GENERALISATION OF GAUSS-CODAZZI EQUATIONS IN A GENERALISED FINSLER SPACE

By

SURENDRA PRATAP SINGH

(Received October 20, 1972)

The theory of hypersurfaces of a generalised *Finsler* space has been developed by *Shamihoke* (1963) on the lines of *Rund* (1956, 1959). He has also obtained two alternative forms of *Gauss-Codazzi* equations. The object of present paper is to generalise the equations of *Gauss and Codazzi* by considering a congruence of curves associated to a hypersurface of a generalised *Finsler* space and to study its special cases. The results obtained in the first case coincide with the generalisation of *Gauss-Codazzi* equations of *Shamihoke* (1963) in the generalised Finsler space. On the other hand the results of third case are same as the generalisation of *Rund* (1956) when the connection parameters $\bar{P}^{*\alpha}_{\beta\gamma}$ are considered to be symmetric in the lower indices, that is, when the space is a Finsler space.

1. Introduction. We consider an n-dimensional generalised Finsler space F_n with local coordinates $x^i (i=1, 2, \cdots, n)$ in which the distance between two neighbouring points $P(x^i)$ and $Q(x^i+dx^i)$ is given by $ds=F(x^i, dx^i)$. The distance function F satisfies the usual conditions imposed upon such function. The symmetric part of the metric tensor $g_{ij}(x, \dot{x})$ is given by

$$g_{(ij)}(x, \dot{x})=\frac{1}{2}\frac{\partial^2 F^2(x, \dot{x})}{\partial \dot{x}^i \partial \dot{x}^j}. \tag{1.1}$$

The covariant derivative of a vector X^i is defined by

$$\delta_k X^i=\frac{\partial X^i}{\partial x^k}+\frac{\partial X^i}{\partial \dot{x}^h}\frac{\partial \dot{x}^h}{\partial x^k}+P^{*i}_{hk}X^h. \tag{1.2}$$

where

$$P^{*i}_{hk}=\Delta^i_{hk}-(C^i_{hp}P^p_{kl}+C^i_{kp}P^p_{hl}-g^{(im)}C_{hkp}P^p_{ml})\dot{x}^l, \tag{1.3}$$

$$P^{*i}_{hk}=\Delta^i_{hk}-C^i_{hm}\Delta^m_{pk}\dot{x}^p.$$

We also have the following relations,

(1.4) $$P^{*i}_{[hk]}=\frac{1}{2}(P^{*i}_{hk}-P^{*i}_{kh})=\Delta^i_{[hk]},$$

$$\dot{x}^i\delta_k g_{(ij)}=\dot{x}^j\delta_k g_{[ij]}=0\ .$$

In a hypersurface F_{n-1} of F_n, the $(n-1)$ parameters u^α are the coordinates and the matrix of projection parameters $B_\alpha{}^i=\partial x^i/\partial u^\alpha$ is supposed to be of rank $(n-1)$. If the directional argument $\dot{x}^i$ be tangential to F_{n-1} and be denoted by $\dot{u}^\alpha$ in terms of the coordinates of F_{n-1}, then

(1.5) $$\dot{x}^i=B_{\alpha i}\dot{u}^\alpha\ .$$

The metric tensor ${}^1g_{\alpha\beta}(u,\dot{u})$ of F_{n-1}, the hypersurface is defined by

(1.6) $${}^1g_{\alpha\beta}(u,\dot{u})=g_{ij}(x,\dot{x})B^{ij}_{\alpha\beta}\ ,$$

where

$$B^{ij}_{\alpha\beta}{}^{\cdots}_{\cdots}=B_\alpha{}^iB_\beta{}^j\cdots\ .$$

From (1.6), we have

(1.7) $$\frac{\partial\,{}^1g_{[\alpha\beta]}(u,\dot{u})}{\partial\dot{u}^\alpha}=0\ .$$

The metric function $F(x,\dot{x})$ of the enveloping space F_n induces a metric function $\bar{F}(u,\dot{u})$ of F_{n-1} which is given by

(1.8) $$\bar{F}(u,\dot{u})=F(x^i(u^\alpha),\ B_\alpha{}^i\dot{u}^\alpha)\ .$$

From (1.6) and (1.8), we deduce

(1.9) $${}^1g_{(\alpha\beta)}(u,\dot{u})=\frac{1}{2}\frac{\partial^2\bar{F}^2(u,\dot{u})}{\partial\dot{u}^\alpha\partial\dot{u}^\beta}\ .$$

The conjugate tensor ${}^1g^{(\alpha\beta)}(u,\dot{u})$ of the metric tensor ${}^1g_{(\alpha\beta)}(u,\dot{u})$ is defined by

(1.10) $${}^1g_{(\alpha\beta)}(u,\dot{u})\,{}^1g^{(\alpha\tau)}(u,\dot{u})=B_\beta{}^\tau\ .$$

Now we may introduce the quantities

(1.11) $$B_i{}^\alpha=g_{(ij)}\,{}^1g^{(\alpha\delta)}B_\delta{}^j$$

and

(1.12) $$B_\alpha{}^iB_i{}^\delta=\delta_\alpha{}^\delta\ ,\quad B_j{}^i=B_\alpha{}^iB_j{}^\alpha\ .$$

One set of normals are defined at each point of F_{n-1} by the system of equations

(1.13) $$g_{(ij)}(x,\dot{x})B_\alpha{}^in^{*j}(x,\dot{x})=0\ ,\quad g_{ij}(x,n^*)n^{*i}n^{*j}=1\ ,$$

which imply

(1.14) $$g_{(ij)}(x, \dot{x})n^{*j}=n_i^{*}\ , \quad n_i^{*}n^{*i}=\psi\ .$$

We have the relations

(1.15) $$B_\alpha{}^i n_i^{*}=0\ , \quad B_i{}^\alpha n^{*i}=0\ .$$

Furthermore, it is easily seen that

(1.16) $$g^{(ij)}(\dot{x}, \dot{x})={}^1g^{(\alpha\beta)}(u, \dot{u})B^{ij}_{\alpha\beta}+\frac{1}{\psi}n^{*i}n^{*j}\ .$$

The induced connection parameters $\bar{P}^{*\alpha}_{\beta\tau}$ of F_{n-1} are given as under

(1.17) $$\bar{P}^{*\alpha}_{\beta\tau}=B_i{}^\alpha(B^i_{\beta\tau}+P^{*i}_{hk}B^{hk}_{\beta\tau})\ ,$$

where

$$B^i_{\beta\tau}=\partial B_\beta{}^i/\partial u^\tau\ .$$

Here we may define a mixed tensor

(1.18) $$I^i_{\alpha\beta}=\delta_\beta{}^0B_\alpha{}^i=B^i_{\alpha\beta}+P^{*i}_{hk}B^{hk}_{\alpha\beta}-\bar{P}^{*\delta}_{\alpha\ \beta}B_\delta{}^i\ .$$

Multiplying (1.18) by $B_i{}^\tau$, we obtain

(1.19) $$B_i{}^\tau I^i_{\alpha\beta}=B_i{}^\tau(B^i_{\alpha\beta}+P^{*i}_{hk}B^{hk}_{\alpha\beta})-\bar{P}^{*\tau}_{\alpha\beta}=0\ .$$

From (1.15) and (1.19) we conclude that

(1.20) $$I^i_{\alpha\beta}=\Omega^{*}_{\alpha\beta}n^{*i}$$

such that

(1.21) $$\Omega^{*}_{\alpha\beta}=\frac{1}{\psi(x, \dot{x})}n_i^{*}(x, \dot{x})I^i_{\alpha\beta}(x, \dot{x})\ ,$$

where $\Omega^{*}_{\alpha\beta}$ are the coefficients of the secondary second fundament form.

