Muonium-antimuonium conversion in models with dilepton gauge bosons

Kuninori Horikawa and Ken Sasaki*
Department of Physics, Yokohama National University, Yokohama 240, Japan

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We examine the magnetic field dependence of the muonium (μ^+e^-)-antimuonium (μ^-e^+) conversion in the models which accommodate the dilepton gauge bosons. The effective Hamiltonian for the conversion due to dileptons turns out to be in the $(V-A)\times(V+A)$ form and as a consequence, the conversion probability is rather insensitive to the strength of the magnetic field. The reduction is less than 20% for up to $B\approx 300$ G and 33% even in the large B limit.

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Muonium M, which is a bound state of μ^+ and e^-, can be transformed to antimuonium $\bar{M}$, a bound state of μ^- and e^+, if there exists a lepton-number-nonconserving interaction [1]. Feinberg and Weinberg [2] studied the M-$\overline{M}$ conversion with a postulated effective Hamiltonian of $(V-A)\times(V-A)$ form. Later, this process was studied within the left-right symmetric models and the models with doubly charged Higgs bosons [3–7]. In these models, the effective Hamiltonian for the conversion is expressed either in the $(V-A)\times(V-A)$ form or in the $(V+A)\times(V+A)$ form. Thus far no M-$\overline{M}$ conversion has been observed [8].

Recently, an interesting class of models which have new $\mathrm{SU}(2)_L$-doublet gauge bosons were proposed as extensions of the standard model [9–12]. In these models each family of leptons $(l^+,\nu_l,l^-)_L$ transforms as a triplet under the gauge group SU(3) and the total lepton number defined as $L=L_e+L_\mu+L_\tau$ is conserved, while the separate lepton number for each family is not. The new gauge bosons $(X^\mp, X^{\mp\mp})$ carry lepton number $L=\pm 2$. Hence, hereafter, we refer to these gauge bosons as dileptons. The gauge group SU(3) will be, for example, an $\mathrm{SU}(3)_l$ in the SU(15) grand unification theory model [10] or an $\mathrm{SU}(3)_L$ in the $\mathrm{SU}(3)_C\times\mathrm{SU}(3)_L\times\mathrm{U}(1)_X$ model [12].

The phenomenology on dilepton gauge bosons has been extensively studied. When the doubly charged dilepton exists, the mixing of muonium and antimuonium is possible through the diagram in Fig. 1 and thus M-$\overline{M}$ conversion takes place [13–15]. In particular, the effective Hamiltonian for the mixing turns out to be in the $(V-A)\times(V+A)$ form. One of the present authors (K.S.) and Fujii and Nakamura calculated the probability for the M-$\overline{M}$ conversion in the models with dileptons and

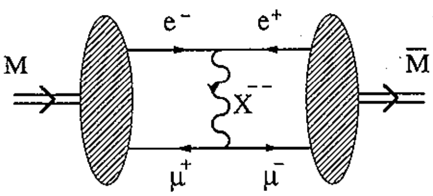

FIG. 1. The doubly charged dilepton exchange diagram for muon-antimuonium conversion. The arrows show the flow of lepton number.

*Electronic address: sasaki@ed.ynu.ac.jp

examined the lower mass bound on the doubly charged dilepton $X^{\pm\pm}$ in [14] but the analysis was done in the case of absence of magnetic fields. Here we consider the M-$\overline{M}$ conversion in static external magnetic fields and study the field dependence of the conversion probability.

The muonium or antimuonium system in the presence of static external magnetic field $\vec{B}$ is described by the Hamiltonian

$$\mathcal{H}_{\rm int}=A\vec{S_e}\cdot\vec{S_\mu}+\mu_B g_e\vec{S_e}\cdot\vec{B}+\mu_B\frac{m_e}{m_\mu}g_\mu\vec{S_\mu}\cdot\vec{B}, \quad (1)$$

where $\vec{S_e}$, m_e, $g_{e^-}=-g_{e^+}$ and $\vec{S_\mu}$, m_μ, $g_{\mu^+}=-g_{\mu^-}$ are spin, mass, the gyromagnetic ratio of electron (or positron), and μ^+ (or μ^-), respectively, and μ_B is Bohr magneton. The first term of Eq. (1) is the source of the $1S$ hyperfine splitting of the muonium (or antimuonium) system and $A=1.846\times10^{-5}$ eV. Taking the magnetic field direction as the spin-quantization axis, we obtain the muonium energy eigenvalues as [16]

