CORRECTION TO "ON RANDOM TRANSLATIONS OF POINT PROCESSES"

By

TOSHIO MORI

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In the above titled article (Yokohama Math. J. Vol. 19, 119-139) the following corrections should be made to complete the proof of existence theorem. The author is indebted to Mr. T. Shiga for pointing out mistakes.

1. The expressions in lines 11 and 12, page 126, should be read

$$\limsup_{j\to\infty} L(t_1, \cdots, t_n, t_1, \cdots, t_n : K_1, \cdots, K_n, G_{1j}\cap K_1^c, \cdots, G_{nj}\cap K_n^c)\ ,$$

and

$$\limsup_{j\to\infty} L(t_1, \cdots, t_n, \max_{1\le i\le n} t_i\, ; K_1, \cdots, K_n, \bigcup_{i=1}^{n} (G_{ij}\cap K_i^c))\ ,$$

respectively.

2. On the same page lines 15-21 should be corrected as follows:
For a finite number of bounded Borel sets $A_1, \cdots, A_n$ let us define

$$L^*(t_1, \cdots, t_n\, ; A_1, \cdots, A_n)=L(s_1, \cdots, s_m\, ; B_1, \cdots, B_m)\ ,$$

where $t_i\ge 0$, B_j's are disjoint bounded Borel sets such that every A_i is a union of some B_j's, and every s_j is the sum of t_i's such that $A_i\supset B_j$. This definition does not depend on the choice of B_j's, and $L^*(\cdot, \cdots, \cdot\, ; A_1, \cdots, A_n)$ is Laplace transform of a probability distribution on R^{n+}. It is easily seen that

$$\text{(3.1a)}\qquad L^*(t_1, \cdots, t_n\, ; A_1, \cdots, A_n)\ge L^*(t_1, \cdots, t_n\, ; A_1', \cdots, A_n')\ ,$$

if A_i and A_i' are bounded Borel sets such that $A_i\subset A_i'$, $1\le i\le n$. It follows from (v) that (3.1) holds if A_i''s are disjoint bounded F_σ. Thus (3.1) replaced L by L^* holds for bounded (not necessarily disjoint) open sets A_i, $1\le i\le n$.

If $A_1, \cdots, A_n$ are disjoint bounded Borel sets then for every i, $1\le i\le n$, there exist an increasing sequence $K_{i1}, K_{i2}, \cdots$ of compact sets and a decreasing sequence $U_{i1}, U_{i2}, \cdots$ of bounded open sets such that $K_{ij}\subset A_i\subset U_{ij}$, $1\le i\le n$, $j=1, 2, \cdots$, and

(3.1b)
$$\begin{aligned}&\lim_{j\to\infty}\int\exp\Big[-\sum_{i=1}^{n}t_i\mu(K_{ij})\Big]P(d\mu)\\&=\lim_{j\to\infty}\int\exp\Big[-\sum_{i=1}^{n}t_i\mu(U_{ij})\Big]P(d\mu)\\&=\int\exp\Big[-\sum_{i=1}^{n}t_i\mu(A_i)\Big]P(d\mu)\,.\end{aligned}$$

In fact let $U_0\supset\bigcup_{i=1}^{n}A_i$ be a bounded open set, $K_0=\bar{U}_0$, and $M_n=\{\mu\,;\,\mu(K_0)\leq n\}$. For any $\varepsilon>0$ there exist n such that $P(M_n)>1-\varepsilon$. Let ν_n be a measure on $\mathfrak{B}$ defined by

$$\nu_n(B)=\int_{M_n}\mu(B)P(d\mu)\,,\qquad B\in\mathfrak{B}\,.$$

For every $\varepsilon>0$ and i there exist a compact set K_i and a bounded open set U_i such that $K_i\subset A_i\subset U_i$ and

$$\nu_n(K_i)+\varepsilon>\nu_n(A_i)>\nu_n(U_i)-\varepsilon\,,\qquad 1\leq i\leq n\,.$$

Hence we have

$$\begin{aligned}&0\leq L^*(t_1,\cdots,t_n\,;K_1,\cdots,K_n)-L^*(t_1,\cdots,t_n\,;A_1,\cdots,A_n)\\&=\int\exp\Big[-\sum_{i=1}^{n}t_i\mu(K_i)\Big]\Big\{1-\exp\Big[-\sum_{i=1}^{n}t_i\mu(A_i\cap K_i^c)\Big]\Big\}P(d\mu)\\&=\int_{M_n}+\int_{M_n^c}<\sum_{i=1}^{n}t_i\int_{M_n}\mu(A_i\cap K_i^c)P(d\mu)+\varepsilon\\&=\sum_{i=1}^{n}t_i\{\nu_n(A_i)-\nu_n(K_i)\}+\varepsilon<\varepsilon\Big(\sum_{i=1}^{n}t_i+1\Big)\,.\end{aligned}$$

This shows the existence of a sequence $\{K_{ij}\}$ satisfying (3.1b). The existence of $\{U_{ij}\}$ is similarly proved.

Since (3.1) holds for A_i replaced by either K_{ij} or U_{ij}, it follows from (3.1a) and (3.1b) that (3.1) holds for A_i. This completes the proof.