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3 Submitted to Computational Materials Science, December 2019  
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6 **First-principles calculation of elastic properties of Cu-Zn intermetallic compounds for**  
7 **improving the stiffness of aluminum alloys**  
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18 Keywords:

19 Intermetallic compound

20 Young's modulus

21 Precipitation

22 First-principles calculation  
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28 **Abstract**

29 In this study, we computed the elastic properties of Cu-Zn binary intermetallic compounds, CuZn,  
30 Cu<sub>5</sub>Zn<sub>8</sub> and CuZn<sub>4</sub>, by first-principles calculation and discussed the capability of the improvement in  
31 stiffness of aluminum alloys by aging treatment. The disordered CuZn<sub>4</sub> with random atom  
32 distribution was emulated for the first time by virtual crystal approximation (VCA) model and  
33 special quasirandom structure (SQS) model with symmetry-based projection (SBP) technique. From  
34 the present calculation results, it was found that Young's modulus of polycrystalline aggregate of  
35 CuZn<sub>4</sub> is almost comparable to the highest counterpart of Cu<sub>5</sub>Zn<sub>8</sub> with lower elastic anisotropy, but  
36 the expected volume fraction of CuZn<sub>4</sub> is much higher than that of Cu<sub>5</sub>Zn<sub>8</sub> after aging treatment.  
37 According to the rule of mixtures for the aluminum matrix and differently oriented intermetallic  
38 compounds, therefore, CuZn<sub>4</sub> was rationally recommended as the most suitable intermetallic  
39 compound for improving the stiffness of Al-Cu-Zn alloys.  
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49 **1. Introduction**

50 Aging treatment is well known as an effective method to improve the strength of aluminum  
51 alloys. Since age-hardening phenomenon was discovered by Wilm in 1906 [1], heat treatment  
52 conditions and alloy compositions have been optimized to obtain higher strength, and thus  
53 significant expansion of industrial application of aluminum alloys has been accomplished. However,  
54 some structural components of industrial products require not only the resistance to plastic  
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3 deformation; i.e. strength, but also the resistance to elastic deformation; i.e. stiffness. Unfortunately,  
4 the stiffness of aluminum is intrinsically lower than that of other elements (Table 1), and thus  
5 commercially compensated by increasing the area or thickness of cross-sectional shapes of the  
6 components. Therefore, if the stiffness of aluminum alloys can be improved, application ranges will  
7 be further expanded because structural components such as automobile parts have low flexibility in  
8 shapes due to limitations of space. To date, temporal changes in Young's modulus during aging  
9 treatment have been reported for Al-Cu [4-8], Al-Li [9] and Al-Zn-Mg alloys [8, 10], but none of the  
10 high-strength alloys exhibited large enough increase in stiffness from pure aluminum  
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17 In general, the stiffness of alloys is represented by elastic moduli such as bulk modulus  $B$ ,  
18 shear modulus  $G$  and Young's modulus  $E$ , and such elastic moduli are believed to follow the rule of  
19 mixtures for the matrix and second phases. Because aging treatment decomposes a supersaturated  
20 solid solution into a precipitate microstructure by exploiting the difference of solubility limit, solute  
21 elements with larger solubility limit at higher temperatures and smaller solubility limit at lower  
22 temperatures become favorable for increasing the volume fraction of the second phase. Copper and  
23 zinc satisfy these requirements in aluminum (Table 1), but elastic moduli of Al-Cu or Al-Zn alloy  
24 had not been sufficiently improved by aging treatment [4-8, 10]. This is partly attributed to the lower  
25 elastic moduli of aluminum-containing intermetallic compounds formed in the two binary systems,  
26 and thus in this study we focused on the ternary system, where precipitates comprise only solute  
27 elements of Cu and Zn. Note that in the Al-Cu-Zn system  $\text{CuZn}$ ,  $\text{Cu}_5\text{Zn}_8$  and  $\text{CuZn}_4$  are expected to  
28 precipitate, and thus the most suitable intermetallic compound for improving the stiffness of the  
29 ternary alloy can be rationally recommended.  
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41 The elastic moduli of intermetallic compounds are measured experimentally, but no elastic  
42 properties have been reported for  $\text{Cu}_5\text{Zn}_8$  and  $\text{CuZn}_4$ . Computational estimation by first-principles  
43 calculation is a powerful tool to estimate elastic moduli as well as elastic constant  $C_{ij}$  or elastic  
44 compliance  $S_{ij}$ . The ability to treat not only ordered structures such as  $\text{CuZn}$  and  $\text{Cu}_5\text{Zn}_8$  but also  
45 disordered structures including  $\text{CuZn}_4$  is another strength of computational estimation. For example,  
46 virtual crystal approximation (VCA) model [11] has been utilized to compute elastic properties of  
47 intermetallic compounds with random atom distribution by assuming virtual atoms with intermediate  
48 properties in case that the lattice sites are stochastically occupied by two or more kinds of atoms.  
49 Although it becomes difficult to compute such an occupation behavior if the elemental atoms are far  
50 from each other on the periodic table, our selected copper and zinc are neighboring elements with  
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3 similar properties, and thus VCA model becomes applicable to the computation of elastic moduli of  
4 disordered CuZn<sub>4</sub>.

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6 In this study, furthermore, a special quasirandom structure (SQS) model [12] was also  
7 employed to emulate random atom distribution in CuZn<sub>4</sub>. In SQS model, a large supercell composed  
8 of several numbers of unit cells is randomly occupied by atoms, and the randomness of the  
9 arrangement is evaluated by the correlation function between several atoms arranged into the  
10 adjacent lattice sites. The correlation function is then repeatedly renewed in conjunction with the  
11 update of the atomic arrangement, resulting in the minimized bias in the arrangement of atoms into  
12 the lattice sites. SQS model has been successfully applied to emulate random atom distribution of  
13 solid solutions [13, 14] or high-entropy alloys [15, 16], and thus elastic moduli of disordered CuZn<sub>4</sub>  
14 are expected to be well estimated even in SQS model.  
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22 In this study, elastic properties of Cu-Zn binary intermetallic compounds were computed by  
23 first-principles calculation. The random atom distribution in CuZn<sub>4</sub> was emulated through VCA or  
24 SQS model, and the elastic moduli were numerically compared with those of CuZn and Cu<sub>5</sub>Zn<sub>8</sub>.  
25 Based on the present calculation results, furthermore, the most suitable intermetallic compound was  
26 rationally recommended for improving the stiffness of Al-Cu-Zn alloy by aging treatment.  
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## 32 33 34 **2. Computational approach**

35 In this study, elastic properties of pure Cu, pure Zn and three intermetallic compounds of  
36 CuZn, Cu<sub>5</sub>Zn<sub>8</sub> and CuZn<sub>4</sub> were computed by first-principles calculation using Cambridge Sequential  
37 Total Energy Package (CASTEP) [17] based on the density functional theory (DFT) and plane-wave  
38 pseudopotential method. The reported lattice constants, mass density and elemental atom distribution  
39 of these compounds are compared in Table 2 and Fig.1. In VCA model, disordered CuZn<sub>4</sub> was  
40 emulated by occupying all the lattice sites within its hexagonal close-packed (HCP) structure ( $c/a =$   
41  $1.568$ ) by virtual atoms, enabling the properties of zinc with a higher occupation probability of 80%  
42 to be strongly reflected by the virtual atoms (Fig.1(c)). In contrast, SQS model was generated using  
43 “mcsqs” code [23] of Alloy Theoretic Automated Toolkit (ATAT) [24]. As illustrated in Fig.1(d),  
44 eleven Cu atoms and forty-three Zn atoms were first arranged in fifty-four sites of  $3 \times 3 \times 3$  supercell  
45 for CuZn<sub>4</sub>, and pairs or triplets of atoms with a distance within  $1.6a$  ( $a$  is lattice constant) were  
46 selected for evaluating the correlation function between those atoms in SQS model.  
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57 The generalized-gradient approximation (GGA) functional of Perdew-Burke-Ernzerhof  
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(PBE) [25] was utilized as the exchange-correlation term in Kohn-Sham equations, and ultrasoft pseudopotentials [26] were assumed under a condition that cutoff energy of plane-wave is set at 600 eV for Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> (VCA) and Zn or at 450 eV for CuZn<sub>4</sub> (SQS). The k-point sampling grid in Brillouin zone generated by Monkhorst-Pack scheme [27] was 48×48×48 for Cu (primitive cell), 10×10×10 for CuZn, 8×8×8 for Cu<sub>5</sub>Zn<sub>8</sub> (primitive cell), 42×42×24 for CuZn<sub>4</sub> (VCA), 4×4×2 for CuZn<sub>4</sub> (SQS) and 44×44×20 for Zn respectively. The geometrical optimization was performed using Broyden-Fletcher-Goldfarb-Shanno (BFGS) method [28] in accordance with convergence criteria of 2×10<sup>-6</sup> eV/atom for energy, 6×10<sup>-3</sup> eV/Å for maximum force or 2×10<sup>-4</sup> Å for maximum displacement. The elastic constants were calculated from variations of stress when elastic strain of a maximum magnitude of 3×10<sup>-3</sup> is applied, whereas the elastic compliances were obtained as inverse matrix of the elastic constants.