$$\begin{aligned}&E_M(1,+1)=(A/4)+P,\quad E_M(1,-1)=(A/4)-P,\\&E_M(1,0)=-(A/4)(1-2\sqrt{1+y^2}),\\&E_M(0,0)=-(A/4)(1+2\sqrt{1+y^2}),\end{aligned} \quad (2)$$

with

$$P=\frac{1}{2}\mu_B B\left(g_{e^-}-g_{\mu^-}\frac{m_e}{m_\mu}\right)\approx 5.76\times10^{-9}B\ (\mathrm{eV/G}),$$

$$y=\frac{1}{A}\mu_B B\left(g_{e^-}+g_{\mu^-}\frac{m_e}{m_\mu}\right)\approx 6.30\times10^{-4}B(1/\mathrm{G}). \quad (3)$$

The corresponding eigenstates are expressed in a "natural" basis $|S^z_\mu S^z_e\rangle$ as

$$\begin{aligned}&|1,+1\rangle_M=|++\rangle_M,\quad |1,-1\rangle_M=|--\rangle_M,\\&|1,0\rangle_M=c\,|-+\rangle_M+\,s\,|+-\rangle_M,\\&|0,0\rangle_M=-s\,|-+\rangle_M+\,c\,|+-\rangle_M,\end{aligned} \quad (4)$$

where $|+-\rangle_M$ means $|S^z_\mu=\frac{1}{2},S^z_e=-\frac{1}{2}\rangle_M$, etc., and

$$c=\frac{1}{\sqrt{2}}\left[1+\frac{y}{\sqrt{1+y^2}}\right]^{1/2},\ s=\frac{1}{\sqrt{2}}\left[1-\frac{y}{\sqrt{1+y^2}}\right]^{1/2}. \quad (5)$$

It is noted that the $(J=1,J_z=0)$ state among the $1S$ triplet and $1S$ singlet state $(J=0,J_z=0)$, which are both energy eigenstates in the absence of external magnetic fields, mix with each other in the presence of $\vec{B}$ and they are not energy eigenstates any more. Thus it is understood that energy eigenstates $|1,0\rangle$ and $|0,0\rangle$ are the states which approach the $(J=1, J_z=0)$ and $(J=0, J_z=0)$ states, respectively, when the magnetic field $\vec{B}$ vanishes. However, $(J=1, J_z=\pm 1)$ states among $1S$ triplet remain as energy eigenstates even in the presence of $\vec{B}$. Energy eigenvalues and the corresponding eigenstates for the antimuonium system in the presence of an external magnetic field $\vec{B}$ are obtained from Eqs. (2) and (4) by interchanging $M \leftrightarrow \overline{M}$, $P \leftrightarrow -P$, $y \leftrightarrow -y$, and $c \leftrightarrow s$.

Now we consider the M-$\overline{M}$ conversion in the presence of static external magnetic fields. First we write down a useful formula for the M-$\overline{M}$ conversion which was derived by Feinberg and Weinberg a long time ago [2]. If there exists an interaction $\mathcal{H}_{M\overline{M}}$ which would yield a matrix element for conversion of M into $\overline{M}$ equal to

$$\langle \overline{M}|\mathcal{H}_{M\overline{M}}|M\rangle = \Delta/2, \tag{6}$$

the mass matrix for the M-$\overline{M}$ system is written as

$$\mathcal{M}_{M\overline{M}} = \begin{pmatrix} E_M & \frac{\Delta}{2} \\ \frac{\Delta}{2} & E_{\overline{M}} \end{pmatrix}. \tag{7}$$

Then the probability for a muonium atom of the state $|M\rangle$ to decay as antimuonium of the state $|\overline{M}\rangle$ at all is

$$P(\overline{M}) = \frac{\Delta^2}{2[\lambda^2 + (E_M - E_{\overline{M}})^2 + \Delta^2]}, \tag{8}$$

where $\lambda = G_F^2 m_\mu^5/192\pi^3$ is the muon decay rate and G_F is Fermi constant.

