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Decision Support

Dynamic focus programming: A new approach to sequential decision problems under uncertainty

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ABSTRACT

A new approach to sequential decision problems under uncertainty named dynamic focus programming is proposed with the focus theory of choice. In dynamic focus programming, there are two distinct evaluation systems: Positive and negative ones. Each possible path consisting of a decision sequence from the initial stage to the final stage and the associated states is examined. In the positive evaluation system, for each decision in the initial stage, if a path starting from it can bring about a relatively low total cost with a relatively high probability, then this path is selected as the positive focus path of this decision; based on the positive focus paths of all initial decisions, a decision maker chooses a most-preferred decision rule. In the negative evaluation system, for each decision in the initial stage, if a path starting from it can bring about a relatively high total cost with a relatively high probability, then this path is selected as the negative focus path of this decision; based on the negative focus paths of all initial decisions, a decision maker chooses a most acceptable decision rule. For a specific sequential decision problem, only one system is activated; as for which one works, it is strongly dependent on decision maker's personality and the framing. We apply dynamic focus programming to a real bidding decision-making problem: We obtain the optimal decision rule and gain the behavioral insights of the decision maker.

1. Introduction

Sequential decision making involves a series of interdependent decisions which are implemented at each sequential stage. Dynamic programming is a powerful vehicle for solving a wide class of sequential decision problems (Bellman, 1957). The stochastic dynamic programming is used for handling sequential decision problem under risk. For comprehensive reviews, readers are referred to Marescot et al. (2013) and Rust (2019) and the book by Schneider (2014). More recent research on this important topic is reported in Chang, Ferris, Kim and Rutherford (2020), Fei, Gülpınar and Branke (2019), Flapper, Gayon and Vercraene (2012), Maggioni, Allevi and Tomasgard (2020), Mak, Cheung, Lam and Luk (2011), Minis and Tatarakis (2011), Misra and Nair (2011), Powell et al. (2014) and Shapiro, Tekaya, da Costa and Soares (2013). A stochastic dynamic programming problem is formulated as a maximization problem of an expectation of discounted sum of utilities over all stages. Theoretically, stochastic dynamic programming allows us to compute optimal decision rules to general sequential decision problems with non-stationary, history-dependent transition probabilities and time non-separable utility function. However, there is a practical limitation to the method which Bellman termed "the curse of dimensionality". To decrease dimensionality of the decision problem, Markov process is generally considered for stochastic dynamic programming. For an infinite horizon case, a stationary Markov process is a basic assumption for stochastic dynamic programming and with this assumption the optimal value function is the solution of the Bellman equation, a fixed point of Bellman operator (Blackwell, 1965). Solving the Bellman equation is the core problem of Markovian decision process with continuous states and decision variables. There is a rich literature on this topic (see, e.g., Bertsekas & Tsitsiklis, 1996; Powell, 2010; Rust, 2017).

Backward induction is a key operation in stochastic dynamic programming. However, there is a long history to question whether people behave according to backward induction. After reviewing the empirical literature on stochastic dynamic programming, Rust concluded "Indeed, introspection suggests that it is impossible that we literally use backward induction calculations in determining our behavior, at least at a conscious level. It seems much more likely that people use some sort of "forward induction" to prune branches of the decision tree that seem unlikely to yield high payoffs rather than methodically assigning values to each node by backward induction calculation" (Rust, 1992, p. 53). Hutchinson

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and Meyer assert “It would hardly seem controversial to suggest that individuals lack the ability to engage in the rather complex process of backward induction presumed by normative decision theory.” (Hutchinson & Meyer, 1994, p. 372). Further, Meyer and Hutchinson affirm “people almost never engage backward induction, the solution used to compute optimal behavior in many dynamic planning problems.” (Meyer & Hutchinson, 2016).

Stochastic dynamic programming is an expected utility-based approach to maximize an expectation of discounted sum of utilities over time. However, there exists a vast amount of empirical evidence to show that people systematically violate the axioms of the expected utility theory and the subjective expected utility theory (see, e.g., Allais, 1953; Ellsberg, 1961; Etner, Jeleva & Tallon, 2012; Kahneman & Tversky, 1979; Starmer, 2000).

This research aims to address sequential decision problems under uncertainty by putting it within the focus theory of choice framework, which offers decision aids based on salient information instead of an expected value. A growing body of evidence has shown that salience (attention-grabbing) information plays a critical role in human decision making (see, e.g., Brandstätter & Korner, 2014; Busse, Lacetera, Pope, Silva-Risso & Sydnor, 2013; Lacetera, Pope & Sydnor, 2012). Guo (2011) argues that an individual evaluates a decision based on some associated event (called the focus of a decision), which is most salient to the decision maker due to its resultant payoff and probability, thereby proposing the one-shot decision theory. The one-shot decision theory has been applied to many decision problems (see, e.g., Guo & Ma, 2014; Wang & Guo, 2017; Zhu & Guo, 2020). To further refine the one-shot decision theory, Guo (2017, 2019) proposes the focus theory of choice that models and axiomatizes the procedural rationality articulated first by Simon (1976) and resolves several well-known anomalies such as the St. Petersburg, Allais, and Ellsberg paradoxes, preference reversals, the event-splitting effect, and the violations of tail-separability, stochastic dominance and transitivity. The core argument of the focus theory of choice is that the most salient event corresponds to the most-preferred decision, where salience depends on the decision-maker’s specific frame of mind and reflects different behavioural patterns in human decision processes. This argument is consistent with the results of the psychological experiments (see, e.g., Fiedler & Glockner, 2012; Stewart, Hermens & Matthews, 2016; Yu, 2015).

The focus theory of choice postulates that a decision maker inherently owns two distinct evaluation systems: Positive and negative. In the positive evaluation system, an event that generates a relatively high payoff with a relatively high probability sticks out as more salient. Similarly, in the negative evaluation system, an event that brings about a relatively low payoff with a relatively high probability has a relatively high salience. Typically, these two systems correspond to different frames of mind and one of them is working for a particular decision situation. As for which one works, it is strongly dependent on the decision maker’s personality traits: Generally, the positive evaluation system is active for an optimistic decision maker and the negative evaluation system is activated when a decision maker is pessimistic. Meanwhile, it can also be strongly influenced by the framing (Kahneman & Tversky, 1984): The negative evaluation system becomes apparent when the problem is negatively framed, or the problem is critical or serious for the decision maker.

In this paper, we employ the focus theory of choice to sequential decision problems under uncertainty. Essentially, a sequential decision is to seek a best current decision from all possible ones considering not only the current reward gained by this decision but also the future ones obtained through its effect on the future states of the system. As such, dynamic focus programming envisages a decision maker’s decision as a two-step procedure. In the first step, the decision maker examines each possible path consisting of a decision sequence from the initial stage to the final stage and the associated states and looks for his/her most salient path by assessing the total reward generated by the decision sequence and the corresponding probability. If a path starting from an initial decision can bring about a relatively high reward or a relatively low reward with a relatively high probability, then it is identified as the positive or negative focus path of this decision. In the second step, the decision maker selects the optimal initial decision by scanning through the focus paths of all possible initial decisions; the optimal initial decision’s focus path is the optimal decision rule.

There are two distinct differences between stochastic dynamic programming and dynamic focus programming. First, instead of calculating the expected utility dynamic focus programming looks for the optimal decision rule corresponding to the most salient path. Second, stochastic dynamic programming employs backward induction whereas dynamic focus programming utilizes forward calculation which is close to human being intuition.

