Dynamic Communication Mechanism Design*

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Abstract

We consider dynamic communication mechanisms in a quasi-linear environment with single-dimensional types. The mechanism designer gradually identifies agents' valuations by iteratively offering prices to agents at different stages. Agents pay the maximum price they accepted if their desirable decision is made. We show that within weakly tight mechanisms, if a communication mechanism is ex-post incentive compatible, then it is a monotone-price mechanism. English auctions are characterized as a class of mechanisms that satisfy ex-post incentive compatibility and efficiency.

Keywords: dynamic communication mechanism, English auction, binary question, monotone price

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1 Introduction

This study considers dynamic indirect mechanism design when the mechanism designer faces communication constraints and only partial information is communicated at once. We consider an environment in which agents have private values and single-dimensional types. The mechanism designer can ask only binary questions whether an agent's own type is higher than a threshold value. The mechanism designer can ask binary questions many times. Agents pay the maximum threshold value they accepted if their preferred decision is made and nothing otherwise. In this environment, we examine how to design dynamic asking-question processes that induce truth-telling.

In this study, the mechanism designer is constrained in two aspects. First, the designer can only use binary questions in a single stage. One of the reasons to take such communication constraints into account is privacy concerns. In standard mechanism design, we generally focus on direct revelation mechanisms according to the revelation principle. However, in various collective decisions in reality, including auctions and voting, individuals often do not want their private information to be made public. For example, business people do not want to reveal their full valuations or costs in auctions because they may include confidential information on their firms.[1] Another reason is that the information to be expressed in direct revelation mechanisms becomes too large for complex problems such as multi-object auctions. For example, when n distinct objects are auctioned, agents need to evaluate and report the valuations of $2^n - 1$ packages of objects. In such a situation, it is important to communicate in many stages and reduce conveyed information as much as possible.

In addition to communication constraints, mechanisms using sequential binary questions are actually used in various collective decision-making problems. English and Dutch (clock) auctions are an example of such mechanisms. In an English clock auction, each bidder decides whether to continue bidding at every clock price. In a Dutch auction, each bidder decides whether to buy at every current price. In public decision-making, sequential binary voting is widely used in legislatures and committees in many countries when there are many alternatives.

[1] See Rothkopf et al. (1990) for example.

Regarding the second constraint, the mechanism designer can use only a simple monetary transfer rule. In some practical situations, monetary transfer rules may not be able to depend on agents' reports in a complex way. In public decision-making, there would be a high legislative cost to approve a complex transfer rule. Furthermore, the government may need to make all people understand the rule. Alternatively, the government or mechanism designer may not be able to commit to a complex transfer rule. Even if such a transfer rule is enforceable, simple transfer rules would be preferable in practice.

Under these restrictions, we answer the following question: in what kind of dynamic question processes is truth-telling an equilibrium? Within a class of processes satisfying weak tightness, we show that if a dynamic binary-question process is ex-post incentive compatible (EPIC), it must be a *monotone-price mechanism.* In a monotone-price mechanism, the mechanism designer offers (personalized) prices to agents in an ascending-price manner. Agents are active as long as they accept the offered prices, and the seller finally chooses an outcome preferable only to the active agents at the termination. Specifically, in a single-object auction, we characterize English auctions as dynamic mechanisms that satisfies efficiency and EPIC. On the other hand, in a simple indivisible public goods environment, truth-telling is an ex-post equilibrium only in a kind of unanimity rule. There is no efficient mechanism that induces ex-post incentive compatibility in the public goods environment.

This result is related to the well-known characterization of the second-price sealed-bid auction in static mechanism design. In a single-object auction, the second-price auction is known as an essentially unique mechanism that satisfies efficiency and dominant-strategy incentive compatibility (DSIC) (Vickrey, 1961; Green and Laffont, 1977; Holmstrom, 1979). An English auction is considered a dynamic (indirect) mechanism that is strategically equivalent to the second-price auction.[2] It is a natural question whether there is another dynamic mechanism, except for the English auction, which is efficient and induces truth-telling in an equilibrium. Our answer is no: English auctions are only dynamic mechanisms that are efficient and EPIC among the class of communication processes.

[2]The strategic equivalence holds in a private value setting.

Recently, Li (2017) characterizes monotone-price mechanisms by a novel notion of obvious strategy-proofness (OSP), which is a strong incentive condition motivated by bounded rational players. A mechanism is OSP if, at any information set where two strategies first diverge, the worst-case payoff under truth-telling is at least as great as the best-case payoff under the deviation. Li (2017) shows that a dynamic indirect mechanism is OSP if and only if it is a monotone-price mechanism, although he does not explicitly characterize English auctions using efficiency. Our result complements Li (2017) in two respects. First, we provide another characterization of monotone-price mechanisms using EPIC, which is a weaker equilibrium condition than OSP. This is done by limiting the class of indirect mechanisms, whereas Li allows arbitrary mechanisms. Second, we explicitly define and characterize English auctions as efficient monotone-price mechanisms.

1.1 Related Literature

In terms of the characterization of English auctions, Li (2017) shows that a dynamic indirect mechanism is OSP if and only if it is a monotone-price mechanism, which explains the superiority of an English auction over a second-price auction in experimental studies. OSP is stronger than DSIC and EPIC. Akbarpour and Li (2019) characterize the revenue-optimal ascending auctions by strategy-proofness and credibility. The credibility requires a mechanism to be incentive compatible for the designer or the auctioneer. Milgrom and Segal (2020) consider the recent spectrum reallocation problem in the U.S. They formulate an allocation problem with single-dimensional types and monotone-price mechanisms similar to ours. They show that an allocation rule is implementable by a monotone-price mechanism if it is monotone, non-bossy, and satisfies a substitutes condition.

