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Mass of the lightest supersymmetric Higgs boson beyond the leading logarithm approximation

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We examine the radiative corrections to the mass of the lightest Higgs boson in the minimal supersymmetric extension of the standard model. We use the renormalization-group-improved effective potential which includes the next-to-leading-order contributions. We find that the higher-order corrections to the lightest Higgs boson mass are non-negligible, adding 3–11 GeV (3–9 GeV) to the result in the leading logarithm approximation for the range of top quark mass 100 GeV $< m_t <$ 200 GeV and for the supersymmetry-breaking scale $M_{\rm SUSY} = 1$ TeV ($M_{\rm SUSY} = 10$ TeV). Also we find that our result is stable under the change of the renormalization parameter t.

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Although the standard model (SM) is highly successful and in excellent agreement with all the measurements at preset energies, it is widely believed that SM is not the final theory for the world of elementary particles. The minimal supersymmetric extension of the standard model (MSSM) is one of the most promising candidates beyond the SM. The MSSM possesses in its physical spectrum three neutral and two charged Higgs bosons, and there exists a tree-level relation which implies that at least one neutral Higgs boson is lighter than the Z^0 mass (M_Z). Radiative corrections to the masses of these Higgs bosons have been calculated by several groups [1–14], who found that they are quite significant, depending strongly on the top quark mass and the scale of supersymmetry breaking ($M_{\rm SUSY}$) or the quark masses. All the above works except Ref. [6] considered the one-loop radiative corrections to Higgs boson masses.

Now that the one-loop corrections have been found to be significantly large, it is quite natural to ask next how large the higher order corrections would be. Indeed, Espinosa and Quirós [6] have analyzed the "two-loop" radiative corrections to the mass of the lightest Higgs boson in the minimal and nonminimal (including a gauge singlet) supersymmetric standard model. They used the effective potential (EP) in the leading logarithm approximation and examined the evolution of the Higgs quartic coupling λ by renormalization group (RG) techniques with the one- and two-loop β functions. They found that the "two-loop" correction is negative and stays within a few percent even in cases where the one-loop correction is larger than the tree-level mass.

Recently there appeared interesting papers [15–17] which discussed the improvement of the EP by using the renormalization group equation (RGE). It was shown there that to improve the EP which satisfies the RGE with up to the two-loop β functions and anomalous dimension γ, one should include the one-loop-level potential with the running parameters into the solution. In this respect, the work of Espinosa and Quirós [6] seems unsatisfactory: they used the EP in the leading logarithm approximation, which is the tree-level potential with the running parameters, and they made use of the two-loop β functions only to determine the evolution of these parameters. In this paper we reanalyze the mass of the lightest Higgs boson (m_ϕ) in the MSSM using the EP improved by the RGE up to the next-to-leading order. We find that new terms which were not considered by Espinosa and Quirós give non-negligible contributions to the m_ϕ. We also find that the predicted values of m_ϕ are stable under the change of the renormalization parameter t when we use the RGE-improved EP which includes the next-to-leading-order contributions.

Two basic assumptions were made in their analysis of the lightest Higgs boson mass [6]: (a) all supersymmetric (SUSY) partners of SM particles have masses of the order of the supersymmetry-breaking scale $M_{\rm SUSY}$; (b) one linear combination H of the two Higgs boson doublets, $H_1 = (H_1^0, H_1^-)^T$ and $H_2 = (H_2^+, H_2^0)^T$,

$$H = H_1 \cos\beta + i\tau_2 H_2^* \sin\beta \tag{1}$$

is light, while the other linear combination, which is orthogonal to the former one, is as heavy as the SUSY partners. Under these assumptions, it is clear that the effective theory below the scale $M_{\rm SUSY}$ is the usual SM with one light Higgs boson doublet H. Throughout the following analyses, we will make the same assumptions (a) and (b). The tree-level Higgs boson potential below $M_{\rm SUSY}$ is then given by

$$V_{\rm tree} = -m^2|H|^2 + \tfrac{1}{6}\lambda|H|^4\ , \tag{2}$$

where

$$\tfrac{1}{3}\lambda = \tfrac{1}{4}(g_1^2 + g_2^2)\cos^2 2\beta\ , \tag{3}$$

and g_1 and g_2 are the gauge coupling constants of $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$, respectively.

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When the neutral component of H acquires a vacuum expectation value $\langle H^0 \rangle = v/\sqrt{2}$, the above tree-level potential (2) gives the physical Higgs boson $\phi \equiv (\mathrm{Re}H^0 - v)/\sqrt{2}$ [which corresponds to the lightest Higgs boson under assumptions (a) and (b)] a squared mass

$$m_\phi^2 = \tfrac{1}{3}\lambda v^2 \ . \tag{4}$$

Also at the symmetry breaking, the top quark and the $Z^0, W^\pm$ gauge bosons gain masses which are given by

$$m_t = h_t v/\sqrt{2} \ , \quad M_Z^2 = \tfrac{1}{4}(g_1^2 + g_2^2)v^2 \ , \quad M_W^2 = \tfrac{1}{4}g_2^2 v^2 \ , \tag{5}$$

where h_t is the Yukawa coupling of ϕ to the top quark. The tree-level relation $m_\phi^2 = M_Z^2 \cos^2 2\beta$ follows from Eqs. (3)–(5).