The covariant derivative of the normal n^{*i} are expressed in the form

(1.22) $$\delta_\beta{}^0n^{*i}=-\psi\,{}^1g^{(\alpha\delta)}\Omega^{*}_{\delta\beta}B_\alpha{}^i+\frac{1}{2\psi}n^{*i}\delta_\beta\psi$$
$$-a^{*}_{jhk}n^{*j}B_\beta{}^k\left[g^{(hi)}(x, \dot{x})-\frac{1}{2\psi}n^{*h}n^{*i}\right],$$

where

$$a^{*}_{jhk}=\delta_k g_{(jh)}(x, \dot{x})\ .$$

2. Gauss-Codazzi Equations

We consider a set of $(n-m)$ congruences of curves such that one curve of which passes through each point of the hypersurface F_{n-1}. Let λ_i be the contravariant components of a unit vector in the direction of the curve of congruence. It can be expressed linearly in terms of $B_\alpha{}^i$ and the set of normals n^{*i} to F_{n-1}. Thus we write

$$\lambda^i = t^\alpha B_\alpha{}^i + \gamma n^{*i} , \tag{2.1}$$

where t^α and γ are the parameters.

Taking mixed covariant derivative of (2.1), it yields

$$\delta_\beta{}^0 \lambda^i = \delta_\beta t^\alpha B_\alpha{}^i + t^\alpha I^i_{\alpha\beta} + \delta_\beta \gamma n^{*i} + \gamma \delta_\beta{}^0 n^{*i} , \tag{2.2}$$

in view of (1.18).
By virtue of (1.20) and (1.22), the equation (2.2) becomes

$$\begin{aligned} \delta_\beta{}^0 \lambda^i = {} & \delta_\beta t^\alpha B_\alpha{}^i + t^\alpha \Omega^*_{\alpha\beta} n^{*i} + \delta_\beta \gamma n^{*i} + \gamma \Big[-\psi^{\,1} g^{(\alpha\delta)} \Omega^*_{\delta\beta} B_\alpha{}^i + \frac{1}{2\psi} n^{*i} \delta_\beta \psi \\ & - a^*_{jhk} n^{*j} B_\beta{}^k g^{(hi)}(x, \dot{x}) + a^*_{jhk} n^{*j} B_\beta{}^k \frac{1}{2\psi} n^{*i} n^{*h} \Big] . \end{aligned} \tag{2.3}$$

Differentiating (2.3) covariantly and making use of (1.18), (1.20) and (1.22), we obtain

$$\begin{aligned} \delta_\tau{}^0 \delta_\beta{}^0 \lambda^i = {} & \delta_\tau \delta_\beta t^\alpha B_\alpha{}^i + \delta_\beta t^\alpha \Omega^*_{\alpha\tau} n^{*i} + \delta_\tau t^\alpha \Omega^*_{\alpha\beta} n^{*i} - \psi t^\alpha \Omega^*_{\alpha\beta}\, {}^1 g^{(\varepsilon\delta)} \Omega^*_{\delta\tau} B_\varepsilon{}^i \\ & - \gamma a^*_{jhk} n^{*j} B_\tau{}^k g^{(hi)} t^\alpha \Omega^*_{\alpha\beta} - \delta_\beta \gamma a^*_{jhk} n^{*j} B_\tau{}^k g^{(hi)} - \delta_\tau \gamma a^*_{jhk} n^{*j} B_\beta{}^k g^{(hi)} \\ & - \frac{\gamma}{2\psi} \delta_\beta \psi a^*_{jhk} n^{*j} B_\tau{}^k g^{(hi)} - \gamma \delta_\tau{}^0 a^*_{jhk} n^{*j} B^k{}_\beta g^{(hi)} - \gamma a^*_{jhk} B_\beta{}^k g^{(hi)} \\ & \times \Big[-\psi^1 g^{(\alpha\delta)} \Omega^*_{\delta\tau} B_\alpha{}^j + \frac{1}{2\psi} n^{*j} \delta_\tau \psi - a^*_{lmp} n^{*l} B_\tau{}^p g^{(mj)} + a^*_{lmp} n^{*l} n^{*m} B_\tau{}^p n^{*j} \frac{1}{2\psi} \Big] \\ & - \gamma a^*_{jhk} n^{*j} \Omega^*_{\beta\tau} n^{*k} g^{(hi)} - \frac{\gamma}{2\psi} a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k a^*_{lmp} n^{*l} B_\tau{}^p g^{(mi)} \\ & - \gamma a^*_{jhk} n^{*j} B_\beta{}^k \delta_\tau{}^0 g^{(hi)} + n^{*i} P_{\beta\tau} - B_\alpha{}^i Q^\alpha_{\beta\tau} , \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} P_{\beta\tau} = {} & t^\alpha \delta_\tau \Omega^*_{\alpha\beta} + t^\alpha \Omega^*_{\alpha\beta} \frac{1}{2\psi} \delta_\tau \psi + t^\alpha \Omega^*_{\alpha\beta} a^*_{jhk} n^{*j} n^{*h} B_\tau{}^k \frac{1}{2\psi} + \delta_\tau \delta_\beta \gamma + \frac{1}{2\psi} \delta_\beta \gamma \delta_\tau \psi \\ & + \delta_\beta \gamma a^*_{jhk} n^{*j} n^{*h} B_\tau{}^k \frac{1}{2\psi} + \frac{1}{2\psi} \delta_\tau \gamma \delta_\beta \psi + \frac{1}{2\psi} \delta_\tau \gamma a^*_{jhk} n^{*j} B_\beta{}^k n^{*h} - \gamma \psi^{\,1} g^{(\alpha\delta)} \Omega^*_{\delta\beta} \Omega^*_{\alpha\tau} \\ & + \frac{1}{2} \gamma \delta_\tau \psi^{-1} \delta_\beta \psi + \frac{\gamma}{2} \psi^{-1} \delta_\beta \psi \left(\frac{1}{2\psi} \delta_\tau \psi + \frac{1}{2\psi} a^*_{jhk} n^{*j} n^{*h} B_\tau{}^k \right) + \frac{1}{2\psi} \gamma \delta_\tau \delta_\beta \psi \end{aligned} \tag{2.5}$$

