In the nuclear, oil and gas sectors or more generally in the energetic industry, the components like vessels and piping are regularly inspected during manufacture and/or operation in order to check whether they are structurally sound and in accordance with the design quality requirements. If flaws (or cracks) are detected, assessments have to be done in order to determine the remaining life of the component containing the flaws and to demonstrate the fitness-for-service (FFS) of the component for continued operation. This FFS demonstration is performed in accordance with FFS Codes providing flaw assessment procedure and acceptance standards. The first step of the flaw assessment is the flaw characterization which aims at determining the flaw geometry for analysis. This key step is done according to flaw characterization rules provided in the FFS Codes and appears as essential for the rest of the assessment. Therefore, the continuous developments and improvements in the flaw characterization are important and necessary. On the other hand, a recent computational method specifically dedicated to Fracture Mechanics was developed since the end of the 1990’s: the Extended Finite Element Method (XFEM). This dissertation presents the improvement and the development of flaw characterization rules related to the subsurface-to-surface flaw recharacterization, the flaw combination for fatigue analyses, the characterization rules of quasi-laminar flaws and the characterization rules of laminar flaws. Each topic is presented in the six chapters contained within this dissertation.

by Valéry Lacroix
Brussels, November 2016
I would like to extend thanks to the many people who so generously contributed to the work presented in this thesis.

First of all, I would like to express my special gratitude and thanks to my supervisor Professor Osamu Umezawa for giving me the opportunity to present this PhD dissertation at Graduate School of Engineering, Yokohama National University. I acknowledge him for his guidance, remarks and support to this dissertation. In addition, his help for filling in all Japanese forms and letters is fully esteemed.

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INTRODUCTION

In the nuclear, oil and gas sectors or more generally in the energetic industry, the components like vessels and piping are regularly inspected during manufacture and/or operation in order to check whether they are structurally sound and in accordance with the design quality requirements. If flaws (or cracks) are detected, assessments have to be done in order to determine the remaining life of the component containing the flaws and to demonstrate the fitness-for-service (FFS) of the component for continued operation.

This fitness-for-service demonstration is performed in accordance with FFS Codes providing flaw assessment procedures and acceptance standards. The concepts of the FFS Codes are based in particular on Fracture Mechanics which is the field of mechanics dealing with the analyses of the cracks in materials.

A large number of FFS Codes and standards have been published in various countries: ASME Code Section XI in USA, A16 (RCC-MR) in France, JSME S NA1 in Japan, etc. All these FFS Codes are continuously evolving to account for the developments in calculation capacities, in material knowledges as well as the updating of regulatory requirements. As far as the ASME Code is concerned, which this dissertation focuses on, the evolutions are carried out and discussed in the ASME Working Groups and Committees during the Code weeks held every three months in USA.

In order to assess a flaw, it has to be characterized i.e., its size, its orientation and its position have to be unequivocally defined from the examination measurements (by ultrasonic test or other). The characterization of a flaw is done according to characterization rules provided in the FFS Codes. The flaw characterization appears as a key step for the rest of the assessment. The developments and the enhancements of the characterization rules are
therefore also essential, justifying on the same time the motivation of the works presented in this dissertation. Most of the topics treated in this dissertation address the improvements of existing characterization rules in particular the subsurface-to-surface rules, the combination rules between multiple flaws and also propose innovative rules in order to overcome some lacks in the current FFS rules.

In parallel, the accurate prediction of the fatigue life of components or structures is more and more of great interest for mechanical engineering applications. So, numerical tools should be able to simulate more accurately and more easily tridimensional fatigue crack growth without any restriction on the complexity of the model. However, the meshing of a crack in the Finite Element Method (FEM), which is still the most widespread numerical method in the industry, is still an important issue in the modelling. Indeed, in the FEM, a crack has to be explicitly meshed in the model in order to properly account for the discontinuities at the crack surfaces. Therefore, when fatigue analyses have performed, a remeshing has to be done at each step of the crack growth. Those kinds of analyses are time consuming and become rapidly unmanageable when multiple flaws have to be considered.

In this frame, since the end of the 1990’s, the computational methods dedicated to Fracture Mechanics have been strongly developed in order to alleviate the main restrictions the FEM for carrying out crack growth analyse in particular the possibility of preserving the mesh during the simulation. Based on the latter possibility, the Extended Finite Element Method (XFEM) was created. The original idea of this approach is to introduce adequate discontinuities within the elements formulation in order not to force the mesh to stick to the discontinuous surfaces of the crack. After a decade of development, this method, specifically dedicated to Fracture Mechanics, has been fully investigated, improved and made robust through a lot of academics and industrial works. XFEM has been also implemented in some FEM software and also in few purely XFEM software. Among those ones, Morfeo
Crack is particularly friendly user and offers a lot of advantages: ease of use, accuracy and stability.

This dissertation presents the findings and the outcomes of four years of research and development achieved as member of ASME Code Working Groups on Flaw Evaluation and Pipe Flaw Evaluation as well as in the frame my professional activities in Belgium at Tractebel (ENGIE). In some cases, both activities were interrelated since the design and the operation of Belgian Nuclear Power Plant have to refer to the rules of the ASME Code. This work can be seen as the outcome of the use of the XFEM with Morfeo Crack software applied to the development and the improvement of the FFS flaw characterization rules. This dissertation is divided in six Chapters:

Chapter 1 presents the theoretical background of the XFEM and the way of implicitly modelling the cracks using the level-sets method. Then, in order to give confidence in the presented results, Chapter 1 also addresses the validation of the XFEM calculations through a benchmark performed in collaboration with Engineering Mechanics Corporation of Columbus.

Chapter 2 introduces the flaw characterization rules of the different FFS Codes and Standards currently in use around the world. This State of the Art depicts the differences in the specific criteria in the flaw characterization procedures in the FFS Codes.

The first step of the flaw characterization consists of characterizing the flaw as surface or subsurface if a flaw is located near the free surface of a component. The re-characterization process from subsurface to surface flaw is addressed in all FFS Codes. Chapter 3 proposes a new subsurface-to-surface proximity rule based on experimental results and equivalent fatigue crack growth rates. Then, analyses are performed on piping and vessels in order to catch the effects of the thickness on the re-characterization of subsurface to surface flaw.

When multiple flaws are simultaneously detected in a component, FFS Codes provide combination rules. In particular, before assessing the flaw
acceptability, fatigue crack growth analyses have to be done according to the combination rules. Chapter 4 deals with the assessment of the remaining lives led by the combination rules of different FFS Codes considering two interacting flaws.

Chapter 5 presents the methodology used to develop the alternative characterization rules for quasi-laminar flaws as well as the rules themselves. The demonstration of the multiple levels of conservatism of the methodology by numerous sensitivity analyses is also showed in this Chapter. These alternative characterization rules for quasi-laminar flaws are a typical example of the development of innovative rules to overcome some lack in the current FFS rules.

Since, as it is explained in Chapter 5, the laminar flaws are a particular case of quasi-laminar flaws, the Chapter 6 proposes new characterization rules for laminar flaws based on results of Chapter 5. The scope of this proposal is to make uniform the combination criteria regardless of the kind of flaws which we would have to deal with.

The outline for this dissertation is as follows

- Chapter 1: The Extended Finite Element Method
- Chapter 2: Flaw Characterization Rules: State of the Art
- Chapter 3: Subsurface to Surface Proximity Rules
- Chapter 4: Combination Rule Assessment for Fatigue Crack Growth Analyses
- Chapter 5: Characterization Rules for Quasi-Laminar Flaws
- Chapter 6: Proposal of Characterization Rules for Laminar Flaws
- Chapter 7: Conclusions and Future Works

Finally, mention must be done that this dissertation mainly focuses of the ASME Code Section XI, although all FFS Code could be concerned by the results presented in this work. This will be repeated at times in the following Chapters.
# LIST OF ABBREVIATIONS AND NOMENCLATURE

## LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>ASME</td>
<td>American Society of Mechanical Engineers</td>
</tr>
<tr>
<td>B&amp;PV</td>
<td>Boiler &amp; Pressure Vessel</td>
</tr>
<tr>
<td>BWR</td>
<td>Boiling Water Reactor</td>
</tr>
<tr>
<td>Emc²</td>
<td>Engineering Mechanics Corporation of Columbus</td>
</tr>
<tr>
<td>EDM</td>
<td>Electric Discharge Machining</td>
</tr>
<tr>
<td>FANC</td>
<td>Federal Agency for Nuclear Control</td>
</tr>
<tr>
<td>FCG</td>
<td>Fatigue Crack Growth</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FEAM</td>
<td>Finite Element Alternating Method</td>
</tr>
<tr>
<td>FFS</td>
<td>Fitness-for-Service</td>
</tr>
<tr>
<td>JIS</td>
<td>Japan Industrial Standards</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>NDE</td>
<td>Non Destructive Examination</td>
</tr>
<tr>
<td>NPP</td>
<td>Nuclear Power Plant</td>
</tr>
<tr>
<td>PWR</td>
<td>Pressurized Water Reactor</td>
</tr>
<tr>
<td>RPV</td>
<td>Reactor Pressure Vessel</td>
</tr>
<tr>
<td>SIF</td>
<td>Stress Intensity Factor</td>
</tr>
<tr>
<td>UT</td>
<td>Ultrasonic Testing</td>
</tr>
<tr>
<td>XFEM</td>
<td>Extended Finite Element Method</td>
</tr>
</tbody>
</table>
NOMENCLATURE

$a$  Flaw depth (mm)

$a_0$  Initial flaw depth (mm)

$\beta_i$  Inclination of flaw $i$ around Y-axis ($^\circ$)

$da/dN$  Fatigue crack growth rate (mm/cycle)

$E$  Young’s modulus (MPa)

$2D_{ij}$  Dimension of the diagonal $j$ of the bounding box of flaw $i$ (mm)

$H$  Distance between flaw bounding boxes in the through-wall direction of a component Z (mm)

$k_h$  Threshold parameter for alignment criterion (-)

$k_s$  Threshold parameter for combination criterion (-)

$K_I$  Mode I SIF (MPa $\sqrt{m}$)

$K_{II}$  Mode II SIF (MPa $\sqrt{m}$)

$K_{III}$  Mode III SIF (MPa $\sqrt{m}$)

$K_{eq}$  Equivalent SIF (MPa $\sqrt{m}$)

$K_0$  Mode I SIF of the reference flaw alone (MPa $\sqrt{m}$)

$K_{1}$  Mode I SIF of the reference flaw in the presence of a second flaw (MPa $\sqrt{m}$)

$K_{eq,0}$  Equivalent SIF of the reference flaw alone (MPa $\sqrt{m}$)

$K_{eq,1}$  Equivalent SIF of the reference flaw in the presence of a second flaw (MPa $\sqrt{m}$)

$\ell$  Flaw length (mm)

$\ell_0$  Initial flaw length (mm)

$N$  Number of load cycles in fatigue crack growth analyses

$n$  Fatigue Crack Growth rate exponent

$\nu$  Poisson’s ratio (-)
\( \sigma_1 \)  Principal stress in the X-direction (MPa)
\( \sigma_2 \)  Principal stress in the Y-direction (MPa)
\( S_1 \)  Distance between flaw bounding boxes in the direction of \( \sigma_1 \) (mm)
\( S_2 \)  Distance between flaw bounding boxes in the direction of \( \sigma_2 \) (mm)
\( S \)  Distance between two flaws (mm)
\( S/a \)  Distance flaw to free surface of the component (mm)
\( S'/a \)  Flaw-to-surface proximity factor
\( T \)  Metal temperature (°C)
\( t \)  Component thickness (mm)
\( \theta_i \)  Point of elliptical crack front (-)
\( \theta_i \)  Inclination of flaw \( i \) around X-axis (°)
\( u(x) \)  Displacement field
\( u^h(x) \)  Approximation of the displacement field
\( w \)  Height of a laminar flaw (mm)
\( \xi \)  Interaction factor between two flaws (-)
\( x \)  Cartesian coordinates in the considered domain
\( X \)  Axis in the direction of the principal stress \( \sigma_1 \)
\( Y \)  Axis in the direction of the principal stress \( \sigma_2 \)
\( Z \)  Axis perpendicular to the principal stresses \( \sigma_1 \) and \( \sigma_2 \)
CHAPTER 1

THE EXTENDED FINITE ELEMENT METHOD

1.1. INTRODUCTION

The extended finite element method (XFEM) [1], [2] is an alternative method to the finite element method (FEM), which allows the introduction of some knowledge (called enrichment) of the solution into the approximation space, using the partition of unity method (PUM) [3]. In particular, discontinuities may be incorporated into the approximation of the unknown field such that the faces and edges of the mesh do not need to match the discontinuity geometry.