### 3. Results

The lattice constants and mass density of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn after geometry optimization are listed in Table 2. The difference between the computed and experimentally reported values is within 1%, and thus the present calculation results appear to be correctly estimated. Note that although the lattice constants of CuZn<sub>4</sub> (SQS) are slightly deviated from the HCP structure; i.e.  $a \neq b$ ,  $\alpha \neq \beta \neq 90^\circ$  and  $\gamma \neq 120^\circ$ , the small deviation can be regarded as allowable error for representing the hexagonal structure by SQS model.

The calculated elastic constant  $C_{ij}$  and elastic compliance  $S_{ij}$  of single crystal of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn are compared in Table 3 and Table 4 with experimentally reported and previously computed values. It can be seen from Table 3 that  $C_{ij}$  of CuZn<sub>4</sub> (SQS) does not satisfy the relationship of hexagonal structures;

$$C_{11} = C_{22}, \quad C_{13} = C_{23}, \quad C_{44} = C_{55}, \quad C_{66} = (C_{11} - C_{12})/2 \quad (1)$$

because the symmetry of crystal structure of CuZn<sub>4</sub> is slightly broken after geometry optimization in SQS model. In this study, therefore, we employed a symmetry-based projection (SBP) technique [49, 50] to modify elastic tensor of CuZn<sub>4</sub>. In SBP technique, elastic tensor is projected to that with a similar but more-symmetric crystal structure, and thus elastic moduli of asymmetrical SQS model can be estimated [14, 16, 51]. The elastic constants calculated from the projected HCP structure,  $\bar{C}_{ij}$  [49, 50];

$$\bar{C}_{11} = \frac{3}{8}(C_{11} + C_{22}) + \frac{1}{4}C_{12} + \frac{1}{2}C_{66}, \quad \bar{C}_{12} = \frac{1}{8}(C_{11} + C_{22}) + \frac{3}{4}C_{12} - \frac{1}{2}C_{66},$$

$$\bar{C}_{13} = \frac{1}{2}(C_{13} + C_{23}), \quad \bar{C}_{33} = C_{33}, \quad \bar{C}_{44} = \frac{1}{2}(C_{44} + C_{55}), \quad (2)$$

are listed in **Table 3**, whereas the corresponding elastic compliances obtained by inverse matrix calculation of  $\bar{C}_{ij}$ ,  $\bar{S}_{ij}$  are listed in **Table 4**. Note that all the calculation results hereafter were estimated by SQS model with SBP technique.

In general, bulk modulus  $B$  and shear modulus  $G$  of polycrystalline aggregate are estimated from elastic constant  $C_{ij}$  and elastic compliance  $S_{ij}$  in Voigt-Reuss-Hill (V-R-H) model [52];

$$B_V = \frac{1}{9}(C_{11} + C_{22} + C_{33}) + \frac{2}{9}(C_{12} + C_{13} + C_{23}), \quad (3)$$

$$B_R = \frac{1}{(S_{11} + S_{22} + S_{33}) + 2(S_{12} + S_{13} + S_{23})}, \quad (4)$$

$$G_V = \frac{1}{15}(C_{11} + C_{22} + C_{33}) - \frac{1}{15}(C_{12} + C_{13} + C_{23}) + \frac{1}{5}(C_{44} + C_{55} + C_{66}), \quad (5)$$

$$G_R = \frac{15}{4(S_{11} + S_{22} + S_{33}) - 4(S_{12} + S_{13} + S_{23}) + 3(S_{44} + S_{55} + S_{66})}, \quad (6)$$

where subscripts V and R denote Voigt and Reuss models, respectively. In V-R-H model, furthermore,  $B$  and  $G$  in Hill model,  $B_H$  and  $G_H$ , are obtained by taking the average of  $B$  or  $G$  in Voigt and Reuss models;

$$B_H = \frac{B_V + B_R}{2}, \quad (7)$$

$$G_H = \frac{G_V + G_R}{2}. \quad (8)$$

Because Young's modulus  $E$  is calculated from  $B$  and  $G$  by

$$E = \frac{9BG}{3B + G}, \quad (9)$$

therefore, elastic moduli of polycrystalline aggregate of **Cu**, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and **Zn** can be estimated in **Table 5**. From the fact that elastic moduli of CuZn<sub>4</sub> in VCA model and SQS model with SBP technique are close each other, the latter model was confirmed to be another calculation method for emulating random atom distribution in CuZn<sub>4</sub>. It was also found from **Table 5** that in any model  $E$  of CuZn<sub>4</sub> is almost comparable to the highest  $E$  of Cu<sub>5</sub>Zn<sub>8</sub>, whereas  $G$  and  $E$  of CuZn significantly differ between Voigt and Reuss models due to the elastic anisotropy of CuZn as described below.

**Fig.2** shows the mole fraction dependence of the calculated elastic moduli in Hill model for polycrystalline aggregate of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present calculation results (red

cross) are compared with literature values of previous computation (green triangle) or experiments (blue square and yellow circle): Cu [29-35], CuZn [29, 36-41], Cu<sub>5</sub>Zn<sub>8</sub> [42, 43] and Zn [29, 44-48]. It can be seen from Fig.2 that  $G_H$  and  $E_H$  possess a non-monotonic variation tendency against mole fraction of Zn, whereas  $B_H$  monotonically decreases with increasing Zn content. The similar monotonic tendency of  $B$  is reported in other alloy systems [53-55], and attributed to the mass density dependence of  $B$ . Because bulk modulus  $B$  is defined by pressure  $P$  and volume  $V$ ;

$$B = -V \frac{\partial P}{\partial V}, \quad (10)$$

therefore, by replacing  $V$  by  $\rho$  the proportional relationship between  $B$  and  $\rho$  can be derived. Fig.3 shows the mass density dependence of  $B_H$  of polycrystalline aggregate for Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. As expected,  $B_H$  was found to increase linearly with  $\rho$  even through the plots of CuZn<sub>4</sub> in VCA model and SQS model with SBP technique, confirming again that the latter model is applicable to the emulation of random atom distribution in CuZn<sub>4</sub>.

Table 6 summarizes the Poisson's ratio  $\nu$ , Young's modulus to shear modulus ratio in Hill model  $B/G$  and universal elastic anisotropy index  $A_U$  of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. Here,  $\nu$  was estimated as a ratio of elastic strains generated perpendicular and parallel to applied stress;

$$\nu = \frac{3B - 2G}{6B + 2G}, \quad (11)$$

whereas  $A_U$  was evaluated as a numerical index to represent the degree of elastic anisotropy of single crystals [56];

$$A_U = 5 \frac{G_V}{G_R} + \frac{B_V}{B_R} - 6. \quad (12)$$

Because  $\nu$  and  $B/G$  have been exploited as empirical parameters that determine the fracture morphology of a material; i.e. ductile fracture is likely to occur when  $B/G > 1.75$  [57] or  $\nu > 0.26$  [58], polycrystalline aggregate of CuZn<sub>4</sub> was suggested to be brittle because CuZn<sub>4</sub> does not meet these criteria (Table 6). Such embrittlement might be the case if CuZn<sub>4</sub> is polycrystalline aggregate, but there will still be a possibility that CuZn<sub>4</sub> embedded into aluminum matrix possesses enough ductility available for commercial use. The expected stiffness of those composite aluminum alloys is evaluated and discussed in 4. Discussion.