The (RGE-unimproved) EP of the SM up to the one-loop level is given by

$$V_1 = V_{(0)} + V_{(1)} \ , \tag{6}$$

$$V_{(0)} = -\tfrac{1}{2}m^2\varphi_c^2 + \tfrac{1}{24}\lambda\varphi_c^4 \ , \tag{7}$$

$$V_{(1)} = -\frac{3}{64\pi^2}(h_t^2\varphi_c^2)^2\left(\ln\frac{h_t^2\varphi_c^2}{2\mu^2} - \frac{3}{2}\right) + \cdots \ , \tag{8}$$

where φ_c is the classical field corresponding to the physical Higgs boson ϕ, and all the Yukawa couplings of ϕ to quarks and leptons except the top quark are neglected. The calculation is performed in the Landau gauge and in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme to obtain the one-loop result $V_{(1)}$, and μ is the renormalization scale. The ellipsis in Eq. (8) represent contributions to the gauge bosons and the would-be Goldstone bosons. Throughout this paper we use the Landau gauge which is the most convenient for our purpose [18] and the $\overline{\mathrm{MS}}$ scheme.

Now we improve the EP by using the RGE. It was recently emphasized by the authors of Refs. [15–17] that in the $\overline{\mathrm{MS}}$ scheme the EP $V(\varphi_c)$ fails to satisfy the usual (homogeneous) RGE unless $V(0)$, a contribution to the "vacuum energy," is suitably dealt with. When we use the RGE-improved EP in the leading order and obtain m_ϕ, the consideration of $V(0)$ term is unnecessary. However, as we shall see below, if we improve the EP by RGE up to the next-to-leading order, $V(0)$ becomes relevant to us and we must take its presence into account. Thus with an appropriate φ_c-independent term being added, the new EP $V(\varphi_c)$ satisfies the following RGE of the usual form:

$$\left(\mathcal{D} - \gamma_\phi\varphi_c\frac{\partial}{\partial\varphi_c}\right)V(\varphi_c, X_i, \mu) = 0 \tag{9}$$

with

$$\mathcal{D} = \mu\frac{\partial}{\partial\mu} + \beta_{X_i}\frac{\partial}{\partial X_i} \ , \tag{10}$$

where $X_i = \lambda, h_t, m^2, g_3, g_2, g_1$, and g_3 is the gauge coupling constant of $\mathrm{SU}(3)_C$. The solution is easily found by the method of characteristics and we get

$$V(\varphi_c, X_i, \mu) = V(\varphi_c(t), X_i(t), \mu(t)) \ , \tag{11}$$

where $X_i(t)$ are running couplings and running mass which are determined by the equations

$$\frac{dX_i(t)}{dt} = \beta_{X_i}(X_j(t)) \ , \quad X_i, X_j = \lambda, h_t, m^2, g_3, g_2, g_1 \tag{12}$$

with the boundary conditions $X_i(0) = X_i$, and

$$\begin{aligned} \xi(t) &= \exp\left\{-\int_0^t \gamma_\phi(t')dt'\right\} \ , \\ \varphi_c(t) &= \varphi_c\xi(t) \ , \\ \mu(t) &= \mu e^t \ . \end{aligned} \tag{13}$$

Then using the result of tree- and one-loop-level EP of Eqs. (7) and (8), we obtain the RGE-improved V as follows:

$$\begin{aligned} V = {} & \Omega(X_i(t), \mu(t)) + V_{(0)}(\varphi_c(t), X_i(t)) \\ & + V_{(1)}(\varphi_c(t), X_i(t), \mu(t)) + \cdots \ , \end{aligned} \tag{14}$$

where Ω is the φ_c-independent term which is added for V to satisfy a RGE of the usual homogeneous form, and the ellipses represent the higher loop contributions.

For later convenience, let us expand the RGE coefficient functions β_{X_i} and γ_ϕ by the number of loops as follows:

$$\begin{aligned} \beta_{X_i} &= \hbar\beta_{X_i}^{(1)} + \hbar^2\beta_{X_i}^{(2)} + \cdots \ , \quad X_i = \lambda, h_t, m^2, g_3, g_2, g_1 \ , \\ \gamma_\phi &= \hbar\gamma_\phi^{(1)} + \hbar^2\gamma_\phi^{(2)} + \cdots \ , \end{aligned} \tag{15}$$

where we have introduced the Planck's constant $\hbar$ so that the power of $\hbar$ counts the number of loops and $\beta_{X_i}^{(n)}$ and $\gamma_\phi^{(n)}$ are the n-loop contributions to β_{X_i} and γ_ϕ, respectively. Similarly, V has the loop expansion

$$V = \Omega + V_{(0)}(\varphi_c(t)) + \hbar V_{(1)}(\varphi_c(t)) + \cdots \ , \tag{16}$$

and we have denoted $\Omega(X_i(t), \mu(t))$ and $V_{(n)}(\varphi_c(t), X_i(t), \mu(t))$ as Ω and $V_{(n)}(\varphi_c(t))$, for short, respectively.

Inserting Eq. (16) into Eq. (9) and picking the terms up to of the order $\hbar$, we find

$$\mathcal{D}\Omega + \hbar\left\{\beta_{X_i}^{(1)}\frac{\partial V_{(0)}}{\partial X_i} - \gamma_\phi^{(1)}\varphi_c\frac{\partial V_{(0)}}{\partial\varphi_c} + \frac{3}{32\pi^2}h_t^4\varphi_c^4\right\} = 0 \ , \tag{17}$$

where the last term in the curly brackets arises from $\mu\partial V_{(1)}/\partial\mu$ when we use the expression of $V_{(1)}$ in Eq. (8) and neglect the contributions of the gauge bosons and the would-be Goldstone bosons to $V_{(1)}$. If we further set $\varphi_c = 0$ in the above equation, we obtain

$$\mathcal{D}\Omega + \hbar\mu\frac{\partial V_{(1)}(\varphi_c = 0)}{\partial\mu} = 0 \ , \tag{18}$$

from which Ω can be determined to the leading order. For the later discussion, however, we do not need the specific form of Ω, and we will see below that the knowledge of Eq. (17) is sufficient for our purpose.

