressure error by ideal gas model from van der Waals is very small that it is negligible. While the accumulator is used in some heavy machines, especially running machine, the average working pressure approaches 30~35MPa, abs that the pressure error is noticeable. The accumulator volume prediction should be calculated considering the actual gas characteristics[50].
4.5 Summary

This chapter investigated how much difference of ideal gas model and van der Waals gas model. Both isothermal change and adiabatic change of accumulator gas have been analyzed. Up to now, most accumulator applications are employing ideal gas model and parameter in calculation equation is adjusted according to experimental data. In this study, a gas model selection example was given as a reference for application in solving engineering problems. Quantitatively, a selection method of employing real gas model in accumulator design is introduced. For higher accuracy, the Soave-Redlich-Kwong equation of state, which is improved from the original van der Waal equation of state, should be analyzed as it has been regarded as more suitable real gas model based on many research analysis and experimental results. The mathematical derivation of its isothermal equation and adiabatic equation will be discussed in next chapter.
Chapter 5

Soave-Redlich-Kwong isothermal & adiabatic equation for gas-loaded accumulator

Since Otto Redlich and Joseph Neng Shun Kwong proposed the Redlich-Kwong equation of state in 1949, which is a two-parameter empirical, algebraic cubic equation of state, that relates pressure, temperature and volume of gases. It has shown accuracy than the van der Waals equation or the ideal gas equation at temperatures above the critical temperature. Although it can be used to model real gas thermodynamics with a good degree of accuracy, the application for modeling the multicomponent calculations often gives poor results. Many revisions and modifications have focused on the Redlich-Kwong equation, for either simulating conditions at lower temperatures including vapor-liquid equilibria, as well as in better improving its accuracy in terms of predicting gas-phase properties of more compounds. The Soave-Redlich-Kwong equation fitted experimental vapor-liquid data well and could predict phase behavior of mixtures in the critical region.

The difference of thermodynamics evaluated by ideal gas model from van der Waals gas model have been discussed in Chapter 4. It shows that the actual gas properties should be considered with increasing of the initial pressure in accumulator. As for now, many real gas models have been presented for modeling the actual gas thermodynamics. According to the equation of state, the isothermal process and adiabatic process can be predicted. However, the famous Soave-Redlich-Kwong equation of state, which is improved from Redlich-Kwong equation of state, is difficult to be used in this prediction as its complexed form. As the Soave-Redlich-Kwong equation of state has been widely employed in applications such as commercial software[52], it is necessary to investigate its isothermal and adiabatic form to more accurately predict the real gas performances. The numerical accumulator model employing the Soave-Redlich-Kwong equation was used in wind turbines, and has been confirmed that it is overall best suited with experimental data and the equation of state is as much as six times faster than the Benedict-Webb-Rubin equation of state[53].

5.1 Thermal time constant

5.1.1 Thermal time constant with BWR equation of state

The isothermal process and adiabatic process in accumulator describe the thermodynamics of accumulator gas during compression and expansion. As the internal energy frequently varies
the temperature may change, therefore the heat exchange will happen between accumulator gas and
ambient. The compression/expansion rate and accumulator type decide the heat exchange rate. Extremely, isothermal process assumes the heat exchange completed occurs and gas temperature keeps in line with ambient temperature. Adiabatic process assumes heat exchange cannot occur, that means all internal energy is depended on work done from hydraulic system. However, neither isothermal process nor adiabatic process exist under actual working conditions. The thermal losses certainly occur in accumulator and it has been described in many research papers[54]–[58]. As the wall boundary layer is not stable, the heat exchange process is unsteady, combined free and forced convection with transition from laminar flow to turbulent flow. The Instead of polytropic assumption, the thermal time constant concept was derived by Otis to calculate thermal loss for cyclical processes in accumulator. The thermodynamic loss during compression and expansion has been analyzed in terms of thermal-time constant and energy storage efficiency by Yutaka Tanaka[59], [60].

The Otis’s thermal time constant concept introduced a thermal time constant $\tau$ to express the properties of time rate of change of gas temperature:

$$\frac{dT}{dt} = \frac{T_v - T}{\tau} - \frac{T}{c_v} \left( \frac{\partial P}{\partial T} \right)_v \frac{dv}{dt},$$

(5-1)

where the definition of thermal time constant is given in Eq. (3-2) in Section 3.1.2. The heat transfer can be expressed by thermal time constant as:

$$h = \frac{mc_v}{\tau A},$$

(5-2)

The thermal time constant $\tau$ should be measured for the specific accumulator and range of operations. As the heat transfer coefficient $h$ and the effective wall area $A$ both change with time, the thermal time constant is not actually constant. For the gas sealed in accumulator, the gas mole number $m$ is constant. It is defined as:

$$m = \frac{V}{v},$$

(5-3)

Therefore the form of $dv/dt$ can be written as

$$\frac{dv}{dt} = \frac{1}{m} \frac{dV}{dt}.$$  

(5-4)

The conditions of gas mole volume which keeps in constant is identical to the constant gas volume condition. Thus,
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

\[
\left( \frac{\partial P}{\partial T} \right)_v = \left( \frac{\partial P}{\partial T} \right)_{v'}.
\] (5-5)

Then the temperature change with time can be written as:

\[
\frac{dT}{dt} = \frac{T_v - T}{\tau} T \left( \frac{\partial P}{\partial T} \right)_{v'} \frac{1}{m} \frac{dV}{dt}.
\] (5-6)