As for elastic anisotropy, on the other hand, it was found from Table 6 that CuZn has a larger  $A_U$ , and thus higher elastic anisotropy than Cu<sub>5</sub>Zn<sub>8</sub> and CuZn<sub>4</sub> because of its large difference between  $G_V$  and  $G_R$  (Table 5). This anisotropy can be visually confirmed in Fig.4, where the

magnitude of Young's modulus  $E$  in each direction is illustrated not only by color-coding according to each color scale but also by the distance from the center of the three-dimensional (3D) space, using SC-EMA (Self-consistent Calculations of Elasticity of Multi-phase Aggregates) software package [59-61]. Here,  $E$  of Cu, CuZn and Cu<sub>5</sub>Zn<sub>8</sub> with cubic structures was calculated [62] by

$$\frac{1}{E} = S_{11} - (2S_{11} - 2S_{12} - S_{44})(l_1^2 l_2^2 + l_2^2 l_3^2 + l_3^2 l_1^2), \quad (13)$$

whereas  $E$  of CuZn<sub>4</sub> and Zn with hexagonal structures was estimated by

$$\frac{1}{E} = (1 - l_3^2)^2 S_{11} + l_3^4 S_{33} + l_3^2 (1 - l_3^2)(2S_{13} + S_{44}) \quad (14)$$

( $l_1$ ,  $l_2$  and  $l_3$  are the directional cosines). It is suggested from "ameboid" shape of the 3D surface in Fig.4(b) that single crystal of CuZn has the highest elastic anisotropy of Young's modulus; i.e. 43.1 GPa, 101.7 GPa and 185.6 GPa in the <100>, <110> and <111> directions, in agreement with the larger elastic anisotropy index  $A_U$  (Table 6). In the case of polycrystalline aggregate, however, smaller elastic moduli were inversely estimated for CuZn than those of Cu<sub>5</sub>Zn<sub>8</sub> and CuZn<sub>4</sub> (Table 5), suggesting that the highest stiffness of optimally aligned grains is averaged out by lower stiffness of the surrounding grains with the different orientations. In the following section, therefore, more elastically isotropic Cu<sub>5</sub>Zn<sub>8</sub> and CuZn<sub>4</sub> are considered to take advantage of their larger elastic moduli in polycrystalline aggregate (Table 5).

#### 4. Discussion

In this study, elastic properties of Cu-Zn binary intermetallic compounds were computed and compared from the viewpoint of not only the magnitude of elastic moduli but also the elastic anisotropy. The present calculation results revealed that Young's modulus of polycrystalline aggregate of CuZn<sub>4</sub> is almost comparable to the highest counterpart of Cu<sub>5</sub>Zn<sub>8</sub> (Table 5) with lower elastic anisotropy (Table 6 and Fig.4). Therefore, the improvement in stiffness of aluminum alloys appears to be accomplished by dispersing CuZn<sub>4</sub> or Cu<sub>5</sub>Zn<sub>8</sub> into the aluminum matrix through aging treatment, because the rule of mixtures consists of Young's moduli of aluminum matrix and differently oriented intermetallic compounds. In this study, the latter was estimated as Young's modulus of polycrystalline aggregate of the compound (Table 5), and thus the expected volume fraction of CuZn<sub>4</sub> or Cu<sub>5</sub>Zn<sub>8</sub> after aging treatment becomes a determining factor to be investigated.

If it is assumed that Cu<sub>x</sub>Zn<sub>y</sub> is formed from a supersaturated solid solution with a concentration of  $Al-k Cu - (y/x) \times k Zn$  ( $k$  is atomic fraction) and all solute atoms are used up for the

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3 formation of  $\text{Cu}_x\text{Zn}_y$ , the maximum volume of  $\text{Cu}_x\text{Zn}_y$ , formed from 1 mol of this alloy can be  
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5 estimated as follows;

$$6 \quad V_p = \frac{N_A v_p k}{n_{\text{Cu-p}}}, \quad (15)$$

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9 where  $n_{\text{Cu-p}}$  is the number of Cu atoms in the unit cell of  $\text{Cu}_x\text{Zn}_y$ ,  $v_p$  is the volume of the unit cell of  
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11  $\text{Cu}_x\text{Zn}_y$  and  $N_A$  is Avogadro number. At this time, the volume of Al matrix can be estimated as  
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13 follows;

$$14 \quad V_{\text{Al}} = \frac{N_A v_{\text{Al}} \left(1 - \frac{x+y}{x} k\right)}{n_{\text{Al-Al}}}, \quad (16)$$

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17 where  $n_{\text{Al-Al}}$  is the number of Al atoms in the unit cell of Al,  $v_{\text{Al}}$  is the volume of the unit cell of Al.  
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19 Therefore, the maximum volume fraction of the precipitated intermetallic compound  $f_p$  can be  
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21 described as follows;

$$22 \quad f_p = \frac{n_{\text{Al-Al}} v_p k}{n_{\text{Al-Al}} v_p k + n_{\text{Cu-p}} v_{\text{Al}} \left(1 - \frac{x+y}{x} k\right)}. \quad (17)$$

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24 This assumption is plausible because solubility limit of copper in aluminum is much lower than that  
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26 of zinc, allowing  $f_p$  to be determined only by mole fraction of copper in the supersaturated solid  
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28 solution. Using  $n_{\text{Al-Al}} = 4$ ,  $n_{\text{Cu-p}}(\text{Cu}_5\text{Zn}_8) = 20$ ,  $n_{\text{Cu-p}}(\text{CuZn}_4) = 0.4$ ,  $v_{\text{Al}} = 66.40 \text{ \AA}^3$  and  $v_p(\text{Cu}_5\text{Zn}_8) =$   
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30  $699.8 \text{ \AA}^3$  and  $v_p(\text{CuZn}_4) = 27.88 \text{ \AA}^3$ , the maximum volume fraction of the two precipitates were  
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32 estimated as  $f_p(\text{Cu}_5\text{Zn}_8) = 4.26 \text{ vol\%}$  and  $f_p(\text{CuZn}_4) = 8.53 \text{ vol\%}$  when  $k = 2.0 \text{ at\%}$  in consistent with  
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34 the fact that smaller mole fraction of copper within  $\text{CuZn}_4$  can lead to the higher volume fraction of  
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36  $\text{CuZn}_4$  than  $\text{Cu}_5\text{Zn}_8$ .

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39 When two phases are arranged in parallel to or in series to tensile direction, Young's  
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41 modulus of a composite material consisting of the two phases,  $E_c$ , can be described by the  
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43 following rule of mixtures, respectively [63, 64];

$$44 \quad E_c = E_1(1 - f_2) + E_2 f_2 \quad (18)$$

$$45 \quad \frac{1}{E_c} = \frac{1 - f_2}{E_1} + \frac{f_2}{E_2}, \quad (19)$$

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47 where the subscripts 1 and 2 denote each phase. The  $E_c$  values obtained from Eq.(18) and (19)  
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49 correspond to upper and lower bounds of elastic moduli for the composite material, respectively.  
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51 Young's modulus of a composite material containing particulate secondary phases takes a value  
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53 between these bounds. By substituting Young's modulus of aluminum,  $E(\text{Al}) = 70.6 \text{ GPa}$  (Table 1),  
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55 for  $E_1$ , and Young's modulus of the precipitated intermetallic compound,  $E(\text{Cu}_5\text{Zn}_8) = 138.9 \text{ GPa}$  or  
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57  $E(\text{CuZn}_4) = 135.0 \text{ GPa}$  (Table 5), for  $E_2$ , therefore, the increment in Young's modulus from pure  
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2 aluminum was found to be 1.5-2.9 GPa by  $\text{Cu}_5\text{Zn}_8$  or 3.0-5.5 GPa by  $\text{CuZn}_4$ . This indicates that the  
3 most suitable intermetallic compound for improving the stiffness of Al-Cu-Zn alloys is high-stiffness  
4 but less-anisotropic  $\text{CuZn}_4$  due to its larger volume fraction after aging treatment. The experimental  
5 assessment of the validity of this guideline will be reported elsewhere using a newly developed  
6 Al-4wt%Cu-20wt%Zn alloy.  
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## 11 12 13 **5. Conclusions**