The remainder of this paper is organized as follows. In Section 2, we propose dynamic focus programming under the positive evaluation system. In Section 3, we propose dynamic focus programming under the negative evaluation system. In Section 4, we analyze a real bidding decision-making problem by using dynamic focus programming: The optimal decision rule is obtained, and the behavioral insights of the decision maker are gained. Section 5 concludes this paper with some remarks.

2. Dynamic focus programming under the positive evaluation system

2.1. The basic settings of dynamic focus programming

A decision maker is faced with the problem in a probabilistic system which evolves through time and should choose a sequence of decisions to achieve a final outcome. The performance of a sequence of decisions is evaluated by a predetermined criterion. Decisions are made at points of time referred to as decision stages. Here we consider discrete decision stages and denote the length of decision stages as T such that the initial decision is at stage 0 while the last decision is at stage $T-1$. $A_t = \{a_{i,t}\}$ $(t = 0,\dots,T-1)$ and $X_t = \{x_{i,t}\}$ $(t = 0,\dots,T)$ are the sets of decisions and states at stage t, respectively where we use the subscript i to stand for a generic element of the sets. Set $X_0 = \{x_0\}$ which means that the current state is x_0. When we choose $a_{k,t}$ at stage t, the possible state at stage $t+1$ is an element of X_{t+1}. The state $x_{i,t}$ at stage t will turn into $x \in X_{t+1}$ at stage $t+1$ with the probability $p_{t+1}(x|x_{i,t}, a_{k,t})$ for a decision $a_{k,t}$. $p_{t+1}(x|x_{i,t}, a_{k,t})$ is called a transition probability. Note that we consider a Markov process because the transition probability depends on the past only through the current state of the system and the decision selected by a decision maker in that state. Denote $c_{t+1}(x_{i,t}, a_{k,t}, x_{j,t+1})$ as the cost generated by the state transition from $x_{i,t}$ to $x_{j,t+1}$ when we choose $a_{k,t}$. The set of the possible costs corresponding to $x_{i,t}$ and $a_{k,t}$ is expressed as $C_{t+1}(x_{i,t}, a_{k,t}) = \{c_{t+1}(x_{i,t}, a_{k,t}, x_{j,t+1})\}$. Given $X_0 = \{x_0\}$, A_t, X_t, $C_{t+1}(x_{i,t}, a_{k,t})$ and $p_{t+1}(x|x_{i,t}, a_{k,t})$ for any $x_{i,t}$, $a_{k,t}$, $t = 0,\dots,T-1$ and X_T, the sequential decision is to choose an optimal decision at each stage to minimize the total cost.

With the initial state x_0, each decision $a_{k,0}$ at stage 0 can generate N possible final outcomes through N paths where

$$N = |X_T| * \prod_{i=1,\dots,T-1} |X_i| * |A_i|. \tag{1}$$

We use $s(t)$ and $a(t)$ to represent a generic state and a generic decision at stage t, then we can denote a path from the initial stage to the final stage as $\{a(0), (s(1), a(1)), \dots, (s(T-1), a(T-1)), s(T)\}$ and use $\langle d \rangle$ to represent the set of all possible paths generated by the initial decision $a(0) = d \in A_0$ from stage 0 to stage T. For $z \in \langle d \rangle$, its total cost denoted as $c(z,d)$ is given as

$$c(z,d) = \sum_{k=1,\ldots,T} \beta^{k-1} c_k(s(k-1), a(k-1), s(k)), \tag{2}$$

where β is the discount factor and specified by $\beta = \frac{1}{1+r}$ with a discount rate r in this paper; its probability denoted as $p(z,d)$ is

$$p(z,d) = \prod_{k=1,\ldots,T} p_k(s(k)|s(k-1), a(k-1)). \tag{3}$$

Satisfaction function. The satisfaction function of $d \in A_0$ for a path $z \in \langle d \rangle$ is defined as

$$u(z,d) = \left(1 - \frac{c(z,d)}{\max\limits_{a\in A_0, y\in\langle a\rangle} c(y,a)}\right) \Bigg/ \left(1 - \frac{\min\limits_{a\in A_0, y\in\langle a\rangle} c(y,a)}{\max\limits_{a\in A_0, y\in\langle a\rangle} c(y,a)}\right). \tag{4}$$

For a given $d \in A_0$, $u(z,d)$ represents the decision maker's satisfaction level about the resulting cost if the path arises as $z \in \langle d \rangle$. Clearly, $u(z,d)$ reaches its maximum at $\min\limits_{a\in A_0, y\in\langle a\rangle} c(y,a)$, that is, amongst all the paths originating from x_0, the path generates the lowest cost has the highest satisfaction level. This satisfaction function is fundamentally different from a utility function. A utility function is used to represent a decision-maker's different risk attitude by using a convex function, a linear function, or a concave function. By contrast, the satisfaction function here is simply a normalized cost function representing relative positions of different costs exogenously determined by the decision maker. It is well known that utilities appear to be extremely sensitive to the adopted elicitation methods and different assessment approaches often lead to distinct utilities (Hershey et al.,1982; Johnson & Schkade, 1989). In addition, eliciting utility is very time-consuming (Goodwin & Wright, 2014). On the other hand, obtaining the satisfaction function is easy because it is simply a value function.

Relative likelihood function. The relative likelihood function of a path $z \in \langle d \rangle$ is defined as

$$\pi(z,d) = p(z,d) / \max_{a\in A_0, y\in\langle a\rangle} p(y,a). \tag{5}$$

Clearly, amongst all paths originating from x_0, the path with the highest probability has the highest relative likely degree of 1. A relative likelihood function is a normalized probability density function for a continuous random variable (or a normalized probability mass function for a discrete random variable) to represent the relative likelihood positions of different outcomes. Instead of using the original values of costs and probabilities, the satisfaction and relative likelihood functions are taken as the basic decision inputs because a mounting body of evidence suggests that relative values are more perceptible (have a higher accessibility) than absolute values and play a more important role in human decision making (Frank, 1985; Solnick & Hemenway, 1998).

2.2. Dynamic focus programming model under the positive evaluation system

Under the positive evaluation system, for $\forall d \in A_0$, we denote $Z_p(d)$ as the set of optimal solutions of the following optimization problem:

$$\max_{z\in\langle d\rangle} \min\{\varphi * \pi(z,d), u(z,d)\} \tag{6}$$

where φ is a positive real number. (6) is derived from the representation theorem of positive foci (Guo, 2019). Note that both $\pi(z,d)$ and $u(z,d)$ are dimensionless and between 0 and 1, and φ works as a scaling factor that directly affects whether a likelihood or a satisfaction arises from the inner minimization operation in (6).

For $z_1, z_2 \in \langle d \rangle$, if $\pi(z_1,d) \ge \pi(z_2,d)$ and $u(z_1,d) \ge u(z_2,d)$, we have $\min\{\varphi * \pi(z_1,d), u(z_1,d)\} \ge \min\{\varphi * \pi(z_2,d), u(z_2,d)\}$. Clearly, for $\forall d \in A_0$, (6) is used to seek a path which has a relatively high relative likelihood degree and generates a relatively high satisfaction level. Increasing φ makes $\varphi * \pi(z,d)$ bigger and allows $u(z,d)$ to arise more easily out of the inner minimization operation in (6), leading to an optimal z therein with a relatively high satisfaction level and a relatively low likelihood. Conversely, decreasing φ makes $\pi(z,d)$ to take a more prominent role in determining the output of (6), resulting in an optimal z with a relatively high likelihood and a relatively low satisfaction level. Hence, φ can be interpreted as a weight that a decision maker balances his/her emphasis on the satisfaction level and the relative likelihood degree. Increasing φ means that the decision maker aims to pursue a higher satisfaction by somewhat sacrificing the relative likelihood. Therefore, φ can be used to measure how optimistic the decision maker is: The higher the value of φ, the more optimistic the decision maker.