To use Eq. (5-6) evaluating actual gas properties, it is necessary to employ a gas model to figure out the term of \((\partial P/\partial T)_v\) in the second part of the right side of the equation. Combining the Benedict-Webb-Rubin equation of state shown in Eq. (1-3), it can be expressed as:

\[
\left( \frac{\partial P}{\partial T} \right)_{v'} = \frac{mR}{V} + \frac{m^3}{V^2} \left( \frac{B_0 R + \frac{2C_0}{T^3}}{V^2} + m^2 b R \frac{V^2}{V^3} - \frac{2m^3 c_0 \left( 1 + \frac{2m^2 \gamma}{V^2} \right) e^\frac{m^2 \gamma}{V^2}}{V^3 T^3} \right)
\] (5-7)

The Benedict-Webb-Rubin equation of state has been shown in noticeable agreement with the result by Jacobsen[21] for nitrogen gas for the completed range of interest for accumulator application[61]. Combining Eq. (5-7) and (5-6) yields,

\[
\frac{dT}{dt} = \frac{T_v - T}{\tau} R \left( \frac{B_0 R + \frac{2C_0}{T^3}}{V^2} + m^2 b R \frac{V^2}{V^3} - \frac{2m^3 c_0 \left( 1 + \frac{2m^2 \gamma}{V^2} \right) e^\frac{m^2 \gamma}{V^2}}{V^3 T^3} \right) \frac{dV}{dt}
\] (5-8)

| Table 5.1 Parameters in Benedict-Webb-Rubin equation of state[62] |
|-----------------|-----------------|------------------|
| Parameter | Nitrogen | Unit |
| \( A_0 \) | 0.15019143 | [MPam⁹/kmol²] |
| \( B_0 \) | 0.55931023×10⁻¹ | [m³/kmol] |
| \( C_0 \) | 0.30769511×10⁻¹ | [MPaK²m⁶/kmol²] |
| \( a_0 \) | 0.26460522×10⁻¹ | [MPam⁹/kmol²] |
| \( b_0 \) | 0.36848198×10⁻² | [m³/kmol²] |
| \( c_0 \) | 0.46882840×10⁻² | [MPaK²m⁶/mol³] |
| \( \alpha \) | 0.79014894×10⁻⁴ | [m⁹/kmol³] |
| \( \gamma \) | 0.56821964×10⁻² | [m⁶/kmol²] |
There are many parameters in Benedict-Webb-Rubin equation of state and that has been a subject for many researchers, here the parameters for nitrogen gas by Asami are employed to compare with other models. The values of parameters are shown in **Table 5.1**.

### 5.1.2 Thermal time constant with the Soave-Redlich-Kwong equation

Here the Soave-Redlich-Kwong equation of state is the study subject. This real gas model has been modified by many researchers. Mehdi Ghanbari and Gholam Reza Check presented new super-critical cohesion parameters for the equation of state by fitting to the Joule-Thomson inversion curve[63]. The expression of Soave-Redlich-Kwong equation of state has been shown in Eq. (1-5)-Eq. (1-8). The parameter \( k \) was modified as the following expression:

\[
k = \begin{cases} 
0.480 + 1.57\omega - 0.176\omega^2; & \frac{T}{T_c} \leq 1 \\
\frac{11}{15}(0.480 + 1.57\omega - 0.176\omega^2); & \frac{T}{T_c} > 1
\end{cases}
\]  \hspace{1cm} (5-9)

The modified parameter \( k \) was fit with a coefficient of \( 11/15 \) for super-critical region. As the operating temperature of nitrogen gas used in gas-loaded accumulator is commonly higher than its critical temperature (126.2 K), thus, the function of \( k \) with \( 11/15 \) is employed. To use Eq. (5-6) evaluating actual gas properties, it is necessary to employ a gas model to figure out the term of \((\partial P / \partial T)_V\) in the second part of the right side of the equation. Combining the Soave-Redlich-Kwong equation of state shown in Eq. (1-3), it can be expressed as:

\[
\left( \frac{\partial P}{\partial T} \right)_V = \frac{mR}{V - mb} \left( \frac{m^2}{V(V + mb)} \right) \frac{\tilde{c}a(T)}{\partial T},
\]  \hspace{1cm} (5-10)

where

\[
\frac{\tilde{c}a(T)}{\partial T} = -0.4274 \frac{kR^2 T_c}{P_c^2} \left( \frac{T}{T_c} \right)^{\frac{1}{2}} \left[ 1 + k \left( 1 - \left( \frac{T}{T_c} \right)^{\frac{1}{2}} \right) \right].
\]  \hspace{1cm} (5-11)

Combining thermal time constant model of Eq. (5-6) with Eq. (5-11), it is possible to mathematically analyze the accumulator gas charge and discharge processes by numerical integration:
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

\[
\frac{dT}{dt} = \frac{T_w - T}{\tau} - \frac{T}{c_v} \left( \frac{R}{V - mb} - \frac{m}{V(V + mb)} \frac{\partial a(T)}{\partial T} \right) dV. \tag{5-12}
\]

The temperature or pressure properties by volume change with time can be evaluated considering the heat transfer characteristics. Both the expression of Benedict-Webb-Rubin equation of state and Soave-Redlich-Kwong equation of state coupled with thermal time constant have been derived.