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15 The elastic properties of the three Cu-Zn binary intermetallic compounds of  $\text{CuZn}$ ,  $\text{Cu}_5\text{Zn}_8$   
16 and  $\text{CuZn}_4$  were computed by first-principles calculations. The random atom distribution of  
17 disordered  $\text{CuZn}_4$  was emulated by virtual crystal approximation (VCA) model and special  
18 quasirandom structure (SQS) model with symmetry-based projection (SBP) technique. Based on the  
19 present calculation results, the most suitable compound for improving the stiffness of Al-Cu-Zn  
20 alloys was recommended as follows.  
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26 1. The single crystal of  $\text{CuZn}$  had higher Young's modulus in a specific direction of  $\langle 111 \rangle$  than  
27  $\text{Cu}_5\text{Zn}_8$  and  $\text{CuZn}_4$ . In the case of polycrystalline aggregate, however, smaller elastic moduli were  
28 inversely estimated for  $\text{CuZn}$ , suggesting that the highest stiffness of optimally aligned grains is  
29 averaged out by lower stiffness of the surrounding grains with the different orientations.  
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33 2. The elastic moduli of disordered  $\text{CuZn}_4$  were identically computed by VCA model and SQS  
34 model with SBP technique, confirming that the latter model is also effective in emulating random  
35 atom distribution in  $\text{CuZn}_4$ .  
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39 3. Young's modulus of polycrystalline aggregate of  $\text{CuZn}_4$  was almost comparable to the highest  
40 counterpart of  $\text{Cu}_5\text{Zn}_8$  with lower elastic anisotropy. This suggests that the most suitable  
41 intermetallic compound for improving the stiffness of Al-Cu-Zn alloys is high-stiffness but  
42 less-anisotropic  $\text{CuZn}_4$  due to its larger volume fraction after aging treatment.  
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## 48 **Acknowledgements**

49  
50 This research was supported by The Light Metals Educational Foundation of Japan. The authors  
51 deeply acknowledge the generous support.  
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3 **Captions**  
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6 **Table 1** Young's modulus of polycrystalline aggregate of Al, Cu, Zn and Fe at room temperature,  
7  $E_{\text{poly}}$  and maximum solubility limit of Cu, Zn and Fe in Al,  $S$ .  
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11 **Table 2** Lattice constants  $a, b, c, \alpha, \beta, \gamma$  and mass density  $\rho$  of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn.  
12 The present calculation results are compared with experimentally reported values.  
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16 **Table 3** Elastic constants  $C_{ij}$  of single crystal of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present  
17 calculation results are compared with experimentally reported and previously computed values.  
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21 **Table 4** Elastic compliances  $S_{ij}$  of single crystal of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present  
22 calculation results are compared with experimentally reported and previously computed values.  
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26 **Table 5** Bulk modulus  $B_{\text{poly}}$ , shear modulus  $G_{\text{poly}}$  and Young's modulus  $E_{\text{poly}}$  of polycrystalline  
27 aggregate of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present calculation results in Voigt, Reuss and  
28 Hill models are compared with experimentally reported and previously computed values.  
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32 **Table 6** Poisson's ratio  $\nu$ , bulk modulus to shear modulus ratio  $B_{\text{H}}/G_{\text{H}}$  and universal elastic  
33 anisotropy index  $A_{\text{U}}$  of Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present calculation results are  
34 compared with experimentally reported and previously computed values.  
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39 **Fig.1** Elemental atom distribution of (a) CuZn, (b) Cu<sub>5</sub>Zn<sub>8</sub> and CuZn<sub>4</sub> in (c) VCA model or (d) SQS  
40 ( $3 \times 3 \times 3$  supercell) model before geometry optimization.  
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44 **Fig.2** Mole fraction dependence of (a) bulk modulus  $B_{\text{H}}$ , (b) shear modulus  $G_{\text{H}}$  and (c) Young's  
45 modulus  $E_{\text{H}}$  of polycrystalline aggregate for Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present  
46 calculation results in Hill model are compared with experimentally reported and previously  
47 computed values.  
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52 **Fig.3** Mass density dependence of bulk modulus  $B_{\text{H}}$  of polycrystalline aggregate for Cu, CuZn,  
53 Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present calculation results in Hill model are compared with  
54 experimentally reported and previously computed values.  
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3 **Fig.4** Directional anisotropy of Young's modulus  $E$  of single crystal for **Cu**, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>,  
4 CuZn<sub>4</sub> and **Zn**. The magnitude of  $E$  in each direction is illustrated not only by color-coding  
5 according to each color scale but also by the distance from the center of the three-dimensional  
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**Table**

Table 1 Young's modulus of polycrystalline aggregate of Al, Cu, Zn and Fe at room temperature,  $E_{\text{poly}}$  and maximum solubility limit of Cu, Zn and Fe in Al,  $S$ .

	$E_{\text{poly}}$ (GPa) [2]	$S$ (at%) [3]
<b>Al</b>	70.6	
<b>Cu</b>	129.8	2.48
<b>Zn</b>	104.5	66.4
<b>Fe</b>	211.4	0.025

Table 2 Lattice constants  $a$ ,  $b$ ,  $c$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$  and mass density  $\rho$  of **Cu**, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and **Zn**. The present calculation results are compared with experimentally reported values.

Phase Space group		$a$ (Å)	$b$ (Å)	$c$ (Å)	$\alpha$ (deg.)	$\beta$ (deg.)	$\gamma$ (deg.)	$\rho$ (g/cm <sup>3</sup> )	
<b>Cu</b> <i>Fm-3m</i> (225)	Present	3.630	-	-	-	-	-	8.824	
	Exp.	3.615	-	-	-	-	-	8.935	[18]
<b>CuZn</b> <i>Pm-3m</i> (221)	Present	2.959	-	-	-	-	-	8.264	
	Exp.	2.958	-	-	-	-	-	8.270	[19]
<b>Cu<sub>5</sub>Zn<sub>8</sub></b> <i>I-43m</i> (217)	Present	8.858	-	-	-	-	-	8.035	
	Exp.	8.878	-	-	-	-	-	7.981	[20]
<b>CuZn<sub>4</sub></b> <i>P6<sub>3</sub>/mmc</i> (194)	Present(VCA)	2.747	-	4.291	-	-	-	7.702	
	Present(SQS)	2.744	2.748	4.291	90.00	90.01	120.99	7.782	
	Exp.	2.738	-	4.294	-	-	-	7.744	[21]
<b>Zn</b> <i>P6<sub>3</sub>/mmc</i> (194)	Present	2.640	-	4.977	-	-	-	7.231	
	Exp.	2.665	-	4.947	-	-	-	7.138	[22]

Table 3 Elastic constants  $C_{ij}$  of single crystal of **Cu**, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and **Zn**. The present calculation results are compared with experimentally reported and previously computed values.

		Elastic constant of single crystal (GPa)									
		$C_{11}$	$C_{12}$	$C_{13}$	$C_{22}$	$C_{23}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	
<b>Cu</b>	Present	170.8	121.5					75.3			
	Cal.	176.0	118.2					81.9			[29]
	Cal.	183.5	125.9					80.9			[30]
	Cal.	171.2	123.1					72.4			[31]
	Cal.	171.1	122.2					75.3			[32]
	Exp. at 4.2 K	176.2	124.9					81.8			[33]
	Exp. at RT	170.0	122.5					75.8			[34]
	Exp. at RT	168.1	121.5					75.1			[35]
<b>CuZn</b>	Present	134.8	104.7					75.4			
	Cal.	126.3	110.5					89.3			[36]
	Cal.	130.2	112.5					83.1			[29]
	Cal.	124.0	108.7					78.6			[37]
	Cal.	123.4	110.9					84.3			[38]
	Exp. at 4.2 K	139.6	109.2					82.3			[39]
	Exp. at RT	131.1	101.5					73.8			[40]
	Exp. at RT	127.1	107.1					80.3			[41]
<b>Cu<sub>5</sub>Zn<sub>8</sub></b>	Present	204.3	59.7					44.4			
	Cal.	195.6	61.5					41.7			[42]
	Cal.	185.3	72.9					60.5			[43]
<b>CuZn<sub>4</sub></b>	Present(VCA)	149.4	52.4	57.9			182.2	58.7			
	Present(SQS)	157.9	49.8	63.8	148.0	63.7	170.3	76.0	63.8	35.9	
	Present (SQS+SBP)	145.1	57.7	63.7			170.3	69.9			
<b>Zn</b>	Present	175.5	43.2	51.2			58.7	36.1			
	Cal.	170.7	29.8	40.4			61.3	44.6			[44]
	Cal.	171.0	37.3	51.9			63.7	41.3			[29]
	Exp. at 4.2 K	179.1	37.5	55.4			68.8	46.0			[45]
	Exp. at 4.2 K	177.0	34.8	52.8			68.5	45.9			[46]
	Exp. at RT	160.9	33.5	50.1			61.0	38.3			[47]
	Exp. at RT	159.0	32.3	48.2			62.1	40.0			[48]

Table 4 Elastic compliances  $S_{ij}$  of single crystal of **Cu**, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and **Zn**. The present calculation results are compared with experimentally reported and previously computed values.