Next, based on the optimal solution to (6), we define the positive focus path of a given $d \in A_0$.

Definition 1. If there exists a unique element in $Z_p(d)$, this element is the positive focus path of d, denoted by $z_p^*(d)$. If there is more than one element in $Z_p(d)$ and $\nexists z \in Z_p(d)$ such that $\pi(z,d) > \pi(z_p^*(d),d)$, $u(z,d) \ge u(z_p^*(d),d)$ or $\pi(z,d) \ge \pi(z_p^*(d),d)$, $u(z,d) > u(z_p^*(d),d)$ for $z_p^*(d) \in Z_p(d)$, then $z_p^*(d)$ is a positive focus path of d.

From Definition 1, we know that dominated paths are excluded and $z_p^*(d)$ is the most favorite path for d. For one d, there may be multiple positive focus paths and the set of $z_p^*(d)$ is denoted as $Z_p^*(d)$.

Next, amongst all the positive focus paths of all $d \in A_0$, we seek the optimal d_p such that

$$\begin{aligned}&\min\left\{\kappa * \pi\left(z_p^*(d_p), d_p\right), u\left(z_p^*(d_p), d_p\right)\right\} \\ &= \max_{z\in Z_p^*} \min\left\{\kappa * \pi\left(z, d^+(z)\right), u\left(z, d^+(z)\right)\right\},\end{aligned} \tag{7}$$

where parameter κ is a positive real number, $Z_p^* = \{z | z \in Z_p^*(d), d \in A_0\}$ and $d^+(z)$ stands for a $d \in A_0$ whose positive focus path is z. (7) is from the representation theorem for an optimal decision in the positive evaluation system (Guo, 2019). The set of d_p is denoted as D_p.

For $\forall d_1, d_2 \in A_0$, if $\exists z_1 \in Z_p^*(d_1)$ such that $\pi(z_1,d_1) \ge \pi(z_2,d_2)$ and $u(z_1,d_1) \ge u(z_2,d_2)$ hold for any $z_2 \in Z_p^*(d_2)$, we have $\min\{\kappa * \pi(z_1,d_1), u(z_1,d_1)\} \ge \min\{\kappa * \pi(z_2,d_2), u(z_2,d_2)\}$. It implies that (7) seeks the initial decision whose positive focus path has a relatively high relative likelihood degree and can generate a relatively high satisfaction level.

Similar to the interpretations of parameter φ, a heightened κ elevates the level of $\pi(z,d^+(z))$ relative to $u(z,d^+(z))$ and leads to finding an initial decision whose positive focus path is relatively high in satisfaction but relatively low in likelihood. Conversely, a reduced κ lowers the level of $\pi(z,d^+(z))$ relative to $u(z,d^+(z))$ and results in an initial decision whose positive focus path is relatively high in likelihood but relatively low in satisfaction. Since (7) is used for identifying decisions based on their positive focus paths, κ can be interpreted as the decision-maker's confidence index on his/her decision: The higher the value of κ, the more confident the decision-maker.

Definition 2. If there is only one element in D_p, this element is the optimal initial decision under the positive evaluation system, denoted as d_p^* and $z_p^*(d_p^*)$ is the optimal decision rule under the positive evaluation system. If there is more than one element in D_p and $\nexists d \in D_p$ such that $\pi(z_p^*(d),d) > \pi(z_p^*(d_p^*),d_p^*)$, $u(z_p^*(d),d) \ge u(z_p^*(d_p^*),d_p^*)$ or $\pi(z_p^*(d),d) \ge \pi(z_p^*(d_p^*),d_p^*)$, $u(z_p^*(d),d) > u(z_p^*(d_p^*),d_p^*)$ for $d_p^* \in$

Table 1
The costs of all the paths of two initial decisions.

$c(z_{1,1},a_1)$	$c(z_{1,2},a_1)$	$c(z_{1,3},a_1)$	$c(z_{1,4},a_1)$	$c(z_{1,5},a_1)$	$c(z_{1,6},a_1)$	$c(z_{1,7},a_1)$	$c(z_{1,8},a_1)$	$c(z_{1,9},a_1)$
2	2	2	3	1.5	1.5	2	2	2
$c(z_{1,10},a_1)$	$c(z_{1,11},a_1)$	$c(z_{1,12},a_1)$	$c(z_{1,13},a_1)$	$c(z_{1,14},a_1)$	$c(z_{1,15},a_1)$	$c(z_{1,16},a_1)$	$c(z_{1,17},a_1)$	$c(z_{1,18},a_1)$
3	3	1.5	2	2	2	3	3	3
$c(z_{2,1},a_2)$	$c(z_{2,2},a_2)$	$c(z_{2,3},a_2)$	$c(z_{2,4},a_2)$	$c(z_{2,5},a_2)$	$c(z_{2,6},a_2)$	$c(z_{2,7},a_2)$	$c(z_{2,8},a_2)$	$c(z_{2,9},a_2)$
3	3	3	4	2.5	2.5	3	3	3
$c(z_{2,10},a_2)$	$c(z_{2,11},a_2)$	$c(z_{2,12},a_2)$	$c(z_{2,13},a_2)$	$c(z_{2,14},a_2)$	$c(z_{2,15},a_2)$	$c(z_{2,16},a_2)$	$c(z_{2,17},a_2)$	$c(z_{2,18},a_2)$
4	4	2.5	1.5	1.5	1.5	2.5	2.5	2.5

D_p, then d_p^* is an optimal initial decision and $z_p^*(d_p^*)$ is the optimal decision rule under the positive evaluation system.

It follows from Definition 2 that d_p^* is the decision which weakly dominates the other ones if there are other elements in D_p.

2.3. An illustrative numerical example

Consider a time-invariant system with finite states. The set of states is $X=\{0,1,2\}$ and the set of decisions is $A=\{a_1,a_2\}$. We set $T=2$ and $x_0=1$. The transition probability matrix associated with a_1 is

$$P_{a_1}=\begin{bmatrix}0.8 & 0.1 & 0.1\\ 0.1 & 0.8 & 0.1\\ 0.1 & 0.1 & 0.8\end{bmatrix}. \tag{8}$$

The transition probability matrix associated with a_2 is

$$P_{a_2}=\begin{bmatrix}0.1 & 0.7 & 0.2\\ 0.2 & 0.1 & 0.7\\ 0.1 & 0.8 & 0.1\end{bmatrix}. \tag{9}$$

The cost generated by a_1 is always 1. Taking the decision a_2, if $x_{t+1}>x_t$, the cost is 0.5, otherwise the cost is 2. The discount rate r is 0. What is the optimal decision rule to minimize the total cost?

