Design of Bilateral Control Based on Complementary Sensitivity Function Using Velocity Information

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Abstract—We propose a bilateral control by design based on complementary sensitivity function. In this control method, we regard force applied by an operator at the master side and contact force from the environment at the slave side as input signals. We are able to determine the transfer function for displacement of the position of the master and slave systems by designing complementary sensitivity function. Thereby this control method is possible to transfer the force at the master side and slave side to the others by feedback only the position information. Adding velocity information in feedback is possible to alleviate the stability condition of the complementary sensitivity function. This paper proposed that the bilateral control can be established in low-order controller.

I. INTRODUCTION

In recent years, many research works on bilateral control have been reported. The bilateral control is able to work position control and force control bi-directionally in the plurality of actuators [1]. This is used for the haptic technology to transfer forces sensory information via the communication network [2].

Haptics is expected to be applied to the various fields machine. Haptics interfaces can synchronize one’s position and force, while providing operators with position and force information from remote place. It is possible to improve the efficiency and accuracy in the operator’s work since haptics technology can transfer force from an environment where a machine contacts to the operator. For example, surgeons can operate medical treatment by using a remote robot, when they are not in the site of treating patients [3]–[5]. Also, this improves a feeling of immersion to the augmented reality using a robot located in the different or virtual place to the operator. Thus it is expected to realize avatars for human.

However, to achieve these goals, it is required to realize high accuracy of haptics and simplification of the control method for the Multi-DOF system. Conventional haptics methods are based on feedback both of position and force information in order to achieve high accuracy of performance. The authors have proposed a new control method based on feedback of position information only without reaction force estimation in order to simplify control system design in haptics [6]. A transfer-function-based controller design for bilateral control system was proposed in the paper. By designing the transfer function arbitrarily, this control method can extend the influence of external force in one system to the others and be synchronized these positions [7], [8].

In this paper, we will show a relaxation of the condition of the control system design proposed in [6], under the assumption that the velocity information is available. As the results, we can design a bilateral control system with low order controllers. Finally, we will show experiment results of controlling the method which we have proposed. Subsequently we compared the force that can be felt in each of the machines as response value of the measured force by disturbance observer (DOB). Thus, we confirm the effectiveness of the control method.
II. DERIVATION OF PROPOSED METHOD

Purpose of the bilateral control system is to realize the law of action and reaction between forces being applied to the two motion systems while their positions are synchronized. We consider MIMO feedback from two position observations of the motion systems to two actuator inputs as shown in Fig. 1. In Fig. 1, the control method uses feedback controllers for achieving their stabilization, and controllers to achieve synchronization of the other position and force. A design of the control system can be derived by a transfer-function-based approach.

In Fig. 1, relation between the external signals \( f \), \( \xi \) and output signals from each subsystems \( y \), \( u \) are described as follows.

\[
y_1 = P_1(f_1 + u_1) \quad (1)
y_2 = P_2(f_2 + u_2) \quad (2)
u_1 = -C_{11}(y_1 + \xi_1) + C_{21}(y_2 + \xi_2) \quad (3)
u_2 = C_{12}(y_1 + \xi_1) - C_{22}(y_2 + \xi_2) \quad (4)
\]

Then, (1)–(4) can be rewritten as follows.

\[
\begin{bmatrix}
y_1 \\
y_2 \\
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
P_1 \\
P_2 \\
C_{11} \\
C_{21}
\end{bmatrix} \begin{bmatrix}
f_1 + u_1 \\
f_2 + u_2 \\
y_1 + \xi_1 \\
y_2 + \xi_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
y_1 \\
y_2 \\
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
P_1 & -C_{11} \\
P_2 & C_{21} \\
C_{11} & C_{12} \\
C_{21} & -C_{22}
\end{bmatrix} \begin{bmatrix}
f_1 + u_1 \\
f_2 + u_2 \\
y_1 + \xi_1 \\
y_2 + \xi_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
y_1 \\
y_2 \\
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
P_1 & -C_{11} \\
P_2 & C_{21} \\
C_{11} & C_{12} \\
C_{21} & -C_{22}
\end{bmatrix} \begin{bmatrix}
f_1 + u_1 \\
f_2 + u_2 \\
y_1 + \xi_1 \\
y_2 + \xi_2
\end{bmatrix}
\]

from the observation noise \( \xi \) to the output \( y \), namely \( Q = G_{y\xi}/A \). It is given as follows.