		Elastic compliance of single crystal (TPa <sup>-1</sup> )								
		$S_{11}$	$S_{12}$	$S_{13}$	$S_{22}$	$S_{23}$	$S_{33}$	$S_{44}$	$S_{55}$	$S_{66}$
<b>Cu</b>	Present	14.35	-5.97					13.29		
	Cal.	12.33	-4.95					12.20		[29]
	Cal.	12.34	-5.02					12.36		[30]
	Cal.	14.65	-6.13					13.82		[31]
	Cal.	14.44	-6.01					13.28		[32]
	Exp. at 4.2 K	13.78	-5.72					12.22		[33]
	Exp. at RT	14.84	-6.21					13.19		[34]
	Exp. at RT	15.12	-6.34					13.31		[35]
<b>CuZn</b>	Present	23.18	-10.14					13.26		
	Cal.	43.15	-20.14					11.20		[36]
	Cal.	38.75	-17.97					12.04		[29]
	Cal.	44.55	-20.81					12.72		[37]
	Cal.	54.30	-25.70					11.86		[38]
	Exp. at 4.2 K	22.86	-10.03					12.15		[39]
	Exp. at RT	23.52	-10.26					13.55		[40]
	Exp. at RT	34.31	-15.69					12.45		[41]
<b>Cu<sub>5</sub>Zn<sub>8</sub></b>	Present	5.64	-1.28					22.51		
	Cal.	6.02	-1.44					23.98		[42]
	Cal.	6.94	-1.96					16.53		[43]
<b>CuZn<sub>4</sub></b>	Present(VCA)	8.19	-2.12	-1.93			6.72	17.04		
	Present(SQS)	8.19	-2.33	-2.15	9.61	-2.80	7.74	13.52	16.10	32.59
	Present (SQS+SBP)	8.94	-2.49	-2.41			7.68	14.31		
<b>Zn</b>	Present	7.64	0.08	-6.74			28.80	27.74		
	Cal.	6.95	-0.15	-4.48			22.22	22.41		[44]
	Cal.	7.79	0.31	-6.60			26.47	24.21		[29]
	Exp. at 4.2 K	7.46	0.39	-6.32			24.72	21.76		[45]
	Exp. at 4.2 K	7.35	0.32	-5.92			23.72	21.79		[46]
	Exp. at RT	8.38	0.54	-7.33			28.43	26.11		[47]
	Exp. at RT	8.24	0.35	-6.66			26.45	25.00		[48]

Table 5 Bulk modulus  $B_{\text{poly}}$ , shear modulus  $G_{\text{poly}}$  and Young's modulus  $E_{\text{poly}}$  of polycrystalline aggregate of **Cu**, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and **Zn**. The present calculation results in Voigt, Reuss and Hill models are compared with experimentally reported and previously computed values.

		$B_{\text{poly}}$ (GPa)			$G_{\text{poly}}$ (GPa)			$E_{\text{poly}}$ (GPa)		
		$B_V$	$B_R$	$B_H$	$G_V$	$G_R$	$G_H$	$E_V$	$E_R$	$E_H$
<b>Cu</b>	Present	137.9	137.9	137.9	55.0	41.3	48.1	145.7	112.6	129.4
	Cal.	137.4	137.4	137.4	60.7	47.3	54.0	158.8	127.3	143.3 [29]
	Cal.	145.1	145.1	145.1	60.1	46.9	53.5	158.3	127.1	142.9 [30]
	Cal.	139.1	139.1	139.1	53.0	40.1	46.6	141.2	109.9	125.7 [31]
	Cal.	138.5	138.5	138.5	55.0	41.1	48.0	145.6	112.2	129.2 [32]
	Exp. at 4.2 K	142.0	142.0	142.0	59.3	43.6	51.5	156.3	118.7	137.8 [33]
	Exp. at RT	138.3	138.3	138.3	55.0	40.4	47.7	145.6	110.4	128.3 [34]
	Exp. at RT	137.0	137.0	137.0	54.4	39.8	47.1	144.1	108.7	126.7 [35]
<b>CuZn</b>	Present	114.7	114.7	114.7	51.3	28.9	40.1	133.8	80.0	107.7
	Cal.	115.8	115.8	115.8	56.7	17.4	37.1	146.3	49.8	100.5 [36]
	Cal.	118.4	118.4	118.4	53.4	19.0	36.2	139.2	54.1	98.5 [29]
	Cal.	113.8	113.8	113.8	50.2	16.7	33.5	131.3	47.7	91.4 [37]
	Cal.	115.1	115.1	115.1	53.1	14.1	33.6	138.0	40.5	91.8 [38]
	Exp. at 4.2 K	119.3	119.3	119.3	55.5	29.8	42.6	144.1	82.4	114.2 [39]
	Exp. at RT	111.4	111.4	111.4	50.2	28.4	39.3	130.9	78.6	105.5 [40]
	Exp. at RT	113.8	113.8	113.8	52.2	21.1	36.6	135.8	59.5	99.2 [41]
<b>Cu<sub>5</sub>Zn<sub>8</sub></b>	Present	107.9	107.9	107.9	55.6	52.5	54.0	142.3	135.6	138.9
	Cal.	106.2	106.2	106.2	51.8	49.1	50.5	133.8	127.7	130.7 [42]
	Cal.	110.4	110.4	110.4	58.8	58.7	58.7	149.8	149.6	149.7 [43]
<b>CuZn<sub>4</sub></b>	Present(VCA)	90.8	89.8	90.3	54.0	53.5	53.8	135.2	133.9	134.6
	Present (SQS+SBP)	92.3	91.6	91.9	55.1	52.4	53.8	137.8	132.1	135.0
<b>Zn</b>	Present	77.9	57.8	67.8	45.2	34.0	39.6	113.7	85.2	99.4
	Cal.	69.3	55.9	62.6	51.4	41.9	46.7	123.6	100.6	112.1 [44]
	Cal.	76.4	61.5	69.0	47.5	36.6	42.1	118.1	91.6	104.9 [29]
	Exp. at 4.2 K	80.4	66.1	73.2	51.1	39.4	45.3	126.5	98.7	112.6 [45]
	Exp. at 4.2 K	78.1	64.9	71.5	51.4	40.2	45.8	126.4	100.1	113.2 [46]
	Exp. at RT	72.2	59.0	65.6	44.7	34.0	39.3	111.1	85.6	98.4 [47]
	Exp. at RT	70.8	59.0	64.9	45.4	35.6	40.5	112.3	89.0	100.6 [48]

Table 6 Poisson's ratio  $\nu$ , bulk modulus to shear modulus ratio  $B_H/G_H$  and universal elastic anisotropy index  $A_U$  of **Cu**, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and **Zn**. The present calculation results are compared with experimentally reported and previously computed values.

		$\nu$	$B_H/G_H$	$A_U$	
<b>Cu</b>	Present	0.34	2.87	1.66	
	Cal.	0.33	2.54	1.42	[29]
	Cal.	0.34	2.71	1.40	[30]
	Cal.	0.35	2.99	1.61	[31]
	Cal.	0.34	2.88	1.69	[32]
	Exp. at 4.2 K	0.34	2.76	1.80	[33]
	Exp. at RT	0.35	2.90	1.81	[34]
	Exp. at RT	0.35	2.91	1.84	[35]
<b>CuZn</b>	Present	0.34	2.86	3.87	
	Cal.	0.36	3.26	11.27	[36]
	Cal.	0.36	3.27	9.03	[29]
	Cal.	0.37	3.40	10.05	[37]
	Cal.	0.37	3.43	13.87	[38]
	Exp. at 4.2 K	0.34	2.80	4.32	[39]
	Exp. at RT	0.34	2.83	3.82	[40]
	Exp. at RT	0.35	3.11	7.39	[41]
<b>Cu<sub>5</sub>Zn<sub>8</sub></b>	Present	0.29	2.00	0.29	
	Cal.	0.29	2.10	0.28	[42]
	Cal.	0.27	1.88	0.01	[43]
<b>CuZn<sub>4</sub></b>	Present (VCA)	0.25	1.68	0.06	
	Present (SQS+SBP)	0.26	1.71	0.26	
<b>Zn</b>	Present	0.26	1.71	2.01	
	Cal.	0.20	1.34	1.37	[44]
	Cal.	0.25	1.64	1.74	[29]
	Exp. at 4.2 K	0.24	1.62	1.70	[45]
	Exp. at 4.2 K	0.24	1.56	1.59	[46]
	Exp. at RT	0.25	1.67	1.79	[47]
	Exp. at RT	0.24	1.60	1.58	[48]

