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LIE DERIVATIVES ON HOMOGENEOUS REAL HYPERSURFACES OF TYPE B IN A COMPLEX PROJECTIVE SPACE

By

U-HANG KI*, SADAHIRO MAEDA** and YOUNG-JIN SUH†

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Abstract. The purpose of this paper is to give some characterizations of homogeneous real hypersurfaces of type B in a complex projective space $P_n(C)$ in terms of Lie derivative.

1. Introduction

Let $P_n(C)$ be an n-dimensional complex projective space with Fubini-Study metric G of constant holomorphic sectional curvature 4, and let M be a real hypersurface of $P_n(C)$. Then M has an almost contact metric structure (ϕ, ξ, η, g) (cf. §1) induced from the complex structure J of $P_n(C)$. R. Takagi [15] classified homogeneous real hypersurfaces in $P_n(C)$. Due to his work, we can see that a homogeneous real hypersurface of $P_n(C)$ is locally congruent to one of the six model spaces of type A_1, A_2, B, C, D and E (cf. Theorem A). In differential geometry of real hypersurfaces it is very interesting to characterize homogeneous ones. In particular, there are many characterizations of homogeneous ones of type A_1 and A_2, since these two examples have a lot of beautiful geometric properties. We here pay attention to the work of Okumura [14]. He showed that a real hypersurface M of $P_n(C)$ is locally congruent to one of homogeneous ones of type A_1 and A_2 if and only if the structure vector ξ is an infinitesimal isometry, that is, $L_\xi g=0$, where L is the Lie derivative. Motivated by this result, the first author, Kim and Lee [3] proved the fact that "M is of type A_1 or type A_2" is equivalent to "$L_\xi A=0$, where A is the shape operator of M".

In this paper, we investigate real hypersurfaces M of $P_n(C)$ in terms of

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Lie derivative on M. Until now there are few results about characterizations of homogeneous real hypersurfaces of type B in $P_n(C)$. The main purpose of this paper is to characterize this manifold in the class of real hypersurfaces on which the structure vector ξ is principal. We have the following:

Theorem 1. *Let M be a real hypersurface of $P_n(C)$. Suppose that ξ is a principal curvature vector with principal curvature $\alpha = 2\cot 2r$ and M satisfies*

$$(L_\xi A)^2 = -c^2\phi^2,$$

where c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B, which lies on a tube of radius r over a complex quadric Q_{n-1}, where $0<r<\pi/4$.

Theorem 2. *Let M be a real hypersurface with constant mean curvature in $P_n(C)$. Suppose that ξ is a principal curvature vector with principal curvature $\alpha = 2\cot 2r$ and M satisfies*

$$(L_\xi S)^2 = -c^2\phi^2,$$

where S is the Ricci tensor of M and c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B, which lies on a tube of radius r over a complex quadric Q_{n-1}, where $0<r<\pi/4$ but $\cot^2 2r \neq n-2$.

We now recall the work of Kimura [7]. He constucted a certain class of non-homogeneous real hypersurfaces (in $P_n(C)$) satisfying $(L_\xi\phi)^2=0$. It is well known that the conditions "$L_\xi\phi=0$", "$\phi A = A\phi$" and "$L_\xi g=0$" are mutually equivalent (cf. [11], [13], [14]). It is also interesting to study non-homogeneous real hypersurfaces. In this paper, we investigate some conditions which are equivalent to condition "$(L_\xi\phi)^2=0$". For this purpose we here define a linear map A^0 on the tangent bundle TM in such a way that

$$A^0X = AX - \eta(X)A\xi - \eta(AX)\xi + \eta(A\xi)\eta(X)\xi$$

for any vector field X in M. Thus for this linear map we have the following (cf §4):

Proposition. *The following conditions are equivalent:*
(i) $(L_\xi\phi)^2=0$,
(ii) $\phi A^0 = A^0\phi$,
(iii) $\nabla_Z\xi$ *is a* $(0, 1)$*-vector for any* $Z\in T^0M^{(0,1)}$.

In connection with condition (iii) in Proposition we recall the following well known fact: Let M be a Kaehler manifold with complex structure J. Then the following are equivalent:

(i) $L_X J=0$,

(ii) $\nabla_Z X$ is a (0, 1)-vector for any (0, 1)-vector Z.