We will now analyze the mass of the lightest Higgs boson using the RGE-improved V. At first, let us consider the boundary conditions for coupling constants. Under our basic assumptions explained before, the relation between the quartic coupling constant λ and the gauge coupling constants g_1 and g_2 given in Eq. (3) should be satisfied at the scale $M_{\rm SUSY}$, that is,

$$\tfrac{1}{3}\lambda(M_{\rm SUSY}) = \tfrac{1}{4}[g_1^2(M_{\rm SUSY}) + g_2^2(M_{\rm SUSY})]\cos^2 2\beta \ . \tag{19}$$

So we choose the renormalization scale μ to be $M_{\rm SUSY}$ and take the parameter t as $t = \ln(\varphi_c/M_{\rm SUSY})$. Then we find that $\mu(t) = \varphi_c$ and the RGE-improved V is given by

$$V = \Omega + V_{(0)}(\varphi_c(t)) + \hbar V_{(1)}(\varphi_c(t)) + O(\hbar^2) \ , \tag{20}$$

with

$$V_{(1)}(\varphi_c(t)) = -\frac{3}{64\pi^2}h_t^4(t)\varphi_c^4(t)\left[\ln\frac{h_t^2(t)\xi^2(t)}{2} - \frac{3}{2}\right] \ , \tag{21}$$

where in $V_{(1)}(\varphi_c(t))$ we only include the top-quark contribution to the one-loop EP, because, with its very large Yukawa coupling, the contribution of the top quark is dominant over those from the gauge bosons and the would-be Goldstone bosons. We will obtain the lightest Higgs boson mass by evaluating $\partial^2 V/\partial\varphi_c(t)^2$ at $\varphi_c(t_v) = v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV under the minimum condition $\partial V/\partial\varphi_c(t) = 0$ at $\varphi_c(t_v) = v$. The value t_v is determined by the equation $\varphi_c(t_v) = v$, which, with help of Eq. (13), is transformed into

$$t_v = \ln\frac{v}{M_{\rm SUSY}} + \int_0^{t_v}\gamma_\phi(t')dt' \ . \tag{22}$$

It is noted that we differentiate V not by φ_c but by the renormalized field $\varphi_c(t)$ and also that we evaluate the differentials at the point of $\varphi_c(t_v) = v$ and not at $\varphi_c = v$.

Since $X_i(t)$, $\varphi_c(t)$, and $\mu(t)$ are functions of t, we find

$$\begin{aligned}\frac{\partial X_i(t)}{\partial\varphi_c(t)} &= \hbar\beta_{X_i}^{(1)}(t)\frac{1}{\varphi_c(t)} + O(\hbar^2) \ , \\ \frac{\partial\mu(t)}{\partial\varphi_c(t)} &= \frac{\mu(t)}{\varphi_c(t)} + O(\hbar) \ .\end{aligned} \tag{23}$$

Thus we obtain

$$\begin{aligned}\frac{\partial\Omega}{\partial\varphi_c(t)} &= \frac{1}{\varphi_c(t)}\left\{\mu(t)\frac{\partial\Omega}{\partial\mu(t)} + \hbar\beta_{X_i}^{(1)}(t)\frac{\partial\Omega}{\partial X_i(t)}\right\} + O(\hbar^2) \ , \\ \frac{\partial V_{(0)}(\varphi_c(t))}{\partial\varphi_c(t)} &= V'_{(0)}(\varphi_c(t)) + \hbar\frac{\beta_{X_i}^{(1)}(t)}{\varphi_c(t)}\frac{\partial V_{(0)}(\varphi_c(t))}{\partial X_i(t)} + O(\hbar^2) \ , \\ \frac{\hbar\partial V_{(1)}(\varphi_c(t))}{\partial\varphi_c(t)} &= -\hbar\frac{3}{16\pi^2}h_t^4(t)\varphi_c^3(t)\left[\ln\frac{h_t^2(t)\xi^2(t)}{2} - \frac{3}{2}\right] + O(\hbar^2) \ .\end{aligned} \tag{24}$$

It is now straightforward to evaluate $\partial V/\partial\varphi_c(t)$ from Eq. (24). Using the relation (17), we eliminate $\mathcal{D}\Omega$ term and find

$$\frac{\partial V}{\partial\varphi_c(t)} = (1 + \hbar\gamma_\phi^{(1)}(t))\left\{-m^2(t)\varphi_c(t) + \tfrac{1}{6}\lambda(t)\varphi_c^3(t)\right\} + \hbar\left\{-\frac{3}{16\pi^2}h_t^4(t)\varphi_c^3(t)\left[\ln\frac{h_t^2(t)\xi^2(t)}{2} - 1\right]\right\} + O(\hbar^2) \ . \tag{25}$$