In Fracture Mechanics, the enrichments are the displacement jump across the crack surface, representing the crack opening, and functions spanning the leading term of the asymptotic expansion of the linear elastic solution in the vicinity of the front, accurately capturing the stress singularity. This way, the method permits the propagation of cracks without the need to remesh the domain between each step of the simulation. XFEM has caught a lot of attention in the computational Fracture Mechanics community since its inception in 1999. Hundreds of academic papers and tens of theses have been published on the XFEM during the last decade, applying the method for example to crack problems with contact [4], with branchments [5], in plates [6], undergoing dynamic propagation [7], but also to other physical phenomena like for example inclusions [8], microstructures [9], phase changes [10], shear bands [11], growing biofilms [12] or fluid dynamics [13].
All those works have led, on one hand, to improve the XFEM and, on the other hand, to acquire good knowledges of XFEM capabilities and limitations.

As a result, this method has been implemented in different pre-existing finite element software like Abaqus [14], Ansys [15], etc., in order to deal more easily with cracked structural components. However, so far as the author know, the use of the so implemented XFEM in FEM software is, up to know, not friendly user and need some caution in results exploitation.

In parallel, some software specifically XFEM-oriented have been developed like “Morfeo Crack” [16]. From the outset of its development, this software was developed on the basis of XFEM formulation. In this way, since the whole implementation of the calculation code has been formerly focused on XFEM, Morfeo Crack has been made very stable and robust, from the pre-processing to the solving tools and the post-processing as well.

The first part of this Chapter presents the theoretical background of the XFEM and the modelling of the cracks in the models using the level-sets method.

Then, since all the XFEM calculations were carried oud with the software Morfeo Crack, the next part of this Chapter addresses the validation of these XFEM calculations. This validation has been done through a significant benchmark performed in collaboration with Engineering Mechanics Corporation of Columbus [17].
1.2. THEORETICAL BACKGROUND OF XFEM

In the classical FEM, the following linear combination is used for the approximation of the displacement field:

\[ u^h(x) = \sum_i \phi_i(x)q_i \tag{1-1} \]

where \( \phi_i(x) \) are the shape functions defined with the help of a mesh of the domain and \( q_i \) are the degrees of freedom. The displacement field is discontinuous across a crack in the FEM by aligning the edges and the faces of the mesh with the crack. The nodes on the crack faces are duplicates in such a way that no shape function in the sum here above extends on both sides of the crack (except for the shape functions associated with the nodes on the front).

The XFEM permits the mesh not to match the crack faces thanks to the addition of a term to the discretization that represents the crack opening [2]. Naturally, this property is desirable for crack propagation simulation since a single mesh may be used for the different steps of the simulation. The approximation of the displacement field is given by:

\[ u^h(x) = \sum_i \phi_i(x)q_i + \sum_{j \in J} \phi_j(x)h(x)a_j \tag{1-2} \]

where \( h(x) \) is the jump function that is equal to +1 on one side of the crack and -1 on the other, and \( a_j \) are additional degrees of freedom. The set of nodes \( J \) over which the second sum in the right-hand side is performed contains all the nodes belonging to an element entirely cut by the crack. In this work, the shape functions are the first-order Lagrange functions. To improve the effectiveness of the displacement approximation, it is desirable to incorporate into the approximation certain enrichment related to the known near-tip displacement field. The leading terms of the asymptotic expansion of the displacement \( u(x) \) near the front under the assumptions of the LEFM are (see for example [18]):

\[ u(x) = K_iQ_i(x) + K_{II}Q_{II}(x) + K_{III}Q_{III}(x) \tag{1-3} \]
where $Q_I(x)$, $Q_{II}(x)$ and $Q_{III}(x)$ are given by:

$$Q_I(x) = \frac{1}{2\mu} \sqrt{\frac{r}{2\pi}} \begin{pmatrix} \cos \left(\frac{\theta}{2}\right) \left[\kappa - 1 + 2\sin^2 \left(\frac{\theta}{2}\right)\right] \\ \sin \left(\frac{\theta}{2}\right) \left[\kappa + 1 - 2\cos^2 \left(\frac{\theta}{2}\right)\right] \end{pmatrix}$$

(1-4)

$$Q_{II}(x) = \frac{1}{2\mu} \sqrt{\frac{r}{2\pi}} \begin{pmatrix} \sin \left(\frac{\theta}{2}\right) \left[\kappa + 1 + 2\cos^2 \left(\frac{\theta}{2}\right)\right] \\ \cos \left(\frac{\theta}{2}\right) \left[-\kappa + 1 + 2\sin^2 \left(\frac{\theta}{2}\right)\right] \end{pmatrix}$$

(1-5)

$$Q_{III}(x) = \frac{1}{\mu} \sqrt{\frac{2r}{\pi}} \begin{pmatrix} 0 \\ \sin \left(\frac{\theta}{2}\right) \end{pmatrix}$$

(1-6)

where $r$ and $\theta$ are the polar coordinates of $x$ in the normal plane to the front passing by $x$ with the angle measured from a line ahead of the crack front as depicted in Figure 1-1.

![Figure 1-1 Definition of the polar coordinates ahead the crack front](image)

$\mu$ is the shear modulus and $\kappa$ is the Kolosov constant defined as:

$$\kappa = \begin{cases} 3 - 4\nu & \text{for plane strain} \\ \frac{3-\nu}{1+\nu} & \text{for plane stress} \end{cases}$$

(1-7)

where $\nu$ is the Poison’s ratio. $K_I$, $K_{II}$ and $K_{III}$ are the stress intensity factors (SIFs) of the opening, sliding and tearing modes respectively as shown in Figure 1-2.
Consequently, a third term is added to the previous approximation of the displacement field:

\[
\mathbf{u}^h(x) = \mathbf{u}_0 + \mathbf{u}_1 + \mathbf{u}_2 = \sum_i \phi_i(x) q_i + \sum_j \phi_j(x) h(x) a_j + \sum_k \phi_k(x) \left( \sum_{l=1}^{n_E} \gamma_l(x) b_{kl} \right)
\] (1-8)

where \( \gamma_l(x) \) are \( n_E \) functions that enrich the near-tip field and \( b_{kl} \) are additional degrees of freedom. It is shown in [19] that the four following functions, called the branch functions, are suitable:

\[
\gamma(x) = \left[ \sqrt{r} \cos \left( \frac{\theta}{2} \right), \sqrt{r} \sin \left( \frac{\theta}{2} \right), \sqrt{r} \sin \left( \frac{\theta}{2} \right) \sin(\theta), \sqrt{r} \cos \left( \frac{\theta}{2} \right) \sin(\theta) \right]
\] (1-9)

This enrichment is applied on the set of nodes \( K \) located within a distance \( r_e \) to the front. This distance is called the enrichment radius.

The XFEM principles are illustrated in Figure 1-3.
1.3. CRACK REPRESENTATION BY THE LEVEL-SET METHOD

The description of discontinuities in the context of the XFEM is often realized by the level-set method [19], so as implemented in the XFEM software “Morfeo Crack” [16] used in this work.

A level-set function is a scalar function within the domain whose zero-level is interpreted as a discontinuity.

Using the level-set method, a crack can be represented with the help of two signed distance functions. The first function is the normal level set \( \phi \), which is negative on one side of the crack surface and positive on the other side. The second function is the tangent level set \( \psi \), which is negative behind the crack front and positive in front of it. Then, the crack surface is defined as the locus where the normal function is zero and the tangent level set is negative and the crack front is defined as the locus where both functions are zero.
This method is directly compatible with the XFEM formulation, as the jump function $h(x)$ corresponds to the sign of the normal level-set $\phi$. Moreover, both functions $\phi$ and $\psi$ are discretized on the same mesh as the displacement field with first-order shape functions.

The implicit crack definition is illustrated in Figure 1-4 for an elliptical crack, shown in blue color:

![Figure 1-4 Level-set functions for crack representation](image)

1.4. VALIDATION OF XFEM CALCULATIONS

In order to validate the models and calculations performed in this work using XFEM software Morfeo Crack, a code-to-code benchmark has been conducted with Engineering Mechanics Corporation of Columbus (Emc²) [17]. While this company also uses XFEM through the “Abaqus” Software [14], it also uses a completely different approach for calculating the SIFs. The finite element alternating method (FEAM) implemented in the “FRAC@ALT” program [20] is a state-of-the-art method for obtaining SIF for 3D surface and subsurface crack problems.
The benchmark consists of multiple cases focused on the SIF calculation of individual as well as interacting elliptical cracks and on the resulting interaction factor. It deals with tilted cracks (single or double inclination) subjected to uniaxial and biaxial loading. In this way, mode I, mode II and mode III SIFs of the cracks are solicited.

1.4.1. Benchmark definition

1.4.1.1. Domain and Loads

The benchmark cases involve one or two cracks in an infinite domain. This domain is modelled by a cube, illustrated in Figure 1-5, in the centre of which the cracks are located. The domain is large compared to the size of the cracks.

![Diagram](image)

Figure 1-5 Infinite domain modelled by a cube of dimensions \( L_x, L_y \) and \( L_z \)

These cracks are either stressed with uniform uniaxial load \( P_y \) applied on the top face of the cube, or with a biaxial load \((P_y, P_z)\) applied on the top and front faces of the cube.
1.4.1.2. Cases with single inclination

The first set of cases includes cracks with a single inclination $\alpha$ around the $Z$-axis as depicted in Figure 1-6. This inclination might be different from one crack to the other. Their centers are situated at a distance $H_x$ and $H_y$ from one another (see Figure 1-7 and Figure 1-8). The cracks have an elliptical shape with semi-axes $(R_{11}, R_{12})$ for crack 1 and $(R_{21}, R_{22})$ for crack 2.

![Rotation of $\alpha$ around Z-axis](image)

Figure 1-6 Two cracks tilted with an angle $\alpha$ around Z axis (3D view)

![c_1=(c_{1x},c_{1y},c_{1z})](image)

Figure 1-7 Two cracks tilted with an angle $\alpha$ around Z axis (XY view)
Based on the previous definitions, the first three cases are described in Table 1-1. It has to be noted that the cracks are either circular with $R_{12}=R_{22}=10$ (for comparison with the Abaqus X-FEM) or quasi-circular with $R_{12}=R_{22}=11$ (for comparison with FRAC@ALT FEAM).

The first case is a single crack case that will be the reference for the calculation of interaction factor. The crack is tilted with an angle of 20° so that it has a projection in the XZ plane which is loaded with a traction of $P_y = 100$ MPa.