Although we use EPIC as the equilibrium concept, we also argue that monotone-price mechanisms satisfy an additional incentive property regarding ex-post perfection. Ex-post *perfect* incentive compatibility requires that truth-telling is an ex-post equilibrium even off the equilibrium paths. An extensive literature on dynamic auction design develops efficient auctions for multiple objects that are EPIC or ex-post perfect incentive compatible. For homogeneous multi-unit auctions, Ausubel

(2004, 2018) and Okamoto (2018) propose the so-called "Ausubel auction," which implements the Vickrey auction outcome in ex-post perfect equilibrium. Dynamic Vickrey-Clarke-Groves mechanisms for heterogeneous objects are studied by Ausubel (2006), Ausubel and Milgrom (2002), and Mishra and Parkes (2007) among others.

Several papers investigate ex-post perfect equilibrium for sequential binary voting procedures under incomplete information. Gershkov et al. (2017) analyze the equilibrium under single-peaked preferences and single-dimensional types. Kleiner and Moldovanu (2017, 2020) characterize the sequential voting procedures in which sincere voting at each round is an ex-post perfect equilibrium.

Our study is also related to the literature on mechanism design and communication complexity. Van Zandt (2007) shows the difference between EPIC and DSIC in dynamic mechanisms under private values. Fadel and Segal (2009) examine the communication costs for determining an allocation and calculating the monetary transfer that induces incentive compatibility. They show that additional cost exists for calculating EPIC monetary transfers other than the communication cost of finding the desirable allocation. Kos (2014) examines a dynamic binary-question mechanism design for a single-object auction with two bidders. He develops a dynamic question protocol that maximizes social welfare when the number of binary questions is exogenously limited. Grigorieva et al. (2007) use the bisection method and propose another dynamic auction that requires less communication than the English auction. Mookherjee and Tsumagari (2014) provide a necessary and sufficient condition for dynamic communication mechanisms being Bayesian incentive compatible. Blumrosen et al. (2007) and Kos (2012) study static single-object auction design under restricted message space.

The rest of this paper is organized as follows. In section 2, we formulate the model and introduce dynamic communication mechanisms. The main results are presented in section 3. We show that EPIC with an additional condition induces monotone-price mechanisms in a general case. We then consider the specific environments of a single-object auction and an indivisible public good. We characterize English auctions as the class of mechanisms satisfying EPIC and efficiency. We also discuss the case in which agents have additional private information regarding their interests.

Section 4 concludes the paper. All proofs are provided in the Appendix.

2 Model

Let $I \equiv \{1, 2, \ldots, n\}$ be the set of all agents. The mechanism designer[3] chooses an outcome x from a finite set of alternatives X and monetary transfer $m = (m_i)_{i \in I}$. Each agent has a quasi-linear utility with an integer-valued valuation function $u_i : X \to \mathbb{Z}$.[4] Agent i's utility is given by $u_i(x) - m_i$. Each agent has dichotomous preferences, that is, he has a non-empty set of *interests* and the same value for each outcome of interest. Let $X_i \subset X$ be a set of i's interests. Agent i's valuation function takes a form of

$$u_i(x) = \begin{cases} v_i & \text{if } x \in X_i \\ 0 & \text{if } x \notin X_i \end{cases}. \tag{1}$$

Given the dichotomous preferences of agents, the set of agents interested in an outcome x is denoted by $I(x)$, that is,

$$I(x) \equiv \{i \in I | x \in X_i\}.$$

For convenience of notation, a set of consecutive integers from a to b is denoted by $[a, b]$. The domain of values is bounded and denoted by $V_i \equiv \{\underline{v}_i, \underline{v}_i + 1, \ldots, \bar{v}_i\} = [\underline{v}_i, \bar{v}_i]$. It is possible for agents to have a negative value or cost $v_i < 0$ for the outcomes of interest.

We assume that each agent's interests X_i and the domain of values V_i are all commonly known to each other, including the mechanism designer. This is natural in problems such as public goods and auctions.[5] The state of the world is simply the vector of values, $v = (v_1, \ldots, v_n) \in V \equiv \times_{i \in I} V_i \subset \mathbb{Z}^n$.

Our environment includes a single-object auction, indivisible public good provision, and a multi-object auction with so-called "single-minded" bidders. A bidder in a multi-object auction is said to be single-minded if he is interested in a particular

[3]We use female pronoun for the mechanism designer and male pronoun for an agent.

[4]Integer valuation is not crucial in our analysis. When valuation is bounded as we assume, it guarantees that the mechanism designer identifies the state of the world in finitely many steps.

[5]We discuss the case in which X_i is private information of agent i in section 3.4.

package of goods and makes bids only for that package (Lehmann et al., 2002; Sano, 2011; Milgrom and Segal, 2020). In addition, a double auction with single-unit demand and supply is also included. In a double auction, a buyer has a positive value $v_i > 0$ of the good, whereas a seller's valuation is denoted by a negative value $v_i < 0$.

2.1 Dynamic Communication Mechanisms

The mechanism designer gradually collects information about the state of the world through a sequence of threshold binary questions. In each round of a mechanism, the designer offers prices (or subsidies) to agents to achieve an outcome of their interests.

A dynamic communication mechanism consists of a sequence of prices, publicly observable, offered to agents at different rounds. For each round $t = 1, 2, \ldots,$ the mechanism designer selects a set of agents $J^t \subseteq I$. The mechanism designer offers a price p_i^t to agent $i \in J^t$. Each agent $i \in J^t$ makes a report $a_i^t \in \{yes, no\}$. The offered prices and reports are publicly observable. The mechanism designer can offer prices finitely many times.

The price vector offered at round t is denoted by $p^t = (p_i^t)_{i \in J^t}$, and the report profile at t is denoted by $a^t = (a_i^t)_{i \in J^t}$. A history up to the end of round t is denoted by $h^t = (p^1, a^1, \ldots, p^t, a^t)$. To formally describe the mechanisms we consider, we define several variables. Let

$$\underline{v}_i(h^t) \equiv \begin{cases} \max_{\tau \le t}\{p_i^\tau | a_i^\tau = yes\} & \text{if } a_i^\tau = yes \text{ for some } \tau \le t \\ \underline{v}_i & \text{otherwise} \end{cases} \tag{2}$$

and

$$\bar{v}_i(h^t) \equiv \begin{cases} \min_{\tau \le t}\{p_i^\tau | a_i^\tau = no\} - 1 & \text{if } a_i^\tau = no \text{ for some } \tau \le t \\ \bar{v}_i & \text{otherwise} \end{cases}. \tag{3}$$

Then, we define the *revealed value set* of agent i at round t as[6]

$$V_i(h^t) \equiv [\underline{v}_i(h^t), \bar{v}_i(h^t)]. \tag{4}$$

Using these notations, a *dynamic communication mechanism (DCM)*, denoted by $\Gamma = \big(\{J^t, p^t\}_t, g, m\big)$, is formally defined as follows:[7] For each round t, $J^t : H^{t-1} \to 2^I$

[6]We will consider mechanisms such that $V_i(h^t)$ is well defined and non-empty.