Further differentiation of $\partial V/\partial\varphi_c(t)$ by $\varphi_c(t)$ gives

$$\begin{aligned}\frac{\partial^2 V}{\partial\varphi_c^2(t)} &= (1 + \hbar\gamma_\phi^{(1)}(t))\{-m^2(t) + \tfrac{1}{2}\lambda(t)\varphi_c^2(t)\} + \hbar\{-\beta_{m^2}^{(1)}(t) + \tfrac{1}{6}\beta_\lambda^{(1)}(t)\varphi_c^2(t)\} \\ &\quad + \hbar\left\{-\frac{9}{16\pi^2}h_t^4(t)\varphi_c^2(t)\left[\ln\frac{h_t^2(t)\xi^2(t)}{2} - 1\right]\right\} + O(\hbar^2) \ .\end{aligned} \tag{26}$$

Using the minimum condition $\partial V/\partial\varphi_c(t) = 0$ at $\varphi_c(t_v) = v$, we finally obtain for the lightest Higgs boson mass in the next-to-leading logarithm approximation

$$\begin{aligned}m^2_{\phi(2\ \rm loop)} &= \left.\frac{\partial^2 V}{\partial\varphi_c^2(t)}\right|_{\varphi_c(t_v)=v} \\ &= \frac{1}{3}\lambda(t_v)v^2 + \hbar v^2\left\{\frac{1}{6}\beta_\lambda^{(1)}(t_v) - \frac{1}{6}\lambda(t_v)\left[\frac{\beta_{m^2}^{(1)}(t_v)}{m^2(t_v)} - 2\gamma_\phi^{(1)}(t_v)\right]\right. \\ &\quad \left. - \frac{3}{8\pi^2}h_t(t_v)^4\left[\ln\frac{h_t^2(t_v)\xi^2(t_v)}{2} - 1\right]\right\} + O(\hbar^2) \ .\end{aligned} \tag{27}$$

The first term in Eq. (27) gives

$$m^2_{\phi(1\ \mathrm{loop})} = \tfrac{1}{3}\lambda(t_v)v^2 , \tag{28}$$

which is the result given by Refs. [1,4], except that the authors of Ref. [1] have evaluated the running coupling $\lambda(t)$ at $t_{m_\phi} = \ln(m_\phi/M_{\mathrm{SUSY}})$. As far as the leading logarithm approximation is concerned, the terms of order $\hbar$ in Eq. (27) are neglected as the higher order effects. An arbitrariness coming from different choice of the parameter t also falls in the higher order corrections although it has no small effect on the predicted values numerically. Espinosa and Quirós have employed the "one-loop" formula $m^2_\phi = \frac{1}{3}\lambda(t)v^2$, which is correct only in the leading logarithm approximation, and computed $\lambda(t)$ at $t_{m_\phi} = \ln(m_\phi/M_{\mathrm{SUSY}})$ using RGE with up to two-loop β functions [6]. However, if we make use of the two-loop RGE coefficient functions for the running parameters and evaluate the lightest Higgs boson mass m^2_ϕ, we should take into account the order-$\hbar$ terms in Eq. (27) which also collect the next-to-leading logarithmic contributions.

The one- and two-loop β functions and anomalous dimension γ_ϕ for the SM, which we will use in this analysis, read as follows [17]. We define the constant A as $A \equiv 16\pi^2$. For the Higgs boson quartic coupling λ,

$$\begin{aligned} A\beta^{(1)}_\lambda &= 4\lambda^2 + 12\lambda h_t^2 - 36h_t^4 - 3\lambda(3g_2^2 + g_1^2) + \tfrac{27}{4}g_2^4 + \tfrac{9}{2}g_2^2g_1^2 + \tfrac{9}{4}g_1^4 , \\ A^2\beta^{(2)}_\lambda &= -\tfrac{26}{3}\lambda^3 - 24\lambda^2h_t^2 + 6\lambda^2(3g_2^2 + g_1^2) + \lambda\{-3h_t^4 + h_t^2(80g_3^2 + \tfrac{45}{2}g_2^2 + \tfrac{85}{6}g_1^2) - \tfrac{73}{8}g_2^4 + \tfrac{39}{4}g_2^2g_1^2 + \tfrac{629}{24}g_1^4\} \\ &\quad +180h_t^6 - h_t^4(192g_3^2 + 16g_1^2) + h_t^2(-\tfrac{27}{2}g_2^4 + 63g_2^2g_1^2 - \tfrac{57}{2}g_1^4) + \tfrac{915}{8}g_2^6 - \tfrac{289}{8}g_2^4g_1^2 - \tfrac{559}{8}g_2^2g_1^4 - \tfrac{379}{8}g_1^6 . \end{aligned} \tag{29}$$

For the top-quark Yukawa coupling h_t,

$$\begin{aligned} A\beta^{(1)}_{h_t} &= \tfrac{9}{2}h_t^3 - h_t(8g_3^2 + \tfrac{9}{4}g_2 + \tfrac{17}{12}g_1^2) , \\ A^2\beta^{(2)}_{h_t} &= h_t\{-12h_t^4 - 2\lambda h_t^2 + h_t^2(36g_3^2 + \tfrac{225}{16}g_2^2 + \tfrac{131}{6}g_1^2) \\ &\quad +\tfrac{1}{6}\lambda^2 - 108g_3^4 + 9g_3^2g_2^2 + \tfrac{19}{9}g_3^2g_1^2 - \tfrac{23}{4}g_2^4 - \tfrac{3}{4}g_2^2g_1^2 + \tfrac{1187}{216}g_1^4\} . \end{aligned} \tag{30}$$