$$+\gamma\delta_\tau{}^0 a^*_{jhk} n^{*j} B_\beta{}^k \frac{1}{2\psi} n^{*h} + \frac{\gamma}{2\psi} a^*_{jhk} \Omega^*_{\beta\tau} n^{*j} n^{*h} n^{*k} + \frac{1}{2}\gamma a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k \delta_\tau \frac{1}{\psi}$$

$$+\frac{\gamma}{\psi} a^*_{jhk} n^{*h} B_\beta{}^k \Big[-\psi\, {}^1g^{(\alpha\delta)} \Omega^*_{\delta\tau} B_\alpha{}^j + \frac{1}{2\psi} n^{*j} \delta_\tau\psi - a^*_{lmp} n^{*l} B_\tau{}^p g^{(mj)}$$

$$+\frac{1}{2\psi} a^*_{lmp} n^{*l} n^{*m} n^{*j} B_\tau{}^p \Big] + \frac{\gamma}{2\psi} a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k \Big(\frac{1}{2\psi}\delta_\tau\psi + \frac{1}{2\psi} a^*_{lmp} n^{*l} n^{*m} B_\tau{}^p \Big)$$

and

(2.6) $$Q^\alpha_{\beta\tau} = \psi\delta_\tau\gamma\, {}^1g^{(\alpha\delta)}\Omega^*_{\delta\beta} + \gamma\delta_\tau\psi\, {}^1g^{(\alpha\delta)}\Omega^*_{\delta\beta} + \gamma\psi\delta_\tau\, {}^1g^{(\alpha\delta)}\Omega^*_{\delta\beta} + \gamma\psi\, {}^1g^{(\alpha\delta)}\delta_\tau\Omega^*_{\delta\beta}$$

$$+\delta_\beta\gamma\psi\, {}^1g^{(\alpha\delta)}\Omega^*_{\delta\tau} - \frac{\gamma}{2}\, {}^1g^{(\alpha\delta)}\Omega^*_{\delta\tau}\, [\delta_\beta\psi + a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k] \,.$$

Interchanging the indices β, τ in (2.4) and subtracting the result from it, we get

(27) $$\delta^0_{[\tau}\delta^0_{\beta]}\lambda^i = \delta_{[\tau}\delta_{\beta]} t^\alpha B_\alpha{}^i - \psi\Omega^*_{\delta[\tau}\Omega^*_{|\alpha|\beta]} t^\alpha\, {}^1g^{(\varepsilon\delta)} B_\varepsilon{}^i - \gamma a^*_{jhk} n^{*j} g^{(hi)} t^\alpha B^k_{[\tau}\Omega^*_{|\alpha|\beta]}$$

$$-\gamma B^k_{[\beta}\delta^0_{\tau]} a^*_{jhk} g^{(hi)} n^{*j} + \gamma a^*_{jhk}\psi g^{(hi)}\, {}^1g^{(\alpha\delta)} B_\alpha{}^j \Omega^*_{\delta[\tau} B^k_{\beta]}$$

$$+\gamma a^*_{jhk} g^{(hi)} a^*_{lmp} n^{*l} g^{(mj)} B^{pk}_{[\tau\beta]} - \frac{\gamma}{2\psi} a^*_{jhk} g^{(hi)} a^*_{lmp} n^{*l} n^{*m} n^{*j} B^{pk}_{[\tau\beta]}$$

$$-\gamma a^*_{jhk} n^{*j} n^{*k} \Omega^*_{[\beta\tau]}\, g^{(hi)} - \frac{\gamma}{2\psi} a^*_{jhk} n^{*j} n^{*h} B^{pk}_{[\tau\beta]} a^*_{lmp} n^{*l} g^{(mi)}$$

$$-\gamma a^*_{jhk} n^{*j} \delta^0_{[\tau} g^{(hi)} B^k_{\beta]} + n^{*i} P_{[\beta\tau]} - B_\alpha{}^i Q^\alpha_{[\beta\tau]} \,.$$

It is easily seen that

(2.8) $$2\delta^0_{[\tau}\delta^0_{\beta]}\lambda^i = \lambda^l B^{kj}_{\beta\tau} \widetilde{K}_{ljk}{}^i - 2\delta_\delta{}^0 \lambda^i \overline{P}^{*\delta}_{[\beta\tau]}$$

and

(2.9) $$2\delta_{[\tau}\delta_{\beta]} t^\alpha = t^\varepsilon \widetilde{K}^\alpha_{\varepsilon\tau\beta} - 2\delta_\delta t^\alpha \overline{P}^{*\delta}_{[\beta\tau]} \,,$$

where

$$\widetilde{K}_{ljk}{}^i = \frac{\delta P^{*i}_{lk}}{\delta x^j} - \frac{\delta P^{*i}_{lj}}{\delta x^k} + P^{*h}_{lk} P^{*i}_{hj} - P^{*h}_{lj} P^{*i}_{hk}$$

and

$$\widetilde{K}^\alpha_{\varepsilon\tau\beta} = \frac{\delta \overline{P}^{*\alpha}_{\varepsilon\beta}}{\delta u^\tau} - \frac{\delta \overline{P}^{*\alpha}_{\varepsilon\tau}}{\delta u^\beta} + \overline{P}^{*\theta}_{\varepsilon\beta} \overline{P}^{*\alpha}_{\theta\tau} - \overline{P}^{*\theta}_{\varepsilon\tau} \overline{P}^{*\alpha}_{\theta\beta} \,.$$

By means of (2.8) and (2.9), the equation (2.7) takes the form

(2.10) $$\frac{1}{2}\lambda^l B^{kj}_{\beta\tau} \widetilde{K}_{ljk}{}^i - \delta_\delta{}^0 \lambda^i \overline{P}^{*\delta}_{[\beta\tau]} = \frac{1}{2} t^\varepsilon \widetilde{K}^\alpha_{\varepsilon\tau\beta} B_\alpha{}^i - \delta_\delta{}^0 t^\alpha \overline{P}^{*\delta}_{[\beta\tau]} B_\alpha{}^i - \psi\Omega^*_{\delta[\tau}\Omega^*_{|\alpha|\beta]} t^\alpha\, {}^1g^{(\varepsilon\delta)} B_\varepsilon{}^i$$

$$-\gamma a^*_{jhk} n^{*j} g^{(hi)} t^\alpha B^k_{[\tau}\Omega^*_{|\alpha|\beta]} - \gamma B^k_{[\beta}\delta^0_{\tau]} a^*_{jhk}\, g^{(hi)} n^{*j} + \gamma a^*_{jhk}\psi\, g^{(hi)}\, {}^1g^{(\alpha\delta)} B_\alpha{}^j \Omega^*_{\delta[\tau} B^k_{\beta]}$$

$$-\frac{\gamma}{2\phi}n^{*l}n^{*m}n^{*j}a^{*}_{jhk}g^{(hi)}a^{*}_{lmp}B^{pk}_{[\tau\beta]}+\gamma a^{*}_{jhk}g^{(hi)}a^{*}_{lmp}n^{*l}g^{(mj)}B^{pk}_{[\tau\beta]}$$

$$-\gamma a^{*}_{jhk}n^{*j}\Omega^{*}_{[\beta\tau]}n^{*k}g^{(hi)}-\frac{\gamma}{2\phi}a^{*}_{jhk}n^{*j}n^{*h}B^{pk}_{[\tau\beta]}a^{*}_{lmp}n^{*l}g^{(mi)}$$

$$-\gamma a^{*}_{jhk}n^{*j}\delta^{0}_{[\tau}g^{(hi)}B^{k}_{\beta]}+n^{*i}P_{[\beta\tau]}-B_{\alpha}{}^{i}Q^{\alpha}_{[\beta\tau]} .$$