[7]See Appendix A for a formal definition as an extensive form game.

determines a set of agents to whom the mechanism designer offers a price, where H^t is the set of all histories up to round t and $H^0 = \{h^0\}$. The price that the mechanism designer offers to agent $i \in J^t(h^{t-1})$ is determined by $p_i^t : H^{t-1} \to \mathbb{Z}$. At round t, the mechanism designer offers $p_i^t(h^{t-1})$ to each agent $i \in J^t(h^{t-1})$. Each selected agent $i \in J^t(h^{t-1})$ makes a report $a_i^t \in \{yes, no\}$. For each round t, each h^{t-1}, and each agent $i \in J^t(h^{t-1})$, the offered price satisfies $p_i^t(h^{t-1}) \in V_i(h^{t-1}) \setminus \{\underline{v}_i(h^{t-1})\}$. The mechanism terminates at round T when $J^{T+1}(h^T) = \emptyset$. A terminal history is denoted by $h = (p^1, a^1, \ldots, p^T, a^T) \in H$, where H is a set of all terminal histories. For each terminal history $h \in H$, the mechanism designer selects an outcome $g(h) \in X$ and collects monetary transfer $m(h) \in \mathbb{Z}^n$ from agents, which is determined by

$$m_i(h) = \begin{cases} \underline{v}_i(h) & \text{if } g(h) \in X_i \\ 0 & \text{if } g(h) \notin X_i \end{cases}. \tag{5}$$

A decision rule g, along with a sequence of prices $\{J^t, p^t\}_t$ is said a *communication protocol.*

A DCM is interpreted as follows. When agent i responds to a price p_i with *yes*, the mechanism designer regards the agent's value as at least as great as p_i: $v_i \geq p_i$. Conversely, when agent i responds with *no*, the mechanism designer considers the agent's value to be less than p_i: $v_i < p_i$.[8] By offering different prices many times, the mechanism designer gradually identifies v_i. For example, if agent i accepts a payment $p_i = 5$ and rejects $p_i = 12$, then the mechanism designer considers i's value to be between 5 and 11.

Hence, from the responses by agents up to round t of a communication protocol, the mechanism designer regards i's value as in his revealed value set $V_i(h^t)$. Any value $\tilde{v}_i \in V_i(h^t)$ is consistent with all the responses from agent i. Let $V(h^t) \equiv \times_{i \in I} V_i(h^t)$, called the *revealed state set at* t, which indicates the set of possible states consistent with a history up to t. It should be noted that $V_i(h^t) \subseteq V_i(h^s)$ and $V(h^t) \subseteq V(h^s)$ for all $t > s$ and that $V(h^t) \neq \emptyset$. As the mechanism designer makes a question to an agent, his revealed value set shrinks and the revealed state set also does so accordingly.

[8] We assume that a negative response against p_i implies a strict preference so that the mechanism designer can certainly identify v_i.

We do not impose any specification on the communication protocol $(\{J^t, p^t\}_t, g)$. The mechanism designer can employ various methods such as ascending price, descending price, and others. For example, in an ascending-price protocol, the mechanism designer starts by offering a sufficiently low price and gradually increases the price as long as an agent responds *yes*. Another protocol is the bisection method, in which the mechanism designer offers the median value in an agent's revealed value set in each round. The bisection method minimizes the number of questions for uniquely specifying the value of i.

We do not specify a termination condition. When the mechanism designer can ask arbitrarily many questions without any cost, she may want to continue asking until the revealed state is uniquely determined or the efficient allocation is identified. For example, a protocol may terminate at T if there is an outcome x such that it is efficient with respect to every $v \in V(h^T)$. When asking questions is costly and the number of questions is restricted, the protocol may need to terminate by an exogenous deadline.

Although we do not specify any details of a communication protocol, we restrict our attention to a particular monetary transfer rule specified by (5). The payment rule requires that when the allocation $g(h)$ is of interest for agent i, then i needs to pay the minimum value in his revealed value set. Equivalently, each agent pays the maximum price to which he said *yes* in the course of questions. In this sense, this payment rule can be regarded as a dynamic version of a "pay-as-bid" transfer rule.

Focusing on a particular payment rule is restrictive from the mechanism design point of view, which seeks payment schemes that incentivize agents. This study specifies the class of dynamic processes that are incentive compatible given a simple transfer scheme. When an arbitrary monetary transfer rule is allowed, the mechanism design problem boils down to a standard static one because the mechanism designer can offer prices in an arbitrary manner and uniquely identify the state. Another justification is that the mechanism designer does not commit to a complex monetary transfer rule. When the mechanism designer cannot commit to a specific monetary transfer rule, she would like to extract surplus from agents as much as possible.[9]

[9]We do not argue the formal notion of the simplicity of a transfer rule or the commitment issue.

Agents observe all past prices and reports. A (pure) strategy $\sigma_i \in \Sigma_i$ of agent i in a DCM Γ is a profile of actions $a_i^t(h^{t-1}) \in \{yes, no\}$ at every decision node h^{t-1} at round t such that $i \in J^t(h^{t-1})$. Agent i reports *sincerely* when his strategy is such that for every information set h^{t-1} that i moves, $a_i^t(h^{t-1}) = yes$ if and only if $p_i^t(h^{t-1}) \leq v_i$.

2.2 Equilibrium Concept

We use ex-post Nash equilibrium for the equilibrium concept.

Definition 1 A DCM is said to be *ex-post incentive compatible (EPIC)* if the sincere reporting by each agent is an ex-post Nash equilibrium. That is, given that the other agents report sincerely, each agent has no incentive to deviate from the sincere strategy under every state $v \in V$.

It would be worth noting some remarks on related equilibrium concepts. First, EPIC is weaker than DSIC. In a static mechanism design with private values, EPIC is equivalent to DSIC; however, it is not the case in dynamic mechanisms. This is because the strategy space is generally larger than the type space in dynamic mechanisms, so that the sincere strategy may not be the best response to strategies not taken by any type.[10] At the same time, EPIC is weaker than OSP. OSP requires that at any information set where two strategies diverge, the worst-case payoff under the sincere strategy is at least as great as the best deviation payoff.

Second, EPIC does not require subgame perfection.[11] Several studies on dynamic auctions and voting, such as in Ausubel (2004, 2006) and Kleiner and Moldovanu (2017), consider the ex-post perfect incentive compatibility, which requires that at any non-terminal history, sincere reporting is an ex-post Nash equilibrium of the associated continuation game.[12] Although we also examine dynamic mechanisms, we

Akbarpour and Li (2019) show that the credibility of a mechanism induces the pay-as-bid transfer rule.

[10]See Van Zandt (2007).

[11]Note that subgame perfection is not required for DSIC or OSP.

[12]The original "Ausubel auction" in Ausubel (2004) was not precisely ex-post perfect incentive compatible. Okamoto (2018) and Ausubel (2018) modify it to hold the property.

do not impose subgame perfection. However, we will confirm that EPIC mechanisms are also ex-post perfect incentive compatible in our model.

2.3 Direct Allocation Rule

For a communication protocol $(\{J^t, p^t\}, g)$, let $\phi : V \to H$ be a mapping from each state to the associated terminal history, assuming sincere reporting by agents. In other words, ϕ is an inverse mapping of $V(h)$. We define a *direct allocation rule* $f : V \to X$ associated with $(\{J^t, p^t\}, g)$ as $f = g \circ \phi$. A direct allocation rule is a mapping from each state $v \in V$ to a corresponding allocation $x \in X$ via a history generated by sincere behavior. For each communication protocol, the associated direct allocation rule f is uniquely determined; however, the converse is not true. In general, for a given allocation rule f, there are multiple protocols that have the direct allocation rule f.

A direct allocation rule is said to be *efficient* (with respect to reports) if for all $v \in V$,[13]

$$f(v) \in \arg\max_{X} \sum_{i \in I} u_i(x). \tag{6}$$

A direct allocation rule f is said to be *monotone* if for all $i \in I$, all $v \in V$, and all $\tilde{v}_i > v_i$,

$$f(v) \in X_i \Rightarrow f(\tilde{v}_i, v_{-i}) \in X_i.$$

Every efficient allocation rule is clearly monotone.

Definition 2 A DCM Γ is said to be *monotone* if the associated direct allocation rule is monotone. A DCM Γ is said to be *efficient* if the associated direct allocation rule is efficient.

Note that the monotonicity is the necessary and sufficient condition for an allocation rule to be implementable in weakly dominant strategy in a static single-object auction (Myerson, 1981). Similarly, given that X_i for each agent is publicly known, the monotonicity is the necessary and sufficient condition for DSIC in static mechanism design.

[13]Ties are broken not randomly, but in an arbitrary predetermined way. The efficient allocation rule is not unique.

3 Result

3.1 General Result

It is straightforward to confirm that EPIC implies monotonicity of the direct allocation rule, as is the case with the static case.

Lemma 1 *If a DCM is EPIC, then it is monotone.*

For every DCM and every history h^t, let $Y(h^t)$ be the set of agents who have never reported *no* in history h^t. That is,

$$Y(h^t) \equiv I \setminus \{i \in I | \exists s \leq t, a_i^s = no\}.$$

Agent $i \in Y(h^t)$ is said to be *active* under h^t, whereas agent $i \notin Y(h^t)$ is said to be *inactive*. The following lemma shows that "winning" agents must be active at the termination.

Lemma 2 *If a DCM is EPIC, then the allocation rule satisfies for all $h \in H$,*

$$I(g(h)) \subseteq Y(h).$$

The intuition of Lemma 2 is simple. If an agent reports *no* for an offer and a preferred outcome is achieved, then by the monotonicity a preferred outcome should be achieved when he reports *yes* for the same offer. When the outcome is preferable regardless of the response to an offer, reporting *no* is the dominant strategy because the payment is strictly reduced, which contradicts incentive compatibility.

Lemma 2 immediately implies that with an additional condition, every EPIC DCM needs to be a monotone-price mechanism.

Definition 3 A DCM is said to be a *monotone-price mechanism* if for every terminal history $h \in H$, the sequence of offered prices for each agent satisfies $p_i^s < p_i^t$ for all $i \in I$ and all $s < t$, and outcome $g(h)$ satisfies $I(g(h)) \subseteq Y(h)$.

We additionally impose the non-redundancy of questions that deal with "economy of communication." We call the condition *weak tightness*, which requires that if an agent is asked a question, he should have a chance of achieving an outcome of interest.

Definition 4 A DCM with an associated direct allocation rule f is *weakly tight* if $i \in J^t(h^{t-1})$, then there exists a state $\tilde{v} \in V(h^{t-1})$ and $f(\tilde{v}) \in X_i$.

Weak tightness requires that if agent i is a mover of a node, then there is a path such that an outcome of i's interest is realized in that subgame. If it does not hold, then i never realizes any outcome of his interest, regardless of the current and future responses. Thus, agent i may be reluctant to make a serious response, and such a question would be redundant.

The following proposition states that, with weak tightness, EPIC characterizes monotone-price mechanisms.

Proposition 1 *If a DCM is EPIC and weakly tight, then it is a monotone-price mechanism. If a DCM is a monotone-price mechanism, it is EPIC.*

The second statement is a corollary of Li (2017), which shows that every monotone-price mechanism is OSP. In Proposition 1, monotone-price mechanisms are characterized by EPIC (and weak tightness), which is a weaker equilibrium condition. This is because we limit attention to a specific class of indirect mechanisms, whereas Li (2017) does not.

It is also worth noting that every monotone-price mechanism is ex-post perfect incentive compatible. That is, sincere reporting is optimal even off the equilibrium paths, which is confirmed as follows. Note that for any strategy profile, the associated terminal history h is consistent with some state $v \in V(h)$. Therefore, every off-path history for agent i is one after agent i's own deviation. Given the weak tightness, at each off-path node of round t, agent i has reported *yes* for $p_i^s > v_i$ at some round $s < t$ and is offered a price $p_i^t > p_i^s > v_i$. Hence, agent i's possible continuation payoff is non-positive, and i earns zero payoff by sincere reporting. Therefore, sincere reporting is optimal even if an agent has deviated from sincere reporting before, and the mechanism is ex-post perfect incentive compatible. This observation is formally stated below.

Observation 1 *Every monotone-price mechanism is ex-post perfect incentive compatible.*

Without the weak tightness, we can construct a non-monotone-price mechanism that is EPIC. Consider a single-object auction with three agents. Each agent's value is located between 0 and 100. The following mechanism is not a monotone-price mechanism but EPIC.

1. At round 1, the mechanism designer offers $p_1^1 = 50$ to agent 1. If agent 1 responds *yes*, the protocol terminates, and the object is allocated to the agent 1 (with payment of 50). Otherwise, the protocol goes to round 2.

2. At round 2, the designer offers $p_1^2 = 25$ to agent 1. If agent 1 responds *yes*, the object is allocated to agent 2 (with no payment). Otherwise, the object is allocated to agent 3 (with no payment).

In this example, a decreasing price does not violate incentive compatibility because the associated responses do not affect the responder's own outcome. In this manner, the mechanism designer can use inactive agents' responses as a tie-breaking device. This type of tie-breaking question can be increasing or decreasing prices, so that a monotone-price mechanism is not necessarily weakly tight. This example implies that weak tightness takes the role of non-bossiness of an allocation rule.

Remark 1 Milgrom and Segal (2020) provide conditions under which an allocation rule can be implemented by a monotone-price mechanism. They show that if an allocation rule is monotone, non-bossy, and satisfies a substitutes condition, then it is implementable by a monotone-price mechanism. Hence, not all monotone allocation rules are implementable in general.

3.2 Single-Object Auctions

Let us focus on a single-object auction problem. The mechanism designer or the seller allocates a single unit of an object to n potential buyers. The set of outcomes is $X = \{0, 1, \dots, n\}$, and the agent i's interests is given by $X_i = \{i\}$. The value of the object for buyer i is $v_i \in V = [0, \bar{v}]$. The value of the object for the seller is assumed to be zero.

We characterize English auctions as a class of DCM satisfying efficiency and EPIC. To show this, we formally define English auctions as DCM. We focus on *perfect information DCM*, in which for all t and all h^{t-1}, $|J^t(h^{t-1})| = 1$. Abusing notations, denote the mover at round t by J^t. Initialize $p_i^0 \equiv 0$ for each agent $i \in I$. To complete notations, we denote $p_i^t = p_i^{t-1}$ when agent i is not a mover at round t, that is, p_i^t indicates the price the most recently offered to i. Denote by $\bar{p}_Y^t$ the highest price among all active agents under history h^t and $(p_i^t)_{i \in Y(h^t)}$. We define English auctions as follows.

Definition 5 A perfect information DCM is an *English auction* if it is a monotone-price mechanism that satisfies the following properties: for each non-terminal history h^{t-1},

1. if there exists an agent i^* such that

$$\{i^*\} = \{j \in Y(h^{t-1}) \mid p_j^{t-1} = \bar{p}_Y^{t-1}\},$$

then the mover $J^t(h^{t-1})$ of round t satisfies $J^t(h^{t-1}) \in Y(h^{t-1}) \setminus \{i^*\}$,

2. if $i = J^t(h^{t-1})$, then $p_i^t(h^{t-1}) \leq \bar{p}_Y^{t-1} + 1$, and

3. it terminates at round T when $|Y(h^{T-1})| = 2$ and $|Y(h^T)| = 1$, and the unique agent $i \in Y(h^T)$ wins the object and pays the price p_i^T at the termination.[14]

The first property states that if agent i^* is the unique highest bidder in the current round, then he is not the mover of the next round. The second property states that the price of the current mover must be at most the tentative highest bid plus a bid increment. The third property states that the auction continues whenever two or more agents are active.

The following theorem states that within weakly tight and perfect information DCM, English auctions are the only mechanisms that are efficient and EPIC. It is a dynamic counterpart of the well-known characterization of the second-price auction (Vickrey, 1961; Green and Laffont, 1977; Holmstrom, 1979).

Theorem 1 *Consider a single-object auction problem and perfect information DCM satisfying weak tightness. A DCM is efficient and EPIC if and only if it is an English auction.*

[14]There is an exception when two or more agents respond *yes* at $p_i^t = \bar{v}$. In such cases, one agent is chosen as the winner and pays $\bar{v}$.

Our definition of English auctions includes two standard forms. One is a so-called Japanese clock auction, in which the auctioneer announces a price in each round and continuously increases it until a single agent remains. Hence, the prices increase while keeping $p_i^t = p^t$ for all $i \in I$.[15] Another is the English ascending-bid auction, in which each agent sequentially submits a new bid that is the tentative highest bid plus the increment.

When there are three or more agents, our definition also includes those that do not appear to be standard English auctions. For example, consider the following "tournament auctions," which consist of two phases. In the first phase, an English clock auction is conducted among $n-1$ agents, and a winner is determined. In the second phase, the winner and the remaining agent compete in an English clock auction, which starts at the winning price of the first phase. The winner of the second phase obtains the object. Thus, our "English auctions" allow various ways in which to increase prices when there are many agents. Nevertheless, once just two active agents remain, the remaining communication protocol is a standard English auction.