For the gauge couplings g_3, g_2, and g_1,

$$\begin{aligned} A\beta^{(1)}_{g_3} &= -7g_3^3 , \\ A^2\beta^{(2)}_{g_3} &= g_3^3(-2h_t^2 - 26g_3^2 + \tfrac{9}{2}g_2^2 + \tfrac{11}{6}g_1^2) , \\ A\beta^{(1)}_{g_2} &= -\tfrac{19}{6}g_2^3 , \\ A^2\beta^{(2)}_{g_2} &= g_2^3(-\tfrac{3}{2}h_t^2 + 12g_3^2 + \tfrac{35}{6}g_2^2 + \tfrac{3}{2}g_1^2) , \\ A\beta^{(1)}_{g_1} &= \tfrac{41}{6}g_1^3 , \\ A^2\beta^{(2)}_{g_1} &= g_1^3(-\tfrac{17}{6}h_t^2 + \tfrac{44}{3}g_3^2 + \tfrac{9}{2}g_2^2 + \tfrac{199}{18}g_1^2) . \end{aligned} \tag{31}$$

For the mass parameter m^2,

$$\begin{aligned} A\beta^{(1)}_{m^2} &= m^2(2\lambda + 6h_t^2 - \tfrac{9}{2}g_2^2 - \tfrac{3}{2}g_1^2) , \\ A^2\beta^{(2)}_{m^2} &= m^2\{-\tfrac{5}{3}\lambda^2 - 12\lambda h_t^2 + 4\lambda(3g_2^2 + g_1^2) - \tfrac{27}{2}h_t^4 \\ &\quad +h_t^2(40g_3^2 + \tfrac{45}{4}g_2^2 + \tfrac{85}{12}g_1^2) - \tfrac{145}{16}g_2^4 + \tfrac{15}{8}g_2^2g_1^2 + \tfrac{157}{48}g_1^4\} . \end{aligned} \tag{32}$$

For the anomalous dimension γ_ϕ,

$$\begin{aligned} A\gamma^{(1)}_\phi &= 3h_t^2 - \tfrac{9}{4}g_2^2 - \tfrac{3}{4}g_1^2 , \\ A^2\gamma^{(2)}_\phi &= \tfrac{1}{6}\lambda^2 - \tfrac{27}{4}h_t^4 + h_t^2(20g_3^2 + \tfrac{45}{8}g_2^2 + \tfrac{85}{24}g_1^2) - \tfrac{271}{32}g_2^4 + \tfrac{9}{16}g_2^2g_1^2 + \tfrac{431}{96}g_1^4 . \end{aligned} \tag{33}$$

When we substitute the expression of the RGE coefficient functions $\beta^{(1)}_\lambda$, $\beta^{(1)}_{m^2}$, and $\gamma^{(1)}_\phi$ into Eq. (27), we find that many terms cancel out and we obtain a rather simple expression,

$$m^2_{\phi(2\ \mathrm{loop})} = \tfrac{1}{3}\lambda v^2 + \hbar v^2\frac{1}{16\pi^2}\left\{\tfrac{1}{3}\lambda^2 + 2\lambda h_t^2 - \tfrac{1}{2}\lambda(3g_2^2 + g_1^2) + \tfrac{9}{8}g_2^4 + \tfrac{3}{4}g_2^2g_1^2 + \tfrac{3}{8}g_1^4 - 6h_t^4\ln\frac{h_t^2\xi^2}{2}\right\} , \tag{34}$$

where it is understood that all the running parameters are the ones evaluated at $t = t_v$. It is interesting to note, in particular, that an h_t^4 term in Eq. (27), which comes from the one-loop EP of Eq. (8), cancels with another h_t^4 term in $\beta_\lambda^{(1)}$ of Eq. (29) and, as a consequence, there appears only one h_t^4 term of the form $h_t^4 \ln(h_t^2\xi^2/2)$ in the order-$\hbar$ contributions in Eq. (34). This makes the higher order corrections to $m^2_{\phi(2\text{ loop})}$ to be a milder one. From the expression of Eq. (34) we expect that the next-to-leading-order corrections give a positive contribution to m_ϕ^2, which will be shown numerically to be true below.

To evaluate $m_{\phi(2\text{ loop})}$ of Eq. (34) numerically, we choose the initial conditions for the gauge couplings $\alpha_i \equiv g_i^2/4\pi$ $(i = 3,2,1)$ at the scale $M_Z = 91.2$ GeV to be

$$\alpha_3(M_Z)= 0.115 \ , \quad \alpha_2(M_Z) = 0.0336 \ , \quad \alpha_1(M_Z)= 0.0102 \ , \tag{35}$$

which are consistent with preset experimental constraints [19–21], and define the Yukawa coupling of the top quark at the scale of its mass m_t as

$$h_t(m_t) = \sqrt{2}m_t/v \quad \text{with} \quad v = 246 \text{ GeV} \ . \tag{36}$$

The fact that the two-loop $\beta_\lambda^{(2)}$, $\beta_{h_t}^{(2)}$, and $\beta_{g_i}^{(2)}$ $(i = 3,2,1)$ are functions of the couplings λ, h_t, and g_i casts Eq. (12) into a very complicated system of coupled differential equations. For given values of m_t and M_{SUSY}, we first solve the system (12) and (22) with β_{X_i} and γ_ϕ given in Eqs. (29)–(33) together with the initial conditions Eqs. (35) and (36), and we obtain the appropriate t_v and $\lambda(t_v)$ so that λ satisfies the boundary condition, Eq. (19), when it evolves from t_v to $t = 0$. At the same time when we find the appropriate t_v and $\lambda(t_v)$, we gain all the information on the parameters which appear on the right-hand side of Eq. (34). This is how we calculate $m_{\phi(2\text{ loop})}$ for given values of m_t and M_{SUSY}.