Cases 2 and 3 are similar to case 1 except that one crack is added to assess the interaction with the first crack. The additional cracks are tilted either with an angle of +20° or -20°.
### Table 1-1 Definition of Benchmark cases with single crack inclination

<table>
<thead>
<tr>
<th>Unit</th>
<th>CASE 1 Single crack</th>
<th>CASE 2 Two cracks</th>
<th>CASE 3 Two cracks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{11} = R_{21}$ (mm)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$R_{12} = R_{22}$ (mm)</td>
<td>10 // 11</td>
<td>10 // 11</td>
<td>10 // 11</td>
</tr>
<tr>
<td>$\alpha_1$ (°)</td>
<td>20°</td>
<td>20°</td>
<td>20°</td>
</tr>
<tr>
<td>$\alpha_2$ (°)</td>
<td>-</td>
<td>20°</td>
<td>-20°</td>
</tr>
<tr>
<td>$H_x$ (mm)</td>
<td>-</td>
<td>$2 + 2<em>R</em>sin(\alpha_1)$</td>
<td>$2 + 2<em>R</em>sin(\alpha_1)$</td>
</tr>
<tr>
<td>$H_y$ (mm)</td>
<td>-</td>
<td>$2 + 2<em>R</em>sin(\alpha_1)$</td>
<td>$2 + 2<em>R</em>sin(\alpha_1)$</td>
</tr>
<tr>
<td>$H_z$ (mm)</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P_y$ (MPa)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$P_z$ (MPa)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L (mm)</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

### 1.4.1.3. Cases with double inclination

The second set of cases includes cracks with an inclination $\alpha$ around the Z-axis but also with an inclination $\theta$ around the Y-axis. The resulting coordinates of the semi-axes extremities are given in Figure 1-9.

$$
\overrightarrow{p_A} = \overrightarrow{c} + R \begin{pmatrix} \sin \alpha \\ \cos \alpha \\ 0 \end{pmatrix}
$$

$$
\overrightarrow{p_B} = \overrightarrow{c} + R' \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
$$

$$
\overrightarrow{p_A'} = \overrightarrow{c} + R \begin{pmatrix} \sin \alpha \cos \theta \\ \cos \alpha \cos \theta \\ -\sin \alpha \sin \theta \end{pmatrix}
$$

$$
\overrightarrow{p_B'} = \overrightarrow{c} + R' \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix}
$$

Figure 1-9 Tilted crack with two inclinations $\alpha$ and $\theta$ (3D view)
Based on the above definitions, the last three cases are described in Table 1-2. In addition to the second crack inclination, the main difference with the first three cases is that they are either uniaxially loaded (cases 4a, 5a and 6a) or biaxially loaded (cases 4b, 5b and 6b). Cases 4a and 4b are single crack cases that will be the reference for the calculation of interaction factor.

Table 1-2 Definition of Benchmark cases with double crack inclination

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>CASES 4 Single crack</th>
<th>CASES 5 Two cracks</th>
<th>CASES 6 Two cracks</th>
</tr>
</thead>
<tbody>
<tr>
<td>R11 = R21</td>
<td>(mm)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>R12 = R22</td>
<td>(mm)</td>
<td>10 // 11</td>
<td>10 // 11</td>
<td>10 // 11</td>
</tr>
<tr>
<td>α1</td>
<td>(°)</td>
<td>20°</td>
<td>20°</td>
<td>20°</td>
</tr>
<tr>
<td>α2</td>
<td>(°)</td>
<td>-</td>
<td>20°</td>
<td>-20°</td>
</tr>
<tr>
<td>θ1 = θ2</td>
<td>(°)</td>
<td>20°</td>
<td>20°</td>
<td>20°</td>
</tr>
<tr>
<td>Hx</td>
<td>(mm)</td>
<td>-</td>
<td>2 + 2<em>R</em>sin(α1)</td>
<td>2 + 2<em>R</em>sin(α1)</td>
</tr>
<tr>
<td>Hy</td>
<td>(mm)</td>
<td>-</td>
<td>2 + 2<em>R</em>sin(α1)</td>
<td>2 + 2<em>R</em>sin(α1)</td>
</tr>
<tr>
<td>Hz</td>
<td>(mm)</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Py</td>
<td>(MPa)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Pz</td>
<td>(MPa)</td>
<td>4a : Pz=0</td>
<td>5a : Pz=0</td>
<td>6a : Pz=0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4b : Pz=100</td>
<td>5b : Pz=100</td>
<td>6b : Pz=100</td>
</tr>
<tr>
<td>L</td>
<td>(mm)</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

### 1.4.2. Comparison of results

1.4.2.1. Stress intensity factor

The first parameter naturally considered for the comparison of the different codes/methodologies is the SIF. Because the cracks are tilted, they are loaded with mode I (crack opening) but also with mode II (crack shearing) and mode III (crack twisting). Therefore, three SIF are provided, one for each mode. Moreover, the combination of these three opening modes is taken into account using an equivalent SIF:

\[
K_{eq} = \sqrt{K_i^2 + K_{II}^2 + \frac{1}{1-\nu}K_{III}^2}
\]  

(1-10)
The SIF are evaluated at every point of the crack front, as a function of an angular parameter ranging from 0° to 360°. They are illustrated in Table 1-3 (comparison with FRAC@ALT FEAM) and in Table 1-4 (comparison with Abaqus XFEM). In order to assess the mean difference between SIF evolutions 1 and 2 over the whole set of points \(i\) of the crack front, a L2-norm is calculated for each case and given below each figure: (comparison with Abaqus XFEM). In order to assess the mean difference between SIF evolutions 1 and 2 over the whole set of points \(i\) of the crack front, a L2-norm is calculated for each case and given below each graph of Figure 1-10 to Figure 1-15:

\[
\text{L2-norm} = \sqrt{\frac{\sum (\text{SIF}_1 - \text{SIF}_2)^2}{\sum (\text{SIF}_2)^2}} \quad (1-11)
\]

The maximum equivalent SIF \(K_{eq,MAX}\) are also compared in Table 1-3 and Table 1-4.

According to the SIF evolutions presented in Figure 1-10 to Figure 1-12, it directly appears that Morfeo XFEM and FRAC@ALT FEAM results are systematically in very good agreement. While the related L2-norm remains below 2%, the \(K_{eq,MAX}\) values (Table 1-3) are even closer to each other (max. 1.23%). This successful comparison of two highly distinctive methods is a validation of the benchmark results.

As far as the Abaqus XFEM results are concerned in Figure 1-13 to Figure 1-15, the L2-norm of the SIF evolutions stays below 4% except for the very latest case. Results in terms of \(K_{eq,MAX}\) (Table 1-4) are a bit more scattered. While results of uniaxially loaded cases only slightly overestimate Morfeo XFEM (~3%), the bi-axial cases deviate a bit more from the other solutions. This behavior could be linked to the limited mesh refinement in Abaqus modeling because Abaqus provides oscillating results when the crack fronts are not modelled. As a result, a smoothing of Abaqus results was carried out after calculations when oscillations were too pronounced which may lead to introduce approximation error in Abaqus solution.
<table>
<thead>
<tr>
<th>$K_{eq,\text{max}}$ (MPa√mm)</th>
<th>Morfeo</th>
<th>FRAC@ALT</th>
<th>Morfeo − FRAC@ALT.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>146.5</td>
<td>146.2</td>
<td>0.20%</td>
</tr>
<tr>
<td>Case 2 – crack 1</td>
<td>145.1</td>
<td>143.9</td>
<td>0.83%</td>
</tr>
<tr>
<td>Case 2 – crack 2</td>
<td>145.4</td>
<td>143.9</td>
<td>1.03%</td>
</tr>
<tr>
<td>Case 3 – crack 1</td>
<td>173.8</td>
<td>174.2</td>
<td>-0.23%</td>
</tr>
<tr>
<td>Case 3 – crack 2</td>
<td>149.2</td>
<td>149.7</td>
<td>-0.34%</td>
</tr>
<tr>
<td>Case 4a</td>
<td>146.5</td>
<td>146.2</td>
<td>0.20%</td>
</tr>
<tr>
<td>Case 4b</td>
<td>196.2</td>
<td>195.4</td>
<td>0.41%</td>
</tr>
<tr>
<td>Case 5a – crack 1</td>
<td>145.8</td>
<td>144</td>
<td>1.23%</td>
</tr>
<tr>
<td>Case 5a – crack 2</td>
<td>145.8</td>
<td>144</td>
<td>1.23%</td>
</tr>
<tr>
<td>Case 5b – crack 1</td>
<td>196.6</td>
<td>195.2</td>
<td>0.71%</td>
</tr>
<tr>
<td>Case 5b – crack 2</td>
<td>197.1</td>
<td>195.1</td>
<td>1.01%</td>
</tr>
<tr>
<td>Case 6a – crack 1</td>
<td>175.1</td>
<td>175.5</td>
<td>-0.23%</td>
</tr>
<tr>
<td>Case 6a – crack 2</td>
<td>149.9</td>
<td>149.7</td>
<td>0.13%</td>
</tr>
<tr>
<td>Case 6b – crack 1</td>
<td>209.5</td>
<td>209</td>
<td>0.24%</td>
</tr>
<tr>
<td>Case 6b – crack 2</td>
<td>203.4</td>
<td>204.1</td>
<td>-0.34%</td>
</tr>
</tbody>
</table>
Table 1-4 Comparison of Stress Intensity Factor results between Morfeo XFEM and Abaqus XFEM

<table>
<thead>
<tr>
<th>$K_{\text{eq,MAX}}$ (MPa√mm)</th>
<th>Morfeo</th>
<th>FRAC@ALT</th>
<th>Morf. – Abaqus Morf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>141.7</td>
<td>145.9</td>
<td>-2.96%</td>
</tr>
<tr>
<td>Case 2 – crack 1</td>
<td>140.5</td>
<td>144.6</td>
<td>-2.92%</td>
</tr>
<tr>
<td>Case 2 – crack 2</td>
<td>140.4</td>
<td>146.2</td>
<td>-4.13%</td>
</tr>
<tr>
<td>Case 3 – crack 1</td>
<td>166.7</td>
<td>166.3</td>
<td>0.24%</td>
</tr>
<tr>
<td>Case 3 – crack 2</td>
<td>144</td>
<td>149.7</td>
<td>-3.96%</td>
</tr>
<tr>
<td>Case 4a</td>
<td>141.9</td>
<td>144.8</td>
<td>-2.04%</td>
</tr>
<tr>
<td>Case 4b</td>
<td>191.7</td>
<td>202.3</td>
<td>-5.53%</td>
</tr>
<tr>
<td>Case 5a – crack 1</td>
<td>140.8</td>
<td>144.3</td>
<td>-2.49%</td>
</tr>
<tr>
<td>Case 5a – crack 2</td>
<td>141.1</td>
<td>144.5</td>
<td>-2.41%</td>
</tr>
<tr>
<td>Case 5b – crack 1</td>
<td>192.5</td>
<td>202.5</td>
<td>-5.19%</td>
</tr>
<tr>
<td>Case 5b – crack 2</td>
<td>192.4</td>
<td>209.5</td>
<td>-8.89%</td>
</tr>
<tr>
<td>Case 6a – crack 1</td>
<td>168</td>
<td>172.6</td>
<td>-2.74%</td>
</tr>
<tr>
<td>Case 6a – crack 2</td>
<td>144.1</td>
<td>148.8</td>
<td>-3.26%</td>
</tr>
<tr>
<td>Case 6b – crack 1</td>
<td>204.1</td>
<td>216.5</td>
<td>-6.08%</td>
</tr>
<tr>
<td>Case 6b – crack 2</td>
<td>199.9</td>
<td>241.1</td>
<td>-20.61%</td>
</tr>
</tbody>
</table>
Figure 1-10 SIF evolution along the crack front
Comparison between Morfeo X-FEM and FRAC@ALT (1/3)
Figure 1-11 SIF evolution along the crack front
Comparison between Morfeo X-FEM and FRAC@ALT (2/3)
L2-norm $K_I=1.87\%$, $K_{II}=2.08\%$, $K_{III}=3.15\%$, $K_{eq}=0.74\%$

L2-norm $K_I=1.33\%$, $K_{II}=2.46\%$, $K_{III}=2.45\%$, $K_{eq}=0.85\%$

L2-norm $K_I=1.32\%$, $K_{II}=1.78\%$, $K_{III}=2.79\%$, $K_{eq}=0.58\%$

Figure 1-12 SIF evolution along the crack front
Comparison between Morfeo X-FEM and FRAC@ALT (3/3)
L2-norm $K_I=4.55\%$, $K_{II}=4.69\%$, $K_{III}=2.17\%$, $K_{eq}=1.86\%$

L2-norm $K_I=5.97\%$, $K_{II}=5.16\%$, $K_{III}=2.55\%$, $K_{eq}=1.97\%$

L2-norm $K_I=8.93\%$, $K_{II}=4.67\%$, $K_{III}=4.97\%$, $K_{eq}=3.35\%$

L2-norm $K_I=7.73\%$, $K_{II}=5.53\%$, $K_{III}=5.04\%$, $K_{eq}=2.74\%$

L2-norm $K_I=5.66\%$, $K_{II}=6.92\%$, $K_{III}=2.16\%$, $K_{eq}=2.58\%$

L2-norm $K_I=4.08\%$, $K_{II}=4.15\%$, $K_{III}=2.06\%$, $K_{eq}=1.59\%$
L2-norm $K_I=3.46\%$, $K_{II}=6.27\%$, $K_{III}=2.42\%$, $K_{eq}=3.05\%$

L2-norm $K_I=7.11\%$, $K_{II}=7.99\%$, $K_{III}=4.45\%$, $K_{eq}=2.38\%$

L2-norm $K_I=11.96\%$, $K_{II}=5.08\%$, $K_{III}=4.89\%$, $K_{eq}=2.45\%$

L2-norm $K_I=5.72\%$, $K_{II}=6.69\%$, $K_{III}=2.87\%$, $K_{eq}=3.30\%$

L2-norm $K_I=12.27\%$, $K_{II}=6.21\%$, $K_{III}=3.86\%$, $K_{eq}=4.23\%$

L2-norm $K_I=8.95\%$, $K_{II}=6.50\%$, $K_{III}=3.26\%$, $K_{eq}=2.63\%$

Figure 1-14 SIF evolution along the crack front
Comparison between Morfeo XFEM and Abaqus XFEM (2/3)
Figure 1-15 SIF evolution along the crack front
Comparison between Morfeo XFEM and Abaqus XFEM (3/3)
1.4.2.2. Interaction Factor

When the distance between two cracks is decreasing, the mechanical interaction between them is increasing and consequently their SIF. This interaction can lead to the fracture of the ligament between the cracks as illustrated in Figure 1-16.