Let us consider this problem by dynamic focus programming. From (1), we know that there are $|X_1|*|A_1|*|X_2|=3*2*3=18$ paths for initial decisions a_1 and a_2, respectively. We have

$$\begin{aligned}\langle a_1\rangle=\{&\{a_1,(0,a_1),0\},\{a_1,(0,a_1),1\},\{a_1,(0,a_1),2\},\\ &\{a_1,(0,a_2),0\},\{a_1,(0,a_2),1\},\{a_1,(0,a_2),2\},\\ &\{a_1,(1,a_1),0\},\{a_1,(1,a_1),1\},\{a_1,(1,a_1),2\},\\ &\{a_1,(1,a_2),0\},\{a_1,(1,a_2),1\},\{a_1,(1,a_2),2\},\\ &\{a_1,(2,a_1),0\},\{a_1,(2,a_1),1\},\{a_1,(2,a_1),2\},\\ &\{a_1,(2,a_2),0\},\{a_1,(2,a_2),1\},\{a_1,(2,a_2),2\}\}\end{aligned}$$

and

$$\begin{aligned}\langle a_2\rangle=\{&\{a_2,(0,a_1),0\},\{a_2,(0,a_1),1\},\{a_2,(0,a_1),2\},\\ &\{a_2,(0,a_2),0\},\{a_2,(0,a_2),1\},\{a_2,(0,a_2),2\},\\ &\{a_2,(1,a_1),0\},\{a_2,(1,a_1),1\},\{a_2,(1,a_1),2\},\\ &\{a_2,(1,a_2),0\},\{a_2,(1,a_2),1\},\{a_2,(1,a_2),2\},\\ &\{a_2,(2,a_1),0\},\{a_2,(2,a_1),1\},\{a_2,(2,a_1),2\},\\ &\{a_2,(2,a_2),0\},\{a_2,(2,a_2),1\},\{a_2,(2,a_2),2\}\}.\end{aligned}$$

Let us take $\{a_1,(1,a_2),2\}$ as an example to explain its meaning. $\{a_1,(1,a_2),2\}$ that is the twelfth element of $\langle a_1\rangle$ shows the path that starting from $x_0=1$ the decision maker chooses a_1, then the state remains the same, subsequently the decision maker chooses a_2, then the state turns into 2.

We use $z_{1,i}$ $(i=1,\dots 18)$ and $z_{2,i}$ $(i=1,\dots 18)$ to stand for the ith element of $\langle a_1\rangle$ and $\langle a_2\rangle$, respectively. Considering (2) and setting $r=0$, we calculate the total cost of each path, for example, $c(z_{1,3},a_1)=c_1(1,a_1,0)+c_2(0,a_1,2)=1+1=2$ and all the results are shown in Table 1. The probability of each path of each initial decision is calculated by (3), for example, $p(z_{1,3},a_1)=p(0|1,a_1)*p(2|0,a_1)=0.1*0.1=0.01$ and all the results are shown in Table 2. With the data in Table 1, the satisfaction level is calculated by (4) and all the results are shown in Table 3. With the data in Table 2, the relative likelihood degree is calculated by (5) and all the results are shown in Table 4.

Using (6) with the data in Tables 3 and 4 and setting φ as 0.5, 1 and 5, respectively, we can obtain $Z_p(a_1)=\{z_{1,8}\}$, $Z_p(a_1)=\{z_{1,12}\}$ and $Z_p(a_1)=\{z_{1,12}\}$ for φ being 0.5, 1 and 5, respectively; $Z_p(a_2)=\{z_{2,15},z_{2,17}\}$, $Z_p(a_2)=\{z_{2,15}\}$ and $Z_p(a_2)=\{z_{2,15}\}$ for φ being 0.5, 1 and 5, respectively. Let us take $\varphi=1$ as an example to show how (6) works. At $\varphi=1$, (6) becomes $\max\limits_{z\in\langle d\rangle}\min\{\pi(z,d),u(z,d)\}$. If $z=z_{1,1}$ and $d=a_1$, from Tables 3 and 4 we have $\min\{\pi(z_{1,1},a_1),u(z_{1,1},a_1)\}$ =min{0.125, 0.8}=0.125. Similarly, setting $d=a_1$ we can obtain the values of $\min\{\pi(z,a_1),u(z,a_1)\}$ for $z=z_{1,2}$, $z=z_{1,3},\dots,z=z_{1,18}$ as 0.016, 0.016, 0.016, 0.109, 0.031, 0.125, 0.8, 0.125, 0.25, 0.125, 0.875, 0.016, 0.016, 0.125, 0.016, 0.125 and 0.016, respectively so that we have $\max\limits_{z\in\langle a_1\rangle}\min\{\pi(z,a_1),u(z,a_1)\}$ =max{0.125, 0.016, 0.016, 0.016, 0.109, 0.031, 0.125, 0.8, 0.125, 0.25, 0.125, 0.875, 0.016, 0.016, 0.125, 0.016, 0.125, 0.016}=0.875 which is the value of $\min\{\pi(z_{1,12},a_1),u(z_{1,12},a_1)\}$. Thus, we have $Z_p(a_1)=\{z_{1,12}\}$ for $\varphi=1$. According to Definition 1, the positive focus path of a_1 is obtained as $z_p^*(a_1)=z_{1,12}$ for φ being 1. Similarly, we can have $z_p^*(a_1)=z_{1,8}$ and $z_p^*(a_1)=z_{1,12}$ for φ being 0.5, and 5, respectively; we can have $z_p^*(a_2)=z_{2,15}$, $z_p^*(a_2)=z_{2,15}$ and $z_p^*(a_2)=z_{2,15}$ for φ being 0.5, 1 and 5, respectively. All the results are listed in Table 5.

From the positive focus paths of different initial decision for various φ's shown in Table 5, we understand that, $z_{2,15}$ is an unique positive focus path which dominates all the other paths of a_2; for a_1, increasing φ from 0.5 to 1, the positive focus path changes from $z_{1,8}$ to $z_{1,12}$ while $u(z_{1,8},a_1)=0.8<u(z_{1,12},a_1)=1$ and $\pi(z_{1,8},a_1)=1>\pi(z_{1,12},a_1)=0.875$. It means that increasing φ leads to a positive focus path with a higher satisfaction level at a lower relative likelihood degree.

Next, let us examine the optimal decision rule under the positive evaluation system for $\kappa=0.1$ and $\kappa=10$ by considering only the positive focus paths obtained for $\varphi=0.5$. Setting $\kappa=0.1$, the right-hand side of (7) becomes $\max\limits_{z\in Z_p^*}\min\{0.1*\pi(z,d^+(z)),u(z,d^+(z))\}$. Since for $\varphi=0.5$ $z_p^*(a_1)=z_{1,8}$, $z_p^*(a_2)=z_{2,15}$, we have Z_p^*=$\{z_{1,8},z_{2,15}\}$, $d^+(z_{1,8})=a_1$ and $d^+(z_{2,15})=a_2$. From Table 3 we know $u(z_{1,8},a_1)=0.8$ and $u(z_{2,15},a_2)=1$. From Table 4 we know $\pi(z_{1,8},a_1)=1$ and $\pi(z_{2,15},a_2)=0.875$. Since $\max\limits_{z\in Z_p^*}\min\{0.1*\pi(z,d^+(z)),u(z,d^+(z))\}$=max{min{0.1 × 1,0.8}, min{0.1 × 0.875, 1}}=max{0.1, 0.0875}=0.1 which corresponds to a_1, we have $D_p=\{a_1\}$ for $\kappa=0.1$. Likewise, we have $D_p=\{a_2\}$ for $\kappa=10$. According to Definition 2, we obtain optimal initial decisions as a_1 and a_2, and the optimal decision rules as $z_{1,8}$ and $z_{2,15}$ for $\kappa=0.1$ and $\kappa=10$, respectively. It follows from the results that increasing κ will lead to an optimal decision

Table 2
The probabilities of all the paths of two initial decisions.

$p(z_{1,1}, a_1)$	$p(z_{1,2}, a_1)$	$p(z_{1,3}, a_1)$	$p(z_{1,4}, a_1)$	$p(z_{1,5}, a_1)$	$p(z_{1,6}, a_1)$	$p(z_{1,7}, a_1)$	$p(z_{1,8}, a_1)$	$p(z_{1,9}, a_1)$
0.08	0.01	0.01	0.01	0.07	0.02	0.08	0.64	0.08
$p(z_{1,10}, a_1)$	$p(z_{1,11}, a_1)$	$p(z_{1,12}, a_1)$	$p(z_{1,13}, a_1)$	$p(z_{1,14}, a_1)$	$p(, z_{1,15}, a_1)$	$p(z_{1,16}, a_1)$	$p(z_{1,17}, a_1)$	$p(z_{1,18}, a_1)$
0.16	0.08	0.56	0.01	0.01	0.08	0.01	0.08	0.01
$p(z_{2,1}, a_2)$	$p(z_{2,2}, a_2)$	$p(z_{2,3}, a_2)$	$p(z_{2,4}, a_2)$	$p(z_{2,5}, a_2)$	$p(z_{2,6}, a_2)$	$p(z_{2,7}, a_2)$	$p(z_{2,8}, a_2)$	$p(z_{2,9}, a_2)$
0.16	0.02	0.02	0.02	0.14	0.04	0.01	0.08	0.01
$p(z_{2,10}, a_2)$	$p(z_{2,11}, a_2)$	$p(z_{2,12}, a_2)$	$p(z_{2,13}, a_2)$	$p(z_{2,14}, a_2)$	$p(z_{2,15}, a_2)$	$p(z_{2,16}, a_2)$	$p(z_{2,17}, a_2)$	$p(z_{2,18}, a_2)$
0.02	0.01	0.07	0.07	0.07	0.56	0.07	0.56	0.07

Table 3
The satisfaction levels of all the paths of two initial decisions.

$u(z_{1,1}, a_1)$	$u(z_{1,2}, a_1)$	$u(z_{1,3}, a_1)$	$u(z_{1,4}, a_1)$	$u(z_{1,5}, a_1)$	$u(z_{1,6}, a_1)$	$u(z_{1,7}, a_1)$	$u(z_{1,8}, a_1)$	$u(z_{1,9}, a_1)$
0.8	0.8	0.8	0.4	1	1	0.8	0.8	0.8
$u(z_{1,10}, a_1)$	$u(z_{1,11}, a_1)$	$u(z_{1,12}, a_1)$	$u(z_{1,13}, a_1)$	$u(z_{1,14}, a_1)$	$u(z_{1,15}, a_1)$	$u(z_{1,16}, a_1)$	$u(z_{1,17}, a_1)$	$u(z_{1,18}, a_1)$
0.4	0.4	1	0.8	0.8	0.8	0.4	0.4	0.4
$u(z_{2,1}, a_2)$	$u(z_{2,2}, a_2)$	$u(z_{2,3}, a_2)$	$u(z_{2,4}, a_2)$	$u(z_{2,5}, a_2)$	$u(z_{2,6}, a_2)$	$u(z_{2,7}, a_2)$	$u(z_{2,8}, a_2)$	$u(z_{2,9}, a_2)$
0.4	0.4	0.4	0	0.6	0.6	0.4	0.4	0.4
$u(z_{2,10}, a_2)$	$u(z_{2,11}, a_2)$	$u(z_{2,12}, a_2)$	$u(z_{2,13}, a_2)$	$u(z_{2,14}, a_2)$	$u(z_{2,15}, a_2)$	$u(z_{2,16}, a_2)$	$u(z_{2,17}, a_2)$	$u(z_{2,18}, a_2)$
0	0	0.6	1	1	1	0.6	0.6	0.6

Table 4
The relative likelihood degrees of all the paths of two initial decisions.

$\pi(z_{1,1}, a_1)$	$\pi(z_{1,2}, a_1)$	$\pi(z_{1,3}, a_1)$	$\pi(z_{1,4}, a_1)$	$\pi(z_{1,5}, a_1)$	$\pi(z_{1,6}, a_1)$	$\pi(z_{1,7}, a_1)$	$\pi(z_{1,8}, a_1)$	$\pi(z_{1,9}, a_1)$
0.125	0.016	0.016	0.016	0.109	0.031	0.125	1	0.125
$\pi(z_{1,10}, a_1)$	$\pi(z_{1,11}, a_1)$	$\pi(z_{1,12}, a_1)$	$\pi(z_{1,13}, a_1)$	$\pi(z_{1,14}, a_1)$	$\pi(z_{1,15}, a_1)$	$\pi(z_{1,16}, a_1)$	$\pi(z_{1,17}, a_1)$	$\pi(z_{1,18}, a_1)$
0.25	0.125	0.875	0.016	0.016	0.125	0.016	0.125	0.016
$\pi(z_{2,1}, a_2)$	$\pi(z_{2,2}, a_2)$	$\pi(z_{2,3}, a_2)$	$\pi(z_{2,4}, a_2)$	$\pi(z_{2,5}, a_2)$	$\pi(z_{2,6}, a_2)$	$\pi(z_{2,7}, a_2)$	$\pi(z_{2,8}, a_2)$	$\pi(z_{2,9}, a_2)$
0.25	0.031	0.031	0.031	0.219	0.063	0.016	0.125	0.016
$\pi(z_{2,10}, a_2)$	$\pi(z_{2,11}, a_2)$	$\pi(z_{2,12}, a_2)$	$\pi(z_{2,13}, a_2)$	$\pi(z_{2,14}, a_2)$	$\pi(z_{2,15}, a_2)$	$\pi(z_{2,16}, a_2)$	$\pi(z_{2,17}, a_2)$	$\pi(z_{2,18}, a_2)$
0.031	0.016	0.109	0.109	0.109	0.875	0.109	0.875	0.109

Table 5
The positive focus paths of different initial decisions under various φ's.

	φ		
Focus path	0.5	1	5
a_1	$z_{1,8}$	$z_{1,12}$	$z_{1,12}$
a_2	$z_{2,15}$	$z_{2,15}$	$z_{2,15}$

rule with a relatively high satisfaction level and a relatively low likelihood, that is, $u(z_{1,8}, a_1) = 0.8 < u(z_{2,15}, a_2) = 1$ and $\pi(z_{1,8}, a_1) = 1 > \pi(z_{2,15}, a_2) = 0.875$.

From this simple numerical example, we understand under the positive evaluation system of dynamic focus programming, each initial decision is associated with its most favorite path (positive focus path) and the optimal initial decision is chosen by examining the positive focus paths of all possible initial decisions and the optimal decision rule is the positive focus path of the optimal initial decision. In so doing, the decision maker's personality and behavioural attributes can be properly accommodated by adjusting the parameter values of φ and κ.

3. Dynamic focus programming under the negative evaluation system

Under the negative evaluation system, for $\forall d \in A_0$, we denote $Z_n(d)$ as the set of optimal solutions of the following optimization problem:

$$\max_{z \in \langle d \rangle} \min\{\theta * \pi(z, d), 1 - u(z, d)\} \tag{10}$$

where θ is a positive real number. (10) is derived from the representation theorem of negative foci (Guo, 2019). Given that $0 \le u(z, d) \le 1$, we know that $0 \le 1 - u(z, d) \le 1$ holds. As such, we can loosely treat $1 - u(z, d)$ as the complement of the satisfaction function or dissatisfaction function. The difference between (10) and (6) is that the satisfaction function $u(z, d)$ is replaced with its complement $1 - u(z, d)$ here. For $z_1, z_2 \in \langle d \rangle$, if $\pi(z_1, d) \ge \pi(z_2, d)$ and $u(z_1, d) \le u(z_2, d)$, we have $\min\{\theta * \pi(z_1, d), 1 - u(z_1, d)\} \ge \min\{\theta * \pi(z_2, d), 1 - u(z_2, d)\}$. Clearly, for $\forall d \in A_0$, (10) is used to seek a path which has a relatively high relative likelihood degree and generate a relatively low satisfaction level. This indicates that the decision maker under the negative evaluation system is more concerned with the bottom line and focuses on the unfavorable outcome that has a relatively low satisfaction level (high cost). This type of decision makers possesses a pessimistic mindset. Increasing θ means that a decision maker is more concerned about a relatively low satisfaction level (high cost). Therefore, θ can be used to measure how pessimistic a decision maker is: The higher the value of θ, the more pessimistic a decision maker.

Definition 3. If there exists a unique element in $Z_n(d)$, this element is the negative focus path of d, denoted by $z_n^*(d)$. If there is more than one element in $Z_n(d)$ and $\nexists z \in Z_n(d)$, such that $\pi(z, d) > \pi(z_n^*(d), d)$, $u(z, d) \le u(z_n^*(d), d)$ or $\pi(z, d) \ge \pi(z_n^*(d), d)$, $u(z, d) < u(z_n^*(d), d)$ for $z_n^*(d) \in Z_n(d)$, then $z_n^*(d)$ is the negative focus path of d.

The definition of the negative focus path indicates that $z_n^*(d)$ is the most concerned path for d. For any $\forall d \in A_0$, there may be multiple negative focus paths and the set of $z_n^*(d)$ is denoted as $Z_n^*(d)$.

Next, we seek d_n satisfying the following equation:

$$\begin{aligned} &\max\{\pi(z_n^*(d_n), d_n), \tau * (1 - u(z_n^*(d_n), d_n))\} \\ &= \min_{z \in Z_n^*} \max\left\{\pi\left(z, d^-(z)\right), \tau * \left(1 - u\left(z, d^-(z)\right)\right)\right\} \end{aligned} \tag{11}$$

where parameter τ is a positive real number, $Z_n^* = \{z | z \in Z_n^*(d), d \in A_0\}$ and $d^-(z)$ stands for a $d \in A_0$ whose negative focus path is z. (11) is derived from the represen-

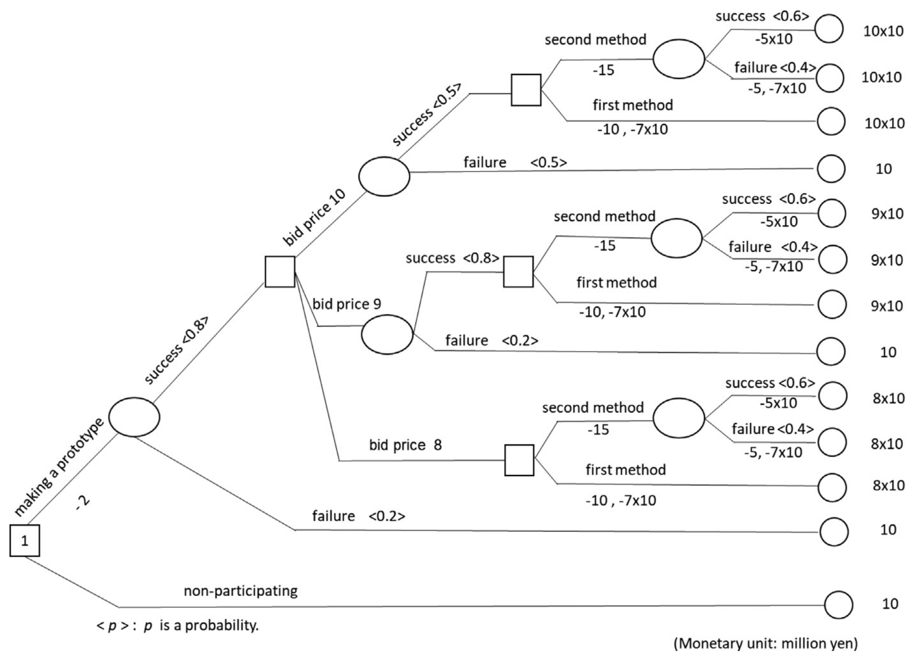

Fig. 1. The decision tree of the bidding problem.

tation theorem for an optimal action under the negative evaluation system (Guo, 2019). The set of d_n is denoted as D_n. For $\forall d_1, d_2 \in A_0$, if $\exists z_1 \in Z_n^*(d_1)$ such that $\pi(z_1) \le \pi(z_2)$ and $u(z_1, d_1) \ge u(z_2, d_2)$ hold for any $z_2 \in Z_n^*(d_2)$, we have $\max\{\pi(z_1), \tau * (1 - u(z_1, d_1))\} \le \max\{\pi(z_2), \tau * (1 - u(z_2, d_2))\}$. It means that (11) is used for seeking the initial decision $d \in A_0$ whose negative focus path has a relatively low relative likelihood degree and can generate a relatively high satisfaction level. In other words, since the negative focus paths are unfavorite ones, the decision maker dislikes their occurrence and, thus, prefers a low chance of occurring. In the meantime, he/she pursues a high satisfaction level (low cost) amongst these unfavorable outcomes.

Since a higher τ makes the dissatisfaction function at the negative focus paths to arise more easily in the inner maximization operation in (11), along with the outer minimization operations for all negative focus paths, we know that increasing τ leads to an initial decision whose negative focus path is relatively high in both satisfaction and likelihood. Choosing an initial decision associated with an unfavorable outcome (negative focus path) at a relatively high likelihood (as a result of increasing τ) implies that the decision maker is prepared and ready to accept the consequence after assessing all negative focus paths. Hence, τ can be regarded as the decision maker's acceptance index on potential losses resulting from his/her decision: The higher the value of τ, the more the acceptance level.

Definition 4. If there exists a unique element in D_n, this element is the optimal initial decision under the negative evaluation system, denoted as d_n^* and $z_n^*(d_n^*)$ is the optimal decision rule under the negative evaluation system. If there is more than one element in D_n and $\nexists d \in D_n$, such that $\pi(z_n^*(d), d) < \pi(z_n^*(d_n^*), d_n^*)$, $u(z_n^*(d), d) \ge u(z_p^*(d_n^*), d_n^*)$ or $\pi(z_n^*(d), d) \le \pi(z_n^*(d_n^*), d_n^*)$, $u(z_n^*(d), d) > u(z_p^*(d_n^*), d_n^*)$ for $d_n^* \in D_n$, then d_n^* is the optimal initial decision and $z_n^*(d_n^*)$ is the optimal decision rule under the negative evaluation system.

It follows from Definition 4 that d_n^* is the decision which weakly dominates the other ones if there are other elements in D_n.

Numerical example.

We apply the negative evaluation system of dynamic focus programming to the numerical example given in Section 2.3. Using

Table 6
The negative focus paths of different initial decisions under various θ's.

Focus path	θ 0.5	1	5
a_1	$z_{1,8}$	$z_{1,10}$	$z_{1,10}$
a_2	$z_{2,17}$	$z_{2,17}$	$z_{2,1}$

(10) with the data in Tables 3 and 4 and setting θ as 0.5, 1 and 5, respectively, we can obtain $Z_n(a_1) = \{z_{1,8}\}$, $Z_n(a_1) = \{z_{1,10}\}$ and $Z_n(a_1) = \{z_{1,10}, z_{1,11}, z_{1,17}\}$ for θ being 0.5, 1 and 5, respectively; $Z_n(a_2) = \{z_{2,17}\}$, $Z_n(a_2) = \{z_{2,17}\}$ and $Z_n(a_2) = \{z_{2,1}, z_{2,8}\}$ for θ being 0.5, 1 and 5, respectively. According to Definition 3, we have $z_n^*(a_1) = z_{1,8}$, $z_n^*(a_1) = z_{1,10}$ and $z_n^*(a_1) = z_{1,10}$ for θ being 0.5, 1 and 5, respectively; we have $z_n^*(a_2) = z_{2,17}$, $z_n^*(a_2) = z_{2,17}$ and $z_n^*(a_2) = z_{2,1}$ for θ being 0.5, 1 and 5, respectively. All the results are listed in Table 6. We can understand that increasing θ can lead to finding a negative focus path with a relatively low satisfaction and a relatively low likelihood, for example, for a_1, increasing θ from 0.5 to 1, the negative focus path changes from $z_{1,8}$ to $z_{1,10}$ while $u(z_{1,8}, a_1) = 0.8 > u(z_{1,10}, a_1) = 0.4$ and $\pi(z_{1,8}, a_1) = 1 > \pi(z_{1,10}, a_1) = 0.25$. Next, let us find out the optimal initial decision for the case $\theta = 1$. Using (11) and setting τ as 0.1 and 10, respectively, we can obtain $D_n = \{a_1\}$ and $D_n = \{a_2\}$ for τ being 0.1 and 10, respectively. According to Definition 4, we obtain the optimal initial decisions as a_1 and a_2, and the optimal decision rules as $z_{1,10}$ and $z_{2,17}$ for τ being 0.1 and 10, respectively. Since increasing τ will emphasize the satisfaction level so that an optimal decision rule with a relatively high satisfaction level is obtained, that is, $u(z_{2,17}, a_2) = 0.6 > u(z_{1,10}, a_1) = 0.4$.

4. Case study: a bidding problem of Murakami machinery manufacturing co., ltd

We use a case study introduced by Fujita and Kumada (2001, p.42-p.43) to illustrate how dynamic focus programming works for solving a real sequential decision-making problem under uncertainty. The case is as follows.

Mr. Miura, a marketing director reports to Mr. Murakami, the president of Murakami Machinery Manufacturing Co., Ltd. that Fu-

Table 7
The costs of all the paths of decision a_1.

$c(z_{1,1},a_1)$	$c(z_{1,2},a_1)$	$c(z_{1,3},a_1)$	$c(z_{1,4},a_1)$	$c(z_{1,5},a_1)$	$c(z_{1,6},a_1)$
−33	−8	−18	−8	−23	2
$c(z_{1,7},a_1)$	$c(z_{1,8},a_1)$	$c(z_{1,9},a_1)$	$c(z_{1,10},a_1)$	$c(z_{1,11},a_1)$	$c(z_{1,12},a_1)$
−8	−8	−13	12	2	−8

jimoto Ink Co., Ltd. is going to purchase 10 advanced printing machines which require some special function. Fujimoto Ink Co., Ltd. will choose one supplier by bidding in two months. The companies which can satisfy the basic requirements are qualified for bidding participation. The President calls a board meeting to discuss this matter. The opinions of each director are recorded below.

R&D director: We have the HJ-3 printing machine. If we reinforce one function of this machine by introducing a new component, then we may meet special requirements of Fujimoto Ink. For that purpose, we need to make a prototype of that component and perform a test to see if it can be manufactured at our technical level. It will cost 2 million yen to make and test the prototype. In my opinion, there is an 80% chance of producing the qualified component.

Manufacturing director: There are two methods to produce the designed printing machine. One is using a machine tool and the other is using a crushing machine. For the first method, it will cost 10 million yen for setup and the direct manufacturing cost per machine is 7 million yen. For the second method, the setup cost is 15 million yen and the direct manufacturing cost per machine is 5 million yen. However, the setup process of the second method is unstable and may fail. The probability of failure is estimated to be 40%. Even if it fails, we can switch to the first method by adding 5 million yen. In this case, the setup cost for the first method is not needed.

Financial director: To complete this large project, another job must be stopped. However, this job can certainly make a profit of 10 million yen.

Marketing director: The bid price can be roughly divided into three cases. One is 10 million yen per machine, then the total bid price is 100 million yen. Given the technical level of the competitors and the enthusiasm for participation, the probability of a successful bidding at this price is 50%. The second is 9 million yen per machine, that is, 90 million yen as the total bid price. The probability of this successful bidding is 80%. The third is 8 million yen per unit, that is, 80 million yen as the total bid price. In this case, it is certainly a successful bidding. If eventually our bid fails, we still can do the job that the finance director mentioned and make a certain profit of 10 million yen.

How should the president make a decision after he knows the opinions of each director?

This sequential decision problem can be described by a decision tree shown in Fig. 1. We use a_1, a_2, a_3, a_4, a_5, a_6 and a_7 to stand for making a prototype, non-participating, bid price 10, bid price 9, bid price 8, first method and second method in Fig. 1, respectively; we use s_1 and s_2 to represent success and failure, respectively. Thus, we have $A_0=\{a_1,\ a_2\}$,

$$\begin{aligned}\langle a_1\rangle = \{&\{a_1,(s_1,a_3),(s_1,a_7),s_1\},\{a_1,(s_1,a_3),(s_1,a_7),s_2\},\\ &\{a_1,(s_1,a_3),(s_1,a_6),s_1\},\{a_1,(s_1,a_3),s_2\},\{a_1,(s_1,a_4),\\ &(s_1,a_7),s_1\},\{a_1,(s_1,a_4),(s_1,a_7),s_2\},\{a_1,(s_1,a_4),\\ &(s_1,a_6),s_1\},\{a_1,(s_1,a_4),s_2\},\{a_1,(s_1,a_5),(s_1,a_7),s_1\},\\ &\{a_1,(s_1,a_5),(s_1,a_7),s_2\},\{a_1,(s_1,a_5),(s_1,a_6),s_1\},\{a_1,\ s_2\}\}.\end{aligned}$$

We use $z_{1,i}$ $(i=1,\dots 12\}$ to stand for the ith element of $\langle a_1\rangle$ which are listed above. Considering (2) and setting $r=0$, we can calculate the total cost of each path of a_1, for example, $c(z_{1,2},a_1)=-100+70+5+15+2=-8$ and all the results are listed in Table 7. The probability of each path of decision a_1 is calculated by (3) and all results are presented in Table 8. There is a unique certain result for a_2; we use z_2 to stand for this unique element of $\langle a_2\rangle$ and have $c(z_2,a_2)=-10$ and $p(z_2,a_2)=1$. The satisfaction level is calculated by (4) and all the results for a_1 are shown in Table 9. The relative likelihood degree of each path for a_1 is calculated by (5) and all the results are shown in Table 10.

Table 8
The probabilities of all the paths of decision a_1.

$p(z_{1,1},a_1)$	$p(z_{1,2},a_1)$	$p(z_{1,3},a_1)$	$p(z_{1,4},a_1)$	$p(z_{1,5},a_1)$	$p(z_{1,6},a_1)$
0.24	0.16	0.4	0.4	0.384	0.256
$p(z_{1,7},a_1)$	$p(z_{1,8},a_1)$	$p(z_{1,9},a_1)$	$p(z_{1,10},a_1)$	$p(z_{1,11},a_1)$	$p(z_{1,12},a_1)$
0.64	0.16	0.48	0.32	0.8	0.2

Table 9
The satisfaction levels of all the paths of decision a_1.

$u(z_{1,1},a_1)$	$u(z_{1,2},a_1)$	$u(z_{1,3},a_1)$	$u(z_{1,4},a_1)$	$u(z_{1,5},a_1)$	$u(z_{1,6},a_1)$
1	0.444	0.667	0.444	0.778	0.222
$u(z_{1,7},a_1)$	$u(z_{1,8},a_1)$	$u(z_{1,9},a_1)$	$u(z_{1,10},a_1)$	$u(z_{1,11},a_1)$	$u(z_{1,12},a_1)$
0.444	0.444	0.556	0	0.222	0.444

Since a_2 generates a certain positive payoff (negative cost), there is no negative focus path of a_2. Therefore, we analyze this decision problem by the positive evaluation system of dynamic focus programming. Using (6) with the data in Tables 9 and 10 and setting φ as 0.1, 1 and 10, respectively, we can obtain $Z_p(a_1)=\{z_{1,7}\}$, $Z_p(a_1)=\{z_{1,9}\}$ and $Z_p(a_1)=\{z_{1,1}\}$ for φ being 0.1, 1 and 10, respectively. According to Definition 1, we have $z_p^*(a_1)=z_{1,7}$, $z_p^*(a_1)=z_{1,9}$ and $z_p^*(a_1)=z_{1,1}$ for φ being 0.1, 1 and 10, respectively shown in Table 11. Clearly, increasing φ can lead to finding a positive focus path with a relatively high satisfaction level and a relatively low likelihood, that is, $u(z_{1,7},a_1)=0.444<u(z_{1,9},a_1)=0.556<u(z_{1,1},a_1)=1$ and $\pi(z_{1,7},a_1)=0.64>\pi(z_{1,9},a_1)=0.48>\pi(z_{1,1},a_1)=0.24$. φ is used as a weight for a decision maker to balance his/her emphasis on the satisfaction level and the relative likelihood degree. Increasing φ means that the decision maker aims to pursue a higher satisfaction by somewhat sacrificing the relative likelihood. Hence, φ can measure how optimistic the decision maker is: The higher the value of φ, the more optimistic the decision maker. Since there is a unique path for a_2, it is straightforward that $z_p^*(a_2)=z_2$ for any φ. Using (4) and (5), we know $u(a_2,z_2)=0.489$ and $\pi(a_2,z_2)=1$.

Next, we seek the optimal decision rule. For $\varphi=0.1$, the positive focus path of a_1 is $z_p^*(a_1)=z_{1,7}$. Since $\pi(z_{1,7},a_1)=0.64<\pi(z_2,a_2)=1$ and $u(z_{1,7},a_1)=0.444<u(z_2,a_2)=0.489$, it follows from Definition 2 that under the positive evaluation system the optimal initial decision is a_2 and the optimal decision rule is z_2 for any κ.

Let us further examine the case of $\varphi=10$. If we set κ as 0.1, then it follows from (7) and Definition 2 that under the positive evaluation system the optimal initial decision is a_2 and optimal decision rule is z_2; if we set κ as 10, then we know that the optimal initial decision is a_1 and the optimal decision rule is $z_{1,1}$. κ is interpreted as the decision-maker's confidence index on his/her decision: The higher the value of κ, the more confident the decision-maker.

We summarize the results as follows: If the president is less optimistic (the case of $\varphi=0.1$), he will not participate the bidding; if he is very optimistic (the case of $\varphi=10$) but less confident (the case of $\kappa=0.1$), he will not participate the biding; if he is very optimistic (the case of $\varphi=10$) and very confident (the case of $\kappa=10$), he will participate the biding. Clearly, such results are intuitively acceptable and can provide insights into the behavior of the president.

Table 10
The relative likelihood degrees of all the paths of decision a_1.

$\pi(z_{1,1}, a_1)$	$\pi(z_{1,2}, a_1)$	$\pi(z_{1,3}, a_1)$	$\pi(z_{1,4}, a_1)$	$\pi(z_{1,5}, a_1)$	$\pi(z_{1,6}, a_1)$
0.24	0.16	0.4	0.4	0.384	0.256
$\pi(z_{1,7}, a_1)$	$\pi(z_{1,8}, a_1)$	$\pi(z_{1,9}, a_1)$	$\pi(z_{1,10}, a_1)$	$\pi(z_{1,11}, a_1)$	$\pi(z_{1,12}, a_1)$
0.64	0.16	0.48	0.32	0.8	0.2

Table 11
The positive focus paths of a_1 under various φ's.

		φ	
Focus path	0.1	1	10
a_1	$z_{1,7}$	$z_{1,9}$	$z_{1,1}$

Let us reexamine this case study by stochastic dynamic programming with the assumption that the decision maker is risk neutral. By backward induction, the expected monetary values of a_3 (bid price 10), a_4 (bid price 9) and a_5 (price 8) are obtained as 17.5, 14 and 5, respectively, and all of them choose a_7 (the second method) at the third stage. Since 17.5>14>5 holds, a_3 will be chosen at the second stage. Then the expected monetary value of a_1 (making a prototype) is calculated as 14 which is larger than the monetary value of a_2 (non-participating), that is, 10. As a result, the decision maker will choose a_1 as his/her optimal initial decision, and the optimal decision rule is $\{a_1, a_3, a_7\}$.

From this case study, we understand that dynamic focus programming and stochastic dynamic programming handle the same sequential decision problem by the fundamentally different ways. Since stochastic dynamic programming utilizes the expected value, it makes more sense if the decision process is repeatable while dynamic focus programing is of scenario-based thinking, it is more suitable for a one-time decision. In stochastic dynamic programming, only a decision sequence can be obtained whereas in dynamic focus programming, a focus path is obtained which provides not only a decision sequence but the reason why such a decision sequence should be chosen. In dynamic focus programming, we can account for the behaviors of the decision makers with different personality traits by simply adjusting the parameters.

5. Conclusions

As a fundamental alternative for modeling and solving sequential decision-making problems under uncertainty, dynamic focus programming is proposed. Different from stochastic dynamic programming that is based on the expected utility theory, dynamic focus programming determines the optimal decision rule according to which initial decision's focus path is the most preferred.

Dynamic focus programming is an axiomatized approach. Guo (2019) proposes the focus theory of choice which models and axiomatizes the procedural rationality of decision-making. The core argument of the focus theory of choice is that the most salient event corresponds to the most-preferred decision. Accordingly, dynamic focus programming claims that the most salient path corresponds to the most-preferred decision rule.

Stochastic dynamic programming utilizes backward induction whereas dynamic focus programing uses forward calculation which is close to human being intuition. In stochastic dynamic programming, only a decision sequence can be obtained whereas in dynamic focus programming, a focus path is obtained. The focus path consists of not only a decision sequence from the initial stage to the final stage but also the associated states. The focus path provides the reason why such a decision sequence should be chosen. In addition, in dynamic focus programming framing effects can be handled by the positive and negative evaluation systems and the decision maker's personality and behavioral attributes can be properly accommodated by adjusting the parameters. Hence, dynamic focus programming makes the complicated decision-making procedure visible and no longer a black box.

For the sake of simplification, we only consider the discrete cases of decision variables and random variables in this research. The same formulas can apply to the case that decision variable is continuous and to the case that random variable is continuous with a conditional probability density function taking the place of a transition probability.

There are several limitations of this research. The parameters φ, κ, θ and τ in the dynamic focus programing are used to reflect the personal traits of the decision makers and should be given by themselves. However, it requires more demanding of cognitive effort. Providing an appropriate approach to help decision makers determine such parameters more easily will be our future research work. As a new approach to the sequential decision problem, the theoretical analysis on the comparison of the computational complexity between dynamic focus programming and stochastic dynamic programming should be done. It will be another future research topic.

The research on dynamic focus programming is at an early stage. Further research can be done from theoretical and applied aspects. This research provides the theoretical base for the further research on sequential decision-making problems under uncertainty, which are commonly encountered in business, economics, and social systems.

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