Multiplying (2.10) by $B_i{}^\sigma$ and $n_i{}^*$ and simplifying with the help of (1.12), (1.14), (1.15), (1.16), (1.20), (1.22) and (2.1), we have respectively

$$\begin{aligned}
(2.11)\quad & \frac{1}{2}(t^\alpha B_\alpha{}^l+\gamma n^{*l})B_i{}^\sigma B^{kj}_{\beta\tau}\tilde{K}^{i}_{ljk}+\gamma[\phi\, {}^1g^{(\sigma\varepsilon)}\Omega^{*}_{\varepsilon\delta}+a^{*}_{jhk}n^{*j}B_\delta{}^k\, {}^1g^{(\sigma\varepsilon)}B_\varepsilon{}^h]\bar{P}^{*\delta}_{[\beta\tau]} \\
& =\frac{1}{2}t^\varepsilon\tilde{K}^{\sigma}_{\varepsilon\tau\beta}-\phi t^\alpha\Omega^{*}_{\delta[\tau}\Omega^{*}_{|\alpha|\beta]}\, {}^1g^{(\sigma\delta)}-\gamma a^{*}_{jhk}n^{*j}t^\alpha B^{k}_{[\tau}\Omega^{*}_{|\alpha|\beta]}\, {}^1g^{(\sigma\varepsilon)}B_\varepsilon{}^h \\
& -\gamma n^{*j}B^{k}_{[\beta}\delta^{0}_{\tau]}a^{*}_{jhk}\, {}^1g^{(\sigma\varepsilon)}B_\varepsilon{}^h+\gamma a^{*}_{jhk}\phi\, {}^1g^{(\sigma\varepsilon)}B^{hj}_{[\varepsilon\alpha]}\, {}^1g^{(\alpha\delta)}\Omega^{*}_{\delta[\tau}B^{k}_{\beta]} \\
& +\gamma a^{*}_{jhk}\, {}^1g^{(\sigma\varepsilon)}B_\varepsilon{}^h a^{*}_{lmp}n^{*l}g^{(mj)}B^{pk}_{[\tau\beta]}-\frac{\gamma}{2\phi}a^{*}_{jhk}\, {}^1g^{(\sigma\varepsilon)}B_\varepsilon{}^h a^{*}_{lmp}n^{*l}n^{*m}n^{*j}B^{pk}_{[\tau\beta]} \\
& -\gamma a^{*}_{jhk}n^{*j}\Omega^{*}_{[\beta\tau]}n^{*k}\, {}^1g^{(\sigma\varepsilon)}B_\varepsilon{}^h-\frac{\gamma}{2\phi}a^{*}_{jhk}n^{*j}n^{*h}B^{pk}_{[\tau\beta]}a^{*}_{lmp}n^{*l}\, {}^1g^{(\sigma\varepsilon)}B_\varepsilon{}^m \\
& -\gamma a^{*}_{jhk}n^{*j}B^{k}_{[\beta}\delta^{0}_{\tau]}g^{(hi)}B_i{}^\sigma-Q^{\sigma}_{[\beta\tau]}
\end{aligned}$$

and

$$\begin{aligned}
(2.12)\quad & \frac{1}{2}[t^\alpha B_\alpha{}^l+\gamma n^{*l}]n_i{}^*B^{kj}_{\beta\tau}\tilde{K}^{i}_{ljk}-\left[\phi t^\alpha\Omega^{*}_{\alpha\delta}+\phi\delta_\delta{}^\gamma+\frac{1}{2}\gamma\delta_i\phi-\frac{\gamma}{2}a^{*}_{jhk}n^{*j}B_\delta{}^k n^{*h}\right]\bar{P}^{*\delta}_{[\beta\tau} \\
& =\phi P_{[\beta\tau]}-\gamma a^{*}_{jhk}n^{*j}n^{*h}t^\alpha B^{k}_{[\tau}\Omega^{*}_{|\alpha|\beta]}-\gamma B^{k}_{[\beta}\delta^{0}_{\tau]}a^{*}_{jhk}n^{*j}n^{*h} \\
& +\gamma a^{*}_{jhk}\phi\, {}^1g^{(\alpha\delta)}B_\alpha{}^j\Omega^{*}_{\delta[\tau}B^{k}_{\beta]}n^{*h}+\gamma a^{*}_{jhk}n^{*h}g^{(mj)}a^{*}_{lmp}n^{*l}B^{pk}_{[\tau\beta]} \\
& -\frac{\gamma}{\phi}a^{*}_{jhk}n^{*h}a^{*}_{lmp}n^{*l}n^{*m}n^{*j}B^{pk}_{[\tau\beta]}-\gamma a^{*}_{jhk}n^{*j}\Omega^{*}_{[\beta\tau]}n^{*h}n^{*k}-\gamma a^{*}_{jhk}n^{*j}B^{k}_{[\beta}\delta^{0}_{\tau]}g^{(hi)}n_i{}^* ,
\end{aligned}$$

which are the generalised equations of Gauss and Codazzi in a generalised Finsler space.

3. Special cases

In this section we shall discuss three special cases of the above generalisation.

1. When λ^i be taken tangential to the hypersurface F_{n-1}, then

$$(3.1)\qquad \lambda^i=B_\alpha{}^i t^\alpha .$$

As previously by suitable covariant differentiations of (3.1), we get

$$(3.2)\qquad \delta_\tau{}^0\delta_\beta{}^0\lambda^i=\delta_\tau\Omega^*_{\alpha\beta}n^{*i}t^\alpha+\Omega^*_{\alpha\beta}t^\alpha\Big[-\psi\,{}^1g^{(\sigma\delta)}\Omega^*_{\delta\tau}B_\sigma{}^i+\frac{1}{2\psi}n^{*i}\delta_\tau\psi-a^*_{jhk}n^{*j}B_\tau{}^k g^{(hi)}$$
$$+\frac{1}{2\psi}a^*_{jhk}n^{*j}B_\tau{}^k n^{*h}n^{*i}\Big]+\Omega^*_{\alpha\beta}n^{*i}\delta_\tau t^\alpha+\delta_\tau\delta_\beta t^\alpha B_\alpha{}^i+\delta_\beta t^\alpha\Omega^*_{\alpha\tau}n^{*i},$$

by virtue of (1.16), (1.18), (1.20) and (1.22).