![Figure 1-16 Mechanical interaction between flaws](image)

Therefore relevant parameter to consider for the comparison of the different codes/methodologies for configuration with multiple cracks is the so-called interaction factor $\xi$. In the case of this benchmark, the interaction factor quantifies the SIF increase of one crack due to the presence of the second one.

The interaction factor is defined as the ratio between $K_{eq,1}$, the SIF of the crack when submitted to the influence of the second flaw, and $K_{eq,0}$, the SIF of the same crack considered as isolated (no influence from other cracks in its vicinity). The SIF is calculated at each point $t$ of the elliptical crack front (see Figure 1-17). Therefore, the interaction factor $\xi$ due to the presence of a second crack can be assessed at each point $t$ of the crack front:

$$\xi(t) = \frac{K_{eq,1}(t)}{K_{eq,0}(t)}$$ (1-12)

However, the most relevant point is the one with the highest SIF as it is the most susceptible to initiate brittle fracture. The interaction factor thus becomes:
\[ \xi = \xi(t^*) \quad \text{where} \quad t^* \mid K_{eq,1}(t^*) = \max_t\left(K_{eq,1}(t)\right) \quad (1-13) \]

Figure 1-17 SIF evolution of one single flaw (left) and the same flaw impacted by another one (right)

The interaction factor has been computed for each case and is compared in Table 1-5 and Table 1-6. Once again, one can notice the very good agreement between the Morfeo XFEM and FRAC@ALT FEAM values (less than 1% discrepancy). This result definitely validates the Morfeo XFEM approach used from XFEM calculation performed in this dissertation.

On the other hand, the comparison with Abaqus XFEM is slightly more scattered (up to 4% discrepancy) but still acceptable.
Table 1-5 Comparison of interaction factor results between Morfeo X-FEM and FRAC@ALT

<table>
<thead>
<tr>
<th>ξ (-)</th>
<th>Morfeo</th>
<th>FRAC@ALT</th>
<th>Morf. −FRAC. Morf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2 / Case 1</td>
<td>0.992</td>
<td>0.984</td>
<td>0.81%</td>
</tr>
<tr>
<td>Case 3 / Case 1</td>
<td>1.187</td>
<td>1.192</td>
<td>-0.42%</td>
</tr>
<tr>
<td>Case 5a / Case 4</td>
<td>0.997</td>
<td>0.991</td>
<td>0.60%</td>
</tr>
<tr>
<td>Case 5b / Case 4</td>
<td>1.006</td>
<td>0.999</td>
<td>0.70%</td>
</tr>
<tr>
<td>Case 6a / Case 4</td>
<td>1.197</td>
<td>1.201</td>
<td>-0.33%</td>
</tr>
<tr>
<td>Case 6b / Case 4</td>
<td>1.071</td>
<td>1.069</td>
<td>0.19%</td>
</tr>
</tbody>
</table>

Table 1-6 Comparison of interaction factor results between Morfeo X-FEM and Abaqus XFEM

<table>
<thead>
<tr>
<th>ξ (-)</th>
<th>Morfeo</th>
<th>FRAC@ALT</th>
<th>Morf. −FRAC. Morf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2 / Case 1</td>
<td>0.992</td>
<td>0.991</td>
<td>0.10%</td>
</tr>
<tr>
<td>Case 3 / Case 1</td>
<td>1.177</td>
<td>1.143</td>
<td>2.89%</td>
</tr>
<tr>
<td>Case 5a / Case 4</td>
<td>0.996</td>
<td>1.037</td>
<td>-4.12%</td>
</tr>
<tr>
<td>Case 5b / Case 4</td>
<td>1.042</td>
<td>1.001</td>
<td>3.93%</td>
</tr>
<tr>
<td>Case 6a / Case 4</td>
<td>1.185</td>
<td>1.192</td>
<td>-0.59%</td>
</tr>
<tr>
<td>Case 6b / Case 4</td>
<td>1.069</td>
<td>1.082</td>
<td>-1.22%</td>
</tr>
</tbody>
</table>
1.5. SUMMARY AND CONCLUSION

The XFEM is an improved technique for modelling cracks in the finite element framework.

In the XFEM formulation, a standard displacement-based approximation is enriched near the crack by incorporating both discontinuous fields and the near tip asymptotic fields through a partition of unity method.

This methodology constructs the enriched approximation from the interaction of the crack geometry with the mesh. This technique, completed by the use of the level-sets method, allows the entire crack to be represented independently of the mesh, and so remeshing is not necessary to model crack growth.

XFEM is therefore very well suited for Fracture Mechanics analysis because its main advantages are the ease of use, the high accuracy (stress singularity captured with the near-tip field enrichment) and the flexibility (no restriction of crack geometry, crack propagation, etc.).

In order to validate XFEM calculations carried out in this work with Morfeo Crack software, a code-to-code benchmark was conducted with Engineering Mechanics Corporation of Columbus (Emc²).

This benchmark included a wide range of 3D cases involving single and interacting cracks submitted to uniaxial and biaxial solicitations.

While involving two complete different calculation methodologies, Morfeo XFEM and FRAC@ALT FEAM simulations led to results in very good agreement, both in terms of stress intensity factors and interaction factor.

Consequently, it can be stated that the Morfeo Crack software used to perform the XFEM calculations in this work is validated.
CHAPTER 2

FLAW CHARACTERISATION RULES: STATE OF THE ART

2.1. INTRODUCTION

In pressure vessel and piping components of power plants, petrochemical plants, etc. flaws might be detected during manufacturing and operation. If flaws are detected, assessments shall be carried out to analyse the origin of the flaw, to evaluate the remaining life of the component containing the flaw, and to determine whether the component shall be repaired/replaced. In other words, assessments are required to demonstrate the fitness-for-service (FFS) of the flawed component for continued operation.

From non-destructive examination (NDE) measurements, the flaw characterization aims at determining flaw size, position and orientation to be used in conjunction with Fracture Mechanics concepts. After characterizing the flaw, the structural integrity of the component containing the flaw has to be assessed. The flaw characterization is therefore the first stage, and consequently the key stage, for the flaw assessment procedures. In a nutshell, the sequence of flaw assessment is as follows (see Figure 2-1):

- flaw characterization to determine flaw geometry for analysis;
- fatigue and stress corrosion cracking growth calculations with reference growth rate curves;
- flaw acceptability assessment using acceptance criteria with appropriate safety factors.
The treatments of detected flaws are performed according to FFS Codes and Standards or guidelines. All are used at each step of a component life by designers, fabricators, inspectors, engineers, and regulators.

A lot of FFS Codes and Standards have been published in various countries. The FFS Codes and Standards considered in this Chapter are ASME B&PV Code Section XI [21] of USA, R6 [22] of UK, RSE-M [23] of France, A16(RCC-MR) [24] of France, SSM [25] of Sweden, A.M.E. [26] of Czech Republic, JSME S NA1 [27] of Japan, FITNET [28] of Europe, BS 7910 [29] of UK, FKM [30] of Germany, API 579/ASME FFS-1 [31] of USA, GB/T-19624 [32] of China, WES 2805 [33] of Japan and HPI S Z101 [34] of Japan. These FFS Codes are summarized in the Table 2-1. The countries with pressure vessels and piping industrial facilities have their own FFS Code or have to refer to one of them. The FFS Codes are split in the Table 2-1 between the nuclear and non-nuclear sectors; FITNET Code covers both sectors.

The objective of this Chapter is to introduce the flaw characterization rules of these FFS Codes and Standards, and to show differences in the specific criteria in the flaw characterization procedures in the FFS Codes.
Table 2-1 Fitness for Service Codes & Standards addressed in this Chapter

<table>
<thead>
<tr>
<th>No</th>
<th>Sector</th>
<th>FFS Codes &amp; Standards</th>
<th>Summary of title</th>
<th>Country</th>
<th>Year of Issue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ASME Code Section XI</td>
<td>Inservice inspection of nuclear power plant</td>
<td>US</td>
<td>2013</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>R6</td>
<td>Integrity of structures containing defects</td>
<td>GB</td>
<td>2012</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>RSE-M</td>
<td>Inservice inspection of PWR nuclear islands</td>
<td>FR</td>
<td>2010</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Nuclear</td>
<td>A16 (RCC-MR)</td>
<td>Defect assessment and leak before break</td>
<td>FR</td>
<td>2010</td>
</tr>
<tr>
<td>5</td>
<td>SSM</td>
<td>Probabilistic procedure for safety assessment</td>
<td>SE</td>
<td>2008</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>A.M.E</td>
<td>Life time assessment for WWER NPPs</td>
<td>CZ</td>
<td>2008</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>JSME</td>
<td>FFS code for nuclear power plants</td>
<td>JP</td>
<td>2012</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Nuclear &amp; non-nuclear</td>
<td>FITNET</td>
<td>Fitness-for-service</td>
<td>EU</td>
<td>2008</td>
</tr>
<tr>
<td>9</td>
<td>Non-nuclear</td>
<td>BS 7910</td>
<td>Acceptability of flaws in metallic structures</td>
<td>GB</td>
<td>2013</td>
</tr>
<tr>
<td>10</td>
<td>FKM</td>
<td>Fracture mechanics proof of strength</td>
<td>DE</td>
<td>2009</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>API 579/ASME FFS-1</td>
<td>Recommended practice for FFS</td>
<td>US</td>
<td>2007</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>GB/T-19824</td>
<td>Assessment for pressure vessels with flaws</td>
<td>CN</td>
<td>2004</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>WES 2805</td>
<td>Fusion welded joint steel structures</td>
<td>JP</td>
<td>2011</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>HPI S Z101</td>
<td>Assessment procedure for pressure equipment</td>
<td>JP</td>
<td>2011</td>
<td></td>
</tr>
</tbody>
</table>
2.2. CHARACTERIZATION RULES FOR A SINGLE FLAW

2.2.1. Flaw orientation

The procedures for determination of the flaw orientation are the same amongst the FFS Codes. The orientation of a planar flaw detected by non-destructive examination is the inclination to the parallel and normal to the pressure retaining surface of the component. Figure 2-2 presents a planar flaw with a length $\ell_o$ subjected to principal stresses $\sigma_1$ and $\sigma_2$. The projected flaw lengths are $\ell_1$ and $\ell_2$, subjected to $\sigma_1$ and $\sigma_2$, respectively. For a first assessment, the actual flaw length $\ell_o$ may be conservatively directly in conjunction with the maximum principal stress [30].