3.3 Indivisible Public Goods

In contrast to the single-object auction, Lemma 2 leads to a negative characterization for an indivisible public good problem. Suppose $X = \{0, 1\}$ and $X_i = \{1\}$ for all $i \in I$. An outcome $x = 1$ indicates providing a public good, such as building a bridge for example, and $x = 0$ indicates not. A class of *static* mechanisms is defined as follows.

Definition 6 In an indivisible public good problem, a *unanimous acceptance mechanism* is the following static mechanism: The mechanism designer selects a set of agents $J \subseteq I$ and offers p_j to each $j \in J$ simultaneously. The allocation is $g = 1$ if and only if all agents in J respond *yes*. When the public good is provided, every agent $j \in J$ pays p_j.

In contrast to a standard unanimous voting rule, the mechanism designer does not need to ask all agents in the economy in this definition. The mechanism designer

[15] Due to the perfect information assumption, actions of agents are made sequentially.

may pick just one agent to determine an allocation. Another difference from the unanimous voting mechanism is that agents need to make payments once they accept.

In the indivisible public good problem, the mechanism designer has no way to utilize a dynamic question process. Every EPIC mechanism is equivalent to a unanimous acceptance mechanism.

Theorem 2 *In the indivisible public good problem, every EPIC DCM is equivalent to a unanimous acceptance mechanism.*

Note that $I(x) \in \{\emptyset, I\}$ in the public goods problem. By Lemma 2, the public good is not provided whenever one agent rejects an offered price in a sequence of questions. Hence, it is immediate that unanimous rules are the only EPIC mechanisms.

3.4 When X_i Is Private Information

We have assumed that the mechanism designer knows each agent's interests X_i. This would be plausible in problems such as single-object auctions and indivisible public goods, but not in other cases such as multi-object auctions with single-minded bidders.

To consider the case in which X_i is private information, let us focus on a multi-unit auction with single-minded agents. Suppose that the seller allocates K units of a homogeneous object. Each agent i demands k_i units and obtains a total value v_i if and only if he wins k_i or more units. The value of any smaller units is zero, and the marginal value for additional units more than k_i is zero.

When there are two or more kinds of information to ask, there are various types of question formats. A simple idea is a two-phase communication protocol. In the first phase, the seller asks each agent how much he demands. In the second phase, a monotone-price mechanism is conducted given the reported demands. Alternatively, the seller may conduct a monotone-price mechanism using nonlinear price vectors. Let $p_i^t(k)$ be the price of k units to bidder i at round t. In each round, the seller chooses a set of agents J^t and offers a personalized price vector

$p^t_j = (p^t_j(1), \ldots, p^t_j(K))$ for each $j \in J^t$. Then, each agent j reports his demand units under the current prices p^t_j. Observing the demands, the seller increases the prices.[16]

When agents initially report their true interests, the rest of the communication mechanism design is the same as the analysis thus far. However, not every monotone-price mechanism induces sincere reporting with respect to interests. In a static mechanism design, Lehmann et al. (2002) show that an allocation rule is implementable in DSIC if for every type profiles $(v_j, k_j)_{j \neq i}$ other than i, the allocation rule is such that if agent i obtains k_i units under a type (v_i, k_i), then he obtains $k'_i < k_i$ units under a type (v_i, k'_i). This property implies that given any type profile of the other agents, the payments of agent i necessary to win k units is non-decreasing in k.[17] Hence, when we focus on monotone-price mechanisms using nonlinear price vectors, a mechanism is EPIC if $p^t_i(k') \leq p^t_i(k)$ for all h^{t-1}, all $i \in J^t(h^{t-1})$, and all $k' < k$. This is a sufficient condition for EPIC mechanisms.

What is necessary and sufficient condition for EPIC is an open question, because there are a wide variety of communication processes when there are many attributes to ask. To extend our results to multi-dimensional types, we need to formally define DCM to accommodate multi-dimensional types.

4 Conclusion

DCM is a class of dynamic indirect mechanisms in which the mechanism designer iteratively asks binary questions and identifies the state of the world after a sequence of questions and responses. We have shown that with the pay-as-bid monetary transfer and weak tightness, every EPIC mechanism must be a monotone-price mechanism. In a single-object auction problem, English auctions are a class of DCM that are efficient and EPIC. This result is a dynamic counterpart of a second-price auction. However, in an indivisible public good problem, efficiency is not achieved. Sincere

[16]Note that these mechanisms use more than two actions in a single round. Such auctions that iteratively offer prices and ask demands are common in dynamic multi-object auction designs. See Ausubel (2004, 2006), Ausubel and Milgrom (2002), and Mishra and Parkes (2007) for example.

[17]For a general case of dichotomous preferences, Mishra and Roy (2013) characterize DSIC in terms of cutoff valuations.

reporting is an ex-post Nash equilibrium only in unanimous acceptance mechanisms.

As we have briefly argued in section 3.4, it is an open question in cases in which agents' interests are private information and, more generally, agents have multi-dimensional types. How to extend the result to multi-dimensional types is beyond the scope of the current study. However, in dynamic multi-object auction design, Ausubel (2004) and Ausubel and Milgrom (2002) provide EPIC ascending-price auctions for homogeneous or heterogeneous substitutes. Another interesting open question is the extension to interdependent values.

A Definition of Extensive Form Games

In this appendix, we formally define DCM as extensive form games with perfect recall. A DCM is a tuple $\Gamma = (\mathcal{H}, \prec, J, p, A, (Q_i)_{i\in I}, g, m)$, where

1. $\mathcal{H}$ is a set of all histories or nodes, and $\prec$ is a partial order on $\mathcal{H}$ that represents precedence.
   (a) The initial node is denoted by $h^0 \in \mathcal{H}$.
   (b) The set of terminal histories is denoted by $H \equiv \{h \in \mathcal{H} |\ \nexists h', h \prec h'\}$.
   (c) We suppose that $(\mathcal{H}, \prec)$ is a binary tree. For every non-terminal history $h \in \mathcal{H} \setminus H$, there are two immediate successors.
   (d) Let $H^t \subset \mathcal{H}$ be the set of histories with depth t. That is,
   $$H^t \equiv \{h \in \mathcal{H} \mid |\{h' \in \mathcal{H} | h' \prec h\}| = t\}.$$
   (e) $(\mathcal{H}, \prec)$ has finite depth. Hence, there exists a number $K \in \mathbb{N}$ and
   $$\mathcal{H} = \bigcup_{t=0}^{K} H^t.$$
   (f) In the main text, a non-terminal history is often denoted by h^t (using superscript), while a terminal history is denoted by h.
2. $J : \mathcal{H} \setminus H \to I$ is a player function, which assigns a player at each non-terminal history.

(a) For each $i \in I$, let $\mathcal{H}_i \equiv \{h \in \mathcal{H} \setminus H \mid J(h) = i\}$ be the set of non-terminal histories that belong to i.

(b) In the main text, a mover at a non-terminal history $h^{t-1} \in H^{t-1}$ is often denoted by $J^t(h^{t-1})$.

3. $p : \mathcal{H} \setminus H \to \mathbb{Z}$ is a price function, which assigns a price offered to the mover $J(h^{t-1})$ at each non-terminal history.[18]

4. In each non-terminal history $h^t \in \mathcal{H} \setminus H$, the set of available actions at h^t is given by $A = \{yes, no\}$.

5. Q_i is an information partition on $\mathcal{H}_i$.

(a) An information set is denoted by $q_i \subset \mathcal{H}_i$.

(b) We suppose that if $h, h' \in q_i$, then there exists some $t > 0$ and $h, h' \in H^t$.

6. $g : H \to X$ is an allocation function, and $m : H \to \mathbb{Z}^n$ is a monetary transfer function.

Given a mechanism Γ, a pure strategy for agent i is a mapping $\sigma_i : V_i \to \{yes, no\}^{\mathcal{H}_i}$ such that $\sigma_i(h) = \sigma_i(h')$ if h and h' are in the same information set.

B Proofs

B.1 Proof of Lemma 1

Suppose that a DCM is EPIC. The associated direct allocation rule is denoted by f. Suppose that there is an agent $i \in I$ and for some $v \in V$, $f(v) \in X_i$. The associated sincere history under v is denoted by h. Because of sincere reporting and the definition of the payment rule, the agent's payment must be $m_i(h) \leq v_i$.

Suppose that there exists $\tilde{v}_i > v_i$ and $f(\tilde{v}_i, v_{-i}) \notin X_i$. When agent i of type $\tilde{v}_i$ behaves as if his type is v_i, then the associated allocation is $f(v) \in X_i$ and the payment is $m_i(h)$. Hence, agent i's deviating payoff is $\tilde{v}_i - m_i(h) > 0$, which is a contradiction. ■

[18] See the main text for the conditions on p.

B.2 Proof of Lemma 2

Suppose that a DCM is EPIC. To have a contradiction, suppose that there exists a history h, and for some $i \in I(g(h))$ and some round t, $a_i^t = no$. Let t be the earliest such round, and consider p_i^t. For any state $v \in V(h)$, sincere reporting indicates $v_i < p_i^t$. Suppose $\tilde{v}_i \geq p_i^t$. By Lemma 1, $f(\tilde{v}_i, v_{-i}) \in X_i$. By the construction of round t, the truthful history up to t is the same between v and $(\tilde{v}_i, v_{-i})$.[19] Hence, under the state $(\tilde{v}_i, v_{-i})$, agent i is offered p_i^t at round t and responds *yes* under sincere reporting. Thus, agent i pays at least p_i^t, and the payoff under sincere reporting is at most $\tilde{v}_i - p_i^t$. If agent i deviates and pretends to have v_i under the state $(\tilde{v}_i, v_{-i})$, the corresponding outcome is $f(v) \in X_i$ and the payment is strictly less than p_i^t, which contradicts EPIC. ■

B.3 Proof of Proposition 1

Suppose that there is a history such that agent i reports *no* at round s and is asked at a later round $t > s$. By the weak tightness, there exists a state $v \in V(h^{t-1})$ and $f(v) \in X_i$. However, this contradicts Lemma 2, which requires $f(v) \notin X_i$. ■

B.4 Proof of Theorem 1

If part. Because an English auction is a monotone-price mechanism, it is EPIC. We will confirm that an English auction chooses the efficient outcome for every $v \in V$ under sincere reporting. To have a contradiction, suppose there exists a state $v \in V$ and an agent not having the highest valuation wins the auction. Let $I^* \subset I$ be the set of agents having the highest value under v. Let h be the associated terminal history. Because every agent $i \in I^*$ loses the auction, i is asked a price $p_i^t \geq v_i + 1$ at some round t and responds *no*. Let agent i be the last agent who responds *no* among I^* under h. By the properties of the auction rule, there exists an active agent $j \neq i$ and $p_j^{t-1} = \bar{p}_Y^{t-1} \geq p_i^t - 1$ at round $t-1$. Because every agent in I^* except for i is inactive after round $t-1$, agent j does not have the highest value. Hence, we have $v_j < v_i \leq p_i^t - 1 \leq p_j^{t-1}$, which contradicts sincere reporting of agent j. Hence,

[19] For any round $s < t$ such that agent i is asked, he always responds *yes* and $p_i^s \leq v_i$.

the auction is efficient.

Only if part. Suppose that a DCM is EPIC and weakly tight. Then, it is a monotone-price mechanism. It is easy to see that the third property of the auction rule holds. When two or more agents remain at the termination, it is clear that the efficient outcome cannot be identified. So $|Y(h)| \leq 1$ for all terminal history h. In addition, if $Y(h^t) = \{i\}$ and the designer asks agent i at round $t+1$, then the object is not allocated to anyone when i responds *no*. Hence, the mechanism terminates immediately if $|Y(h^t)| = 1$.