Permuting the indices β, τ in the equation (3.2) and subtracting the result from it. Simplifying the obtained result with the help of (1.22), (2.8), (2.9) and (3.1), we have

$$(3.3)\qquad B^{lkj}_{\alpha\beta\tau}t^\alpha\tilde{K}^i_{ljk}-2\Omega^*_{\alpha\delta}t^\alpha n^{*i}\bar{P}^{*\delta}_{[\beta\tau]}=2\delta_{[\tau}\Omega^*_{|\alpha|\beta]}n^{*i}t^\alpha-\frac{2}{\psi}t^\alpha B_\alpha{}^i\,{}^1g^{(\sigma\delta)}\Omega^*_{\sigma[\beta}\Omega^*_{|\delta|\tau]}$$
$$+\frac{1}{\psi}n^{*i}t^\alpha\Omega^*_{\alpha[\beta}\delta_{\tau]}\psi-2t^\alpha a^*_{jhk}n^{*j}g^{(hi)}\Omega^*_{\alpha[\beta}B^k_{\tau]}+\frac{1}{\psi}t^\alpha a^*_{jhk}n^{*j}n^{*h}n^{*i}\Omega^*_{\alpha[\beta}B^k_{\tau]}$$
$$+2n^{*i}\Omega^*_{\alpha[\beta}\delta_{\tau]}t^\alpha+t^\epsilon\tilde{K}^\alpha_{\epsilon\tau\beta}B_\alpha{}^i+2\delta_{[\beta}t^\alpha\Omega^*_{|\alpha|\tau]}n^{*i}.$$

Considering t^α as an arbitrary vector in the hypersurface F_{n-1} and multiplying (3.3) by $B_i{}^\epsilon$ and $n_i{}^*$. Simplifying the result in view of (1.12), (1.14), (1.15) and (1.16), we obtain

$$(3.4)\qquad \frac{1}{2}\tilde{K}^\epsilon_{\alpha\beta\tau}-\frac{1}{2}B^{lkj}_{\alpha\beta\tau}\tilde{K}^i_{lkj}B_i{}^\epsilon=\psi\,{}^1g^{(\epsilon\delta)}\Omega^*_{\alpha[\beta}\Omega^*_{|\delta|\tau]}-a^*_{jhk}n^{*j}\,{}^1g^{(\epsilon\delta)}B_\delta{}^h\Omega^*_{\alpha[\beta}B^k_{\tau]}$$

and

$$(3.5)\qquad -\frac{1}{2}B^{lkj}_{\alpha\beta\tau}\tilde{K}^i_{lkj}n_i{}^*-\psi\Omega^*_{\alpha\delta}\bar{P}^{*\delta}_{[\beta\tau]}=\psi\delta_{[\tau}\Omega^*_{|\alpha|\beta]}+\frac{1}{2}\Omega^*_{\alpha[\beta}\delta_{\tau]}\psi$$
$$-\frac{1}{2}a^*_{jhk}n^{*j}n^{*h}\Omega^*_{\alpha[\beta}B^k_{\tau]},$$

which are the same as the generalisations of the equations of Gauss and Codazzi respectively as obtained by Shamihoke (1963) (p. 142, 143).

(ii) When λ^i has no components along $B_\alpha{}^i$, then

$$(3.6)\qquad \lambda^i=\gamma n^{*i}.$$

Thus for this case, if we put $t^\alpha=0$ in the equations (2.11) and (2.12), they reduce to the following forms respectively

$$(3.7)\qquad \frac{1}{2}\gamma n^{*l}B_i{}^\sigma B^{kj}_{\beta\tau}\tilde{K}^i_{ljk}+\gamma[\psi\,{}^1g^{(\sigma\epsilon)}\Omega^*_{\epsilon\delta}+a^*_{jhk}n^{*j}B_\delta{}^k\,{}^1g^{(\sigma\epsilon)}B_\epsilon{}^h]\bar{P}^{*\delta}_{[\beta\tau]}$$
$$=-\gamma B^k_{[\beta}\delta^0_{\tau]}a^*_{jhk}n^{*j}\,{}^1g^{(\sigma\epsilon)}B_\epsilon{}^h+\gamma a^*_{jhk}\psi\,{}^1g^{(\sigma\epsilon)}B_\epsilon{}^h\,{}^1g^{(\alpha\delta)}B_\alpha{}^j\Omega^*_{\delta[\tau}B^k_{\beta]}$$
$$+\gamma a^*_{jhk}\,{}^1g^{(\sigma\epsilon)}B_\epsilon{}^h a^*_{lmp}n^{*l}g^{(mj)}B^{pk}_{[\tau\beta]}-\frac{\gamma}{2\psi}a^*_{jhk}\,{}^1g^{(\sigma\epsilon)}B_\epsilon{}^h a^*_{lmp}n^{*l}n^{*m}n^{*j}B^{pk}_{[\tau\beta]}$$

$$-\gamma a^*_{jhk}n^{*j}\Omega^*_{[\beta\tau]}\,n^{*k}\,{}^1g^{(\sigma\iota)}B_\iota{}^h-\frac{\gamma}{2\phi}\,a^*_{jhk}n^{*j}n^{*h}B^{pk}_{[\tau\beta]}\,a^*_{lmp}n^{*l}\,{}^1g^{(\sigma\iota)}B_\iota{}^m$$

$$-Q^\sigma_{[\beta\tau]}-\gamma a^*_{jhk}n^{*j}B^k_{[\beta}\delta^0_{\tau]}\,g^{(hi)}B_i{}^\sigma$$

and

$$\frac{1}{2}\gamma n^{*l}n^*{}_iB^{kj}_{\beta\tau}\tilde{K}^i_{ljk}-\left[\phi\delta_\delta\gamma+\frac{\gamma}{2}\delta_\delta\phi-\frac{\gamma}{2}\,a^*_{jhk}n^{*j}n^{*h}B_\delta{}^k\right]\bar{P}^{*\delta}_{[\beta\tau]}\tag{3.8}$$

$$=\phi P^1_{[\beta\tau]}-\gamma B^k_{[\beta}\delta^0_{\tau]}a^*_{jhk}n^{*j}n^{*h}+\gamma a^*_{jhk}\phi\,{}^1g^{(\alpha\delta)}B_\alpha{}^jn^{*h}\Omega^*_{\delta[\tau}B^k_{\beta]}$$

$$+\gamma a^*_{jhk}n^{*h}a^*_{lmp}n^{*l}\,g^{(mj)}B^{pk}_{[\tau\beta]}-\frac{\gamma}{\phi}\,a^*_{jhk}n^{*h}a^*_{lmp}n^{*l}n^{*m}n^{*j}B^{pk}_{[\tau\beta]}$$

$$-\gamma a^*_{jhk}n^{*j}n^{*h}n^{*k}\Omega^*_{[\beta\tau]}-\gamma a^*_{jhk}n^{*j}B^k_{[\beta}\delta^0_{\tau]}\,g^{(hi)}n_i{}^*\,,$$

where

$$P^1_{\beta\tau}=\delta_\tau\delta_\beta\gamma+\frac{1}{2\phi}\,\delta_\beta\gamma\delta_\tau\phi+\frac{1}{2\phi}\,\delta_\beta\gamma a^*_{jhk}n^{*j}n^{*h}B_\tau{}^k+\frac{1}{2\phi}\,\delta_\tau\gamma\delta_\beta\phi$$

$$+\delta_\tau\gamma a^*_{jhk}n^{*j}n^{*h}\frac{1}{2\phi}B_\beta{}^k-\gamma\phi\,{}^1g^{(\alpha\delta)}\Omega^*_{\delta\beta}\Omega^*_{\alpha\tau}+\frac{\gamma}{2}\,\delta_\tau\frac{1}{\phi}\,\delta_\beta\phi$$

$$+\frac{\gamma}{2}\phi^{-1}\delta_\beta\phi\left(\frac{1}{2\phi}\,\delta_\tau\phi+\frac{1}{2\phi}\,a^*_{jhk}n^{*j}n^{*h}\,B_\tau{}^k\right)+\frac{\gamma}{2\phi}\,\delta_\tau\delta_\beta\phi$$

$$+\gamma\delta_\tau{}^0a^*_{jhk}n^{*j}\,\frac{1}{2\phi}\,n^{*h}B_\beta{}^k+\frac{\gamma}{2\phi}a^*_{jhk}\Omega^*_{\beta\tau}n^{*j}n^{*h}n^{*k}+\frac{1}{2}\gamma a^*_{jhk}n^{*j}n^{*h}\,B_\beta{}^k\delta_\tau\frac{1}{\phi}$$

$$+\frac{\gamma}{\phi}\,a^*_{jhk}n^{*h}B_\beta{}^k\left[-\phi\,{}^1g^{(\alpha\delta)}\Omega^*_{\delta\tau}B_\alpha{}^j+\frac{1}{2\phi}\,n^{*j}\delta_\tau\phi-a^*_{lmp}n^{*l}B_\tau{}^pg^{(mj)}\right.$$

$$\left.+\frac{1}{2\phi}\,a^*_{lmp}n^{*l}n^{*m}n^{*j}B_\tau{}^p\right]+\frac{\gamma}{2\phi}\,a^*_{jhk}n^{*j}n^{*h}B_\beta{}^k\left(\frac{1}{2\phi}\,\delta_\tau\phi+\frac{1}{2\phi}\,a^*_{lmp}n^{*l}n^{*m}B_\tau{}^p\right).$$

The equations (3.7) and (3.8) represent the alternative forms of the equations of Gauss and Codazzi respectively.

(iii) When t^α and λ^i are tangential the defining equation of becomes

$$\dot{x}=B_\alpha{}^i\dot{u}^\alpha\,.$$

Differentiating the above equation covariantly twice and a simplifying by means of (1.16), (1.18), (1.20) and (1.22), we get

$$\delta_\tau{}^0\delta_\beta{}^0\dot{x}^i=\delta_\tau\Omega^*_{\alpha\beta}n^{*i}\dot{u}^\alpha+\Omega^*_{\alpha\beta}\left[-\phi^1g^{(\sigma\delta)}\Omega^*_{\delta\tau}B_\sigma{}^i+\frac{1}{2\phi}\,n^{*i}\delta_\tau\phi-a^*_{jhk}n^{*j}B_\tau{}^kg^{(hi)}\right.\tag{3.9}$$

$$\left.+\frac{1}{2\phi}a^*_{jhk}n^{*j}B_\tau{}^kn^{*h}n^{*i}\right]\dot{u}^\alpha+\Omega^*_{\alpha\beta}n^{*i}\delta_\tau\dot{u}^\alpha+\Omega^*_{\alpha\tau}n^{*i}\delta_\beta\dot{u}^\alpha+B_\alpha{}^i\delta_\tau\delta_\beta\dot{u}^\alpha\,.$$

We also have

$$\delta_{[\tau}\delta_{\beta]}\dot{u}^{\alpha}=\frac{1}{2}\,\dot{u}^{\theta}\widetilde{K}^{\alpha}_{\theta\tau\beta} \tag{3.10}$$

and

$$\delta^{0}_{[\tau}\delta^{0}_{\beta]}\dot{x}^{i}=\frac{1}{2}\,\dot{x}^{l}B^{kj}_{\beta\tau}\widetilde{K}^{i}_{ljk}-\delta_{\delta}{}^{0}\dot{x}^{i}\bar{P}^{*\delta}_{[\beta\tau]}\,. \tag{3.11}$$

Subtracting the result obtained on permuting the indices τ, β in (3.9) and simplifying by virtue of (1.5), (1.12), (1.20), (3.10) and (3.11), we have

$$\begin{aligned}\frac{\dot{x}^{l}}{2}\,B^{kj}_{\beta\tau}\widetilde{K}^{i}_{ljk}-(\Omega^{*}_{\alpha\delta}\dot{u}^{\alpha}n^{*i}+\dot{u}^{\alpha}\bar{P}^{*\sigma}_{\alpha\delta}B_{\sigma}{}^{i})\bar{P}^{*\delta}_{[\beta\tau]}&=\delta_{[\tau}\Omega^{*}_{|\alpha|\beta]}n^{*i}\dot{u}^{\alpha}\\ &-\psi\Omega^{*}_{\alpha[\beta}\Omega^{*}_{|\delta|\tau]}\dot{u}^{\alpha}\,{}^{1}g^{(\sigma\delta)}B_{\sigma}{}^{i}+\frac{1}{2\psi}\,n^{*i}\dot{u}^{\alpha}\Omega^{*}_{\alpha[\beta}\delta_{\tau]}\psi-a^{*}_{jhk}n^{*j}g^{(hi)}\dot{u}^{\alpha}\Omega^{*}_{\alpha[\beta}B^{k}_{\tau]}\\ &+a^{*}_{jhk}\,\frac{n^{*j}}{2\psi}\,n^{*h}n^{*i}\dot{u}^{\alpha}\Omega^{*}_{\alpha[\beta}B^{k}_{\tau]}+\frac{1}{2}\,B_{\alpha}{}^{i}\dot{u}^{\theta}\widetilde{K}^{\alpha}_{\theta\tau\beta}\;.\end{aligned} \tag{3.12}$$

Multiplying (3.12) by $B_{i}{}^{\varepsilon}$ and $n^{*}{}_{i}$ and noting (1.12) (1.14), (1.15) and (1.16), we obtain

$$\begin{aligned}\frac{1}{2}\,B^{lkj}_{\alpha\beta\tau}\widetilde{K}^{i}_{ljk}B_{i}{}^{\varepsilon}\dot{u}^{\alpha}-\bar{P}^{*\varepsilon}_{\alpha\delta}\dot{u}^{\alpha}\bar{P}^{*\delta}_{[\beta\tau]}&=-\psi\,{}^{1}g^{(\varepsilon\delta)}\Omega^{*}_{\alpha[\beta}\Omega^{*}_{|\delta|\tau]}\dot{u}^{\alpha}\\ &-a^{*}_{jhk}n^{*j}\Omega^{*}_{\alpha[\beta}B^{k}_{\tau]}\,{}^{1}g^{(\varepsilon\delta)}B_{\delta}{}^{h}\dot{u}^{\alpha}-\frac{1}{2}\,\dot{u}^{\alpha}\widetilde{K}^{\varepsilon}_{\alpha\tau\beta}\end{aligned} \tag{3.13}$$

and

$$\begin{aligned}\frac{1}{2}\,B^{lkj}_{\alpha\beta\tau}\widetilde{K}^{i}_{ljk}n^{*}{}_{i}\dot{u}^{\alpha}-\psi\Omega^{*}_{\alpha\delta}\dot{u}^{\alpha}\bar{P}^{*\delta}_{[\beta\tau]}&=\psi\dot{u}^{\alpha}\delta_{[\tau}\Omega^{*}_{|\alpha|\beta]}+\frac{1}{2}\,\dot{u}^{\alpha}\Omega^{*}_{\alpha[\beta}\delta_{\tau]}\psi\\ &-\frac{1}{2}\,a^{*}_{jhk}n^{*j}n^{*h}\Omega^{*}_{\alpha[\beta}B^{k}_{\tau]}\dot{u}^{\alpha}\;,\end{aligned} \tag{3.14}$$

respectively.

Taking $\dot{u}^{\alpha}$ as an arbitrary vector and simplifying on account of (1.10), (1.11) and (1.22), the above equations yield

$$\frac{1}{2}\,\widetilde{K}_{\alpha\sigma\tau\beta}-\psi\Omega^{*}_{\sigma[\tau}\Omega^{*}_{|\alpha|\beta]}-\bar{P}^{*\varepsilon}_{\alpha\delta}\,{}^{1}g_{(\varepsilon\sigma)}\bar{P}^{*\delta}_{[\beta\tau]}=\frac{1}{2}\,B^{hlkj}_{\sigma\alpha\beta\tau}\widetilde{K}_{lhjk}-n^{*j}B_{\sigma}{}^{h}\Omega^{*}_{\alpha[\beta}\delta^{0}_{\tau]}g_{(jh)} \tag{3.15}$$

and

$$\frac{1}{2}\,B^{lkj}_{\alpha\beta\tau}\widetilde{K}_{lhjk}n^{*h}-\psi\Omega^{*}_{\alpha\delta}\bar{P}^{*\delta}_{[\beta\tau]}=\psi\delta_{[\tau}\Omega^{*}_{|\alpha|]\beta}+\frac{1}{2}\,\Omega^{*}_{\alpha[\beta}\delta_{\tau]}\psi-\frac{1}{2}\,n^{*j}n^{*h}\Omega^{*}_{\alpha[\beta}\delta^{0}_{\tau]}\,g_{(jh)}\,, \tag{3.16}$$

which are another generalisations of the equations of Gauss and Codazzi in the generalised Finsler space.

Remark 3.1. We know that the connection parameters $\bar{P}^{*\alpha}_{\beta\tau}$ are not symmetric in their lower indices in the generalised Finsler space, if we consider them to be symmetric the space will no longer remain a generalised Finsler space and it is transferred to ordinary Finsler space. Under such consideration the equations (3.15) and (3.16) reduce to the generalisations of the equations of Gauss and Codazzi as obtained by Rund (1956).

4. Umbilical Hypersurface

A point P of the hypersurface F_{n-1} is said to be an umbilical point, if

$$\Omega^*_{\alpha\beta}(u, u') = k_n{}^{*}\,{}^1g_{\alpha\beta}(u, u') \tag{4.1}$$

for all directions $\dot{u}^\alpha$ of F_{n-1} at P, where $k_n{}^*$ is the secondary normal curvature. Differentiating (4.1) covariantly, we have

$$\delta_\tau \Omega^*_{\alpha\beta} = {}^1g_{\alpha\beta}\delta_\tau k_n{}^* + k_n{}^* \bar{a}^*_{\alpha\beta\tau}\ , \tag{4.2}$$

where

$$\bar{a}^*_{\alpha\beta\tau} = \delta_\tau\,{}^1g_{\alpha\beta}(u, u')\ .$$

The Gauss and Codazzi equation for a umbilical hypersurface of a generalised Finsler space are expressed by virtue of (4.1) and (4.2) as follows:

$$\begin{aligned}
&\frac{1}{2}(t^\alpha B_\alpha{}^l + \gamma n^{*l})B_i{}^\sigma B^{kj}_{\beta\tau}\tilde{K}^i_{ljk} + \gamma[\psi\,{}^1g^{(\sigma\iota)}k_n{}^*\,{}^1g_{\iota\delta} + a^*_{jhk}n^{*j}B_\delta{}^k\,{}^1g^{(\sigma\iota)}B_\iota{}^h]\bar{P}^{*\delta}_{[\beta\tau]} \\
&= \frac{1}{2}t^\iota\tilde{K}^\sigma_{\iota\tau\beta} - \psi t^\alpha\,{}^1g^{(\sigma\delta)}k_n{}^*\,{}^1g_{\delta[\tau}\,{}^1g_{|\alpha|\beta]} - \gamma a^*_{jhk}n^{*j}t^\alpha k_n{}^*B^k_{[\tau}\,{}^1g_{|\alpha|\beta]}\,{}^1g^{(\sigma\iota)}B_\iota{}^h \\
&\quad - \gamma B^k_{[\beta}\delta^0_{\tau]}a^*_{jhk}n^{*j}\,{}^1g^{(\sigma\iota)}B_\iota{}^h + \gamma a^*_{jhk}\psi\,{}^1g^{(\sigma\iota)}B_\iota{}^h\,{}^1g^{(\alpha\delta)}B_\alpha{}^jk_n{}^*\,{}^1g_{\delta[\tau}B^k_{\beta]} \\
&\quad + \gamma a^*_{jhk}\,{}^1g^{(\sigma\iota)}B_\iota{}^h a^*_{lmp}n^{*l}g^{(mj)}B^{pk}_{[\tau\beta]} - \frac{\gamma}{2\psi}a^*_{jhk}\,{}^1g^{(\sigma\iota)}B_\iota{}^h a^*_{lmp}n^{*l}n^{*m}n^{*j}B^{pk}_{[\tau\beta]} \\
&\quad - \gamma a^*_{jhk}n^{*j}k_n{}^*\,{}^1g_{[\beta\tau]}n^{*k}\,{}^1g^{(\sigma\iota)}B_\iota{}^h - \frac{\gamma}{2\psi}a^*_{jhk}n^{*j}n^{*h}B^{pk}_{[\tau\beta]}a^*_{lmp}n^{*l}\,{}^1g^{(\sigma\iota)}B_\iota{}^m \\
&\quad - \gamma a^*_{jhk}n^{*j}B^k_{[\beta}\delta^0_{\tau]}\,g^{(hi)}B_i{}^\sigma - Q^{1\sigma}_{[\beta\tau]}\ ,
\end{aligned} \tag{4.3}$$

together with

$$\begin{aligned}
&\frac{1}{2}[t^\alpha B_\alpha{}^l + \gamma n^{*l}]n^*{}_iB^{kj}_{\beta\tau}\tilde{K}^i_{ljk} - \Big[\psi t^\alpha k_n{}^*\,{}^1g_{\alpha\delta} + \psi\delta_\delta\gamma + \frac{\gamma}{2}\delta_\delta\psi \\
&\quad - \frac{\gamma}{2}a^*_{jhk}n^{*j}B_\delta{}^kn^{*h}\Big]\bar{P}^{*\delta}_{[\beta\tau]} = \psi P^{11}_{[\beta\tau]} - \gamma a^*_{jhk}n^{*j}n^{*h}t^\alpha k_n{}^*B^k_{[\tau}\,{}^1g_{|\alpha|\beta]} \\
&\quad - \gamma B^k_{[\beta}\delta^0_{\tau]}a^*_{jhk}n^{*j}n^{*h} - \gamma a^*_{jhk}\psi\,{}^1g^{(\alpha\delta)}B_\alpha{}^jk_n{}^*\,{}^1g_{\delta[\tau}B^k_{\beta]}n^{*h}
\end{aligned} \tag{4.4}$$

$$-\gamma a^*_{jhk} n^{*h} a^*_{lmp} n^{*l} g^{(mj)} B^{pk}_{[\tau\beta]} - \frac{\gamma}{\psi} a^*_{jhk} n^{*h} a^*_{lmp} n^{*l} n^{*m} n^{*j} B^{pk}_{[\tau\beta]}$$
$$-\gamma a^*_{jhk} n^{*j} k_n^{*} \, {}^1g_{[\beta\tau]} n^{*h} n^{*k} - \gamma a^*_{jhk} n^{*j} B^k_{[\beta} \delta^0_{\tau]} g^{(hi)} n_i^* \,,$$

where

$$P^{11}_{\beta\tau} = t^\alpha({}^1g_{\alpha\beta}\delta_\tau k_n^* + k_n^* \bar{a}^*_{\alpha\beta\tau}) + \frac{t^\alpha}{2\psi} k_n^{*}\,{}^1g_{\alpha\beta}\delta_\tau\psi + t^\alpha k_n^{*}\,{}^1g_{\alpha\beta} a^*_{jhk} n^{*j} n^{*h} \frac{1}{2\psi} B_\tau{}^k$$
$$+\delta_\tau\delta_\beta\gamma + \frac{1}{2\psi}\delta_\beta\gamma\delta_\tau\psi + \delta_\beta\gamma a^*_{jhk} n^{*j} n^{*h} \frac{1}{2\psi} B_\tau{}^k + \frac{1}{2\psi}\delta_\tau\gamma\delta_\beta\psi + \delta_\tau\gamma a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k \frac{1}{2\psi}$$
$$-\gamma\psi\,{}^1g^{(\alpha\delta)} k^{*2}_n \,{}^1g_{\delta\beta}\,{}^1g_{\alpha\tau} + \frac{\gamma}{2}\delta_\tau\psi^{-1}\delta_\beta\psi + \frac{\gamma}{2}\psi^{-1}\delta_\beta\psi\left(\frac{1}{2\psi}\delta_\tau\psi + \frac{1}{2\psi} a^*_{jhk} n^{*j} n^{*h} B_\tau{}^k\right)$$
$$+\frac{\gamma}{2\psi}\gamma\delta_\tau\delta_\beta\psi + \gamma\delta_\tau{}^0 a^*_{jhk} n^{*j} \frac{1}{2\psi} n^{*h} B_\beta{}^k + \frac{\gamma}{2\psi} n^{*j} n^{*h} n^{*k} k_n^{*}\,{}^1g_{\beta\tau} a^*_{jhk}$$
$$+\frac{\gamma}{2} a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k \delta_\tau \frac{1}{\psi} + \frac{\gamma}{\psi} a^*_{jhk} n^{*h} B_\beta{}^k \Big[-\psi\,{}^1g^{(\alpha\delta)} k_n^{*}\,{}^1g_{\delta\tau} B_\alpha{}^j$$
$$+\frac{1}{2\psi} n^{*j}\delta_\tau\psi - a^*_{lmp} n^{*l} B_\tau{}^p g^{(mj)} + a^*_{lmp} n^{*l} n^{*m} n^{*j} \frac{1}{2\psi} B_\tau{}^p \Big]$$
$$+\frac{\gamma}{2\psi} a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k \left(\frac{1}{2\psi}\delta_\tau\psi + \frac{1}{2\psi} a^*_{lmp} n^{*l} n^{*m} B_\tau{}^p\right)$$

and

$$Q^{1\alpha}_{\beta\tau} = \psi\delta_\tau\gamma\, g^{(\alpha\delta)} k_n^* g_{\delta\beta} + \gamma\delta_\tau\psi\,{}^1g^{(\alpha\delta)} k_n^* g_{\delta\beta} + \gamma\psi\delta_\tau\,{}^1g^{(\alpha\delta)} k_n^{*}\,{}^1g_{\delta\beta}$$
$$+\gamma\psi\,{}^1g^{(\alpha\delta)}({}^1g_{\delta\beta}\delta_\tau k_n^* + k_n^* \bar{a}^*_{\delta\beta\tau}) + \delta_\beta\gamma\psi\,{}^1g^{(\alpha\delta)} k_n^{*}\,{}^1g_{\delta\tau}$$
$$-\frac{\gamma}{2}\,{}^1g^{(\alpha\delta)} k_n^{*}\,{}^1g_{\delta\tau}[\delta_\beta\psi + a^*_{jhk} n^{*j} n^{*h} B_\beta{}^k]\,.$$

The Gauss and Codazzi equations for umbilical hypersurfaces of the special cases may be easily obtained on the same lines as mentioned above.

The alternative forms of Gauss Codazzi equations for another set of normals n^i can also be determined on the same pattern for the hypersurface in a generalised Finsler space.

Acknowledgement: The author is highly grateful to Dr. *B. B. Sinha* for his kind help and guidance in the preparation of present paper.

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Department of Mathematics
Faculty of Science
Banaras Hindu University
Varanasi-5, INDIA