### 5.2 Derivation of isothermal and adiabatic model of SRK

#### 5.2.1 Soave-Redlich-Kwong isothermal equation

As the temperature in isothermal process is constant, thus,

\[
mRT = mRT_0. \tag{5-13}
\]

Combing the Soave-Redlich-Kwong equation of state of Eq. (1-5), the initial pressure of gas in accumulator can be expressed by:

\[
P_0 = \frac{mRT_0}{V_0 - mb} - \frac{a(T)_{T=T_0} m^2}{V_0(V_0 + mb)}. \tag{5-14}
\]

Therefore, the pressure change depending on volume can be described as:

\[
P = \frac{V_0 - mb}{V - mb} \left( P_0 + \frac{a(T)_{T=T_0} m^2}{V_0(V_0 + mb)} - \frac{a(T)_{T=T_0} m^2}{V(V + mb)} \right). \tag{5-15}
\]

As the value of temperature \( T \) keeps constant, above equation is relatively simple for calculation for modeling the accumulator.

The isothermal expansion process is simulated by ideal gas model, van der Waals gas model and Soave-Redlich-Kwong gas model. The calculation result of \( PV \) diagram is shown in Figure 5.1. The accumulator gas volume varies from 2.7L to 5L which gives a 34MPa, abs initial gas pressure. The black dot line is ideal isothermal equation result, the purple dash line is van der Waals isothermal equation result and the red solid line is Soave-Redlich-Kwong isothermal equation result. It indicates that the simple ideal gas model gives visible pressure difference compared with the other two gas models. As the high gas pressure in accumulator,
the molecular volume has significant influence on gas thermodynamics distinguished from ideal gas assumption. Meanwhile, the gas pressures modelled by van der Waals gas model and Soave-Redlich-Kwong gas model are different as well. It indicates the temperature-dependent parameter $a(T)$, which models the gas characteristics more detailly instead of constant factor $a_v$ in van der Waal equation of state, has significant influence on gas properties.

**Figure 5.1** Isothermal expansion process in accumulator

**Figure 5.2** Isothermal compression process in accumulator
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

The isothermal compression process is simulated as well by ideal gas model, van der Waals gas model and Soave-Redlich-Kwong gas model. The calculation result of $PV$ diagram is shown in Figure 5.2. The accumulator gas volume varies from 5L to 2.7L which gives a 10MPa, abs initial gas pressure. The black dot line is ideal isothermal equation result, the purple dash line is van der Waals isothermal equation result and the red solid line is Soave-Redlich-Kwong isothermal equation result. The pressure modelled by ideal gas model shows slightly different compared with van der Waals gas model, however, the pressure difference of Soave-Redlich-Kwong gas model is more visible compared with the other two gas models.

5.2.2 Soave-Redlich-Kwong adiabatic equation[64]

The Soave-Redlich-Kwong adiabatic equation is derived in this section. The thermodynamics of the gas follows the first law of thermodynamics. As the same deriving method as the adiabatic equation of van der Waals gas model. The differential form of internal energy $U$ with respect to volume expressed in Eq. (4-12) couples with Soave-Redlich-Kwong equation of state of Eq. (1-5) yields,

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{m^2 a(T)}{V(V + mb)} - \frac{Tm^2}{V(V + mb)} \frac{\partial a(T)}{\partial T}, \quad (5-16)$$

Substituting above equation and the specific heat expression of Eq. (4-8) to the total differential of internal energy of Eq. (4-7) yields:

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dT + \left(\frac{\partial U}{\partial T}\right)_V dV = mC_v dT + \frac{m^2}{V(V + mb)} \left[ a(T) - T \frac{\partial a(T)}{\partial T} \right] dV. \quad (5-17)$$

Combining Eq. (4-6) with Eq. (5-17) to delete $dU$ and solving for the total differential of entropy $dS$ yield:

$$TdS - PdV = mC_v dT + \frac{m^2}{V(V + mb)} \left[ a(T) - T \frac{\partial a(T)}{\partial T} \right] dV, \quad (5-18)$$

$$dS = \frac{mC_v}{T} dT + \frac{1}{T} \left[ P + \frac{m^2}{V(V + mb)} \left( a(T) - T \frac{\partial a(T)}{\partial T} \right) \right] dV. \quad (5-19)$$

Substituting the Soave-Redlich-Kwong equation of state of Eq. (1-5) into Eq. (5-19), the differential form of entropy $dS$ can be expressed by:
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

\[ dS = \frac{mC_v}{T} dT - \frac{mR}{V - mb} dV + \left( \frac{1}{V} - \frac{m^2}{V + mb} \right) m \frac{\partial a(T)}{\partial T} dV. \]  
(5-20)

The entropy \( S \) can be obtained by integrating both sides of the above equation:

\[ S = mC_v \left[ \ln \left( \frac{V}{V + mb} \right)^{\frac{R}{V^2}} \left( \frac{V}{V + mb} \right)^{\frac{1}{V - mb}} \right] + S_0. \]  
(5-21)

This equation is adiabatic equation expressed by Soave-Redlich-Kwong gas model. For adiabatic change of gas, the entropy is constant. Thus, the term in the logarithm in above equation is constant. This formula only includes two variables, gas temperature \( T \) and gas volume \( V \). Once one variable is defined, the other variable can be calculated if the constant value is calculated by initial gas conditions.