Flaws not oriented in a single plane (i.e., non-planar flaws) are resolved into two planar flaws by projecting the flaw area into planes normal to the principal stresses.

![Figure 2-2 Projected flaw length on a principal stress plane](image-url)
Laminar flaws are particular case of planar flaws. Those may appear as results of inclusions such as Manganese Sulphur (MnS), hydrogen flakes, etc., in steels at manufacturing stage. A laminar flaw is a subsurface flaw parallel, or nearly parallel, to the rolling direction of the plate, the applied stress being parallel to the rolling direction. The laminar flaws are addressed only in the ASME Code Section XI [21] and API 579/ASME FFS-1 [31]. Figure 2-3 presents a laminar flaw in the wall thickness $t$ as well as the relevant dimensions to use in FFS Code [21] and [31]:

![Figure 2-3 Definition of a laminar flaw](image)

In accordance with the ASME Code Section XI, if the angle $\alpha$ between the direction parallel to the wall thickness and the axis of the laminar flaw is less than 10 degrees, it is classified as laminar flaw. If the angle $\alpha$ is less than 20 degrees, the flaw is named quasi-laminar [35]. The definition in [35] specifies that “planar indications oriented within 20 degrees of a plane parallel to the surface of the component shall be considered quasi-laminar flaw”. Therefore, it has to be noted that according to this definition, quasi-laminar flaws include any laminar flaws. The latter remark is important for the Chapter 6 of this dissertation.

In API 579/ASME FFS-1 Code, if $L_h \leq 0.09\max(w_S, w_C)$, the flaw is considered laminar flaw, where $L_h$ is the flaw height in the thickness direction, $w_S$ is the flaw dimension in the longitudinal direction and $w_C$ is the flaw dimension in the width direction as illustrated in Figure 2-3. When $w_S >$
where $w_C$, the angle of $L_h/w_S = 0.09$ corresponds to $\alpha = 5.2$ degrees. Therefore the maximum inclination of the laminar flaw given by the API 579/ASME FFS-1 is almost one half of the inclination given by the ASME Section XI Code (i.e., 10 degrees).

### 2.2.2. Flaw idealization

The procedures for flaw idealization are almost the same amongst the FFS Codes, except for the A.M.E. Code.

The flaw is bounded by a rectangle. Afterward, for assessment by Fracture Mechanics, all surface flaws are characterized as semi-elliptical surface flaws and all subsurface flaws are characterized as elliptical flaws. Figure 2-4 illustrates the characterization of a surface and a subsurface flaw. Their dimensions are the depth $a$ and the length $\ell$ for a semi-elliptical shape for a surface flaw, and the depth $2a$ and the length $\ell$ for an elliptical shaped subsurface flaw. In some FFS Codes, subsurface flaws are also called embedded flaws. The length $\ell$ is also often given as $2c$. 
The A.M.E. [26] Czech Code provides a complex and unique flaw characterization procedure explained here below:

Surface flaw sizes are determined by three methods based on:
- flaw area $F$ or;
- flaw area $F$ and flaw length $\ell$ or;
- maximum flaw depth $\delta$ and flaw length $\ell$.

When the flaw area $F$ method is used, the flaw depth $a = \frac{1}{\pi\sqrt{1.6 + 0.4\pi}}F$, and flaw length $\ell = 5a$. When using the flaw area $F$ and the maximum length, the flaw depth is $a = \frac{4F}{\pi\ell}$, and flaw length is $\ell = 2c$. When the maximum depth $\delta$ and length $\ell$ are used, the flaw depth $\delta = a$, and the flaw length $\ell = 2c$.

Subsurface flaw sizes are determined by methods based on:
- flaw area $F$ or;
• maximum flaw depth $\nu$ and flaw length $\ell$.

When the flaw area $F$ method is used, the flaw depth $a = \sqrt{F/2\pi}$, and the flaw length is $\ell/2 = c = \sqrt{2F/\pi}$. When the maximum depth $\delta$ and length $\ell$ are used, the characterized subsurface flaw depth $\nu$ and length $\ell$ become $\nu = 2a$, and $\ell = 2c$, respectively.

2.2.3. Replacement of Surface Flaw with a Through-wall Flaw

In addition to the aforementioned procedures for flaw idealization, some FFS Codes replace a surface flaw with a through-wall flaw according to Fracture Mechanics analyses or according to criteria dealing with the depth of the flaw. Figure 2-5 shows the replacement principle of a surface flaw with a through-wall flaw. The length $\ell_{tw}$ of the through-wall flaw is larger than the length $\ell$ of the initial surface flaw.

![Surface flaw and Through-wall flaw](image)

Figure 2-5 Replacement of a Surface Flaw with a Trough-wall Flaw

In the R6 [22], BS 7910 [29] and FITNET [28] Codes, a surface flaw is replaced with a through-wall flaw if ligament failure at the tip of the surface flaw is predicted to occur by local yielding. In the R6 Code, the re-characterized flaw length $\ell_{tw} = \max(2\ell, \ell+t)$, whichever is greater, for ligament failure by a brittle mechanism, and $\ell_{tw} = \ell+t$ for ligament failure by ductile mechanism, where $t$ is the wall thickness. For the BS7910 and FITNET Codes, the re-characterized flaw length $\ell_{tw} = \ell+t$. 
The FKM Code [30] gives two types of re-characterized through-wall flaw lengths: $\ell_{tw} = \max(2\ell, \ell+t)$ for brittle fracture and $\ell_{tw} = \ell+t$ for ductile fracture, when components are evaluated for global failure. When local ligament failure is predicted to occur prior to global failure, the original surface flaw remains characterized as a surface flaw. Therefore, the FKM Code uses surface flaw characterization rules for either global failure or local ligament failure. The re-characterization of flaw length $\ell_{tw}$ is then used for the assessment of fracture.

For the replacement with a through-wall flaw, the GB/T 19624 [32], the WES 2805 [33] and API 579/ASME FFS-1 [31] Codes deal directly with the depth of the surface flaw; without accounting of the ligament failure mechanism. In the GB/T 19624 Code, if the depth $a$ of the surface flaw is such that $a/t \geq 0.7$, the flaw is re-characterized as a through-wall flaw with a length $\ell_{tw} = \ell+2a$.

In the WES 2805 Code, if $a/t \geq 0.8$, the surface flaw is transformed to a through-wall flaw with a length $\ell_{tw} = \ell$. In the API 579/ASME FFS-1 Code, if $a/t > 0.8$ the surface flaw is re-characterized to a through-wall flaw with a length $\ell_{tw} = \ell+2(t-a)$.

Table 2-2 summarizes all those conditions for surface flaws replacement with a through-wall flaw as well as the re-characterized flaw length $\ell_{tw}$ of the trough-wall flaw.
2.2.4. Proximity Rule for a Subsurface Flaw

If a subsurface flaw is detected close to the free surface of the component, FFS Codes provide a flaw-to-surface proximity rule to determine whether the flaw should be considered as a subsurface flaw, or transformed to a surface flaw. Figure 2-6 illustrates the transformation of a subsurface flaw to a surface flaw, where $t$ is the wall thickness, $a$ is the half depth of the subsurface flaw, $\ell$ is the length of the subsurface flaw, $a_s$ is the transformed surface flaw depth, $\ell_s$ is the length of the transformed surface flaw, and $S$ is the ligament i.e., the distance from the subsurface flaw to the free surface. This transformation procedure is used at NDE stage to determine the flaw acceptance by comparison with the allowable flaw size tables, for crack growth calculations and for the fracture assessment as well.
The reason for the transformation from the subsurface to the surface flaw is the mechanical interaction between the flaw and the free surface of the component that can lead to ligament failure. Indeed, when the subsurface flaw comes close to the free surface, the SIF at the ligament side is higher than at the opposite side of the subsurface flaw due to stress concentration. Initiation of failure of the component is expected to occur at the ligament of the subsurface flaw. Therefore, from the safety point of view, the subsurface flaw located near the free surface should be conservatively transformed by a surface flaw when ligament becomes small.

The procedures of the re-characterization from the transformation of a subsurface flaw to a surface flaw are the same in all FFS Codes. However, the specific criteria for the rules on transforming subsurface flaws to surface flaws are different.

Flaw-to-surface proximity rules in FFS Codes are presented in Table 2-3. The definition of the depth $a_s$ of the transformed surface flaw is the same in all FFS Codes, that is $a_s = 2a + S$. However, the rules that determine the threshold value of the ligament distance $S$ for transformation as well as the surface flaw length $\ell_s$ corresponding to the original subsurface flaw length $\ell$ differ between various FFS Codes.
Table 2-3 Flaw-to-Surface Proximity Rules for Subsurface Flaws

<table>
<thead>
<tr>
<th>FFS Codes &amp; Standards</th>
<th>Flaw-to-Surface Proximity Rule</th>
<th>Flaw length $\ell_s$ after subsurface to surface transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>R6</td>
<td>If ligament failure is expected</td>
<td>$\ell_s = \max(2\ell, 2a+S+\ell)$</td>
</tr>
<tr>
<td>BS 7910, FITNET</td>
<td>If ligament failure is expected</td>
<td>$\ell_s = 2a+S+\ell$</td>
</tr>
<tr>
<td>FKM</td>
<td>$S/a &lt; 1.0$</td>
<td>$\ell_s = \frac{\ell}{\ell}$ if $a/\ell &lt; 0.5$ $\ell_s = 2(2a+S)$ if $a/\ell &gt; 0.5$</td>
</tr>
<tr>
<td>RSE-M</td>
<td>$S/a &lt; 1.0$</td>
<td>$\ell_s = \frac{\ell}{\ell}$ if $a/\ell &lt; 0.5$ $\ell_s = 2(2a+S)$ if $a/\ell &gt; 0.5$</td>
</tr>
<tr>
<td>GB/T-19624</td>
<td>$S/a &lt; 0.8 \leq \frac{1-2(a+S)}{a}$</td>
<td>$\ell_s = \ell$</td>
</tr>
<tr>
<td>HPIS Z101</td>
<td>$S/a \leq 0.60 - 0.51(a/\ell)^{0.5}$; if $SF = 4.0$ $S/a \leq 0.75 - 0.64(a/\ell)^{0.5}$; if $SF = 3.5$</td>
<td>$\ell_s = 2S+\ell$</td>
</tr>
<tr>
<td>ASME, JSME</td>
<td>$S/a &lt; 0.4$</td>
<td>$\ell_s = \ell$ if $a/\ell_s \leq 0.5$ $\ell_s = 2(2a+S)$ if $a/\ell_s &gt; 0.5$</td>
</tr>
<tr>
<td>SSM</td>
<td>$S/a &lt; 0.4$</td>
<td>$\ell_s = \ell$ for $a/\ell_s \leq 0.5$</td>
</tr>
<tr>
<td>A16</td>
<td>$S/a &lt; 0.4$</td>
<td>$\ell_s = \ell$</td>
</tr>
<tr>
<td>WES 2805</td>
<td>$S/a &lt; 0.25$</td>
<td>$\ell_s = \ell$</td>
</tr>
<tr>
<td>A.M.E.</td>
<td>$S/a &lt; 0.11$</td>
<td>$\ell_s = \ell$</td>
</tr>
<tr>
<td>API 579</td>
<td>$S/t &lt; 0.2$</td>
<td>$\ell_s = 2S+\ell$</td>
</tr>
</tbody>
</table>

The R6 [22], BS 7910 [29] and FITNET [28] Codes do not give the specific criteria $S$ for transformation. When ligament failure is predicted to occur by local yielding, the subsurface flaw is transformed to a surface flaw. This implies user’s assessment to determine the distance $S$. The transformed flaw lengths $\ell_s$ are nevertheless different, that is the maximum of $(2\ell, 2a+S+\ell)$ for the R6 Code and $\ell_s = 2a+S+\ell$ for the BS7910 and the FITNET Codes.

The FKM [30] Code provides a flaw-to-surface proximity rule which is if $S/a < 1.0$, the subsurface flaw is transformed to a surface flaw. The transformed flaw length $\ell_s$ of the surface flaw is $\ell_s = \max(2\ell, 2a+S+\ell)$, for the brittle fracture mode, and $\ell_s = 2a+S+\ell$ for ductile fracture mode.