\[
Q_1 = \frac{(A - 1 - C_{22}P_2)/A}{(1 - Q_1)P_1 - Q_2P_2 - Q_1Q_2} \quad (14)
Q_2 = \frac{C_{21}P_1/A}{(1 - Q_1)P_1 - Q_2P_2 - Q_1Q_2} \quad (15)
Q_3 = \frac{C_{12}P_2/A}{(1 - Q_1)P_1 - Q_2P_2 - Q_1Q_2} \quad (16)
Q_4 = \frac{(A - 1 - C_{11}P_1)/A}{(1 - Q_1)P_1 - Q_2P_2 - Q_1Q_2} \quad (17)
\]

By using these parameters \( Q_1, Q_2, Q_3, \) and \( Q_4, (5) \) can be rewritten as follows.

\[
\begin{bmatrix}
y_1 \\
y_2 \\
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
(1 - Q_1)P_1 & Q_2P_2 & -Q_1 & Q_2 \\
Q_3P_1 & (1 - Q_4)P_2 & Q_3 & -Q_4 \\
-Q_1 & P_2Q_2 & -Q_1 & P_2Q_2 \\
Q_3P_1 & P_2Q_2 & Q_3 & Q_4
\end{bmatrix} \begin{bmatrix}
f_1 \\
f_2 \\
\xi_1 \\
\xi_2
\end{bmatrix}
\]

(18)

In addition, each controller, \( C_{11}, C_{12}, C_{21}, \) and \( C_{22}, \) can be obtained as follows by solving (6), (14)-(17) with respect to them.

\[
C_{11} = \frac{1}{P_1} \frac{(1 - Q_4)P_1 + Q_2Q_3}{(1 - Q_1)(1 - Q_4) - Q_2Q_3} \quad (19)
C_{12} = \frac{1}{P_2} \frac{(1 - Q_4)P_1 + Q_2Q_3}{(1 - Q_1)(1 - Q_4) - Q_2Q_3} \quad (20)
C_{21} = \frac{1}{P_1} \frac{Q_2}{(1 - Q_1)(1 - Q_4) - Q_2Q_3} \quad (21)
C_{22} = \frac{1}{P_2} \frac{(1 - Q_1)Q_3 + Q_2Q_3}{(1 - Q_1)(1 - Q_4) - Q_2Q_3} \quad (22)
\]

Each controller is parameterized by the complementary sensitivity functions \( Q_1, Q_2, Q_3, \) and \( Q_4, \) depending on the design of the complementary sensitivity functions \( Q_1, Q_2, Q_3, \) and \( Q_4, \) it is able to perform wide range of response in the control system.

III. DESIGN METHOD OF \( Q \)

The control target of bilateral control are synchronizing the position between the master and slave systems and satisfying the law of action and reaction in force applied to the master and slave systems. In other words, the position error between the master and slave systems should be zero. And the master or slave system is affected by the equivalent force applied to the other system. These goals are expressed by following equations.

\[
\lim_{t \to \infty} y_1 - y_2 = 0 \quad (23)
\lim_{t \to \infty} u_1 - f_2 = 0 \quad (24)
\lim_{t \to \infty} u_2 - f_1 = 0 \quad (25)
\]

If we satisfy (24)(25), The force applied to system 2(slave) occurs in the system 1(master). Namely, operator touching the system 1(master) can feel the force. Thus, the conditions (24)(25) are very important, because they represent a virtual law of action and reaction between the master and slave systems. These conditions can be equivalently converted to

<table>
<thead>
<tr>
<th>parameters</th>
<th>meaning</th>
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<tbody>
<tr>
<td>( u_n )</td>
<td>Controller’s output</td>
</tr>
<tr>
<td>( P_n )</td>
<td>The nominal plant model</td>
</tr>
<tr>
<td>( C_n )</td>
<td>Controller’s function</td>
</tr>
<tr>
<td>( f_n )</td>
<td>External force</td>
</tr>
<tr>
<td>( \xi_n )</td>
<td>Observation noise</td>
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</table>
TABLE II: The conditions that Q must be satisfied

<table>
<thead>
<tr>
<th>conditions</th>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
</tbody>
</table>

practical conditions First, (5) can be rewritten as follows.