To compare the Soave-Redlich-Kwong adiabatic equation with van der Waals adiabatic equation of Eq. (4-25), another form is to be derived. The gas temperature can be expressed by the Soave-Redlich-Kwong equation:

\[ T = \frac{V - mb}{mR} \left( P + \frac{a(T)m^2}{V(V + mb)} \right). \]  
(5-22)

Substituting Eq. (5-22) into Eq. (5-21) yields:

\[ S = mC_v \ln \left[ \frac{1}{mR} \left( P + \frac{a(T)m^2}{V(V + mb)} \right) \left( \frac{V}{V + mb} \right)^{\frac{1}{V - mb}} \left( \frac{V}{V + mb} \right)^{\frac{1}{V - mb}} \right] + S_0. \]  
(5-23)

As the change of heat energy \( dQ = 0 \) in adiabatic process, it is known that the change of entropy \( dS = 0 \) according to Eq. (4-5), which means the entropy \( S \) is constant for adiabatic condition. The entropy keeps constant in Eq. (5-23) means that the term inside the natural logarithm brackets is constant. Therefore, the following formula is valid for adiabatic process:

\[ \left( P + \frac{a(T)m^2}{V(V + mb)} \right) \left( V - mb \right)^{\frac{1}{V - mb}} \left( \frac{V}{V + mb} \right)^{\frac{1}{V - mb}} = \text{const}. \]  
(5-24)

According to this expression form and substituting the definition of adiabatic exponent \( G \) of Eq. (4-26) into above equation, the Soave-Redlich-Kwong adiabatic equation can be written as:
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

\[
\left( P + \frac{a(T)m^2}{V(V + mb)} \right) \left( V - mb \right)^G \left( \frac{V}{V + mb} \right)^{\frac{1}{\frac{\partial a(T)}{\partial T}}} = \text{const}. \tag{5-25}
\]

This is the expression of Soave-Redlich-Kwong adiabatic process. Once the initial conditions were known, it is possible to calculate pressure or temperature thermodynamics by this correlation. The gas volume expression term \((V - mb)^G\) in above equation is identical to that of the van der Waals adiabatic equation. Compared with the Redlich-Kwong equation of state, the term \(a_{RK}/T^{0.5}\) was replaced with a more general temperature-dependent term \(a(T)\). The pressure term of the Soave-Redlich-Kwong adiabatic equation is slightly different from the pressure term of the Redlich-Kwong equation of state and van der Waals equation of state. However, both adiabatic equations in van der Waals gas model in Eq. (4-25) and Soave-Redlich-Kwong gas model in Eq. (5-25) include the product of a pressure term and a volume term. Once the initial conditions \(P_0\), \(T_0\) and \(V_0\) was known, the constant \(C\) in Eq. (5-25) can be calculated:

\[
C = \left( P_0 + \frac{a(T_0)m^2}{V_0(V_0 + mb)} \right) \left( V_0 - mb \right)^G \times \left( \frac{V_0}{V_0 + mb} \right)^{\frac{1}{\frac{\partial a(T)}{\partial T}}} \bigg|_{T=T_0}. \tag{5-26}
\]

Eq. (5-25) can be rewritten as:

\[
P + \frac{a(T)m^2}{V(V + mb)} = C \left( V - mb \right)^G \left( \frac{V}{V + mb} \right)^{\frac{1}{\frac{\partial a(T)}{\partial T}}}. \tag{5-27}
\]

The pressure \(P\) can be expressed by:

\[
P = -\frac{a(T)m^2}{V(V + mb)} + \frac{C}{\left( V - mb \right)^G \left( \frac{V}{V + mb} \right)^{\frac{1}{\frac{\partial a(T)}{\partial T}}}}. \tag{5-28}
\]

In the Soave-Redlich-Kwong adiabatic equation in above equation, an additional term \(Z\) is included:

\[
Z = \left( \frac{V}{V + mb} \right)^{\frac{1}{\frac{\partial a(T)}{\partial T}}}. \tag{5-29}
\]

As the term \(a(T)\) is included in Eq. (5-28), it is necessary to calculate temperature \(T\) during the processes for calculating accumulator compression and expansion processes by the Soave-
Redlich-Kwong adiabatic equation. A function can be built according to Soave-Redlich-Kwong equation:

\[
g(T) = T - \frac{V - mb}{mR} \left( P + \frac{a(T)m^2}{V(V + mb)} \right)
\]  

(5-30)

The value of temperature \( T \) can be obtained by computing the root of the nonlinear equation:

\[
g(T) = 0
\]

(5-31)

Combining the solution of this equation with the Soave-Redlich-Kwong adiabatic equation of Eq. (5-28), it is possible to calculate the pressure of adiabatic process if the corresponding volume is known.

Adiabatic expansion process calculated by the Soave-Redlich-Kwong adiabatic equation is carried out and the results are shown in Figure 5.3. The accumulator gas is expanded from 2.67L to 5L. The initial pressure condition at the point A is 34MPa, abs., and 293.5K in gas temperature. The top plot presents the \( PV \) diagram. When the gas is expanding, the gas pressure decreases to the point B. The \( TV \) diagram is shown in the middle plot, that the temperature decreases as the internal energy is transferred to work done to outside. In the bottom diagram, the additional term \( Z \) of Eq. (5-29) is plotted. The term \( Z \) shows around 0.9 during this process.
Figure 5.3  Adiabatic expansion process of accumulator calculated by the Soave-Redlich-Kwong adiabatic equation
Figure 5.4  Parameter properties in Soave-Redlich-Kwong adiabatic equation
Values of parameters $a(T)$, $b$ and $\partial a(T)/\partial T$ of Soave-Redlich-Kwong adiabatic equation are plotted in Figure 5.4. The parameter values of $a_v$ and $b_v$ of van der Waals equation shown in Eq. (1-2) are constant. Instead of $a_v$, the parameter $a(T)$ of Soave-Redlich-Kwong equation of state is a temperature-dependent variable. Although the parameter $b$ of Soave-Redlich-Kwong equation of state is constant that the value is $2.683 \times 10^{-5}$, however, the value of $b_v$ of van der Waals equation is $3.001 \times 10^{-5}$. The negative value of $\partial a(T)/\partial t$ decrease from the initial point A to the final point B.