In Fig. 1 we plot $m_{\phi(2\text{ loop})}$ as a function of m_t for $M_{\text{SUSY}} = 1$ TeV, $\cos^2 2\beta = 1$ and $\cos^2 2\beta = 0$ along with $m_{\phi(1\text{ loop})}$ in the leading logarithm approximation. Figure 2 shows the case for $M_{\text{SUSY}} = 10$ TeV. Since it is suggested that m_t is not too excessively large in MSSM [22], we have studied $m_{\phi(2\text{ loop})}$ for m_t from 100 to 200 GeV. The curves for $\cos^2 2\beta = 1 (\cos^2 2\beta = 0)$ can be considered as upper (lower) bounds for the lightest Higgs boson mass in the MSSM. From Figs. 1 and 2 we observe that the next-to-leading-order effects are non-negligible. They add 3–11 GeV (3–9 GeV) to the result in the leading logarithm approximation for the range of top quark mass 100 GeV$< m_t <$ 200 GeV and for $M_{\text{SUSY}} = 1$ TeV ($M_{\text{SUSY}} = 10$ TeV). These rather large corrections come from the order-$\hbar$ terms of Eq. (34), since without those terms we could indeed recover the result of Ref. [6], namely, the higher order corrections being negative and negligible for the considered range of parameters. Contrary to the conclusion of Espinosa and Quirós, our result shows that the higher order corrections turn out to be positive and non-negligible when top quark is very heavy.

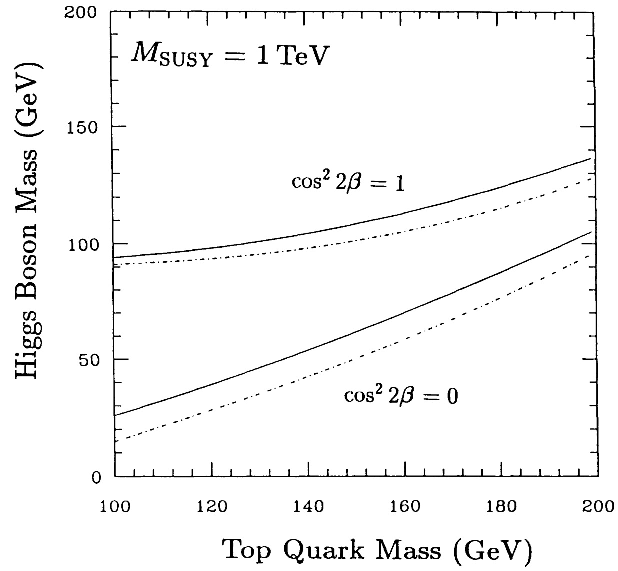

FIG. 1. Values of the Higgs boson mass as a function of m_t, for $M_{\text{SUSY}} = 1$ TeV and $\cos^2 2\beta = 1$ (upper two lines) and $\cos^2 2\beta = 0$ (lower two lines). The solid and dash-dotted lines denote the next-to-leading-order and the leading-order results, respectively.

It is to be noted that our result in the leading logarithm approximation differs numerically from those of Refs. [1,6] because we evaluated the running coupling $\lambda(t)$ at t_v given by Eq. (22), instead of at $t_{m_\phi} = \ln(m_\phi/M_{\text{SUSY}})$. In other words, we have made a different choice of t from the ones made in the above references. In the case of $\cos^2 2\beta = 1$, for example, our predicted values for $m_{\phi(1\text{ loop})}$ are smaller than those calculated in Refs. [1,6] by 0–6 GeV (0–10 GeV) for the range 100 GeV$< m_t <$

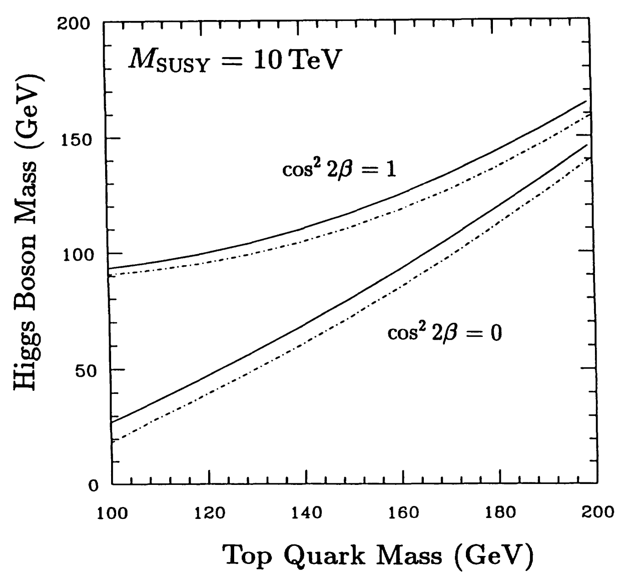

FIG. 2. The same as in Fig. 1, but considering the case for $M_{\text{SUSY}} = 10$ TeV.

200 GeV and for $M_{\rm SUSY} = 1$ TeV ($M_{\rm SUSY} = 10$ TeV). In the leading logarithm approximation, the predicted values for the lightest Higgs boson mass are rather sensitive to the choice of the renormalization parameter t. However, in the next-to-leading logarithm approximation, it is not possible to change the definition of t without modifying the $O(\hbar)$ terms of EP. In consequence, the result is stable under the change of t. This is a well-known issue which often arises in the renormalization group approach, and will be discussed in more detail in the second and third comments below.