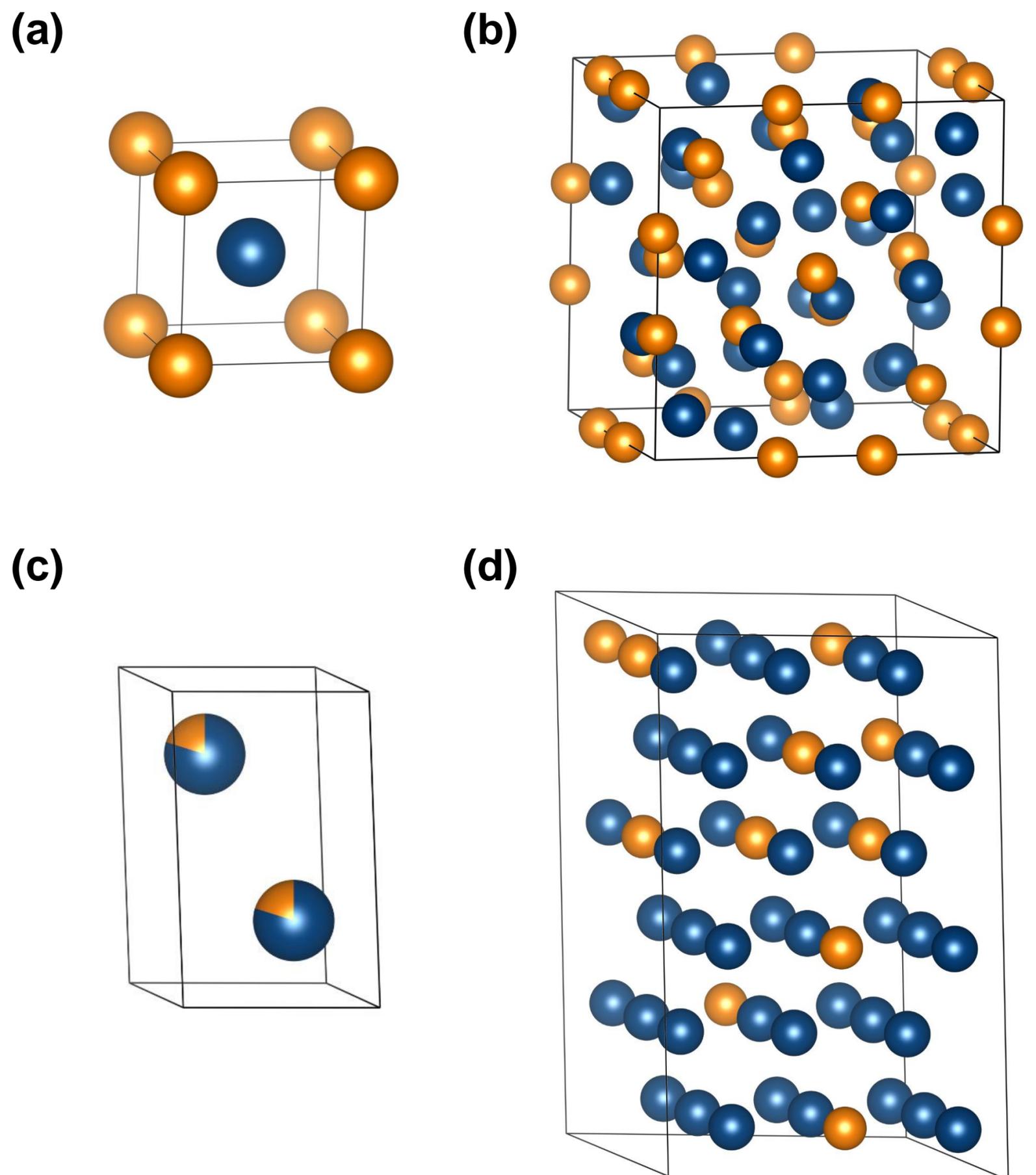


Fig.1 Elemental atom distribution of (a) CuZn, (b)  $\text{Cu}_5\text{Zn}_8$  and  $\text{CuZn}_4$  in (c) VCA model or (d) SQS ( $3 \times 3 \times 3$  supercell) model before geometry optimization.

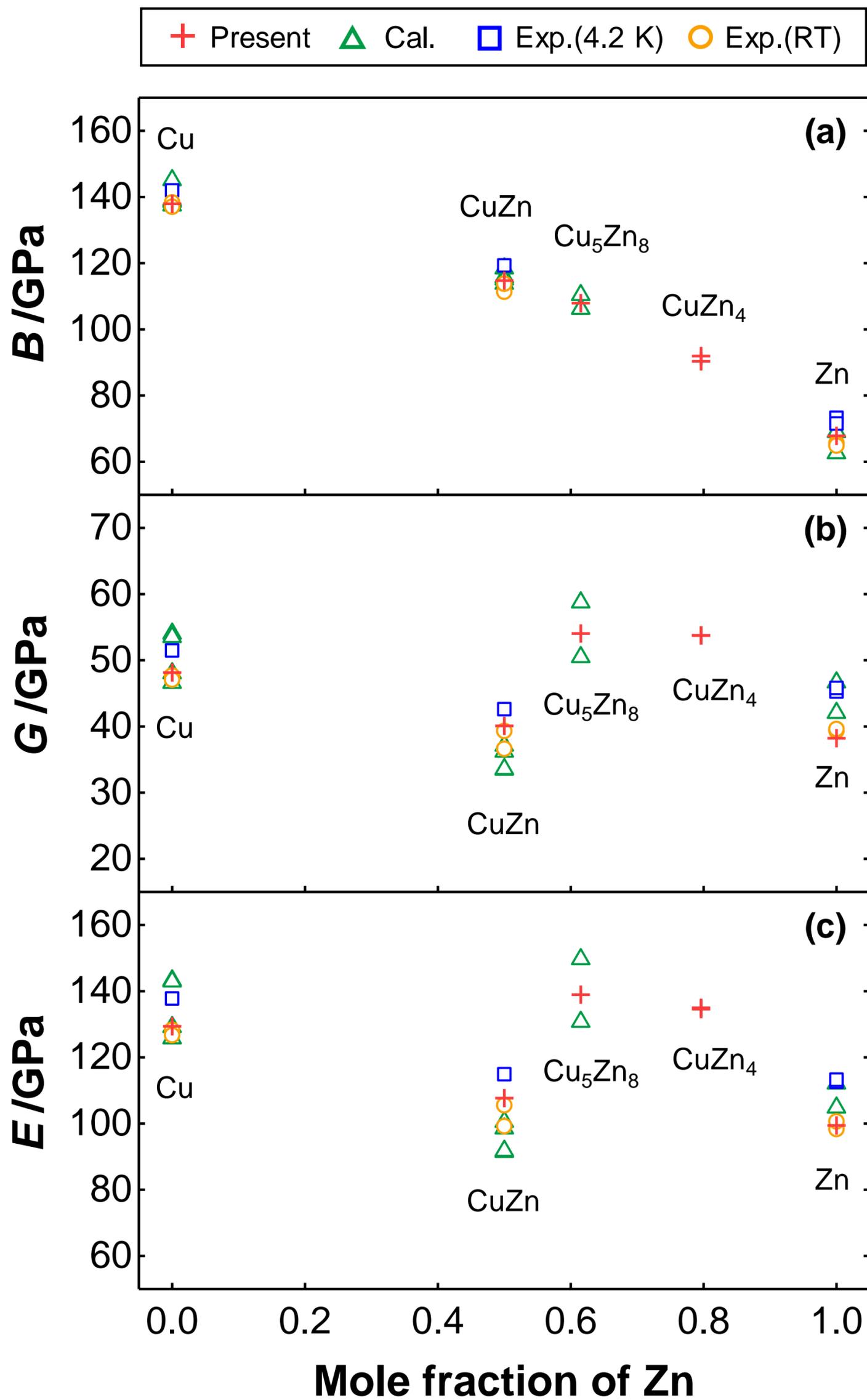


Fig.2 Mole fraction dependence of (a) bulk modulus  $B_H$ , (b) shear modulus  $G_H$  and (c) Young's modulus  $E_H$  of polycrystalline aggregate for Cu, CuZn, Cu<sub>5</sub>Zn<sub>8</sub>, CuZn<sub>4</sub> and Zn. The present calculation results in Hill model are compared with experimentally reported and previously computed values.