The adiabatic expansion process is simulated by ideal gas model, van der Waals gas model and Soave-Redlich-Kwong gas model. The calculation result of $PV$ diagram is shown in Figure 5.5. The accumulator gas volume varies from 2.7L to 5L which gives a 34MPa, abs initial gas pressure. The black dot line is ideal adiabatic equation result, the purple dash line is van der Waals adiabatic equation result and the red solid line is Soave-Redlich-Kwong adiabatic equation result. The result indicates that the simple ideal gas model gives visible pressure difference compared with the other two gas models. The gas pressures modelled by van der Waals gas model and Soave-Redlich-Kwong gas model are different as well. It is well known
that the Soave-Redlich-Kwong equation of state has more accuracy for modeling gas thermodynamics than any other gas models. It is predictable that the Soave-Redlich-Kwong adiabatic equation are more accurate than van der Waals adiabatic equation.

The adiabatic compression process is simulated by ideal gas model, van der Waals gas model and Soave-Redlich-Kwong gas model. The calculation result of $PV$ diagram is shown in Figure 5.6. The accumulator gas volume varies from 5L to 2.7L which gives a 10MPa, abs initial gas pressure. The black dot line is ideal isothermal equation result, the purple dash line is van der Waals isothermal equation result and the red solid line is Soave-Redlich-Kwong isothermal equation result. It indicates that the simple ideal gas model gives visible pressure difference compared with the other two gas models. The gas pressures modelled by van der Waals gas model and Soave-Redlich-Kwong gas model are very closed. It is plausible to use van der Waals gas model from this calculation, however, the calculation result by van der Waals isothermal equation is different from that of Soave-Redlich-Kwong isothermal equation as shown in Figure 5.1.
5.2.3 Consideration of constant volume specific heat of nitrogen gas

The completed Soave-Redlich-Kwong adiabatic equation can be written as:

\[
G = 1 + \frac{R}{C_v} \quad \text{and} \quad Z = \left( \frac{V}{V+mb} \right)^{1 - \frac{\beta(T)}{V/V+mb}}.
\]

Both term \( G \) and term \( Z \) include the constant volume specific heat \( C_v \). It is used to describe the quantity of heat required to raise the temperature of one mole of a gas through 1K when the volume is kept constant. It should be declared that the constant parameter \( C_v \) is not a constant, that its value changes according to gas temperature and pressure\[65\]. Din has a detailed investigation about nitrogen gas thermodynamic factors. The value of \( C_v \) is shown in Table 5.2. The pressure of nitrogen gas in accumulator is related to the hydraulic pressure of the hydraulic system, which is normally not exceed 40MPa (\( \approx 400\,\text{Atm} \)). The temperature of nitrogen gas in accumulator is between 250K to 400K. To understand how the constant volume specific heat impact the accumulator thermodynamics, the variable value of \( C_v \) is considered. Within the reasonable range of temperature and pressure of accumulator gas, the minimum value and maximum value of \( C_v \) is 20.6 and 22.9, respectively.

The accumulator charge process is calculated that gas volume varies from 7.4L to 5L which gives a 4.4MPa, abs initial gas pressure and 293K in gas temperature as shown in Figure 5.7. Different values of \( C_v \) are given to highlight its influence on gas thermodynamics in accumulator. The pressure has a 0.1MPa distinction and temperature has a 5K distinction while \( C_v \) is set as 20.6 and 22.9. Distinction of the parameter \( G \) and \( Z \) are also shown in the diagram.

The accumulator discharge process is calculated that gas volume varies from 2.7L to 5L which gives a 34MPa, abs initial gas pressure and 293K in gas temperature as shown in Figure 5.8. Different values of \( C_v \) are given to highlight its influence on gas thermodynamics in accumulator. The pressure has a 0.5MPa distinction and temperature has a 8K distinction while \( C_v \) is set as 20.6 and 22.9. Distinction of the parameter \( G \) and \( Z \) are also shown in the diagram.

The slight distinctions of gas thermodynamics according to the factor \( C_v \) have been shown, moreover, the real value of \( C_v \) is variable depended on gas pressure and gas temperature. The actual distinctions will be smaller than above results. Consequently, the variation of \( C_v \) can be neglected for applying Soave-Redlich-Kwong adiabatic equation in hydraulic system.
Table 5.2 Specific heat at constant volume for nitrogen gas \([J/(mol\cdot K)]\) [66]

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Figure 5.7 Adiabatic compression process in accumulator adjusted by $C_v$. 
Figure 5.8  Adiabatic expansion process in accumulator adjusted by $C_v$. 
5.3 Comparison of gas models

In this chapter the Soave-Redlich-Kwong has been discussed, especially its isothermal and adiabatic equations have been derived. The equations of state have been developed considering the thermodynamics of real gas from the original ideal gas model. The derived gas model will be compared with other gas model and the experimental results. The experiment data are referred to the experimental result by Miyashita et al[67]. The experiment configuration is shown in Figure 5.9. The laboratory for experiment was kept in room temperature. The bladder-type accumulator (a) was connected with the piston-type accumulator though a control valve (e) and a throttle valve (l). The hydraulic power was supplied by the hydraulic pump (f) coupled with a relief valve (k). While open the valve (g) and close the valves (h), (e), (i) and (j), the pumped hydraulic oil flows into the bladder-type accumulator (a) and keep the condition for seven hours to reach a steady temperature condition in accumulator. Then extract the piston-type accumulator by opening the valve (j). Closing the valves (g) and (j) and opening the valve (e) then the pressurized hydraulic oil in bladder-type accumulator (a) will flows into the piston-type accumulator (b). The piston displacement can be measured by potentiometer (d) and the gas volume in the piston-type accumulator can be calculated. According to the volume of oil charged in the piston-type accumulator (b), the gas volume in the bladder-type accumulator (a) can be calculated. Then record the data including piston displacement, ambient temperature and gas pressure in both accumulators.

(a) bladder-type accumulator, (b) piston-type accumulator, (c) sensor, (d) potentiometer, (e) valve, (f) hydraulic pump, (g)-(j) stop valve, (k) relief valve, (l) throttle valve

Figure 5.9 An experimental circuit using a bladder-type and a piston-type accumulators[67]
The piston-type accumulator was charged and discharged rapidly. The processes are near adiabatic operations in the experiments. The settled initial conditions for experiment and calculation by gas models are shown in Table 5.3. The Otis’s thermal time constant model, which considers the heat losses, is also compared. As the heat losses are considered in this model, it is foreseeable that the calculation result by this model will be in consonance with the experimental data. In Otis’s paper, the thermal time constant model was coupled with the Benedict-Webb-Rubin equation of state[35]. This equation of state will be compared with Soave-Redlich-Kwong equation of state and experimental data. The thermal time constant \(\tau\) is assumed to be 8 seconds for the Otis’s models coupled with the Benedict-Webb-Rubin equation and the Soave-Redlich-Kwong equation. According to Otis’s research, the value of \(\tau\) were determined referring to experimental results.

<table>
<thead>
<tr>
<th>Initial condition</th>
<th>Value</th>
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<tr>
<td>Initial volume (V_0)</td>
<td>7.4 L</td>
</tr>
<tr>
<td>Final volume (V_t)</td>
<td>5 L</td>
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<tr>
<td>Initial temperature (T_0)</td>
<td>301 K</td>
</tr>
<tr>
<td>Expansion period</td>
<td>10s</td>
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</table>

The \(PV\) diagram is shown in Figure 5.10. During the accumulator charging, the gas volume decreases, and the gas pressure increases. The blue dot is calculation result of van der Waals adiabatic equation. The red dash line is calculation result of Soave-Redlich-Kwong adiabatic equation. The purple dot line is calculation result of thermal time constant model coupled with Soave-Redlich-Kwong equation of state. The green short-dash line is calculation result of thermal time constant model coupled with Benedict-Webb-Rubin equation of state. The thermal time constant model gives consistency to experimental result, coupled by both Soave-Redlich-Kwong equation and Benedict-Webb-Rubin equation. The gas pressure varies near adiabatic conditions. But it is difficult to completely build an adiabatic experimental condition because the thermal energy in accumulator will transfer to ambient through the vessel. Therefore, it cannot be higher than the pressure estimated by adiabatic gas model. The adiabatic equations give the boundary line.
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

Figure 5.10  Piston-type accumulator test and calculation in compression process

Table 5.4  Initial conditions for bladder-type accumulator test and calculation in expansion process

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<td>Initial volume</td>
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<tr>
<td>Final volume</td>
<td>5 L</td>
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<tr>
<td>Initial temperature</td>
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</tr>
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<td>Expansion period</td>
<td>10 s</td>
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The bladder-type accumulator was also studied by charging and discharging rapidly as near adiabatic process. The settled initial conditions for experiment and calculation by gas models are shown in Table 5.4. The thermal time constant $\tau$ is assumed to be 15 seconds for the Soave-Redlich-Kwong equation of state and for the Benedict-Webb-Rubin equation of state. The value of $\tau$ was also determined referring to experimental results. The $PV$ diagram is shown in Figure 5.11. During the accumulator discharging, the gas volume increases, and the gas pressure decreases. The blue dot is calculation result of van der Waals adiabatic equation. The red dash line is calculation result of Soave-Redlich-Kwong adiabatic equation. The purple dot line is calculation result of thermal time constant model coupled with Soave-Redlich-Kwong equation of state. The green short-dash line is calculation result of thermal time constant
model coupled with Benedict-Webb-Rubin equation of state. For the Soave-Redlich-Kwong equation of state, its adiabatic equation and its thermal time constant model gives consistency to experimental result. As heat exchange is considered in thermal time constant models, this model cannot simulate pure adiabatic process. However, in some operated conditions the thermodynamic properties vary very similarly as adiabatic process, the physical process occurs so rapidly that there is no enough time for the transfer of thermal energy in accumulator, thus the Soave-Redlich-Kwong adiabatic equation can be employed to estimate the process. Adiabatic assumption of gas equation of state, which describes gas thermodynamic properties without transfer of heat, provides a conceptual basis in thermodynamics. As the comparison result shows, this adiabatic equation can be used as a reference to study thermal time constant model combined with various gas models. The Soave-Redlich-Kwong adiabatic equation shows accurate calculated results in accumulator compression and expansion processes. The proposed Soave-Redlich-Kwong adiabatic equation is successfully validated from the comparison results[64].

For commercial applications, as the accumulator working in hydraulic system is operated between isothermal process and adiabatic process. The gas thermodynamics in accumulator vary between isothermal change and adiabatic change. Especially the adiabatic process, the energy received from the hydraulic system is almost transferred to internal energy of gas in accumulator and does not have enough time for heat exchange, which cases the gas pressure directly increase and reaches maximum than other processes. According to operating conditions, accurate prediction of accumulator thermodynamics and design of accumulator size for efficient operation are significant. In this section, a lack of mathematical expression for the SRK equation of state, that is, a lack of SRK adiabatic equation, has been solved by this research. This contribution is important for mathematical modeling of accumulator. According to previous calculation equation of thermal time constant model, which was combined by Benedict-Webb-Rubin equation of state, the new thermal time constant model coupled with Soave-Redlich-Kwong equation was presented as well in this section.

The measured temperature is compared with the evaluated value modelled by Soave-Redlich-Kwong adiabatic equation, as shown in Figure 5.12. The temperature in isothermal process keeps constant. The comparison indicates the experimental data is very closed to the calculation result of Soave-Redlich-Kwong adiabatic equation. As the quick discharge of the bladder-type accumulator, the internal energy does not have enough time for heat exchange between the gas and surrounding. The internal energy change comparison is shown in Figure 5.13. Before the discharge process, the compressed gas has higher temperature than ambient temperature. In isothermal discharge process, the internal energy is transferred to work done to hydraulic system and heat energy exchanged though vessel wall. The calculation result of internal energy change to heat by the experimental data is very closed to adiabatic line, which is zero.
Figure 5.12  Temperature comparison of experimental with evaluation by Soave-Redlich-Kwong equation of state in bladder-type accumulator

Figure 5.13  Internal energy change comparison of experimental with evaluation by Soave-Redlich-Kwong equation of state in bladder-type accumulator
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

As it is important to accurately size an accumulator, the Soave-Redlich-Kwong equation of state is proper to evaluate an accumulator by calculating the two ultimate states which are isothermal process and adiabatic process. To compare the Soave-Redlich-Kwong equation of state with the van der Waals equation of state and the ideal gas model, a calculation model is built, and the initial conditions have been shown in Table 5.5. The accumulator is compressed in an adiabatic process by 20MPa, abs. initial pressure. Assuming 10L hydraulic oil flows into an accumulator, the accumulator volume is settled as 20L, 30L and 40L. The maximum gas volume is approximated as the accumulator volume. The initial temperature is 301K and the compression process is operated 10 seconds.

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</tbody>
</table>

The $PV$ diagram is shown in Figure 5.14. While the 40L accumulator is compressed from 40L to 30L, the ideal gas pressure is lower than that modelled by the van der Waals gas model and Soave-Redlich-Kwong gas model. These two gas real gas models are very similar. If the maximum gas pressure in accumulator should not exceed to 30MPa, abs., the capacity for hydraulic oil is 8L calculated by real gas model, but not 10L calculated by ideal gas model. While the 30L accumulator is compressed from 30L to 20L, a 2MPa pressure difference of van der Waals gas model and Soave-Redlich-Kwong gas model can be seen from the diagram. While the 20L accumulator is compressed from 20L to 10L, the pressure difference of van der Waals gas model and Soave-Redlich-Kwong gas model is more visible, which will approach to 10MPa. The largest accumulator can reach to 450L in market and the largest gas pressure can reach to 100MPa, abs. as for now[32]. Thus, the Soave-Redlich-Kwong equation of state, which shows more accuracy than other gas model is more suitable for sizing the accumulator.

The $TV$ diagram is shown in Figure 5.15. While the 40L accumulator is compressed from 40L to 30L, the ideal gas temperature is lower than that modelled by the van der Waals gas model and Soave-Redlich-Kwong gas model. Too, the gas temperature modelled by Soave-Redlich-Kwong gas is lower than that of van der Waals gas. While the 30L accumulator is compressed
from 30L to 20L, a 12K temperature difference of van der Waals gas model and Soave-Redlich-Kwong gas model can be seen from the diagram. While the 20L accumulator is compressed from 20L to 10L, the temperature difference of van der Waals gas model and Soave-Redlich-Kwong gas model is more visible, which will approach to 30K.

**Figure 5.14**  Pressure properties evaluated by different gas models

**Figure 5.15**  Temperature properties evaluated by different gas models
The \( \text{UP} \) diagram is shown in **Figure 5.16**. As there is no heat exchange in adiabatic process, the work done \( W \) is equal to the internal energy \( U \) according to Eq. (4-1). While the 40L accumulator is compressed from 40L to 30L, the ideal gas energy is lower than that modelled by van der Waal gas model and Soave-Redlich-Kwong gas model. The gas energy modelled by Soave-Redlich-Kwong gas is lower than that of van der Waals gas. While the 30L accumulator is compressed from 30L to 20L, a 20K temperature difference of van der Waals gas model and Soave-Redlich-Kwong gas model can be seen from the diagram. While the 20L accumulator is compressed from 20L to 10L, the temperature difference of van der Waals gas model and Soave-Redlich-Kwong gas model is more visible, which will approach to 120K.

![Internal energy properties evaluated by different gas models](image)

**Figure 5.16** Internal energy properties evaluated by different gas models

This chapter derived isothermal equation and adiabatic equation of Soave-Redlich-Kwong gas model. It gives high-accuracy boundary line of gas change. They are mathematical equations for evaluate gas change state in gas-loaded accumulator, which can be employed to design accumulator in different hydraulic systems.

### 5.4 Application of SRK adiabatic equation in accumulator in hydraulic system

For confirming the derived equations in above section, a sample hydraulic system including an accumulator is to be simulated by ideal gas model and SRK gas model as shown in **Figure 5.17**.

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The simulation is operated in 30 seconds that the accumulator is compressed (15 seconds) and expanded (15 seconds) rapidly, which is closed to adiabatic change. As the AMESim does not employing the SRK adiabatic equation, the thermal time constant is set as 1000s to approximate to adiabatic process. The accumulator is compressed by hydraulic fluid which is driven by a hydraulic pump. The maximum pressure in pipeline is adjusted by a relief valve. During the expansion of accumulator, a throttle valve is connected represents hydraulic actuator driven by accumulator power. While the same initial conditions are given, the accumulator performance in ideal gas model and SRK gas model is compared.

The accumulator thermodynamic performance is shown in Figure 5.17. In SRK adiabatic evaluation, the accumulator pressure reaches maximum pressure (relief pressure) spent shorter time than that in ideal adiabatic evaluation. The gas volume change has a 3L difference which means the hydraulic fluid volume flows in accumulator is different. Also, the different charging process cases discharging process different between two models. As the accumulator thermodynamic performance is different, it impacts the pump in main drive circuit, and the power outlet side (throttle valve), as shown in Figure 5.18. Both power resource circuit and power output circuit are influenced by accumulator performance. It is possible to impact the selection of hydraulic components and the actuator displacement or velocity. Thus, accurately calculate the ultimate pressure of accumulator is important and SRK adiabatic equation is useful for application in engineering. The derived SRK adiabatic equation gives a calculation expression for evaluating limited pressure in accumulator. Not only in accumulator, another enclosed gas reservoirs can employ this equation for evaluating the gas thermodynamic
Chapter 5 Soave-Redlich-Kwong adiabatic equation for gas-loaded accumulator

characteristics as well.

**Figure 5.18** Accumulator gas properties during compression and expansion in hydraulic system

**Figure 5.19** Hydraulic pressure in delivery side of pump and throttle valve
5.5 Summary

The Soave-Redlich-Kwong equation of state is applied for modeling accumulator in this chapter. The thermal time constant equation expressing the heat exchange of the accumulator is coupled with this equation of state. Moreover, the isothermal and adiabatic expressions of Soave-Redlich-Kwong equation are derived for modeling the ultimate internal variation of accumulator gas. According to the form of the equation, a further analysis is proposed focusing on the constant volume specific heat $C_V$ of nitrogen gas in the accumulator. The result shows that within the scope of temperature and pressure in hydraulic systems, the variation of $C_V$ can be neglected for hydraulic design. By instead of ideal adiabatic gas equation, the novel Soave-Redlich-Kwong adiabatic shows distinction of gas thermodynamics, which has been confirmed that the difference will impact the accumulator gas pressure, then impact the estimation of hydraulic performance. Simulations indicated that more accurate gas model is critical for designing a proper accumulator for a specific hydraulic system.
Chapter 6

Conclusion and recommendations

6.1 Conclusion

The subject of this work is to show how the gas thermodynamics influence the gas-loaded accumulator performances in hydraulic system and compare the ideal gas model with real gas model. In addition, the well-known equation of state Soave-Redlich-Kwong, which expresses the actual gas properties for adiabatic condition, is derived. The study results are shown as follows:

1. The heat loss in accumulator will cause a pressure drop in hydraulic system. The accumulator can be used for energy regeneration system in different configurations. Its obvious effect on energy saving is shown considering the real gas properties. The real gas compression properties are distinguished from ideal gas as the heat transfer and compressibility effects cannot be negligible.

2. The thermodynamics of accumulator gas modelled by ideal gas model and van der Waals gas model are compared, both for isothermal process and adiabatic process. The pressure difference is large enough that it has a significant influence on accumulator performance which is serving in hydraulic system. In isothermal process, the compressed gas in accumulator has an increasing temperature, then internal energy transfers to surrounding by heat exchange. However, while the accumulator operation process trends towards adiabatic from isothermal, the pressure difference becomes larger.

3. This study gives detailed discussion of the difference between ideal gas model and van der Waals gas model in different pressure level. For solving engineering problems of accumulator application in hydraulic system, a quantitative reference is produced. The boundary thermodynamic properties of hydraulic accumulator gas can be evaluated based on this study.

4. Mathematical expressions for describing isothermal process and adiabatic process in accumulator by Soave-Redlich-Kwong equation of state are derived assuming that the constant volume specific heat $C_v$ is constant in this thesis. They are compared with previous gas models. The derived expressions give a precise and efficient evaluation of boundary thermodynamic trends of accumulator gas. The mathematical equations can be employed for designing accumulator and evaluate the hydraulic system performance.
6.2 Recommendations for future work

This thesis gives detailed analysis of gas models in isothermal process or adiabatic process. However, the heat loss happens during accumulator compression or expansion. Either bladder-type accumulator, piston-type accumulator or other types accumulator, the heat exchange between gas and vessel wall cannot be avoided. Thus, more detailed gas model that considers heat exchange should be discussed according to the accumulator structures. As the heavy weight of accumulator which is made of thick steel, even with a small volume, the new lightweight accumulator is being designed in market. The new-structure carbon fiber composite accumulator made of a thin aluminum vessel (liner) wrapped with carbon fiber composite. The study of energy density and power density should be discussed for the newly accumulator. The heat loss should also be considered due to the new structure. New concept named open-accumulator was introduced for improving the energy density considering the limited gas volume in accumulator[68]. As the heavy mass of the accumulator, sometimes it is difficult to be installed in hydraulic systems, the newly steelhead composite offers lightweight vessels[69]. As heat exchange including heat transfer and heat conduction occurs in accumulator, the installation method will influence the heat exchange properties. Thus, accumulator installed in different hydraulic systems including vertical installation and horizontal installation should be the object of heat loss study in gas-loaded accumulator in future work.
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