The French Code RSE-M [23] provides the same threshold distance $S$ as the FKM Guideline i.e., $S/a < 1.0$. The length $\ell_s$ of the transformed surface flaw
depends on the subsurface flaw aspect ratio \( a/\ell \). If \( a/\ell < 0.5 \), \( \ell_s = \max(\ell, 2(2a + S)) \). If \( a/\ell \geq 0.5 \), \( \ell_s = 2(2a + S) \), corresponding to a semi-circular flaw.

The GB/T-19624 Code [32] gives a ligament threshold expressed by \( S/a < 0.8 < (t - (2a + S))/a \). The ligament distance considers therefore the back of the subsurface flaw i.e., the thickness \( t \) of the component. The transformed flaw length is the same as the original length, that is \( \ell_s = \ell \).

The HPIS Z 101 Code [34] provides the ligament distance \( S \) as a function of the initial flaw aspect ratio \( a/\ell \) and a Safety Factor (SF) depending on the type of the component. If \( SF = 4.0 \), the criterion for flaw transformation is given by \( S/a < (0.60 - 0.51 (a/\ell)^{0.5}) \), and if \( SF = 3.5 \), \( S/a < (0.75 - 0.64(a/\ell)^{0.5}) \). The length of the transformed surface flaw \( \ell_s = 2S + \ell \).

The ASME Code Section XI [21] and JSME Code S NA1 [27] give the same flaw-to-surface proximity rules. If \( S/a < 0.4 \), a subsurface flaw is transformed to a surface flaw. The transformed flaw length \( \ell_s = \ell \) for \( a_s/\ell_s \leq 0.5 \), and \( \ell_s = 2(2a + S) \) for \( a_s/\ell_s > 0.5 \). The latter equation means that when the transformed aspect ratio is greater than 0.5, the transformed surface flaw is always a semi-circular flaw.

The SSM Code [25] provides procedure for transformation of a subsurface flaw to a surface flaw identical as in the ASME and JSME Codes. However, if the transformed flaw aspect ratio \( a_s/\ell_s \leq 0.5 \), the transformed surface flaw length is the same with the original flaw length, that is \( \ell = \ell_s \). If \( a_s/\ell_s > 0.5 \), the transformed length \( \ell_s \) of the surface flaw is not given in the SSM Code.

The A16 Code [24] provides a proximity rule that is similar to the rule in the ASME, JSME and SSM Codes i.e., if \( S/a < 0.4 \), a subsurface flaw is transformed to a surface flaw. However, the flaw length is the original length, \( \ell_s = \ell \), regardless of the flaw aspect ratio.

According to the WES 2085 Code [33] a subsurface flaw is transformed to a surface flaw if \( S/a < 0.25 \). The length of the surface flaw is \( \ell_s = \ell \).
For the A.M.E. [26] Czech Code, a subsurface flaw is transformed to a surface flaw if $S/a < 0.11$. This is the smallest threshold for $S$ among FFS Codes. The transformed length of the surface $\ell_s = \ell$ for all flaw aspect ratios.

In the API 579/ASME FFS-1 Code, a subsurface flaw is transformed to a surface flaw if the ratio $S/t < 0.2$, regardless of the flaw depth $a$, where $t$ is the wall thickness. The length of the transformed surface flaw $\ell_s = 2S + \ell$. The API 579/ASME FFS-1 rule is the only proximity criterion based only on the wall thickness $t$.

Through the presentation of all those subsurface to surface proximity rules, it can be seen that, although the concepts of transformation from a subsurface to a surface flaw are the same amongst FFS Codes, the specific criteria are different. Therefore, it is expected that the assessed remaining lives of the components containing the same size subsurface flaws could significantly differ amongst FFS Codes.

### 2.3. CHARACTERIZATION RULES FOR MULTIPLE FLAWS

#### 2.3.1. Principles of Evaluation Procedures for Multiple Flaws

Multiple discrete flaws are often detected in pressure components and piping during inspection. This is e.g. the case when flaws induced by stress corrosion cracking are found in nuclear plant piping. When multiple flaws are assessed, FFS Codes provides two rules in order to account for the potential mechanical interaction between the flaws: an alignment rule and a combination rule. The evaluation procedure for multiple non-aligned flaws is illustrated in Figure 2-7, where the horizontal distance between the flaws is $S$, vertical distance between the flaws is $H$ (i.e. distance between flaw planes) and the direct distance between the nearest tips of the two flaws is $D$. 

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When two flaws (Flaws #1 and #2 in Figure 2-7) are close to each other, it is determined whether they are aligned flaws or non-aligned flaws in compliance with a flaw alignment rule on \( H \) or \( D \). When these flaws are characterized non-aligned flaws, the two flaws are separate flaws and are evaluated independently. When the two flaws are characterized aligned flaws, an additional verification is performed to determine if the two flaws should be combined or not combined. In compliance with a flaw combination rule, the aligned flaws are characterized single combined flaw or two independent aligned flaws. In the latter case, the two flaws are evaluated independently.

![Flowchart](image)

Figure 2-7 Evaluation Procedure for Multiple Flaws

Almost all FFS Codes provide alignment and combination rules for flaw assessment. However, the specific criteria of the alignment and combination rules differ amongst FFS Codes.

### 2.3.2. Alignment Rules for Non-Aligned Flaws

Non-aligned flaws are also called off-set, parallel or non-coplanar flaws. If the detected multiple flaws are non-aligned flaws i.e. not in the same plane,
alignment rules are applied to determine whether the non-aligned flaws should be treated as non-aligned or as aligned (i.e., coplanar) flaws. Figure 2-8 shows two non-aligned subsurface flaws and two non-aligned surface flaws, where $S$ is the horizontal distance between the flaws, $H$ is the vertical distance between the flaws and $D$ is the direct distance between the tips of the two flaws. When $H$ or $D$ is small, these two flaws should be classified as aligned flaws. On the other hand, when $H$ or $D$ is large, these two flaws are non-aligned flaws and they are treated as separate flaws. The aligned/non-aligned flaws classification is performed using alignment rules provided in FFS Codes i.e. using threshold values on $H$ or $D$.

Although the concepts of the alignment rules in the FFS Codes are the same, however, the specific criteria and methodologies are different between these Codes. Table 2-4 shows the alignment rules provided in FFS Codes.
## Table 2-4 Comparison of Alignment Rules in FFS Codes

<table>
<thead>
<tr>
<th>FFS Codes &amp; Standards</th>
<th>Alignment Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ASME, SSM</strong></td>
<td>$H \leq 12.5$ mm</td>
</tr>
<tr>
<td><strong>JSME</strong></td>
<td>( H \leq 12.5 \text{ mm} ) for NDE flaw characterization ( H \leq 10 \text{ mm}, \text{ if } S &lt; 5 \text{ mm} ) ( H &lt; S, \text{ if } S &gt; 5 \text{ mm} ) ( H \leq 12.5 \text{ mm} ) for fracture assessment</td>
</tr>
<tr>
<td><strong>API 579, HPI S</strong></td>
<td>$H \leq 0.5(\ell_1 + \ell_2)$</td>
</tr>
<tr>
<td><strong>WES 2805</strong></td>
<td>$H \leq \max(\ell_1, \ell_2)$ for through wall flaws ( H \leq \max(\ell_1, \ell_2, 2a_2) ) for through wall and surface flaws ( H \leq \max(2a_1, 2a_2, \ell_1, \ell_2) ) for subsurface and surface flaws</td>
</tr>
<tr>
<td><strong>FITNET</strong></td>
<td>$H \leq \min(\ell_1, \ell_2)$</td>
</tr>
<tr>
<td><strong>R6, BS 7910, FKM, A16</strong></td>
<td>$D \leq 0.5(\ell_1 + \ell_2)$</td>
</tr>
<tr>
<td><strong>RSE-M</strong></td>
<td>Overlap of interaction parallelepipeds</td>
</tr>
</tbody>
</table>

The ASME Code [21] and SSM Code [25] provide the simplest flaw alignment criterion. If the vertical distance $H$ between two flaws is less than or equal to 12.5 mm (= 1/2 in.), the two flaws are classified as aligned flaws, regardless of the depths and lengths of the flaws.

The JSME Code [27] provides the same criterion as the ASME and SSM Codes of $H \leq 12.5$ mm for flaw characterization at the NDE stage and for the fracture assessment. However, for subcritical crack growth calculations (fatigue and SCC), the alignment criterion on $H$ is a function of the horizontal distance $S$ between the flaws.

The API 579/ASME FFS-1 [31] and HPIS Codes [34] give alignment rules depending on the flaw lengths: if the vertical separation distance $H$ is less
than or equal to the average of the two flaw lengths, \( \ell_1 \) and \( \ell_2 \), the two flaws are classified as aligned flaws.

The WES 2805 Code [33] provides an alignment rule applicable only for fatigue crack growth calculations. The alignment rule for fracture assessment is given in Table 2-5, together with the flaw combination rule. The alignment rule of the WES 2805 Code for the fatigue crack growth calculations is based on the flaw dimensions, and depends on the types of the flaws to assess. For two through-wall flaws, the vertical separation distance \( H \) is compared with the maximum flaw length, \( \ell_1 \) or \( \ell_2 \). For the alignment of a through-wall and surface flaw, \( H \) is compared with the maximum length or depth of \( \ell_1 \), \( \ell_2 \), or \( 2a_2 \). For the case of two surface or two subsurface flaws, \( H \) is compared with the maximum of \( 2a_1 \), \( 2a_2 \), \( \ell_1 \) or \( \ell_2 \), where the flaw dimensions are defined in Figure 2-8 for surface and subsurface flaws.

The FITNET Code [28] gives an alignment rule based on the minimum flaw lengths. If the vertical distance \( H \leq \min(\ell_1, \ell_2) \), the two flaws are classified as aligned flaws. It clearly appears that the vertical distance \( H \) for flaw alignment can be significantly different between the WES 2805 and FITNET Codes procedures.

The R6 [22], BS 7910 [29], FKM [30] and A16 [24] Codes provides the same flaw alignment rule using the direct separation distance \( D \) between the tips of the two flaws: if \( D \) is less than or equal to the average of the flaw lengths, the two flaws are classified as aligned flaws.

The RSE-M Code [23] provides a methodology based on interaction parallelepipeds enveloping the flaws. The volume of the parallelepipeds depends on the component containing the flaws. The more critical is the component (e.g. pressure vessel nozzles, Class 1 auxiliary systems, etc.), the larger becomes the volume. When the two interaction parallelepipeds are overlapping, the two flaws are directly combined. The two combined flaws are considered as a single flaw with the dimensions based on the initial flaw sizes.
The presentation of the FFS Codes flaw alignment rules has highlighted the differences in the provided criteria. The ASME, SSM and JSME Codes provide alignment criteria based on constant values for the distance between the flaws. The API 579/ASME FFS-1, HPI S, WES 2805 and FITNET Codes use alignment criteria comparing the vertical distance $H$ and the flaw dimensions. The R6, BS 7910, FKM and A16 Codes use a direct separation distance $D$ between the two flaws. The RSE-M Code as for it is based on interaction parallelepipeds. Therefore, it is expected that the remaining live predictions and fracture calculations of the components containing multiple non-aligned flaws could lead to significantly different results amongst FFS Codes.

2.3.3. Combination Rules for Aligned Flaws

When multiple discrete flaws are in the same plane or two non-aligned flaws are classified as aligned according to the aforementioned alignment rules, the flaws are considered as potentially interacting. Then the flaws have to be classified as combined flaw or independent aligned flaws. This is done on the basis of the distance between the adjacent flaws using the flaw combination criteria, as described below. The flaw combination procedures are used at NDE stage to determine the flaw size during inspection, for subcritical crack growth calculations as well as for the fracture assessment of the component.

All existing FFS Codes provide combination rules for the aligned flaws. However, the combination criteria are different between these Codes. It has to be noted that some FFS Codes use the flaw aspect ratio $a/\ell$ as a parameter determining the combination rule to select. After flaw combination, the combined flaw depth and length differ among the FFS Codes. Table 2-5 shows the flaw combination rules provided in FFS Codes. Definitions of $a_1$, $a_2$, $\ell_1$, $\ell_2$ in Figure 2-9 are the flaw depths and lengths, respectively, $S$ is the distance between two surface flaws, and $S_1$ and $S_2$ are horizontal and vertical distances between the subsurface flaws, respectively. These dimensions are depicted in Figure 2-9.
Figure 2-9 Dimensions used in conjunction with FFS Code combination rules in Table 2-5
The ASME [21] and JSME [27] Codes provide flaw combination rules based on flaw depths, \( a_1 \) and \( a_2 \). In the case of surface flaws, two surface flaws are combined to be a single surface flaw if the distance \( S \leq 0.5 \max(a_1, a_2) \). In the case of subsurface flaws, two subsurface flaws are combined if \( S_1 \leq \max(a_1, a_2) \) and if \( S_2 \leq \max(a_1, a_2) \). However, for subcritical crack growth assessment, the combination of the two flaws is not required. That means that the growths of the two flaws are calculated independently until they contact each other i.e., until \( S = 0 \). This is because the maximum stress intensity factors due to operating cyclic loading are small compared with those for monotonic fracture loading; moreover the acceleration of the flaw growth due to the proximity of the adjacent flaw is generally small until the two flaws connect. Furthermore, the remaining component life at the distance \( S \) is negligible in

<table>
<thead>
<tr>
<th>FFS Codes &amp; Standards</th>
<th>Surface flaws</th>
<th>Subsurface flaws</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASME, JSME</td>
<td>( S \leq 0.5 \max(a_1, a_2) )</td>
<td>( S_1 \leq \max(a_1, a_2) ), ( S_2 \leq \max(a_1, a_2) )</td>
</tr>
<tr>
<td>R6 BS 7910</td>
<td>( S \leq \min(\ell_1, \ell_2) ) for ( a_1/\ell_1 &gt; 0.5 ) and if ( S \leq 0.5 \max(\ell_1, \ell_2) )</td>
<td>( S_1 \leq \min(\ell_1, \ell_2) ) for ( a_1/\ell_1 &gt; 0.5 ), ( S \leq \max(\ell_1, \ell_2) ) for ( a_1/\ell_1 &lt; 0.5 )</td>
</tr>
<tr>
<td>FITNET</td>
<td>( S \leq \min(\ell_1, \ell_2) ) or ( S \leq 0.5 \max(\ell_1, \ell_2) )</td>
<td>( S_1 \leq \min(\ell_1, \ell_2) ) for ( a_1/\ell_1 &gt; 0.5 ), ( S \leq \max(\ell_1, \ell_2) ) for ( a_1/\ell_1 &lt; 0.5 )</td>
</tr>
<tr>
<td>FKM</td>
<td>( S \leq \min(\ell_1, \ell_2) )</td>
<td>( S_1 \leq \min(\ell_1, \ell_2) ), ( S \leq (a_1 + a_2) )</td>
</tr>
<tr>
<td>SSM</td>
<td>( S \leq \min(\ell_1, \ell_2) )</td>
<td>( S_1 \leq \min(\ell_1, \ell_2) ), ( S \leq \max(2a_1, 2a_2) )</td>
</tr>
<tr>
<td>GB/T19624</td>
<td>( S \leq \min(\ell_1, \ell_2) )</td>
<td>( S_1 \leq \min(\ell_1, \ell_2) ), ( S \leq \min(2a_1, 2a_2) )</td>
</tr>
<tr>
<td>WES 2805</td>
<td>( S \leq 0.5 \min(\ell_1, \ell_2) ) if ( S \geq 0 ) ( H \leq 0.75 \min(\ell_1, \ell_2) )</td>
<td>( S_1 \leq 0.5 \min(\ell_1, \ell_2) ), ( S \leq \max(a_1, a_2) ) ( H \leq 1.5 \min(\ell_1, \ell_2) ), ( 0.5 \min(\ell_1, \ell_2) )</td>
</tr>
<tr>
<td>API 579 HPI S</td>
<td>( S \leq 0.5(\ell_1 + \ell_2) )</td>
<td>( S_1 \leq 0.5(\ell_1 + \ell_2) ), ( S \leq (a_1 + a_2) )</td>
</tr>
<tr>
<td>A.M.E.</td>
<td>( S &lt; (a_1^2 + a_2^2)^{1/2} ) ( a_2 ), where ( a_1 &gt; a_2 )</td>
<td>( S_{1,2} &lt; (a_1^2 + a_2^2)^{1/2} ) ( a_2 ), where ( a_1 &gt; a_2 )</td>
</tr>
<tr>
<td>RSE-M</td>
<td>Overlap of interaction rectangles</td>
<td>Overlap of interaction rectangles</td>
</tr>
</tbody>
</table>

Table 2-5 Comparison of Combination Rules in FFS Codes
compared to the total life of the component. The combination criterion of $S = 0$ for subcritical crack growth is supported by experimental data as it is presented in Chapter 4 of this dissertation.

The R6 [22], BS 7910 [29] and FITNET [28] Codes have similar flaw combination criteria. For surface flaws, the distance $S$ is compared to the minimum flaw length when the aspect ratio is $a/\ell > 0.5$; when the aspect ratio is $a/\ell \leq 0.5$, the distance $S$ is compared to maximum flaw depth. For subsurface flaws, $S_1$ is the same as with the case of the surface flaws. However, $S_2$ is compared to the average of the flaw depths.

For the FKM [30], SSM [25] and GB/T 19624 [32] Codes the combination criteria are the same for surface flaws: $S$ is compared to the minimum flaw length. However, for the subsurface flaws the criterion on the vertical distance $S_2$ is slightly different amongst these three FFS Codes. The FKM Code $S_2$ criterion is based on the sum of the two flaw depths, the SSM Code $S_2$ criterion is based on the maximum flaw depth and the GB/T 19624 Code $S_2$ criterion is based on the minimum flaw depth.

The combination rules provided by the WES 2805 Code [33] apply only for brittle fracture assessment. As mentioned in section 2.3.2 Alignment Rules for Non-Aligned Flaws, Table 2-5 gives combination and alignment rules for brittle fracture assessment. The criteria on distances $S$, $S_1$ and $S_2$ are one half of those of the GB/T 19624 Code.

In the API 579/ASME FFS-1 [31], HPI S [34] and A16 [24] Codes, the distances $S$ for surface flaws and $S_1$ for subsurface flaws are both compared with the average of the flaw lengths $\ell_1$ and $\ell_2$. For subsurface flaws, $S_2$ is compared with the sum of the flaw depths $a_1$ and $a_2$.

The A.M.E. Code [26] provides complex flaw combination criteria. Flaw depths and lengths are determined by flaw areas, or maximum depths and lengths, as explained in section 2.2.2 Flaw idealization. The A.M.E. Code states that if the distance $S_{1,2}$ of both contours of the two flaws satisfies $S_{1,2} <$
\[(a_1^2 + a_2^2)/9a_2,\] then the two flaws are characterized as one combined flaw, where \(a_1 > a_2\).

The RSE-M Code [23] provides a methodology based on interaction rectangles enveloping the flaws, similarly to the parallelepipeds in the RSE-M flaw alignment rule. The area of the rectangle depends on the component containing the flaws. The more critical is the component, the larger becomes the area. When the two interaction rectangles are overlapping, the two flaws are combined. The two combined flaws are considered as a single flaw with the dimensions based on the initial flaw sizes.

The presentation of the FFS Codes flaw combination rules has highlighted the differences in the provided criteria. The ASME, SSM and JSME Codes provide alignment criteria based on constant values for the distance between the flaws. The API 579/ASME FFS-1, HPI S, WES 2805 and FITNET Codes use alignment criteria comparing the vertical distance \(H\) and the flaw dimensions. The R6, BS 7910, FKM and A16 Codes use a direct separation distance \(D\) between the two flaws. The RSE-M Code as for it is based on interaction parallelepipeds. Therefore, it is expected that the assessed remaining lives and fracture calculations of the components containing multiple non-aligned flaws could significantly differ amongst FFS Codes.

The presentation of the FFS Codes flaw combination rules has highlighted the differences in the provided criteria. The criteria of the FFS are based either on the flaw depths, or on the flaw lengths. A.M.E. and RSE-M Codes provide unique and more complex combination criteria. Therefore, it is expected that the remaining live predictions and fracture calculations of the components containing multiple aligned flaws could lead to significantly different results amongst FFS Codes.
2.4. CHARACTERIZATION RULES FOR MULTIPLE LAMINAR FLAWS

2.4.1. Grouping of Laminar Flaws

According to the ASME Code Section XI [21], definition of laminar flaws is described as “planar indications oriented within 10 degree of a plane parallel to the surface of the component shall be considered laminar flaws”. If there are two or more laminar flaws, those are projected to a single plane and if the separation distance $S$ of the projected flaws is lower than or equal to 25.4 mm (= 1 in.) they should be combined as shown in Figure 2-10. The criteria for combining multiple laminar flaws is therefore based on a fixed distance $S$ of 25.4 mm (= 1 in.) measured along the direction of the principal stresses $\sigma_1$ and $\sigma_2$. After flaw combination, the combined laminar flaw is evaluated with lengths of $\ell$ and $w$ as shown in Figure 2-10. The ASME Code states that the area of a laminar flaw shall be 0.75 times the area of the square or rectangle of dimensions $\ell$ and $w$ that contains the detected area of those flaws that either overlap or are within a distance $S$ of 25.4 mm.
The API 579/ASME FFS-1 Code [31] also provides combination rules for laminar flaws. If there are multiple laminar flaws on the same plane without any indication of through thickness cracking, and if the spacing \( L_S \leq 2t_c \), then the laminar flaws shall be combined into a single laminar flaw for assessment purpose, as shown at the top of Figure 2-11, where \( L_S \) is the direct distance between the laminar flaws and \( t_c \) is the wall thickness considering metal loss and future corrosion loss. If there are multiple laminar flaws at different depths in the wall thickness (i.e., on different planes) and if the spacing \( L_S \leq 2t_c \), then the laminar flaws are evaluated using the criterion for local metal loss. The dimensions of the metal loss are \( w_H \), \( w_S \) and \( w_C \) as shown at the
bottom of Figure 2-11, where $w_H$ is the damage thickness in the thickness direction, $w_S$ is the metal loss in the longitudinal direction and $w_C$ is the metal loss in the width direction. Moreover, the distance $L_W$ between any edge of the laminar flaws and the closest welded joint should meet $L_W \geq \max(2t_c, 25.4 \text{ mm})$. The API 579/ASME FFS-1 Code provides many other conditions to satisfy like distance to a structural discontinuity, etc.

![Diagram of laminar flaws and weld joint](image)

Figure 2-11 Grouping of Laminar Flaws provided by the API 579/ASME FFS-1 Code

### 2.4.2. Alternative Characterization Rules for Quasi-Laminar Flaws

A large number of quasi-laminar indications were detected in specific nuclear power reactor vessels (RPVs) like Belgian RPVs of Doel 3 and Tihange 2 [36]. These indications were caused by hydrogen flaking induced during the manufacturing process. Before this concern, the ASME Code Section XI did not address such a kind of flaws in such large densities. The ASME Code Committees, through the works presented further in this dissertation, have recently developed the Code Case N-848 providing a characterization methodology for a large number of quasi-laminar flaws with a focus on the flaw combination [35].
According to the definition of the Code Case N-848, quasi-laminar flaws are defined as “planar indications oriented within 20 degree of a plane parallel to the surface of the component shall be considered quasi-laminar flaws”. Each quasi-laminar flaw is sized by the minimum bounding box that fully contains the area of the flaw. Figure 2-12 shows the bounding boxes of two quasi-laminar flaws, where $S_1$ and $S_2$ are the distances between the boxes along the direction of principal stresses $\sigma_1$ and $\sigma_2$, respectively. $H$ is the distance between the boxes along the thickness direction and $2D_{ij}$ are the diagonals of the boxes ($i, j = 1, 2$).