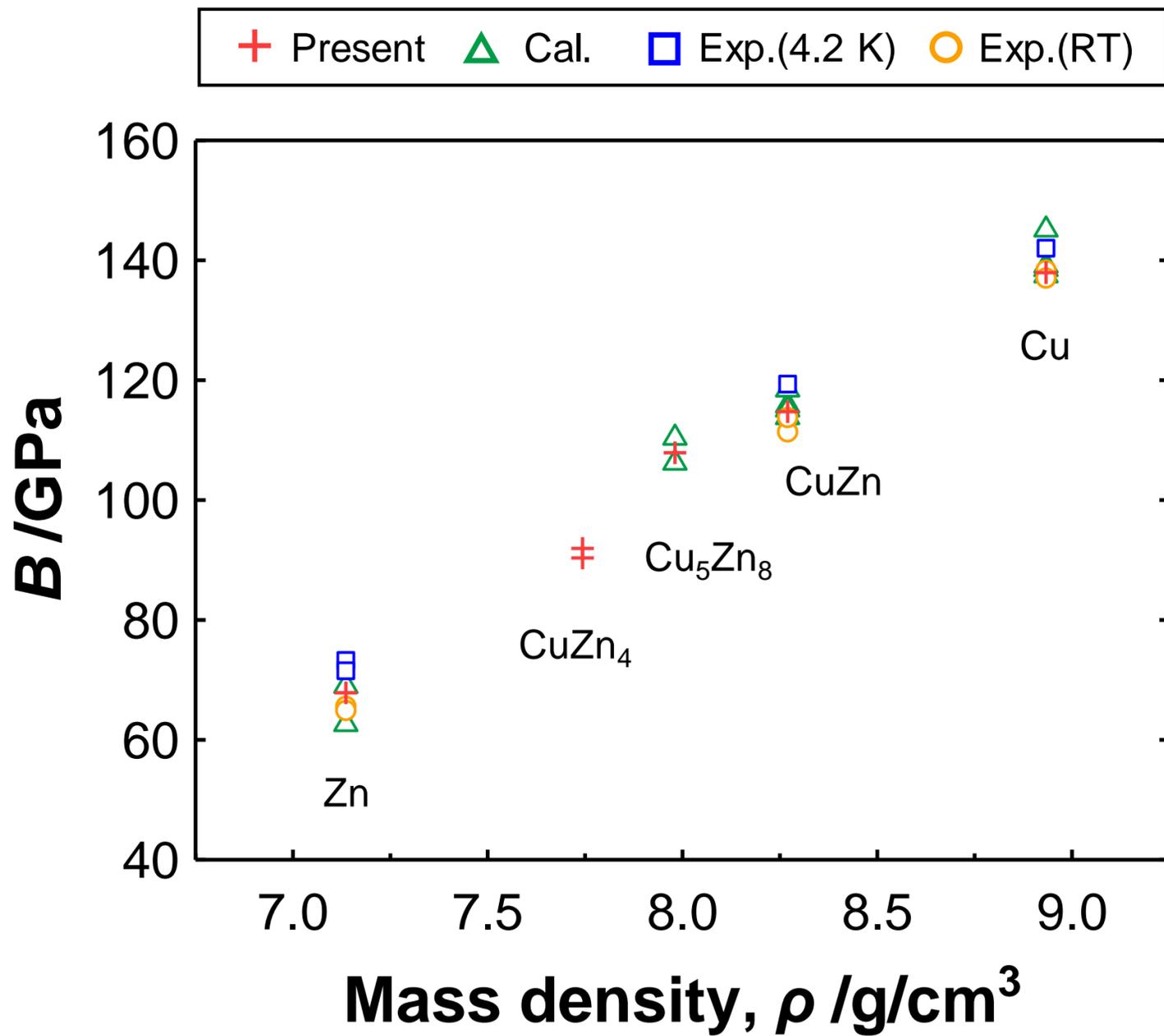
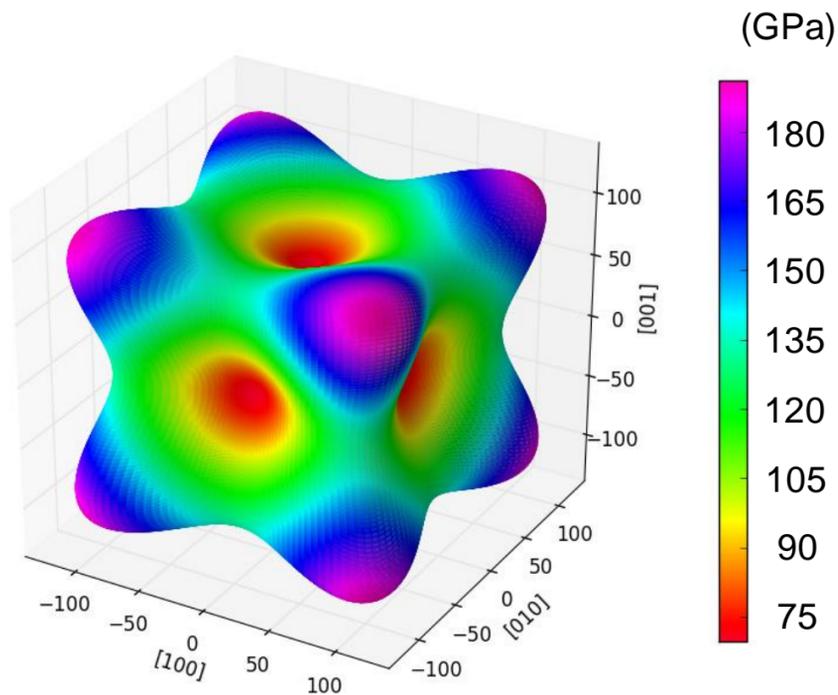
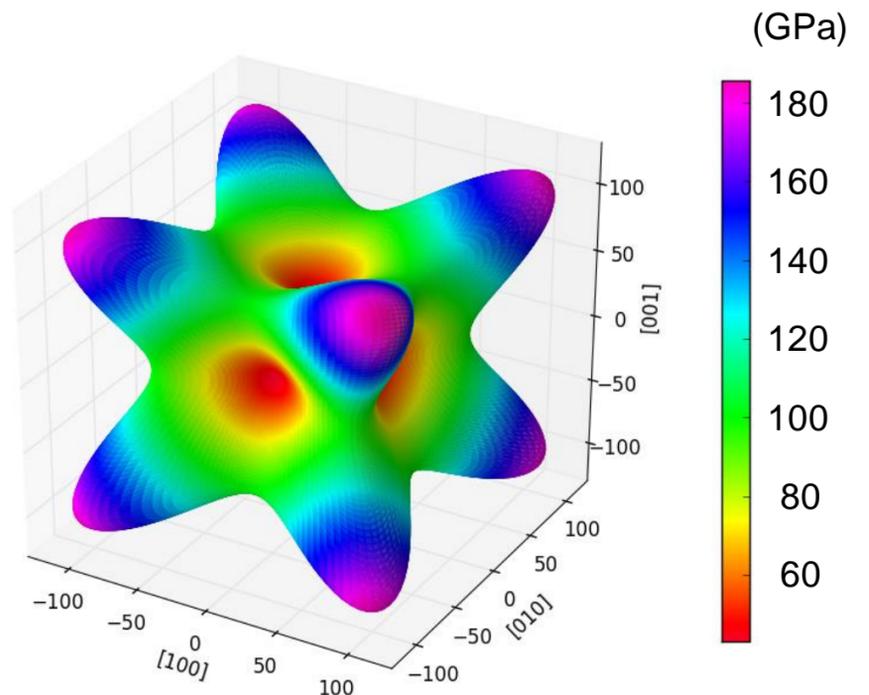


Fig.3 Mass density dependence of bulk modulus  $B_H$  of polycrystalline aggregate for Cu, CuZn,  $\text{Cu}_5\text{Zn}_8$ ,  $\text{CuZn}_4$  and Zn. The present calculation results in Hill model are compared with experimentally reported and previously computed values.