2. Preliminaries

Let M be an orientable real hypersurface of $P_n(C)$ and let N be a unit normal vector field on M. The Riemannian connections $\tilde{\nabla}$ in $P_n(C)$ and ∇ in M are related by the following formulas for any vector fields X and Y on M:

$$\tilde{\nabla}_X Y=\nabla_X Y+g(AX, Y)N, \tag{1.1}$$

$$\tilde{\nabla}_X N=-AX, \tag{1.2}$$

where g denotes the Riemannian metric of M induced from the Fubini-Study metric G of $P_n(C)$ and A is the shape operator of M in $P_n(C)$. An eigenvector X of the shape operator A is called a principal curvature vector. Also an eigenvalue λ of A is called a principal curvature. In what follows, we denote by V_λ the eigenspace of A associated with eigenvalue λ. It is known that M has an almost contact metric structure induced from the complex structure J on $P_n(C)$, that is, we define a tensor field ϕ of type (1, 1), a vector field ξ and a 1-form η on M by

$$g(\phi X, Y)=G(JX, Y)$$

and

$$g(\xi, X)=\eta(X)=G(JX, N).$$

Then we have

$$\phi^2 X=-X+\eta(X)\xi, \qquad g(\xi, \xi)=1, \qquad \phi\xi=0. \tag{1.3}$$

It follows from (1.1) that

$$(\nabla_X\phi)Y=\eta(Y)AX-g(AX, Y)\xi, \tag{1.4}$$

$$\nabla_X\xi=\phi AX. \tag{1.5}$$

Let $\tilde{R}$ and R be the curvature tensors of $P_n(C)$ and M, respectively. Since the curvature tensor $\tilde{R}$ has a nice form, we have the following Gauss and Codazzi equations:

$$\begin{aligned} g(R(X, Y)Z, W)=&\ g(Y, Z)g(X, W)-g(X, Z)g(Y, W)+g(\phi Y, Z)g(\phi X, W)\\ &-g(\phi X, Z)g(\phi Y, W)-2g(\phi X, Y)g(\phi Z, W)\\ &+g(AY, Z)g(AX, W)-g(AX, Z)g(AY, W), \end{aligned} \tag{1.6}$$

$$(\nabla_X A)Y-(\nabla_Y A)X=\eta(X)\phi Y-\eta(Y)\phi X-2g(\phi X, Y)\xi. \tag{1.7}$$

From (1.3), (1.5), (1.6) and (1.7) we get

$$SX=(2n+1)X-3\eta(X)\xi+hAX-A^2X, \tag{1.8}$$

$$\begin{aligned}(\nabla_X S)Y=&-3\{g(\phi AX, Y)\xi+\eta(Y)\phi AX\}+(Xh)AY\\&+(hI-A)(\nabla_X A)Y-(\nabla_X A)AY,\end{aligned} \tag{1.9}$$

where $h=trace\,A$, S is the Ricci tensor of type (1, 1) on M and I is the identity map.

In the following, we use the same terminology and notations as above unless otherwise stated. Now we prepare without proof the following in order to prove our theorems:

Theorem A ([15]). *Let M be a homogeneous real hypersurface of $P_n(C)$. Then M is a tube of radious r over one of the following Kaehler submanifolds:*

(A_1) *hyperplane $P_{n-1}(C)$, where $0<r<\pi/2$,*
(A_2) *totally geodesic $P_k(C)$ $(1\leqq k\leqq n-2)$, where $0<r<\pi/2$,*
(B) *complex quadric Q_{n-1}, where $0<r<\pi/4$,*
(C) *$P_1(C)\times P_{(n-1)/2}(C)$, where $0<r<\pi/4$ and $n(\geqq 5)$ is odd,*
(D) *complex Grassmann $G_{2,5}(C)$, where $0<r<\pi/4$ and $n=9$,*
(E) *Hermitian symmetric space $SO(10)/U(5)$, where $0<r<\pi/4$ and $n=15$.*

Theorem B ([5]). *Let M be a real hypersurface of $P_n(C)$. Then M has constant principal curvatures and ξ is a principal curvature vector if and only if M is locally congruent to a homogeneous real hypersurface.*

Proposition A ([12]). *Assume that ξ is a principal curvature vector and the corresponding principal curvature is α. Then α is locally constant. In addition,*

$$A\phi X=\frac{\alpha\lambda+2}{2\lambda-\alpha}\phi X$$

holds for any $X(\perp\xi)\in V_\lambda$.