\[
\begin{bmatrix}
  y_1 - y_2 \\
  u_1 + u_2
\end{bmatrix}
= \frac{1}{A}
\begin{bmatrix}
  G_{1f} & G_{1\xi} \\
  G_{uf} & G_{u\xi}
\end{bmatrix}
\begin{bmatrix}
  F \\
  \Xi
\end{bmatrix}
\] (26)

where

\[
T =
\begin{bmatrix}
  y_1 - y_2 \\
  u_1 + u_2
\end{bmatrix}
\] (27)

\[
G_{1f} =
\begin{bmatrix}
  (1 - Q_1 - Q_3)P_1 & (Q_2 + Q_4 - 1)P_2 \\
  -Q_1 + \frac{P_3}{P_1}Q_3 & \frac{P_3}{P_1}Q_2 - Q_4
\end{bmatrix}
\] (28)

\[
G_{1\xi} =
\begin{bmatrix}
  Q_1 - Q_3 & Q_2 + Q_4 \\
  \frac{Q_1}{P_1} + \frac{P_3}{P_1}Q_3 & \frac{P_3}{P_1}Q_2 - \frac{Q_1}{P_1}
\end{bmatrix}
\] (29)

\(T\) is defined as a vector of the control target consisting of \(y_1 - y_2\) and \(u_1 + u_2\).

A. Conditions for \(Q\)

The complementary sensitivity functions \(Q_1, Q_2, Q_3,\) and \(Q_4\) are related to relative degree involved in the controller design internal stability of control system, and realization of the control target of bilateral control. Therefore, we derive the conditions to be satisfied by the complementary sensitivity functions. We regard the plant models in the haptic motion control as the double integrator having double unstable poles at the origin, as follows.

\[P_i = \frac{1}{M_i s^2} \quad (i = 1, 2)\] (30)

where \(M_i\) is mass of the mover. The relative degrees and the unstable poles of the plant model are related to the condition 2 ~ 7 in Table II. External force are assumed to be step function having an unstable pole at the origin as well. Therefore, the sensitivity functions must have triple zeros at the origin (Conditions 6 and 7 in Table II).

Conditions 8 ~ 11 in Table II are derived from the conditions (24)(25). Signals \(u_1 - f_2\) and \(u_2 - f_1\) appeared in these conditions can be rewritten as (31). Therefore the conditions (24)(25) are equivalent to the conditions 8 ~ 11 in Table II. By satisfying these conditions, we are able to eliminate influence of external forces to \(u_1 - f_2, u_2 - f_1\).

![Bode plot of Q1-Q](image.png)

Fig. 2: Bode plot of Q1-Q

Therefore the proposed method realizes a part of the control target.

\[
\begin{bmatrix}
  u_1 - f_2 \\
  u_2 - f_1
\end{bmatrix}
= \frac{1}{A}
\begin{bmatrix}
  G_{uf} & G_{u\xi} \\
  F & \Xi
\end{bmatrix}
\begin{bmatrix}
  f_2 \\
  f_1
\end{bmatrix}
\] (31)

Then, Table II summarizes the derived conditions on the complementary sensitivity function. The degree of the denominator of a function having an triple zero and relative degrees of the function is greater than or equal to the relative degree of the plant model are the lowest forth order. The complementary sensitivity functions satisfying these conditions were determined as follows. \(Q_2, Q_3\) are designed as a fourth-order filter. Fig. 2(a) shows a bode plot of \(Q_2, Q_3\).

\[
Q_1 = Q_4 = 0 
\] (32)

\[
Q_2 = Q_3 = \frac{a_2 s^2 + a_1 s + a_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \] (33)

Thus (26) is rewritten as (34) because the complementary sensitivity functions are determined as (32), (33).

\[
\begin{bmatrix}
  y_1 - y_2 \\
  u_1 + u_2 \\
  u_1 \\
  u_2
\end{bmatrix}
= \begin{bmatrix}
  (1 - Q_3)P_1 & -(1 - Q_2)P_2 & -Q_3 & Q_2 \\
  \frac{P_3}{P_1}Q_3 & \frac{P_3}{P_1}Q_2 & \frac{Q_1}{P_1} & \frac{Q_2}{P_1} \\
  0 & \frac{P_3}{P_1}Q_3 & 0 & \frac{Q_1}{P_1} \\
  \frac{P_3}{P_1}Q_3 & 0 & \frac{Q_1}{P_1} & \xi_1 & \xi_2
\end{bmatrix}
\begin{bmatrix}
  f_1 \\
  f_2 \\
  \xi_1 \\
  \xi_2
\end{bmatrix}
\] (34)

We can assume that the external forces \(f_1, f_2\) contain low-frequency components only. On the other hand, the observation noises \(\xi_1, \xi_2\) contain high-frequency components. The sensitivity function \(1 - Q\) is a high-pass filter (HPF), and complementary sensitivity function \(Q\) is a low-pass filter (LPF).
in (34). Therefore, \( y_1 - y_2 \) can be converged to zero because it is not affected by external forces and observation noises. Currently, we think about the system of \( P_1 = P_2 \). In low-frequency domain, \( u_1 + u_2 \) is almost same as \( f_1 + f_2 \) because it is only affected by external forces. On the other hand, in high-frequency domain, the observation noises \( \xi_1, \xi_2 \) are not able to affected by LPF. Similarly, \( u_1 \) can be converged to \( f_2 \), and \( u_2 \) can be converged to \( f_1 \).