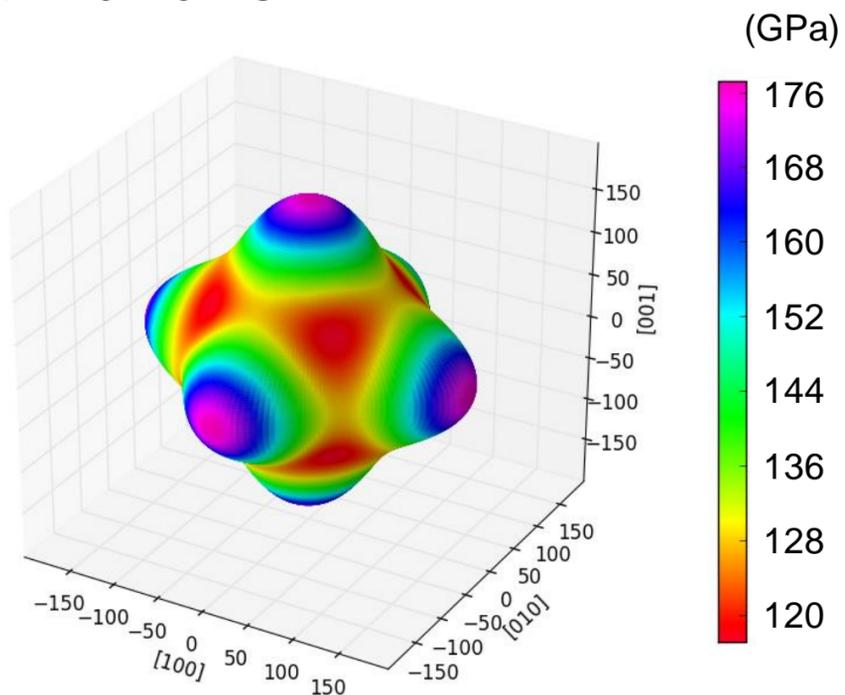
(a) Cu,  $A_U = 1.66$



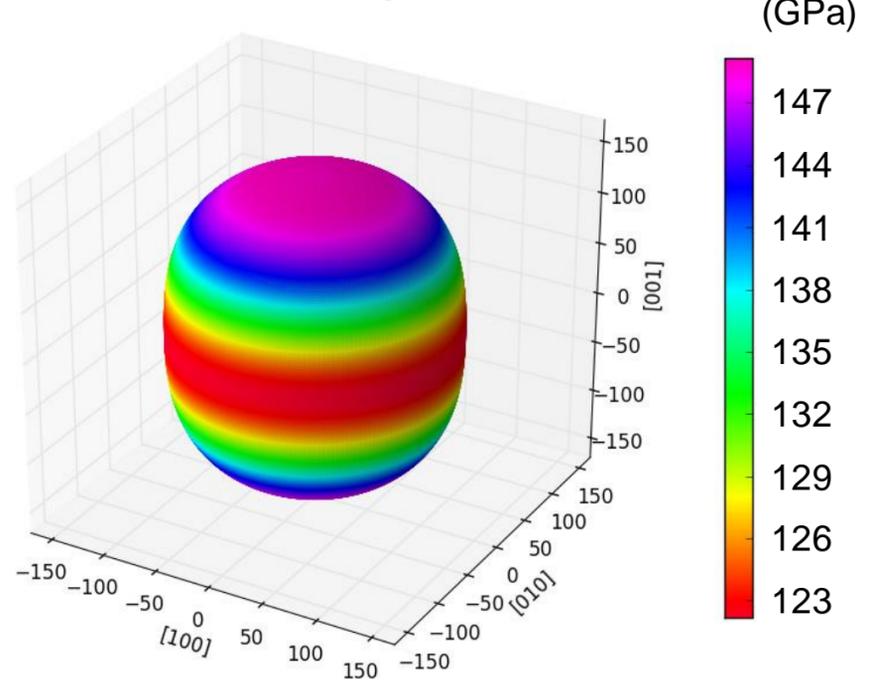
(b) CuZn,  $A_U = 3.87$



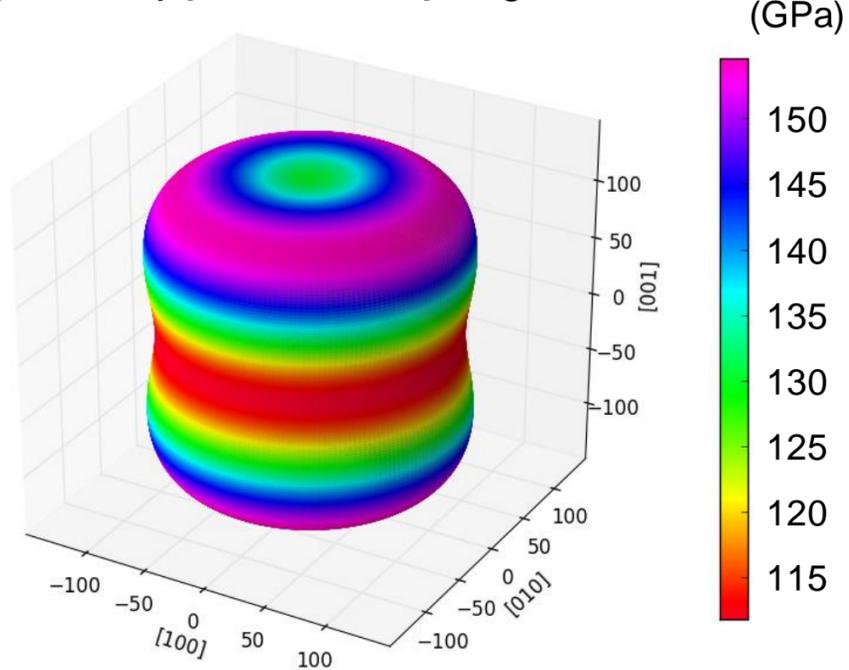
(c)  $\text{Cu}_5\text{Zn}_8$ ,  $A_U = 0.29$



(e)  $\text{CuZn}_4$  (VCA),  $A_U = 0.06$



(e)  $\text{CuZn}_4$  (SQS+SBP),  $A_U = 0.26$



(f) Zn,  $A_U = 2.01$

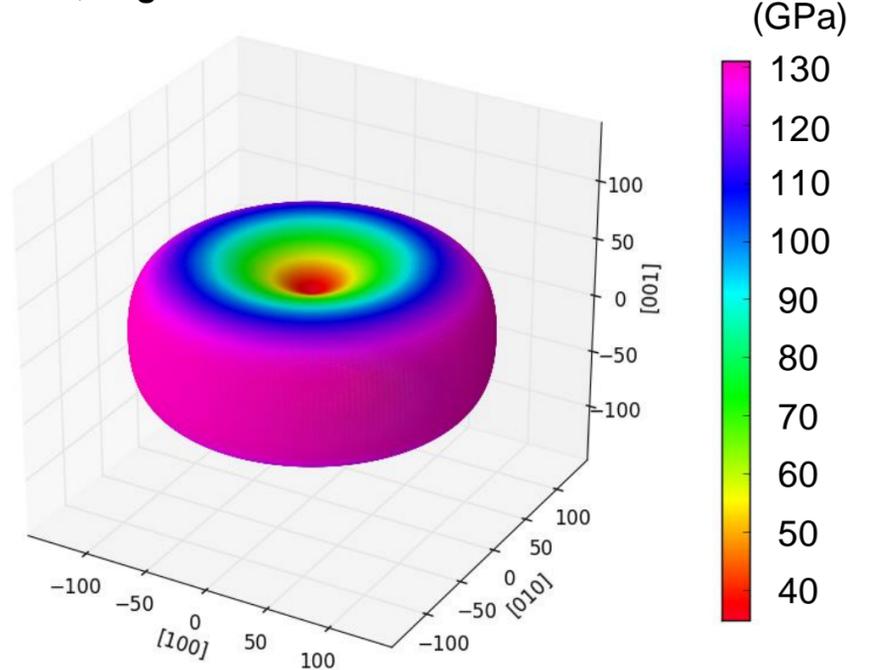


Fig.4 Directional anisotropy of Young's modulus  $E$  of single crystal for **Cu**, CuZn,  $\text{Cu}_5\text{Zn}_8$ ,  $\text{CuZn}_4$  and **Zn**. The magnitude of  $E$  in each direction is illustrated not only by color-coding according to each color scale but also by the distance from the center of the three-dimensional space.

### **Author Contributions Section**

Conceptualization, H.I. and S.H.; Methodology, H.I. and S.H.; Investigation, H.I.; Writing – Original Draft, H.I.; Writing –Review & Editing, S.H.; Funding Acquisition, S.H.; Resources, S.H.; Supervision, S.H.

### **Credit Author Statement**

Conceptualization, H.I. and S.H.; Methodology, H.I. and S.H.; Investigation, H.I.; Writing – Original Draft, H.I.; Writing –Review & Editing, S.H.; Funding Acquisition, S.H.; Resources, S.H.; Supervision, S.H.

**Declaration of interests**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: