Stability of Equilibria in Games with Communication

Eisaku Ohtsuka

1. Introduction: Cooperative and Noncooperative Games

Among recent advances in game theory is a more precise distinction between cooperative and noncooperative games. When the distinction was first introduced by Nash (1951), cooperative games were defined as games which allow free communication and binding agreements, while noncooperative games were defined as games where neither is possible. Recently, this definition has been criticized (see e.g. Harsanyi and Selten, 1988). Cooperative games are now usually defined as games which allow binding agreements, and noncooperative games as games which do not allow them unless suitable commitment moves are explicitly included in their extensive form. The possibility and degree of communication allowed in a game is now included in the specification of the game. If we re-examine the classical cooperative solution concepts in the light of this new model, then we may say that some of those concepts, such as the core (Gillies, 1959) and stable sets (von Neumann and Morgenstern, 1944), are not quite cooperative in nature, that is, they really assume noncooperative situations where players form coalitions communicating freely with each other. For, in cooperative games, any agreement is, by definition, binding, so that it does not have to be self-enforcing or self-stabilizing. This is true especially in a prescriptive context, that is, the mediator can prescribe a solution which he thinks appropriate without worrying about defection. On the other hand, the concepts just mentioned are defined so as to make the solution self-enforcing in some sense.

Recently the problem of communication has attracted more attention, and has been analyzed in noncooperative contexts. Unfortunately, it seems to me that no fully satisfactory solution concept for such situations has been proposed as yet: the Nash equilibrium concept often fails to work in such situations. Meyerson (1984, 1986) andForges (1985) proposed the concept of "communication equilibria" employing Aumann's correlational equilibrium concept (1974). However, they considered the situations where communication is totally controlled by a central authority, namely the "principal", and where no private or personal communication is permitted. In the following sections, more general cases will be discussed and it will be shown that sometimes no satisfactory solution can be defined for situations involving communication.

2. Treatment of Communication

In dealing with communication, the first thing to consider is how to model the situation. Communication is a sequential process of exchanging messages or signals, so that one may try to model it as an extensive game where each message is a move available at some information set.
Unfortunately this will not work in some cases. Suppose that players 1 and 2 are now playing a game of figure 2.1, where player 1 moves first and tells player 2 what he chose before player 2 moves. Then an appropriate extensive representation of the game will be as in figure 2.2, where L “R”, for example, stands for player 1’s strategy of choosing L and of telling player 2, “I have chosen R”. If the player 2’s strategies means that he responds to the messages, “L” and “R”, by choosing 1 and r, respectively. Table 2.1 is the normal form of the game, in which equilibrium points are starred.

\[
\begin{array}{c|cc|cc|c}
2 & 0 & 0 & 1 & 2 \\
1 & 0 & 0 & 1 & 2 \\
L & r & I & r & L \\
R & & & & \\
\end{array}
\]

Figure 2.1

\[
\begin{array}{c|cc|cc|c}
2 & 0 & 0 & 1 & 2 \\
1 & 0 & 0 & 2 & 1 \\
L & L & R & L & R \\
\eta_1 & & & & \\
\end{array}
\]

Figure 2.2

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>l</td>
<td>r</td>
</tr>
<tr>
<td>l</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>r</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>r</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.1
and (uniformly) perfect equilibrium points are double-starred.

In my view, the only natural and intuitively acceptable solution of this game is (L="L", 1r), once we consider the fact that the natural meaning of "L" is "I have made move L," and would normally never be interpreted as "I have made move R" by the recipient of this message, at least as long as the recipient has reasons to think that the sender had meant to tell the truth. On the other hand, traditional game theory would have to define the solution as a mixed-strategy equilibrium point, assigning the same probabilities to the messages "L" and "R". (Of course, as in the game of figure 2.3, for example, if the recipient of the message has good reasons to distrust the message received then he will simply disregard it in deciding his future moves, and the question of how to interpret the message will be irrelevant.) But the mixed-strategy equilibrium assigning messages "L" and "R" equal probabilities would be a very inefficient outcome.

![Figure 2.3](image)

To avoid these difficulties, I will not model communication explicitly in terms of specific moves. Rather, I will model it merely implicitly, assuming that truthful communication will use words in their usual meaning, whereas untruthful communication, of course, can use any verbal messages whatever. This approach was taken also by Meyerson (1984, 1986) and Forges (1985).

A communication is characterized by a number of assumptions such as who talks to whom, what he can say, how, when, and so on. All these assumptions may be crucial to the outcome. This makes proper analysis of communication rather difficult. In what follows, I will discuss only simple situations where only who talks to whom is specified by the rules of a game. This is done by postulating the existence of specific communication groups. The possibility and consequences of one-way communication will not be considered here. More specific assumptions will be explained below.

3. Communication Groups

**Definition:** A subset M of the players in N is called a communication group if the players in M can openly communicate with one another, that is, if any message or agreement addressed to a communication group is shared by all members in the group. For convenience, I will sometimes call an agreement made in a communication group M other than N a sub-agreement, and will call one comprising the entire set N a global agreement.

In speaking of a communication group, I am visualizing a situation like a meeting where participants have to address the whole assembly and no private conversations are permitted. Any
kind of messages can be exchanged within a communication group. If a communication group, say, A, includes another communication group, B, then the members of the latter group may have a separate meeting, and the content of their talk may or may not be revealed to the members in A\B, depending on group B's decision. Suppose, for example, that \{i, j, k\} is a communication group. Then players i and j cannot make a secret agreement excluding player k unless \{i, j\} is also defined as a communication group. Suppose \{i, j\} is also a communication group, then player i (or j) may leak their secret conversation to k only when \{i, k\} (or \{j, k\}) is also defined as a communication group. In this case, player i (or j) will be said to join the two communication groups, \{i, j\} and \{i, k\} (or \{j, k\}).

*Definition:* Communication groups A and B are said to be joined if \(A \cap B \neq \emptyset\). A player in \(A \cap B\) is called a joiner of communication groups A and B.

The situation in a communication group is still noncooperative. That is, an agreement reached in a communication group may or may not be observed by its members. The existence of a joiner is very crucial in forming an agreement in a communication group because he might be a defector and try to manipulate the agreements in several different groups he joins while looking after his own interest.

Note that, if communication groups A and B are joined, then the union of A and B should also be a communication group. This is because a message made in one of the joined groups can be shared by the other group through the joiners. Of course, the message can still remain secret to the latter group if the former wants this because the former is a separate communication group by itself.

As a convention, each player himself forms a communication group as a singleton set. If the communication groups of a game are all singleton, then we obtain a game where all communication is prohibited.

*Definition:* The collection of all communication groups in game G is called a communication structure of the game, which is denoted by CS(G). To sum up the preceding arguments using CG,

1. If \(A \in \text{CS}(G)\), then \(A \subset N\).
2. If \(i \in N\) then \(\{i\} \in \text{CS}(G)\).
3. If \(A, B \in \text{CS}(G)\) and \(A \cap B \neq \emptyset\), then \(A \cup B \in \text{CS}(G)\).

4. Communication and Self-enforceability

In a communication group, any agreement, if there is one, is assumed to be as profitable as

$$
\begin{array}{|c|c|c|}
\hline
 & 1 & r \\
\hline
L & * & 2 \circledast 0 & 0 \\
\hline
R & 0 & 2 \circledast & 2 \\
\hline
\end{array}
$$
possible to its members as long as it is self-enforcing. The self-enforceability requirement is inevitable because the situation is noncooperative.

In the game of table 4.1, if communication between the players is permitted, then no doubt the outcome will be (L, 1) or (R, r) but the mixed strategy equilibrium point is the only sensible solution when the players cannot communicate. Also in the game of table 4.2, the only efficient equilibrium point, (R, r) will be presumably the outcome if they can talk to each other.2) In some other cases, however, this principle may be more questionable. In the game of table 4.3, the situation is so symmetric and the two players’ interests are so conflicting that it may seem unlikely for them to agree on either one of the two efficient equilibrium points, (L, 1) and (R, r), which are much better for both players than the mixed-strategy equilibrium point is. In what follows, even in the case of such conflicts of interest, I will assume that an efficient and self-enforcing outcome can be achieved. In this example, the players may agree to solve the problem by flipping a coin if it is available to them: they may make use of a correlated equilibrium point.

### Table 4.2

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>8</td>
<td>*</td>
</tr>
<tr>
<td>R</td>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>

Though Meyerson (1984) and Forges (1985) accepted Aumann’s correlated equilibrium concept as an appropriate solution concept for the games with communication, under the model here proposed, its availability is not generally guaranteed to the players in a communication group. Correlated equilibria obviously cannot be used in games permitting no communication. But even if communication is permitted, correlated equilibria can be used only if the players can use a random device when all of them are present. In contrast, if they can talk only over the phone then they will be unable to use correlated equilibria. In this thesis, I will not discuss such equilibria any further.

A Nash equilibrium point is usually interpreted as a self-enforcing strategy combination in the sense that, once the players agree on any given equilibrium, no player will have a positive incentive to deviate from his equilibrium strategy as long as he expects the other players to follow their own equilibrium strategies. This definition of self-enforceability is not sufficient once communication is allowed in a game.
Table 4.4

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>3</td>
<td>L</td>
</tr>
<tr>
<td>R</td>
<td>2</td>
<td>R</td>
</tr>
<tr>
<td>L</td>
<td>3</td>
<td>L</td>
</tr>
<tr>
<td>R</td>
<td>2</td>
<td>R</td>
</tr>
</tbody>
</table>

For example, the game of table 4.4 has two pure strategy equilibrium points, namely, $E_1=(L_1, R_2, L_3)$ and $E_2=(R_1, R_2, R_3)$, where $L_i$ and $R_i$ denote player $i$'s strategies $L$ and $R$, respectively. These equilibrium points are starred in the table. (Here player 1 chooses between going up (L) or going down (R). Player 2 chooses between going to the left (L) or to the right (R). Player 3 chooses between going outward (R) or going inward (L).) The former is efficient but not perfect and the latter is not efficient but perfect.

Suppose that $CS=2^N$, that is, players can freely communicate with each other. Secret communications between any two players are also possible. Then, neither of these points are quite stable in the following sense.

Assume that the players initially agreed upon $E_1=(L_1, R_2, L_3)$. Then players 1 and 2 will violate the agreement by secretly defecting to $(R_1, L_2)$ because, given that player 3 takes $L$, the game player 1 and 2 are now playing is a section $G|L_3$ of the original game as shown in table 4.5, where $(R_1, L_2)$ is the only efficient equilibrium and they collectively have a plain incentive to move to it.

An agreement on $E_2=(R_1, R_2, R_3)$ will be also violated. Once player 1 and 3 believed that player 2 takes $R$, they can consider themselves in $G|R_2$, which is shown in table 4.6. They now have an incentive to move to $(L_1, L_3)$ which is the only efficient equilibrium point of the new situation.

To make the point clearer and more precise, I introduce the following definitions.

**Definition:** Suppose that in game $G = \{N, S, U\}$ all players in the communication group $I \in CS(G)$ assume that the players outside $I$ will use the strategy combination $s_{-I}$. Then I will say that on this assumption (to be called $I$'s current assumption) the strategy combination $s_I \in S_I$ is **vulnerable** to some strategy combination $s_J$ of another communication group.
When an agreement is vulnerable, we must obviously expect defections from the postulated assumption about the outside players' strategies and/or the agreement to more profitable agreements.

This definition of vulnerability can also be applied to the set \(N\) of all players. In this case, there is no player outside of \(N\), and all communication groups are joined to \(N\) because they are proper subsets of \(N\). So that the vulnerability of a point, \(s\), in a game simply means that there is \(s'\), \(J \in \text{CS}(G)\), \(J \neq N\), that satisfies (3) and (4) above.

**Definition:** An equilibrium point of game \(G\) is *communication robust* if it is not vulnerable at all.

The robustness provide a definition of self-enforceability under communication. this concept is a variation of Aumann's strong equilibria (Aumann, 1959, 1985) which he defined as follows.

**Definition:** An equilibrium is *strong* if no coalition of players can all gain by a simultaneous deviation while the players outside the coalition maintain their strategies.

The difference between those two concepts lies in the ability or inability of the players in any coalition to commit themselves to cooperation: the definition of strong equilibria allows players in a communication group to commit themselves while that of communication robustness does not.

Suppose an equilibrium point lacks robustness. Under any realistic predictive theory, a significant number of defections must be expected. In a prescriptive context, players may feel uneasy to accept it even though it is a equilibrium point. In any case, robustness is a desirable property. In a two-person game, robustness is always satisfied by all equilibrium points. But, unfortunately, robust equilibria often fail to exist when the number o players exceeds 2. Relaxation of the condition of
robustness is desirable if it can be accomplished in an acceptable way.

In fact, in the above example, E2 is not as unstable as one may think. Suppose that E2 is agreed upon. Then players 1 and 3 may want to defect to (L1, L3) as stated above. Then, player 1 now has incentive to tell player 2 about player 3's defection and will induce him to agree on (R1, L2), to which L3 is no longer the best response. By knowing this, player 1 and 3 will not make a secret agreement in the first place. In this case, player 1 plays a crucial role as a joiner between communication groups {1, 3} and {1, 2} to make the secret agreement between players 1 and 3 unstable. Hence the following definition.

**Definition:** A sub-agreement s1 is plausible under communication group I's current assumption s−1 ∈ S−1 if s1 is not vulnerable to any plausible sub-agreement made by a communication group, J, that is joined to I.

Plausibility is defined recursively. Obviously, a sub-agreement is plausible if it is not vulnerable at all. Note that an infinite recurrence may occur in some cases. In those cases, no sub-agreement in the recurrent circle is plausible. Table 4.7 shows such an example.

![Table 4.7](image)

This game has only one pure strategy equilibrium point, namely, (L1, L2, L3). Suppose CS=2^N. Further suppose that this pure strategy equilibrium point is initially agreed upon. Then player 1 and 2 have incentive to collectively move to (R1, R2) expecting to increase their payoff to 3 from the current payoff 2. The equilibrium point is vulnerable to (1, 2). But once they consented to defect, player 2 will confidentially tell this defection to player 3, and induce him to take R3 to improve their payoff. That is, the sub-agreement, (R1, R2), is vulnerable to (2, 3). Player 3 will accept player 2’s offer because he can expect the payoff 1 which is better than the current payoff 0. We are now at (R1, R2, R3). This point cannot be stable, either. Again, player 2 will violate this sub-agreement. He will contact player 1 and induce him to enter a secret sub-agreement (L1, L2) where players 1 and 2 can expect payoffs 1 and 5, respectively, under the assumption that player 3 takes R. Thus, the sub-agreement, (R2, R3), is vulnerable to (1, 2). Now, we are at (L1, L2, R3), and this is player 1’s turn to cheat: he has an incentive to tell this to player 3, expecting that player 3 change his strategy to L and that his payoff be improved to 2. We returned to the initial point (L, L, L) and this process may go on indefinitely.
One may argue that players 1 and 2 could stop this recurrent process at the first step, or that any joiner can stop the process. But the problem is that the remainder will be left anxious and skeptical. Suppose that players 1 and 2 decided to keep the original agreement because defection will cause indeterminacy. This arrangement seems very reasonable, especially from player 1’s point of view. Then, player 3 will not be secretly contacted by player 2. However, this fact does not assure him that the other player would not defect. Player 2 might have just stopped going on the second step of the process. Thus, player 3 will be in doubt about his own best strategy choice.

Definition: An equilibrium point is dependable if it is not vulnerable to any plausible sub-agreement.

Point $E_1$ of the game of table 4.4 is not dependable, hence not robust, either. On the other hand, $E_2$ of the same game is not robust as seen before, but dependable. The only pure strategy equilibrium point, $(L, L, L)$ of the above example is dependable because the sub-agreement it is vulnerable to cannot be regarded as plausible due to the indeterminacy.

Dependability provides a weaker condition of self-enforceability than communication robustness does. The equilibrium points which are dependable but not communication robust are not so stable as the points which are communication robust so that significantly more defections from the former are expected than from the latter.

Communication robustness and dependability are generalizations of the self-enforceability embodied in the Nash equilibrium concept. In fact, whenever the communication structure includes only singleton sets and $N$, a Nash equilibrium point is trivially communication robust, and thus dependable. Furthermore, for a strategy combination to be dependable or to be communication robust, it must be at least an equilibrium point because, by convention, individual players as a singleton set always form communication groups by themselves. In particular, the Nash equilibrium points of two person games are all communication robust.

Even though we relaxed the conditions of self-enforceability under communication with dependability concept, we still cannot guarantee the existence of strategy combinations satisfying them if $|N|>2$. In the game of table 4.8, for example, no Nash equilibrium is dependable when $CS=2^N$.

**Table 4.8**

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>P2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>P3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>P2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>P3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>P2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>P3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Let $\sigma=[\sigma_1(L), \sigma_2(L), \sigma_3(L)]$ and $\text{CS}=2^N$. The Nash equilibria of the game are $\sigma^1=[1, 1, 1], \sigma^2=[2/3, 2/3, 1]$ and $\sigma^3=[1/2, 1/2, 1/2]$. It is easy to show that the first two are not dependable because they are both vulnerable to an plausible sub-agreement, $(R_1, R_2)$, of $\{1, 2\} \in \text{CS}$. That is, players 1 and 2 will reach an agreement to use their strategies, $R_1$ and $R_2$, respectively, when they expect that player 3 will take $L$, and this agreement is plausible under the current assumption because neither player 1 nor 2 has incentive to defect with player 3. Refer to table 4.9 for the payoff table of $G|L_3$.

An agreement on $\sigma^3$ is not dependable because it is vulnerable to a sub-agreement, $(R_1, R_2)$, of $\{1, 2\} \in \text{CS}$, which is plausible under the assumption that player 3 takes $L$ with probability $1/2$. See table 4.10, which shows the payoff table of $G|\sigma^3$.

That equilibrium points satisfying them do not always exist need not be considered a fatal objection to the proposed concepts of dependability and communication robustness. On the contrary, I even think it is an argument in favor of these concepts. The nonexistence of such equilibrium points in a game suggests that a significant defection from any equilibrium will be observed, and that no solution concept may be valid in the game. In a predictive context, this is a positive statement which can be empirically tested. In other words, if such a defection is observed, then we can attribute it to the nonexistence of such equilibria. One can also infer that players' behavior will be more diversified in a game with dependable equilibria yet without communication-robust equilibria than a game with communication-robust equilibria.

### 5. Implicit Communication

Consideration of communication in a game gives rise also to another question. Does a communication really have to be implemented even though there is only one reasonable way to do it? When what would be said is obvious from the circumstances, isn’t actually saying it really redundant? Suppose that you saw a car accident and the driver bleeding. It is almost sure that he is in trouble. So you will not ask him if he is OK or what he wants. He is not OK and he wants to be taken to a hospital. There is no need for a communication to decide to call police and ambulance. By applying this consideration to game situations, we may assert the following statement.

Communication has no effect on players' decision in a game if one can uniquely predict what messages would be sent. In this case, the solution of the game with communication will be also the solution of the game without communication.

To put it more precisely,

When a game has only one communication-robust equilibrium point on the supposition that $\text{CS}(G)=2^N$, then it is the solution of the game. If, on the supposition that $\text{CS}(G)=2^N$,.
the game has no communication-robust equilibrium but has only one dependable solution, then it is the solution of the game.

I will call this assertion the principle of implicit communication. As a matter of fact, this principle has been commonly assumed by game theorists when the Nash equilibrium concept or the payoff dominance solution is rationalized.

6. Conclusion

In traditional game theory that depends heavily on the concept of Nash equilibrium, communication which plays crucial role in real world competitive decision processes has not been properly treated. This situation is problematic because Nash equilibrium concept implicitly assumes rational expectation, which can only be realized through communication among players. It is now obvious that a suitable framework for the analysis of communication is necessary to make the theory more realistic and practical. In this paper I tried to present a possible direction to set up such a framework by proposing some new concepts: communication robustness, dependability and implicit communication, and showed that the predictive power of equilibrium analysis could be very low as no dependable equilibrium might be found in a game. This result may offer a possible explanation to the fact that the game theory often fails to correctly predict outcomes of real world competitive situations.

注

1) Actually, there are other ways to formulate the problem, but such indeterminacy which causes unnatural results will persist.

2) One may argue that no explicit communication is necessary to achieve this solution because the result of the communication is so obvious in this example. This point will be discussed in the following section.

3) The outcome of communication is so obvious that one may not need explicit communication for joint defection. This point will be discussed in the next section.

References