The magnetic field dependence of the M-$\overline{M}$ conversion has been studied in the case when the effective Hamiltonian for M-$\overline{M}$ transition is written in the $(V-A)\times(V-A)$ form or $(V+A)\times(V+A)$ form [16,17]:

$$\mathcal{H}_{M\overline{M}} = \frac{G_{M\overline{M}}}{\sqrt{2}}[\overline{\mu}\gamma_\lambda(1\mp\gamma_5)e][\overline{\mu}\gamma^\lambda(1\mp\gamma_5)e] + \text{H.c.} \tag{9}$$

The effective Hamiltonian in this form arises in the left-right symmetric models and the models with doubly charged Higgs bosons [3–7]. In Refs. [16,17] the probabilities of a muonium in the $|1,\pm1\rangle$, $|1,0\rangle$, and $|0,0\rangle$ states to decay as antimuonium were given as

$$P^{(1,\pm1)}_{(\mp\mp)}(\overline{M}) = \delta^2/2[\lambda^2 + 4P^2 + \delta^2] \tag{10}$$

for the $|1,+1\rangle$ and $|1,-1\rangle$ states, and

$$P^{(1,0)}_{(\mp\mp)}(\overline{M}) = P^{(0,0)}_{(\mp\mp)}(\overline{M}) = \delta^2/2[(1+y^2)\lambda^2 + \delta^2] \tag{11}$$

for the $|1,0\rangle$ and $|0,0\rangle$ states, where

$$\delta = 16 G_{M\overline{M}}/\sqrt{2}\pi a^3, \tag{12}$$

and a is the Bohr radius of the muonium $(m_r\alpha)^{-1}$ with $m_r^{-1} = m_\mu^{-1} + m_e^{-1}$. Thus the assumption that each state is produced with equal weight at the beginning gives

$$P^{\text{tot}}_{(\mp\mp)}(\overline{M}) = \frac{\delta^2}{4[\lambda^2 + 4P^2 + \delta^2]} + \frac{\delta^2}{4[(1+y^2)\lambda^2 + \delta^2]}, \tag{13}$$

for the "total" probability of a muonium to decay as antimuonium.

The magnetic field dependences of $P^{\text{tot}}_{(\mp\mp)}(\overline{M})$, $\frac{1}{2}P^{(1,1)}_{(\mp\mp)}(\overline{M})$, $\frac{1}{4}P^{(1,0)}_{(\mp\mp)}(\overline{M})$, and $\frac{1}{4}P^{(0,0)}_{(\mp\mp)}(\overline{M})$ are shown in Fig. 2 (dashed lines), where the probabilities are normalized by $P^{\text{tot}}_{(\mp\mp)}(\overline{M})|_{B=0}$ and $G_{M\overline{M}}$ is taken to be $0.1G_F$. The probability $P^{(1,\pm1)}_{(\mp\mp)}(\overline{M})$ becomes negligibly small when B is in the order of 10^{-1} G [Fig. 2(b)], since the presence of static external magnetic fields breaks the degeneracy between the $|1,+1\rangle_M$ and $|1,+1\rangle_{\overline{M}}$ states ($|1,-1\rangle_M$ and $|1,-1\rangle_{\overline{M}}$) and the generated energy difference severely suppresses the conversion. On the other hand, the $|1,0\rangle_M$ and $|1,0\rangle_{\overline{M}}$ states ($|0,0\rangle_M$ and $|0,0\rangle_{\overline{M}}$) remain degenerate and thus the conversion persists up to the fields in the order of 10^3 G. In the limit of large B, the $|1,0\rangle_M$ state becomes a pure $|-+\rangle_M$ while the $|1,0\rangle_{\overline{M}}$ state becomes a pure $|+-\rangle_{\overline{M}}$, and thus the matrix element ${}_{\overline{M}}\langle 1,0|\mathcal{H}_{M\overline{M}}|1,0\rangle_M$ vanishes. Hence the probability $P^{(1,0)}_{(\mp\mp)}(\overline{M})$ reduces to 0 in this limit [Fig. 2(c)]. By the same reasoning, $P^{(0,0)}_{(\mp\mp)}(\overline{M})$ vanishes in the large B limit [Fig. 2(d)]. Finally we see from Fig. 2(a) that in the case of the effective Hamiltonian being in the $(V-A)\times(V-A)$ form or $(V+A)\times(V+A)$ form and $G_{M\overline{M}} = 0.1G_F$, the M-$\overline{M}$ conversion probability is reduced to 50% at a field strength as low as 0.26 G, to 35.8% at $B = 1$ kG, and to 1.2% at $B = 1$ T. The dependence of the normalized probabilities on the coupling strength $G_{M\overline{M}}$ is negligibly small for $G_{M\overline{M}} < 1G_F$.