3. Proof of Theorem 1

Let us suppose that the structure vector field ξ is principal with corresponding principal curvature α. For any $X\in TM$ from (1.5) and (1.7) we have

$$\begin{aligned}(L_\xi A)X&=L_\xi(AX)-AL_\xi X=[\xi, AX]-A[\xi, X]\\&=\nabla_\xi(AX)-\nabla_{AX}\xi-A(\nabla_\xi X-\nabla_X\xi)\\&=(\nabla_X A)\xi+\phi X-\phi A^2X+A\phi AX\\&=\nabla_X(A\xi)-A(\nabla_X\xi)+\phi X-\phi A^2X+A\phi AX\\&=\alpha\phi AX+\phi X-\phi A^2X.\end{aligned} \tag{2.1}$$

Hence we have

$$(L_\xi A)\xi=0,$$

from which

$$(L_\xi A)^2\xi=-c^2\phi^2\xi$$

holds for any c. Now let us consider any X in TM orthogonal to ξ such that $AX=\lambda X$. Substituting in (2.1), we get

$$(L_\xi A)X=(-\lambda^2+\alpha\lambda+1)\phi X.$$

From this and Proposition A we have

$$(L_\xi A)^2X=(-\lambda^2+\alpha\lambda+1)\left\{-\left(\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)^2+\alpha\frac{\alpha\lambda+2}{2\lambda-\alpha}+1\right\}\phi^2X \tag{2.2}$$

for any X in V_λ $(=\{X:\ AX=\lambda X,\ X\perp\xi\})$. Thus from (2.2) and the assumption

$$(L_\xi A)^2=-c^2\phi^2$$

we know that λ is constant. Therefore the manifold M satisfying the hypothesis of our Theorem 1 must be a homogeneous real hypersurface which is a tube of radius r (cf. Theorem A and Theorem B).

The rest of the proof is to check

$$(L_\xi A)^2=-c^2\phi^2,\qquad c\neq 0$$

for homogeneous ones:

Let us notice that $\cot r$, $-\tan r$ are solutions for the quadratic equation $\lambda^2-\alpha\lambda-1=0$, because we have put $\alpha=2\cot 2r$ (cf. [1]). Note that for a case where M is of type A_1, A_2, C, D or E this manifold M has principal curvatures $\cot r$ and $-\tan r$ (cf. [16]). Accordingly, from (2.2) we know that each of them does not satisfy

$$(L_\xi A)^2=-c^2\phi^2,\qquad c\neq 0.$$

Next let us consider for a case where M is of type B. Let $x=\cot r$ (>1). Then M has three distinct constant principal curvatures

$$\lambda_1=\frac{1+x}{1-x}\qquad\text{with multiplicity}\quad n-1,$$

$$\lambda_2=\frac{x-1}{x+1}=\frac{\alpha\lambda_1+2}{2\lambda_1-\alpha}\qquad\text{with multiplicity}\quad n-1$$

and

$$\alpha=\frac{x^2-1}{x}(\neq 0)\qquad\text{with multiplicity } 1$$

(cf. [16]). From this we can calculate the following

$$-\lambda_1^2+\alpha\lambda_1+1=-\frac{(1+x^2)^2}{x(1-x)^2}$$

and

$$-\lambda_2^2+\alpha\lambda_2+1=\frac{(x^2+1)^2}{x(x+1)^2}.$$

And hence a computation yields that $(L_\xi A)^2=-c^2\phi^2$ holds, when

$$c^2=\left\{\frac{(x^2+1)^2}{x(x^2-1)}\right\}^2(\neq 0).$$

So we can conclude that a real hypersurface of type B satisfies the assumption of Theorem 1. This completes the proof of Theorem 1.

Remarks. (1) Due to the proof of Theorem 1, we get

Corollary 1. *Let M be a real hypersurface of $P_n(C)$. Suppose that ξ is a principal curvature vector with principal curvature $\alpha=2\cot 2r$ and M satisfies*

$$(\nabla_\xi A)^2=-c^2\phi^2,$$

where c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B with radius r, where $0<r<\pi/4$.