Figure 1 shows that the next-to-leading-order corrections are large for $\cos^2 2\beta = 0$ especially when $M_{\rm SUSY} =$ 1 TeV. In the case of $\cos^2 2\beta = 0$, the boundary condition for $\lambda(t)$ at $M_{\rm SUSY}$ is $\lambda(t=0) = 0$. When $M_{\rm SUSY} = 1$ TeV, the "evolution time" is not long enough for $\lambda(t)$ to grow from the initial value $\lambda = 0$, and thus a term of the form $-6h_t^4 \ln(h_t^2\xi^2/2)$ and gauge-coupling-constant terms in the order-$\hbar$ contributions in Eq. (34) give relatively large corrections compared with the leading $\frac{1}{3}\lambda(t_v)v^2$.

A few comments are in order. The first comment concerns the definition of mass. The "mass" we have calculated is not the on-shell mass. It might be necessary to consider the correction coming from the wave function renormalization in order to make a realistic prediction for the Higgs boson mass. However, this effect is expected to be small [2,23].

Secondly, we have taken the parameter t as $t = \ln(\varphi_c/M_{\rm SUSY})$ to derive Eq. (27). But the physics should not depend on the choice of t. For example, as was stressed in Refs. [16,17], the "natural choice" of t may be given by the equation

$$2\mu^2(t) = 2\mu^2 e^{2t} = h_t^2(t)\varphi_c^2(t) \ . \tag{37}$$

Then the RGE-improved EP which we deal with will be

$$\tilde V = \Omega(X_i(t),\mu(t)) - \tfrac{1}{2}m^2(t)\varphi_c^2(t) + \tfrac{1}{24}\lambda(t)\varphi_c^4(t) + \hbar\left\{\frac{9}{128\pi^2}h_t(t)^4\varphi_c^4(t)\right\} \ . \tag{38}$$

Since the expression of the order-$\hbar$ term of the above equation is different from the one we analyzed previously, we may think at first sight that we would obtain a different result for $m_{\phi(2\ \rm loop)}$. In fact we follow the same procedure as before, i.e., first differentiate $\tilde V$ by $\varphi_c(t)$, use the relation (17) and eliminate $\mathcal{D}\Omega$ term, evaluate $\partial^2\tilde V/\partial\varphi_c(t)^2$ at $\varphi_c(\tilde t_v) = v$ under the minimum condition $\partial\tilde V/\partial\varphi_c(t) = 0$ at $\varphi_c(\tilde t_v) = v$, and we obtain the expression

$$\tilde m^2_{\phi(2\ \rm loop)} = \tfrac{1}{3}(\tilde t_v)v^2 + \hbar v^2\left\{\tfrac{1}{6}\beta_\lambda^{(1)}(\tilde t_v) - \tfrac{1}{6}\lambda(\tilde t_v)\left[\frac{\beta^{(1)}_{m^2}(\tilde t_v)}{m^2(\tilde t_v)} - 2\gamma_\phi^{(1)}(\tilde t_v)\right] + \frac{3}{8\pi^2}h_t(\tilde t_v)^4\right\} + O(\hbar^2) \ . \tag{39}$$

A logarithmic term of the form $-(3/8\pi^2)\hbar v^2 h_t^4 \ln(h_t^2\xi^2/2)$ which appeared in Eq. (27) is missing. However, we should note that the definition of t has been altered. Remembering that we have chosen the renormalization scale μ to be $M_{\rm SUSY}$, we must now evaluate the curvature of EP at the value of $\tilde t_v$, which is determined by

$$\tilde t_v = \ln\frac{v}{M_{\rm SUSY}} + \ln\frac{h_t(\tilde t_v)}{\sqrt{2}} \ . \tag{40}$$

Expanding $\lambda(\tilde t_v)$ around t_v, we find

$$\begin{aligned}\lambda(\tilde t_v) &\approx \lambda(t_v) + \hbar\beta_\lambda^{(1)}(t_v)[\tilde t_v - t_v] \\ &\approx \lambda(t_v) - \hbar\frac{9}{8\pi^2}h_t^4(t_v)\ln\frac{h_t^2(t_v)\xi^2(t_v)}{2} + \cdots \ .\end{aligned} \tag{41}$$

Then substituting the above expression for $\lambda(\tilde t_v)$ into Eq. (39), we will obtain essentially the same result as Eq. (27) except for the h_t^2 and other nondominant terms. In fact, we have calculated the mass $\tilde m_{\phi(2\ \rm loop)}$ with the above choice of t, Eq. (37), and found the difference between the two numerical results for $m_{\phi(2\ \rm loop)}$ and $\tilde m_{\phi(2\ \rm loop)}$ being less than 1 GeV for $m_t < 200$ GeV and $M_{\rm SUSY} <$ 10 TeV.

Thirdly, we often found in the literature, such as in Ref. [6], that the Higgs boson running mass is evaluated at a scale $\mu(t)$ equal to its mass. In principle, it is possible that we may choose the parameter t such that the improved effective potential takes the minimum value at $\varphi_c(t_{m_\phi}) = v$ with $t_{m_\phi} = \ln(m_\phi/M_{\rm SUSY})$, that is, at the scale $\mu(t_{m_\phi}) = m_\phi$. Again we expect that the calculated value with this choice of t will be very close to the result for $m_{\phi(2\ \rm loop)}$ for the following reason. Suppose we find an appropriate function $E(\varphi_c)$ and choose the parameter t as $t = \ln(E(\varphi_c)/M_{\rm SUSY})$ so that the improved effective potential be minimum at $\varphi_c(t_{m_\phi}) = v$.