We have designed the bilateral control in this method. The proposed method is able to realize high accuracy of performance in bilateral control, and to satisfy the internal stability condition for the control system.

IV. UTILIZATION OF ADDITIONAL VELOCITY INFORMATION

In the previous section, it was confirmed that the proposed control system can be realized by using the fourth-order Q filter. As the result, 6th order controller was designed. Complementary sensitivity functions \( Q_1, Q_2, Q_3, \) and \( Q_4 \) are determined so that they satisfy the conditions in Table II. If we can relax these conditions, the order of the controller can be reduced.

We consider to design the complementary sensitivity function as a third-order filter.

\[
Q' = Q_2' = Q_3' = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + b_2 s^2 + b_1 s + b_0} \quad (35)
\]

Fig. 2(b) shows a bode plot of this third-order filter. However, it does not satisfy the conditions 2 ~ 5 in Table II. Since the transfer functions in (34) must be stable and proper, the relative degree of the complementary sensitivity function \( Q \) must be equal or greater than that of the controlled plant \( P \) and the sensitivity function \( 1 - Q \) must have triple zeros at the origin so that they can cancel both unstable double poles of the plant \( P \) and unstable pole of the step external force \( f \).

If the conditions 2 ~ 5 in Table II are not fulfilled, namely, the relative degree of \( Q \) is less than that of the controlled plant \( P \), the relative degree of the controller becomes negative and the controller is unrealizable by the standard way. However if the velocity information is available as the feedback signal, it is possible to relax the conditions 2 ~ 5 in Table II as follows.

The controller transfer functions (19)–(22) can be rewritten in (36)(37) by using the low-order complementary sensitivity function (35).

\[
C_{11} = \frac{Q'^2}{P_1(1 - Q'^2)}; \quad C_{22} = \frac{Q'^2}{P_2(1 - Q'^2)} \quad (36)
\]

\[
C_{12} = \frac{Q'}{P_2(1 - Q'^2)}; \quad C_{21} = \frac{Q'}{P_1(1 - Q'^2)} \quad (37)
\]

In details,

\[
C_{11} = M_1 \frac{b_2 s^4 + 2b_1 b_2 s^3 + (2b_0 b_2 + b_1^2) s^2 + 2b_0 b_1 s + b_0^2}{s^4 + 2b_2 s^3 + 2b_1 s^2 + 2b_0 s} \quad (38)
\]

\[
C_{12} = \frac{M_2}{s^4 + b_2 s^3 + b_1 s^2 + b_0 s} \times \left( b_2 s^4 + (b_2^2 + b_1) s^3 + (2b_1 b_2 + b_0) s^2 + 2b_0 b_2 + b_1^2 \right) \quad (39)
\]

\[
C_{21} \quad \text{and} \quad C_{22} \quad \text{can be obtained in the same manner. The controllers} \quad C_{11} \quad \text{in (38) and} \quad C_{22} \quad \text{are proper functions. However, the relative degree of the controller} \quad C_{12} \quad \text{in (39) and} \quad C_{21} \quad \text{become negative. Hence, by taking the quotient, (39) can be converted to}
\]

\[
C_{12} = M_2 (b_2 s - D) + \frac{(b_0 + 2b_2 D) s^3 + (b_1 + 2b_1 D) s^2 + (2b_0 b_1 + 2b_0 D) s + b_0^2}{s^4 + 2b_2 s^3 + 2b_1 s^2 + 2b_0 s} \quad (40)
\]

\[
D = b_2^2 - b_1 \quad (41)
\]

In (40), the first term can be realized by using the velocity information because it contains a differentiator. And the second term and third terms can be realized by the standard method using the position information. Also it is possible to obtain similar representations of the other controllers.

\[
C_{21} = M_1 (b_2 s - D) + \frac{(b_0 + 2b_2 D) s^3 + (b_1 + 2b_1 D) s^2 + (2b_0 b_1 + 2b_0 D) s + b_0^2}{s^4 + 2b_2 s^3 + 2b_1 s^2 + 2b_0 s} \quad (42)
\]

\[
C_{22} = M_2 \frac{b_2 s^4 + 2b_1 b_2 s^3 + (2b_1 b_2 + b_1^2) s^2 + 2b_0 b_1 s + b_0^2}{s^4 + 2b_2 s^3 + 2b_1 s^2 + 2b_0 s} \quad (43)
\]

By this method, we can implement our proposed method with the third-order complementary sensitivity function.