(2) Kimura and the second author classified real hypersurfaces M satisfying $\nabla_\xi A=0$ in $P_n(C)$ (cf. [8]).

4. Proof of Theorem 2

From the assumption we can put $A\xi=\alpha\xi$. Now for any $X\in TM$ let us calculate the following

$$\begin{aligned}(L_\xi S)X&=L_\xi(SX)-S(L_\xi X)=[\xi, SX]-S[\xi, X]\\ &=\nabla_\xi(SX)-\nabla_{SX}\xi-S(\nabla_\xi X-\nabla_X\xi)\\ &=(\nabla_\xi S)X-\phi ASX+S\phi AX.\end{aligned}\tag{3.1}$$

Since ξ is an eigenvector of S and h is constant, we find that

$$(L_\xi S)\xi=L_\xi(S\xi)-S(L_\xi\xi)=0,$$

from which

$$(L_\xi S)^2\xi=-c^2\phi^2\xi$$

holds for any c.

Now let us consider any X in TM orthogonal to ξ such that $AX=\lambda X$.

Then Equation (1.8) yields the following:

$$\phi ASX=\lambda(2n+1+h\lambda-\lambda^2)\phi X,$$

$$S\phi AX=\lambda S\phi X=\lambda\left\{2n+1+h\cdot\frac{\alpha\lambda+2}{2\lambda-\alpha}-\left(\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)^2\right\}\phi X.$$

Next we compute $(\nabla_\xi S)X$. Since we have assumed that the mean curvature of M is constant, from (1.9) for any $X\in V_\lambda=\{X\in TM:\ AX=\lambda X\}$ it follows that

$$\begin{aligned}(\nabla_\xi S)X&=(hI-A)(\nabla_\xi A)X-\lambda(\nabla_\xi A)X\\&=\frac{\alpha}{2}\left(\lambda-\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)\left(h-\lambda-\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)\phi X\end{aligned}$$

where we have used the equation of Codazzi (1.7) and Proposition A. Substituting these equations into (3.1), we get

$$(L_\xi S)X=\left(\frac{\alpha}{2}-\lambda\right)\left(\lambda-\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)\left(h-\lambda-\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)\phi X$$

for any $X\in V_\lambda$ so that

$$\begin{aligned}(L_\xi S)^2X=&\left(\frac{\alpha}{2}-\lambda\right)\left(\frac{\alpha}{2}-\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)\left(\lambda-\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)\\&\times\left(\frac{\alpha\lambda+2}{2\lambda-\alpha}-\lambda\right)\left(h-\lambda-\frac{\alpha\lambda+2}{2\lambda-\alpha}\right)^2\phi^2X.\end{aligned}\tag{3.2}$$

Thus from the assumption of Theorem 2 we know that (3.2) implies λ is constant. Accordingly, by Theorem B the manifold M satisfying the hypothesis of Theorem 2 must be homogeneous. Therefore M is locally congruent to a homogeneous one of type A_1, A_2, B, C, D or E. First let us consider for a case where M is of type A_1, A_2, C, D or E. Then the same discussion as in the proof of Theorem 1 asserts that our manifold M does not satisfy the hypothesis of Theorem 2.

Next it remains only to find a nonzero constant c for a case where M is of type B. Then making use of the method given in the proof of Theorem 1, we can calculate the right hand side of (3.2) as follows:

$$\frac{\alpha}{2}-\lambda_1=-\frac{(x+1)(x^2+1)}{2x(1-x)},\qquad \frac{\alpha}{2}-\lambda_2=\frac{(x-1)(x^2+1)}{2x(1+x)},$$

$$\lambda_1-\lambda_2=\frac{2(1+x^2)}{(1+x)(1-x)},\quad\text{and}\quad h-\lambda_1-\lambda_2=\frac{4(n-2)x}{(1+x)(1-x)}+\frac{x^2-1}{x}.$$

Summing up these formulas, we know that $(L_\xi S)^2=-c^2\phi^2$ holds, when

$$c^2=\left(\frac{1+x^2}{2x}\right)^2\left\{\frac{2(1+x^2)}{(1+x)(1-x)}\right\}^2\left\{\frac{4(n-2)x}{(1+x)(1-x)}+\frac{x^2-1}{x}\right\}^2.$$

Solving

$$\frac{4(n-2)x}{(1+x)(1-x)}+\frac{x^2-1}{x}=0,$$

we get

$$x=\cot r=\sqrt{n-1}+\sqrt{n-2} \quad (>1)$$

so that $\cot^2 2r=n-2$. This completes the proof of Theorem 2.

Remarks. (3) By virtue of the proof of Theorem 2, we have

Corollary 2. *Let M be a real hypersurface with constant mean curvature in $P_n(C)$. Suppose that ξ is a principal curvature vector with principal curvature $\alpha=2\cot 2r$ and M satisfies $(\nabla_\xi S)^2=-c^2\phi^2$, where S is the Ricci tensor of M and c is nonzero locally constant. Then M is locally congruent to a homogeneous real hypersurface of type B with radius r, where $0<r<\pi/4$ but $\cot^2 2r\neq n-2$.*

(4) The homogeneous real hypersurface M of type B in the case of $\cot^2 2r=n-2$ satisfies $L_\xi S=\nabla_\xi S=0$. In addition, the M is the only tube over Q_{n-1} which is pseudo-Einstein (cf. [1]).

(5) The first author ([2]) proved that $P_n(C)$ $(n\geqq 3)$ does not admit a real hypersurface M with parallel Ricci tensor.

(6) The second author classified real hypersurfaces M satifying $\nabla_\xi S=0$ (in $P_n(C)$) under the condition that ξ is a principal curvature vector and that focal map ϕ_r has constant rank on M (cf. [10]).

5. Proof of Proposition

A real hypersurface which is foliated by a totally geodesic complex hypersurface $P_{n-1}(C)$ of $P_n(C)$ is said to be a ruled real hypersurface. Moreover, it is known that this manifold M is non-complete non-homogeneous real hypersurface and its structure vector field ξ is not principal. Recently it is known that a real hypersurface M is locally congruent to a ruled real hypersurface if and only if the shape operator A of M satisfies

$$g(AX, Y)=0 \tag{4.1}$$

for any vector fields X, Y in T^0M, where T^0M denotes the orthogonal distribution defined by

$$T^0M=\{X\in TM:\ X\perp\xi\}$$

(cf. [6]). Then (4.1) can be rewritten as

$$g(A(X-\eta(X)\xi),\ Y-\eta(Y)\xi)=0$$

for any vector fields X, Y in TM. Thus we can see that (4.1) is equivalent to

$$AX-\eta(X)A\xi-\eta(AX)\xi+\eta(A\xi)\eta(X)\xi=0$$

for any $X\in TM$. From this formula we can define a linear map A^0: $TM\to TM$ by

$$A^0X=AX-\eta(X)A\xi-\eta(AX)\xi+\eta(A\xi)\eta(X)\xi \tag{4.2}$$

for any vector field X in TM. Then $A^0=0$ if and only if a real hypersurface M is locally congruent to a ruled real hypersurface. Also from (4.2) we know that

$$A^0\xi=0. \tag{4.3}$$

On the other hand, from (1.3), (1.4) and (1.5) we get

$$(L_\xi\phi)X=\eta(X)A\xi-AX-\phi A\phi X$$

for any $X\in TM$. Then a direct calculation shows that

$$(L_\xi\phi)^2X=(\phi A-A\phi)^2X+\eta(X)\{\eta(A\xi)A\xi-A^2\xi-\phi A\phi A\xi\}-g(\phi A\xi,\ X)\phi A\xi.$$

And hence $(L_\xi\phi)^2=0$ is equivalent to

$$(\phi A-A\phi)^2X=g(\phi A\xi,\ X)\phi A\xi+\eta(X)\{\phi A\phi A\xi+A^2\xi-\eta(A\xi)A\xi\} \tag{4.4}$$

for any $X\in TM$. Then from (4.4), using the symmetry of $(\phi A-A\phi)^2$ and (1.3), we can verify

$$(\phi A-A\phi)X=\eta(X)\phi A\xi+g(\phi A\xi,\ X)\xi$$

for any $X\in TM$ (cf. [9]). This, together with (4.2), implies that (i) and (ii) are equivalent to each other. Thus we are now in a position to prove that (ii) and (iii) are equivalent to each other. Now let us denote by T^0M^C a complexification of T^0M. Then we have

$$T^0M^C=T^0M^{(1,0)}\oplus T^0M^{(0,1)}$$

with respect to ϕ, where

$$T^0M^{(1,0)}=\{Z\in T^0M^C:\ \phi Z=\sqrt{-1}Z\}=\{X-\sqrt{-1}\phi X:\ X\in T^0M\},$$
$$T^0M^{(0,1)}=\{Z\in T^0M^C:\ \phi Z=-\sqrt{-1}Z\}=\{X+\sqrt{-1}\phi X:\ X\in T^0M\}.$$

For any $Z(=X+\sqrt{-1}\phi X)\in T^0M^{(0,1)}$, from (1.5) we have

$$\nabla_Z\xi=\nabla_X\xi+\sqrt{-1}\nabla_{\phi X}\xi=\phi AX+\sqrt{-1}\phi A\phi X\in T^0M^C,$$

where $X\in T^0M$. For any X in TM let us substitute (4.2) into the right hand side of the above equation. Then

$$\nabla_Z\xi=\phi A^0X+\sqrt{-1}\phi A^0\phi X$$

for any $Z(=X+\sqrt{-1}\phi X)$.

(ii)$\Rightarrow$(iii): From the condition (ii) and (4.3) it follows that

$$\begin{aligned}\nabla_Z\xi&=\phi A^0X+\sqrt{-1}\phi^2A^0X=\phi A^0X+\sqrt{-1}(-A^0X+\eta(A^0X)\xi)\\&=\phi A^0X-\sqrt{-1}A^0X.\end{aligned}$$

From this, applying ϕ, we get

$$\begin{aligned}\phi(\nabla_Z\xi)&=\phi^2A^0X-\sqrt{-1}\phi A^0X=-A^0X+\eta(A^0X)\xi-\sqrt{-1}\phi A^0X\\&=-\sqrt{-1}(\phi A^0X-\sqrt{-1}A^0X).\end{aligned}$$

This shows that $\nabla_Z\xi$ is a (0, 1)-vector with respect to ϕ.

(iii)$\Rightarrow$(ii): By assumption we have have for any X in T^0M

$$\phi(\phi A^0X+\sqrt{-1}\phi A^0\phi X)=-\sqrt{-1}(\phi A^0X+\sqrt{-1}\phi A^0\phi X).$$

This, combined with (1.3) and (4.3), shows that

$$-A^0X-\sqrt{-1}A^0\phi X=-\sqrt{-1}\phi A^0X+\phi A^0\phi X.$$

Thus for any X in T^0M we have

$$A^0\phi X=\phi A^0X.$$

Therefore we can conclude that

$$A^0\phi=\phi A^0.$$

This completes the proof of Proposition.

Remarks. (7) By virtue of Proposition and the papers [4] and [9] we know that any condition in Proposition is equivalent to the following:

(i) $g((\phi A-A\phi)X, Y)=0$ for any X, Y in T^0M,

(ii) $(L_\xi g)(X, Y)=0$ for any X, Y in T^0M,

(iii) $g((L_\xi\phi)X, Y)=0$ for any X, Y in T^0M,

(iv) $(\phi A-A\phi)X=\eta(X)\phi A\xi-g(\phi A\xi, X)\xi$ for any X in TM.

(8) It is known that real hypersurfaces of type A_1 and A_2 and ruled real hypersurfaces satisfy the above equivalent conditions. Moreover, it can be easily verified that a real hypersurface M (with principal structure vector field ξ) satisfying one of the above equivalent conditions is locally congruent to one of type A_1 and A_2 (cf. [11]). But the classification problem of real hypersurfaces satisfing the above equivalent condition remains still open.

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U-Hang Ki
Department of Mathematics
Kyungpook University
Taegu, 702-701, Korea

Sadahiro Maeda
Department of Mathematics
Shimane University
Matsue, Shemani 690
Japan

Young-Jin Suh
Department of Mathematics
Kyungpook University
Taegu, 702-701, Korea