Next we consider the M-$\overline{M}$ conversion in models with dileptons. The gauge interaction of dileptons with leptons is given by [18]

$$\begin{aligned}\mathcal{L}_{\text{int}} = &-\frac{g_{3l}}{2\sqrt{2}}X_\mu^{++}l^T C\gamma^\mu\gamma_5 l - \frac{g_{3l}}{2\sqrt{2}}X_\mu^{--}\overline{l}\gamma^\mu\gamma_5 C\overline{l}^T \\ &+\frac{g_{3l}}{2\sqrt{2}}X_\mu^{+}l^T C\gamma^\mu(1-\gamma_5)\nu_l \\ &+\frac{g_{3l}}{2\sqrt{2}}X_\mu^{-}\overline{\nu_l}\gamma^\mu(1-\gamma_5)C\overline{l}^T,\end{aligned} \tag{14}$$

where $l = e, \mu, \tau$, and C is the charge-conjugation matrix. The gauge coupling constant g_{3l} is given approximately by $g_{3l} = 1.19e$ for the SU(15) grand unified theory (GUT) model [10] and by $g_{3l} = g_2 = 2.07e$ for the $\text{SU}(3)_L \times \text{U}(1)_X$ model [12], where e and g_2 are the electric charge and the $\text{SU}(2)_L$ gauge coupling constant, respectively. It is noted that the vector currents that couple to doubly charged dileptons $X^{\pm\pm}$ vanish due to Fermi statistics. Through the doubly charged-dilepton-exchange diagram illustrated in Fig. 1, we obtain the following effective Hamiltonian for the M-$\overline{M}$ conversion:

$$\mathcal{H}^{\text{di}}_{M\overline{M}} = \frac{G^{\text{di}}_{M\overline{M}}}{\sqrt{2}}[\overline{\mu}\gamma_\lambda(1-\gamma_5)e][\overline{\mu}\gamma^\lambda(1+\gamma_5)e] + \text{H.c.}, \tag{15}$$

where $G^{\text{di}}_{M\overline{M}}/\sqrt{2} = -g_{3l}^2/(8M^2_{X^{\pm\pm}})$ and $M_{X^{\pm\pm}}$ is the doubly charged dilepton mass. This form is obtained from Eq.(14) and with help of the Fierz transformation. It should be noted that the above effective Hamiltonian is in the $(V-A)\times(V+A)$ form. The most stringent lower mass bound for the doubly charged dileptons at present

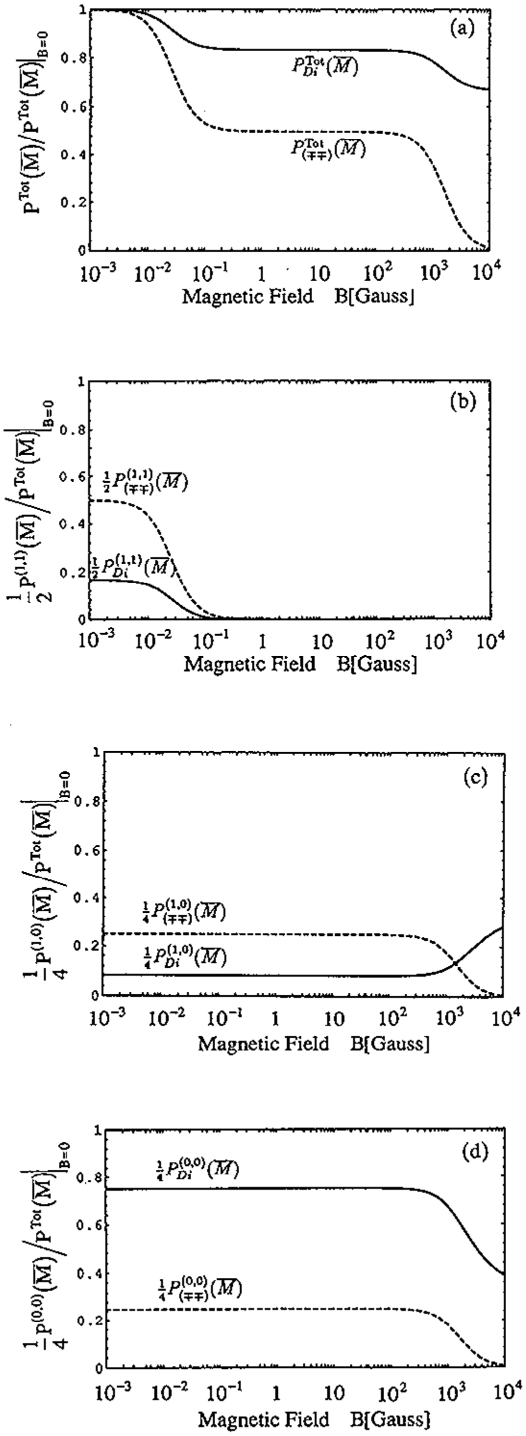

FIG. 2. The magnetic field dependence of the M-$\overline{M}$ conversion probabilities in models with dileptons (solid lines) and in models with an effective $(V\mp A)\times(V\mp A)$ type-Hamiltonian (dashed lines): (a) $P_{\rm di}^{\rm tot}(\overline{M})$ and $P_{(\mp\mp)}^{\rm tot}(\overline{M})$; (b) $\frac{1}{2}P_{\rm di}^{(1,1)}(\overline{M})$ and $\frac{1}{2}P_{(\mp\mp)}^{(1,1)}(\overline{M})$; (c) $\frac{1}{4}P_{\rm di}^{(1,0)}(\overline{M})$ and $\frac{1}{4}P_{(\mp\mp)}^{(1,0)}(\overline{M})$; (d) $\frac{1}{4}P_{\rm di}^{(0,0)}(\overline{M})$ and $\frac{1}{4}P_{(\mp\mp)}^{(0,0)}(\overline{M})$. The probabilities are normalized by $P_{\rm di}^{\rm tot}(\overline{M})|_{B=0}$ or $P_{(\mp\mp)}^{\rm tot}(\overline{M})|_{B=0}$, and $G_{M\overline{M}}^{\rm di}=0.1G_F$ and $G_{M\overline{M}}=0.1G_F$ are assumed. In the large B limit, the normalized probabilities $P_{\rm di}^{\rm tot}(\overline{M})$, $\frac{1}{4}P_{\rm di}^{(1,0)}(\overline{M})$, and $\frac{1}{4}P_{\rm di}^{(0,0)}(\overline{M})$ approach the values 0.67, 0.33, and 0.33, respectively.

is $(M_{X^{\pm\pm}}/g_{3l}) > 340$ GeV (95% C.L.) [18]. This gives $G_{M\overline{M}}^{\rm di} < 0.13G_F$.

With this effective Hamiltonian, we find that the matrix elements for conversion of M into $\overline{M}$ are given in a "natural" basis $|S_\mu^z S_e^z\rangle$ as

$$
\begin{aligned}
{}_{\overline{M}}\langle ++|\mathcal{H}_{M\overline{M}}^{\rm di}|++\rangle_M &= {}_{\overline{M}}\langle --|\mathcal{H}_{M\overline{M}}^{\rm di}|--\rangle_M = \frac{\hat{\delta}}{2},\\
{}_{\overline{M}}\langle +-|\mathcal{H}_{M\overline{M}}^{\rm di}|+-\rangle_M &= {}_{\overline{M}}\langle -+|\mathcal{H}_{M\overline{M}}^{\rm di}|-+\rangle_M = -\frac{\hat{\delta}}{2},\\
{}_{\overline{M}}\langle +-|\mathcal{H}_{M\overline{M}}^{\rm di}|-+\rangle_M &= {}_{\overline{M}}\langle -+|\mathcal{H}_{M\overline{M}}^{\rm di}|+-\rangle_M = \hat{\delta},
\end{aligned}
$$

$$
\text{other elements} = 0, \tag{16}
$$

where

$$
\hat{\delta} = -8G_{M\overline{M}}^{\rm di}/\sqrt{2}\pi a^3. \tag{17}
$$

Since $\mathcal{H}_{M\overline{M}}^{\rm di}$ is in the $(V-A)\times(V+A)$ form, the matrix elements ${}_{\overline{M}}\langle ++|\mathcal{H}_{M\overline{M}}^{\rm di}|++\rangle_M$ and ${}_{\overline{M}}\langle +-|\mathcal{H}_{M\overline{M}}^{\rm di}|+-\rangle_M$ take different values, and ${}_{\overline{M}}\langle +-|\mathcal{H}_{M\overline{M}}^{\rm di}|-+\rangle_M$ and ${}_{\overline{M}}\langle -+|\mathcal{H}_{M\overline{M}}^{\rm di}|+-\rangle_M$ do not vanish.

In terms of the "energy eigenstates," the matrix elements for M-$\overline{M}$ conversion are written as

$$
\begin{aligned}
{}_{\overline{M}}\langle 1,\pm1|\mathcal{H}_{M\overline{M}}^{\rm di}|1,\pm1\rangle_M &= \hat{\delta}/2,\\
{}_{\overline{M}}\langle 1,0|\mathcal{H}_{M\overline{M}}^{\rm di}|1,0\rangle_M &= \left(1-\frac{1}{2\sqrt{1+y^2}}\right)\hat{\delta},\\
{}_{\overline{M}}\langle 0,0|\mathcal{H}_{M\overline{M}}^{\rm di}|0,0\rangle_M &= -\left(1+\frac{1}{2\sqrt{1+y^2}}\right)\hat{\delta}.
\end{aligned} \tag{18}
$$

It is interesting to note that neither ${}_{\overline{M}}\langle 1,0|\mathcal{H}_{M\overline{M}}^{\rm di}|1,0\rangle_M$ nor ${}_{\overline{M}}\langle 0,0|\mathcal{H}_{M\overline{M}}^{\rm di}|0,0\rangle_M$ vanishes in the large B (i.e., large y) limit.

Again using the formula (8), we obtain the following probabilities of a muonium to decay as antimuonium in the models with dileptons:

$$
P_{\rm di}^{(1,\pm1)}(\overline{M}) = \hat{\delta}^2/2[\lambda^2+4P^2+\hat{\delta}^2] \tag{19}
$$

for the $|1,\pm1\rangle_M$ states,

$$
P_{\rm di}^{(1,0)}(\overline{M}) = \frac{(2-1/\sqrt{1+y^2})^2\hat{\delta}^2}{2[\lambda^2+(2-1/\sqrt{1+y^2})^2\hat{\delta}^2]} \tag{20}
$$

for the $|1,0\rangle_M$ state, and finally

$$
P_{\rm di}^{(0,0)}(\overline{M}) = \frac{(2+1/\sqrt{1+y^2})^2\hat{\delta}^2}{2[\lambda^2+(2+1/\sqrt{1+y^2})^2\hat{\delta}^2]} \tag{21}
$$

for the $|0,0\rangle_M$ state.

As before we assume that each state is produced with equal weight at the beginning, and we obtain

$$P_{\rm di}^{\rm tot}(\overline{M}) = \frac{\hat{\delta}^2}{4[\lambda^2+4P^2+\hat{\delta}^2]} + \frac{(2-1/\sqrt{1+y^2})^2\hat{\delta}^2}{8[\lambda^2+(2-1/\sqrt{1+y^2})^2\hat{\delta}^2]} + \frac{(2+1/\sqrt{1+y^2})^2\hat{\delta}^2}{8[\lambda^2+(2+1/\sqrt{1+y^2})^2\hat{\delta}^2]} \quad (22)$$

for the "total" probability of a muonium to decay as antimuonium. In the limit of $B=0$, we have

$$P_{\rm di}^{\rm tot}(\overline{M})|_{B=0} = \frac{3\hat{\delta}^2}{8[\lambda^2+\hat{\delta}^2]} + \frac{9\hat{\delta}^2}{8[\lambda^2+9\hat{\delta}^2]} \approx \frac{3\hat{\delta}^2}{2\lambda^2}, \quad (23)$$

which is the result first obtained in Ref. [14].

In Fig. 2 we plot in solid lines the magnetic field dependence of $P_{\rm di}^{\rm tot}(\overline{M})$, $\frac{1}{2}P_{\rm di}^{(1,1)}(\overline{M})$, $\frac{1}{4}P_{\rm di}^{(1,0)}(\overline{M})$, and $\frac{1}{4}P_{\rm di}^{(0,0)}(\overline{M})$. They are all normalized by $P_{\rm di}^{\rm tot}(\overline{M})|_{B=0}$ and we take $G_{M\overline{M}}^{\rm di} = 0.1G_F$. As in the case of $P_{(\mp\mp)}^{(1,\pm1)}(\overline{M})$, the probability $P_{\rm di}^{(1,\pm1)}(\overline{M})$ becomes negligibly small when B reaches the order of 10^{-1} G since the magnetic field breaks the degeneracy of the $|1,+1\rangle_M$ and $|1,+1\rangle_{\overline{M}}$ states [Fig. 2(b)]. However, the B dependences of $P_{\rm di}^{(1,0)}(\overline{M})$ and $P_{\rm di}^{(0,0)}(\overline{M})$ are quite different from those of $P_{(\mp\mp)}^{(1,0)}(\overline{M})$ and $P_{(\mp\mp)}^{(0,0)}(\overline{M})$ [Figs. 2(c) and 2(d)]. First, up to $B \approx 1$ kG, the M-$\overline{M}$ conversion through the channel $|0,0\rangle_M \to |0,0\rangle_{\overline{M}}$ is much prefered; therefore, $P_{\rm di}^{(0,0)}(\overline{M})$ gives a dominant contribution to $P_{\rm di}^{\rm tot}(\overline{M})$. Second, $P_{\rm di}^{(1,0)}(\overline{M})$ and $P_{\rm di}^{(0,0)}(\overline{M})$ remain finite and reach the same value in the large B limit. This is due to the fact that the matrix elements ${}_{\overline{M}}\langle 1,0|\mathcal{H}_{M\overline{M}}^{\rm di}|1,0\rangle_M$ and ${}_{\overline{M}}\langle 0,0|\mathcal{H}_{M\overline{M}}^{\rm di}|0,0\rangle_M$ do not vanish and become equal in magnitude in the large B limit when the effective Hamiltonian is in the $(V-A)\times(V+A)$ form. We see from Figs. 2(c) and 2(d) that the normalized probability $\frac{1}{4}P_{\rm di}^{(1,0)}(\overline{M})/P_{\rm di}^{\rm tot}(\overline{M})|_{B=0}$ starts to increase around $B = 1$ kG while $\frac{1}{4}P_{\rm di}^{(0,0)}(\overline{M})/P_{\rm di}^{\rm tot}(\overline{M})|_{B=0}$ starts to decrease and that both approach the value 0.33 in the large B limit. We find that $P_{\rm di}^{\rm tot}(\overline{M})$ is rather insensitive to the static external magnetic field. In fact Fig. 2(a) shows that $P_{\rm di}^{\rm tot}(\overline{M})$ is lowered to 83% in the region 0.2 G$< B <$ 300 G and only to 67% in the large B limit. At $B = 1$ kG (1 T) the reduction is 21.4% (32.9%). Again the dependence of the normalized probabilities on the coupling strength $G_{M\overline{M}}^{\rm di}$ is negligibly small for $G_{M\overline{M}}^{\rm di} < 1G_F$.

In conclusion, we have studied the magnetic field dependence of the M-$\overline{M}$ conversion in the models with dileptons. We have found that the conversion is rather insensitive to the strength of the magnetic fields. If an experiment is performed in a magnetic field of 1 T and if a bound for the conversion probability $P(\overline{M}) < 10^{-10}$ is gained [17], then a bound for the coupling strength $G_{M\overline{M}} < 1.8\times10^{-2}G_F$ is obtained for the usual $(V\mp A)\times(V\mp A)$-type Hamiltonian. On the other hand, the models with dileptons give a more stringent bound $G_{M\overline{M}}^{\rm di} < 2.8\times10^{-3}G_F$.

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