V. SIMULATION AND EXPERIMENTAL RESULTS

Simulations and experiments were carried out to evaluate performance of the proposed control with and without velocity information. In the simulation and experiments, a pair of ac servo motors with end-effectors are utilized as controlled plants. Human force \( f_{man} \) is imposed in the master system. Then, reaction force from environment is applied to the slave system when the end-effector of the slave system interacts with the environment. The parameters of the control system are determined as shown in Table III. A spring and damper model is adopted as the environment in the simulations. Coefficients of the spring and damper are \( D = 100[N \cdot s / m], K = 1000[N / m] \). The plant model is a double integrator given by (30).

Fig. 3(a)–(d) shows simulation results of the control system with and without velocity information. The case of using the velocity information is better than the case of not using the velocity information in synchronization accuracy both of position and force. However, this result can not be said is synchronized with good precision. Because cut off frequency is not sufficiently high. Fig. 3(e)–(h) is simulation results in

<table>
<thead>
<tr>
<th>TABLE III: Parameters of simulation and experiment</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>------------------------</td>
</tr>
<tr>
<td>( M_1 )</td>
</tr>
<tr>
<td>( M_2 )</td>
</tr>
<tr>
<td>( \omega_c )</td>
</tr>
<tr>
<td>( \omega_LF )</td>
</tr>
<tr>
<td>( a_0 )</td>
</tr>
<tr>
<td>( a_1 )</td>
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<tr>
<td>( a_2 )</td>
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<tr>
<td>( b_0 )</td>
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<tr>
<td>( b_1 )</td>
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<tr>
<td>( b_2 )</td>
</tr>
<tr>
<td>Sampling time</td>
</tr>
<tr>
<td>Resolution precision of the rotary encoder</td>
</tr>
</tbody>
</table>
$\omega_c = 200\pi [\text{rad/s}]$, it found that the maximum position error is quite small.

The experimental setup are shown in Fig. 4. In these experiments, the soft object (balloon) and the hard object (A5052 of aluminum alloy object) are used as manipulated object. The parameters of the controller are given as shown in Table III. Here, response of human and reaction force are estimated by the reaction force observer, but they are not used in the control system. $f_{\text{man}}$ and $f_{\text{exp}}$ are the estimated human force and the reaction force. Fig. 5 shows experimental results with soft and hard environment.

In soft environment, experimental results are shown in Fig. 5(a)(c). From the result of position and position error, it can be seen that the position of the slave system follows the master system. The position error arises only when the acceleration is big, and it is substantially zero when the master is not moving. The reason of the error is cut-off frequency $\omega_c$ being not sufficiently high, and nominal error of the inertia. Regarding response of force, estimated human force $f_{\text{man}}$ and input of the slave system $u_2$, estimated reaction force from soft environment $f_{\text{exp}}$ and input of the master system $u_1$ are almost synchronized. Thus it proves the operator can feel the reaction force from the environment in small error. It can be also seen that sum of action and reaction force $(u_1 + u_2)$ are almost zero. This means that the control system satisfies the law of action and reaction.

In hard environment, experimental results are shown in Fig. 5(b)(d). The results in hard environment and those in soft environment are similar. But, it is seen that error occurs in the position between the master and the slave when the slave is in contact with the environment from the response of position. Hard environment can not deform when force is applied. Thus high frequency components of the step-like force is occurred. It can not be controlled if the controller of the cut-off frequency was not high enough. Regarding response of force, from the results of the error between the response of the force $f_{\text{man}}$, $f_{\text{exp}}$ and command $u_1$, $u_2$, an error occurs at the moment of contact to the environment, but the error becomes zero when applied force is constant. From these results, we can confirm the proposed method performs precise bilateral control.

VI. CONCLUSION

In this paper, design of the bilateral control based on complementary sensitivity function is proposed. The complementary sensitivity functions are determined from the conditions
derived from internal stability of the system and convergence criteria of the control target of the bilateral system. The proposed method was established both in the cases of the position feedback and the position and velocity feedback. In the latter case, these conditions were relaxed and low-order controllers were obtained. Through the simulations, we confirmed the proposed control realizes suitable control properties with small error in position and force control both in the soft and hard environments.

For the future work, we will increase the cut off frequency in the controller of the proposed method, since it will improve the precision of position control and force control.

VII. ACKNOWLEDGMENT

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