Now, suppose that the first property does not hold for some h^{t-1}. Agent i is the unique active agent facing the highest current price of round $t-1$ and the mover at round t. Then,

$$p_i^t > p_i^{t-1} = \bar{p}_Y^{t-1} > p_j^{t-1}$$

for all $j \in Y(h^{t-1}) \setminus \{i\}$. Then a state v such that $v_i = p_i^{t-1}$, $v_j = p_j^{t-1}$ for each $j \in Y(h^{t-1}) \setminus \{i\}$, and $v_k < p_k^{t-1}$ for each $k \in I \setminus Y(h^{t-1})$ is in the revealed state set $V(h^{t-1})$. The efficiency implies that agent i wins the auction, but he responds *no* at round t and loses, which is a contradiction. Hence, the first property holds.

Next, suppose that the second property does not hold for some h^{t-1}. Suppose that agent i is the mover at round t and $p_i^t \geq \bar{p}_Y^{t-1} + 2$. Because $p_i^{t-1} \leq \bar{p}_Y^{t-1}$, a state v such that $v_i = \bar{p}_Y^{t-1} + 1$, $v_j = p_j^{t-1}$ for each $j \in Y(h^{t-1}) \setminus \{i\}$, and $v_k < p_k^{t-1}$ for each $k \in I \setminus Y(h^{t-1})$ is in the revealed state set $V(h^{t-1})$. The efficiency implies that agent i wins the auction, but he responds *no* at round t and loses, which is a contradiction. Hence, the second property holds. ■

B.5 Proof of Theorem 2

By Lemma 2 and $I(x) \in \{\emptyset, I\}$, the allocation $g(h) = 1$ only if no agent reports *no* in the history h. By definition of DCM, a terminal history such that no agent reports *no* is uniquely determined and denoted by h^*. Let $J \subseteq I$ be the set of agents making a report in h^* and let $\bar{p}_j$ be the maximum price offered to i in h^*. Then, the direct allocation rule is described as

$$f(v) = \begin{cases} 1 & \text{if } (\forall j \in J), v_j \geq \bar{p}_j \\ 0 & \text{otherwise} \end{cases}.$$

This allocation rule is clearly obtained by a unanimous voting among J with offered prices $(\bar{p}_j)_{j \in J}$. ■

References

[1] Akbarpour, M., and S. Li, "Credible Auctions: A Trilemma," *Econometrica*, 88 (2019), 425-467.

[2] Ausubel, L.M., "An Efficient Ascending-Bid Auction for Multiple Objects," *American Economic Review*, 94 (2004), 1452-1475.

[3] Ausubel, L.M., "An Efficient Dynamic Auction for Heterogeneous Commodities," *American Economic Review*, 96 (2006), 602-629.

[4] Ausubel, L.M., "An Efficient Ascending-bid Auction for Multiple Objects: Reply," *American Economic Review*, 108 (2018), 561-563.

[5] Ausubel, L.M., and P. Milgrom, "Ascending Auctions with Package Bidding," *Frontier of Theoretical Economics*, 1 (2002), 1-42.

[6] Blumrosen, L., N. Nisan, and I. Segal, "Auctions with Severely Bounded Communication," *Journal of Artificial Intelligence Research*, 28 (2007), 233-266.

[7] Fadel, R., and I. Segal, "The Communication Cost of Selfishness," *Journal of Economic Theory*, 144 (2009), 1895-1920.

[8] Gershkov, A., B. Moldovanu, and X. Shi, "Optimal Voting Rules," *Review of Economic Studies*, 84 (2017), 688-717.

[9] Green, J., and J-J. Laffont, "Characterization of satisfactory mechanisms for the revelation of preferences for public goods," *Econometrica*, 45 (1977), 427-438.

[10] Grigorieva, E., P. J-J. Herings, R. Muller, and D. Vermeulen, "The Private Value Single Item Bisection Auction," *Economic Theory*, 30 (2007), 107-118.

[11] Holmstrom, B., "Groves' Scheme on Restricted Domains," *Econometrica*, 47 (1979), 1137-1144.

[12] Kleiner, A., and B. Moldovanu, "Content-based Agendas and Qualified Majorities in Sequential Voting," *American Economic Review*, 107 (2017), 1477-1506.

[13] Kleiner, A., and B. Moldovanu, "Voting Agendas and Preferences Trees: Theory and Practice," working paper (2020).

[14] Kos, N., "Communication and Efficiency in Auctions," *Games and Economic Behavior*, 75 (2012), 233-249.

[15] Kos, N., "Asking Questions," *Games and Economic Behavior*, 87 (2014), 642-650.

[16] Lehmann, D., L.I. O'Callaghan, and Y. Shoham, "Truth Revelation in Approximately Efficient Combinatorial Auctions," *Journal of ACM*, 49 (2002), 577-602.

[17] Li, S., "Obviously Strategy-Proof Mechanisms," *American Economic Review*, 107 (2017), 3257-3287.

[18] Milgrom, P., and I. Segal, "Clock Auctions and Radio Spectrum Reallocation," *Journal of Political Economy*, 128 (2020), 1-31.

[19] Mishra, D., and D. Parkes, "Ascending Price Vickrey Auctions for General Valuations," *Journal of Economic Theory*, 132 (2007), 335-366.

[20] Mishra, D., and S. Roy, "Implementation in Multidimensional Dichotomous Domain," *Theoretical Economics*, 8 (2013), 431-466.

[21] Mookherjee, D., and M. Tsumagari, "Mechanism Design with Communication Constraints," *Journal of Political Economy*, 122 (2014), 1094-1129.

[22] Myerson, R., "Optimal Auction Design," *Mathematics of Operations Research*, 6 (1981), 58-73.

[23] Okamoto, N., "An Efficient Ascending-bid Auction for Multiple Objects: Comment," *American Economic Review*, 108 (2018), 555-560.

[24] Rothkopf, M.H., T.J. Teisberg, and E.P. Kahn, "Why Are Vickrey Auctions Rare?" *Journal of Political Economy*, 98 (1990), 94109.

[25] Sano, R., "Incentives in Core-Selecting Auctions with Single-Minded Bidders," *Games and Economic Behavior*, 72 (2011), 602-606.

[26] Van Zandt, T., "Communication Complexity and Mechanism Design," *Journal of the European Economic Association*, 5 (2007), 543-553.

[27] Vickrey, W., "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16 (1961), 8-37.