Now the RGE-improved EP to be dealt with will be

$$\hat V = \Omega(X_i(t),\mu(t)) - \tfrac{1}{2}m^2(t)\varphi_c^2(t) + \tfrac{1}{24}\lambda(t)\varphi_c^4(t) + \hbar\frac{3}{64\pi^2}h_t^4(t)\varphi_c^4(t)\left[\ln\frac{h_t^2(t)\varphi_c^2(t)}{2\mu^2(t)} - \frac{3}{2}\right] \ , \tag{42}$$

and $\mu(t) = E(\varphi_c)$. Again we follow the same procedures as before, and evaluate the curvature of $\hat V$ at $\varphi_c(t_{m_\phi}) = v$ under the minimum condition $\partial\hat V/\partial\varphi_c(t) = 0$ at $\varphi_c(t_{m_\phi}) = v$. Since this time

$$\begin{aligned}\frac{\partial X_i(t)}{\partial\varphi_c(t)} &= \hbar\beta^{(1)}_{X_i}(t)\frac{w(\varphi_c)}{\varphi_c(t)} + O(\hbar^2) \ , \\ \frac{\partial\mu(t)}{\partial\varphi_c(t)} &= \mu(t)\frac{w(\varphi_c)}{\varphi_c(t)} + O(\hbar) \ ,\end{aligned} \tag{43}$$

where

$$w(\varphi_c) \equiv \varphi_c\frac{\partial t}{\partial\varphi_c} = \frac{\varphi_c}{E(\varphi_c)}\frac{\partial E(\varphi_c)}{\partial\varphi_c} \ , \tag{44}$$

we obtain, for m_ϕ^2,

$$m_\phi^2 = \tfrac{1}{3}\lambda(t_{m_\phi})v^2 + \hbar v^2 \left[\left\{ \tfrac{1}{6}\beta_\lambda^{(1)}(t_{m_\phi}) - \tfrac{1}{6}\lambda(t_{m_\phi}) \left(\frac{\beta_{m^2}^{(1)}(t_{m_\phi})}{m^2(t_{m_\phi})} - 2\gamma_\phi^{(1)}(t_{m_\phi}) \right) \right\} w(\widehat{\varphi_c}) + \frac{3}{8\pi^2} h_t(t_{m_\phi})^4 \left\{ \ln \frac{h_t^2(t_{m_\phi})v^2}{2m_\phi^2} - w(\widehat{\varphi_c}) \right\} \right] + O(\hbar^2) , \quad (45)$$

where $\widehat{\varphi_c}$ is the value of φ_c at which $E(\widehat{\varphi_c}) = m_\phi$ holds. We expect that $w(\widehat{\varphi_c})$ is close to 1 and since it appears in the $O(\hbar)$ terms, we can safely set $w(\widehat{\varphi_c}) = 1$ in good approximation. Again expanding $\lambda(t_{m_\phi})$ around t_v, we find this time

$$\lambda(t_{m_\phi}) \approx (t_v) + \hbar\beta_\lambda^{(1)}(t_v)[t_{m_\phi} - t_v] \approx \lambda(t_v) - \hbar\frac{9}{8\pi^2}h_t^4(t_v)\ln\frac{m_\phi^2\xi^2(t_v)}{v^2} + \cdots . \quad (46)$$

Substituting the above expression for $\lambda(t_{m_\phi})$ into Eq. (45), changing the parameter t_{m_ϕ} in the $O(\hbar)$ terms into t_v and setting $w(\widehat{\varphi_c}) = 1$, we will obtain again essentially the same result as Eq. (27) except for the h_t^2 and other nondominant terms.

Finally, there is an argument that $\varphi_c(t)$ takes the value v at the scale M_Z and $\varphi_c(t)$ at the minimum point for the RGE-improved effective potential V is different from v. Still in this case we expect that the predicted value will be very close to the numerical result for $m_{\phi(2\ \mathrm{loop})}$ by the following observations. First we choose the parameter t such that the improved effective potential takes the minimum value at $\varphi_c(t_Z) = v$ with $t_Z = \ln(M_Z/M_{\mathrm{SUSY}})$, that is, at the scale $\mu(t_Z) = M_Z$. Repeating the same procedures as we have done in the third comment, we find that with this choice of t the calculated value is expected to be very close to the one for $m_{\phi(2\ \mathrm{loop})}$. Next we alter the value of $\varphi_c(t)$ at the minimum point from v. This alteration, however, will bring about a change of the evaluation point for t from t_Z and thus bring about changes for the values of the running parameters which conspire, as before, to compensate the effect caused by the change of $\varphi_c(t)$. And we will obtain the result which is numerically very close to the one for $m_{\phi(2\ \mathrm{loop})}$.

In conclusion we have examined the mass of the lightest Higgs boson in the MSSM beyond the leading logarithm approximation. We have made use of the EP improved by RGE up to the next-to-leading order. We have found that the next-to-leading-order corrections to the Higgs boson mass are non-negligible, adding 3–11 GeV (3–9 GeV) to the values predicted by the RGE approach in the leading logarithm approximation for the range 100 GeV$<$ $m_t < 200$ GeV and for $M_{\mathrm{SUSY}} = 1$ TeV ($M_{\mathrm{SUSY}} = 10$ TeV). We also found that the predicted values of m_ϕ are stable under the change of the renormalization parameter t when we use the RGE-improved EP which includes the next-to-leading-order contributions.

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