Figure 2-12 Configuration and Determination of Relevant Dimensions of Multiple Quasi-Laminar Flaws

If all three of the following criteria are met:

\[
H \leq 0.85 \min(D_{11}, D_{12}, D_{21}, D_{22}) \quad (2-1)
\]
\[
S_1 \leq 2 \min(D_{11}, D_{12}, D_{21}, D_{22}) \quad (2-2)
\]
\[
S_2 \leq 2 \min(D_{11}, D_{12}, D_{21}, D_{22}) \quad (2-3)
\]

the multiple quasi-laminar flaws are combined into a single flaw. If the boxes are overlapping in any one direction, the grouping criterion in that direction is met. The combined single flaw is then sized by the minimum bounding box that contains the individual boxes that meet the grouping criteria. The criteria are also applicable to laminar flaws as it is detailed in Chapter 6 of this
dissertation. Figure 2-13 shows the dimensions of the combined flaw after quasi-laminar flaw grouping; they are given as:

\[ \ell_1 = \ell_{11} + S_2 + \ell_{12} \]  
\[ \ell_2 = \ell_{21} + S_1 + \ell_{22} \]  
\[ 2d = 2d_1 + H + 2d_2 \]

For the purpose of subcritical crack growth and fracture assessment, the bounding box is resolved into two rectangular planar flaws corresponding to the faces of the box normal to the principal stresses. For example, as shown in Figure 2-13, the applied stress \( \sigma_1 \) acts on the flaw size with dimensions \( 2d \) and \( \ell_1 \), and the stress \( \sigma_2 \) acts on the flaw size with dimensions \( 2d \) and \( \ell_2 \).

Those criteria were developed by two dimensional XFEM analyses [36] assessing SIF interaction between inclined flaws. However, it has to be mentioned that these proximity criteria developed using 2D calculations are
very conservative. Therefore, more realistic proximity criteria based on 3D XFEM calculations have been developed [37] and used as technical basis for the revision of the Code Case N-848. This revision has been presented to the ASME Committees in November 2015.
2.5. SUMMARY AND CONCLUSIONS

A lot of FFS Codes and Standards are available throughout the world. The flaw characterization from an NDE measurement is the first step in the flaw assessment procedures provided by the FFS Codes.

The flaw indication is sized by a bounding rectangle. Flaw depth and length are determined by the sides of the rectangle. For analytical evaluation, the flaw is considered as a semi-elliptical or an elliptical shape.

After sizing the flaw, if a subsurface flaw is located close to the free surface of the component, the subsurface flaw is transformed to a surface flaw in accordance to flaw-to-surface proximity rules. When multiple flaws are close to each other, the flaws are first classified as aligned or non-aligned using alignment rules and are then classified as combined or not combined using combination rules.

Almost all FFS Codes provide flaw characterization procedures and flaw-to-surface proximity rules for a single flaw, and alignment and combination rules for multiple flaws. However, the criteria related to these rules are differ significantly amongst the FFS Codes. Therefore, it is expected that the remaining live predictions and fracture calculations of the components containing the same original flaws could lead to significantly different results depending on which FFS Code is used.

This information was presented through the courtesy of Dr. Kunio Hasegawa, the lead author of the Chapter 19 of “Global Applications of the ASME B&PV Codes and Standards”.
3.1. INTRODUCTION

Flaw characterization is important to assess integrity of a structural component, because it is one of the initial steps for flaw evaluation procedures in fitness-for-service (FFS) Codes and Standards. If the characterization of the flaw is not appropriate, the predicted results are unreliable, even if precise calculations for fatigue crack growth, stress corrosion crack growth and fracture criteria are provided.

When a subsurface flaw is located near the component free surface, the first step consists of characterizing the flaw as surface or subsurface according to subsurface-to-surface flaw proximity rules. The re-characterization process from subsurface to surface flaw is addressed in all fitness-for-service (FFS) Codes. However, the specific criteria for the rules on transforming subsurface flaws to surface flaws are different among the FFS Codes as already mentioned in Chapter 2 of this work. In addition, the proximity factors in the rules are defined by constant values, regardless of the flaw aspect ratios $a/\ell$. The differences of the stress intensity factors between subsurface flaws and transformed surface flaws were analysed in [38]. In addition, flaw-to-surface proximity rules in different FFS were introduced in reference [39]. As far as it is known, these criteria and procedures were determined by engineering judgment.
This re-characterization concept is essential and important for subsurface flaws in the flaw assessment procedures. It is applied for three steps of the flaw assessment: at service inspection for flaw characterization, at subcritical crack growth calculation, such as fatigue crack growth and at ductile/brittle fracture assessment.

In order to determine remaining life of a component containing a subsurface flaw, fatigue crack growth assessment is necessary due to thermo-mechanical cycles during plant operation [40]. That is, fatigue crack growth morphology such as subsurface flaw, surface flaw after penetration of component free surface, through-wall flaw and leakage of coolant are required for comprehensive understanding of structural integrity. Critical crack size should be evaluated during the process of the flaw growth. However, as far as the author is aware, the fatigue crack growth morphology from subsurface to surface flaws has still not been clearly investigated.

Therefore, fatigue crack growth experiments were performed on flat plate specimens with subsurface flaws containing various flaw aspect ratios [41]. It was found that the re-characterization from subsurface to surface flaw depends on aspect ratio of the subsurface flaw. It is suggested that the proximity rules defined by constant value in FFS Codes could be not suitable and should need to be updated.

In the first part of this Chapter stress intensity factors for subsurface flaws are calculated by XFEM analysis, and the interactions between subsurface flaw and component free surface deduced. Based on the interactions, equivalent fatigue crack growth rates for subsurface flaws are clarified, by reference to the experimental data. A more suitable flaw-to-surface proximity rule, based on experiments, is then deduced from the equivalent fatigue crack growth rates [43].
As a second step, the remaining fatigue lives for subsurface flaws are compared using the proposed new proximity rule and the current rule provided in the ASME Code Section XI [21]. This comparison is done on thin wall component like piping [44] and on thick wall components like reactor pressure vessels (RPVs) [45]. Different flaw aspect ratios, flaw sizes and distances from subsurface flaws to the free surface of the components are taken into account. Results highlight that the current ASME Code Section XI subsurface-to-surface proximity rule should be updated according to the thickness of the component.

Therefore, the last part of this Chapter deals with additional XFEM calculations on thick wall piping and thin wall vessels in order to define at best the limit for the thickness-dependence of the fatigue lives. As a result, a new subsurface-to-surface proximity rule depending on the type or on thickness of the component is proposed [46].
3.2. FLAW-TO-SURFACE PROXIMITY RULE PROPOSAL BASED ON EQUIVALENT FATIGUE CRACK GROWTH RATE

3.2.1. Flaw Proximity Rules for Subsurface to Surface Flaws

Figure 3-1 illustrates the transformation of subsurface flaw (Flaw A) located close to the component free surface to a surface flaw, where $a$ is the half flaw depth, $\ell$ is the length for the subsurface flaw, $a_s$ is the flaw depth, $\ell_s$ is the length for the transformed surface flaw and $S$ is the ligament distance from the subsurface flaw to the component free surface. The reason for the transformation is high stress acting in the ligament, which can lead to the ligament failure. In fact, the stress intensity factor at Point 1 is generally always higher than that at Point 2. That is, when the subsurface flaw approaches the component free surface, the stress intensity factor at Point 1 increases higher than the one at Point 2 due to the interaction by the component free surface. Initiation of failure of the component is expected to occur at the ligament of the subsurface flaw. Therefore, the subsurface flaw located near the free surface should be conservatively replaced by a surface flaw from the safety point of view.
It has to be reminded that the proximity rules for re-characterization from subsurface to surface are different among the FFS Codes. The locations at the transformation from subsurface to surface flaws are different i.e. the limit value of the proximity factor $Y$, and the transformed flaw length $\ell_s$ are also different among the FFS Codes. Those rules in $Y$ and $\ell_s$ are tabulated in Table 2-3 of Chapter 2 of this dissertation for various Codes and Standards. However, in this work we essentially focus on the ASME Code Section XI characterization rules.

According to the ASME Code Section XI [21], this transformation from subsurface to surface flaw is performed as follows;

When $S$ and $a$ satisfy the following equation:

$$Y = S/a < 0.4$$

(3-1)
the subsurface flaw is treated as surface flaw; where $Y$ is the flaw-to-surface proximity factor. The surface flaw is then re-characterized using the following dimensions (see Figure 3-1):

$a_s$ is the crack depth for the surface flaw:

$$a_s = 2a + S$$  \hspace{1cm} (3-2)

$\ell_s$ is the crack length for the surface flaw:

$$\ell_s = \ell \quad \text{for } a_s/\ell_s \leq 0.5$$  \hspace{1cm} (3-3)

$$\ell_s = 2(2a + S) \quad \text{for } a_s/\ell_s > 0.5$$

This rule in $\ell_s$ means that if the transformed aspect ratio $a_s/\ell_s$ is greater than 0.5, the transformed surface flaw is always a semi-circular flaw.

It is important to note that the limit value (0.4) for the flaw-to-surface proximity factor $Y$ does not depend on the length $\ell$ of the flaw. In other words, for a given flaw depth $a$, the flaw-to-surface proximity factor $Y$ is the same regardless of the aspect ratio $a/\ell$ of the flaw. However, it is easy to physically apprehend that the remaining fatigue life is not the same for an elongated flaw (aspect ratio $a/\ell << 0.5$) as for a circular flaw ($a/\ell = 0.5$) even if they have the same proximity factor $Y$ (i.e. same depth $a$ and same ligament $S$). This fact has been exhibited through fatigue crack growth experiments: the sensitivity with respect to the intersection and penetration of the flaw in the component free surface highly depends on its aspect ratio [47].

### 3.2.2. Proximity Factor estimated by experiments

In order to understand the behaviour of the transformation from subsurface to surface flaws, fatigue crack growth tests were conducted by K. Hasegawa et al. The details of the experimental procedures are described in [47] and are summarized here below.
The specimens were JIS (Japan Industrial Standards) G3106 SM 490A carbon steel flat plates with subsurface flaws. The subsurface flaws were manufactured by joining two plates containing the same semi-elliptic surface EDM (Electric Discharge Machining) notch. The plate assembly was done by brazing bond. The width of the EDM is about 0.3mm.

The specimens were submitted to 3 Hz cyclic tensile test at ambient temperature. The maximum applied load $P_{\text{max}}$ is 320kN and the minimum applied load $P_{\text{min}}$ is 0kN. The nominal applied tensile stress due to $P_{\text{max}}$ is about 222MPa. This stress voluntarily corresponds to low SIF in order to properly observe the flaw evolution without failure of the remaining ligament to free surface. The stress ratio $R = P_{\text{min}}/P_{\text{max}} = K_{\text{min}}/K_{\text{max}} = 0$.

Figure 3-2 illustrates the principle of the fatigue crack growth tests on plate with subsurface flaws and depicts the fracture surface of one specimen (FE 25-2 correspond to an initial aspect ratio $a/\ell = 0.25$).

In order to observe the fatigue crack growth behaviour, beach marks were produced from time to time by changing the stress ratio $R$. These beach marks are highlighted in Figure 3-2 for specimen FE25-2. The final shape of the crack is clearly delimited by the brittle fracture area.

Fatigue crack growth rate for the specimens receiving the brazing process was obtained by 0.5TCT (0.5 inch thickness compact tension) specimens at ambient temperature. Fatigue crack growth rate was obtained as:
\[ \frac{da}{dN} = 2.10 \times 10^{-11} (\Delta K_I)^{4.83} \] (3-4)

where \( \frac{da}{dN} \) is the fatigue crack growth rate in mm/cycle and \( \Delta K_I \) is the stress intensity factor range in MPa√m. The stress intensity factor range \( \Delta K_I \) is \( K_{I,\text{max}} - K_{I,\text{min}} \); \( K_{I,\text{max}} \) and \( K_{I,\text{min}} \) being the stress intensity factors led by the loads \( P_{\text{max}} \) and \( P_{\text{min}} \) respectively.

As can be seen in Figure 3-2, fatigue crack growth amount in the thickness direction at the lower part of the specimen is greater than that at the upper part of the specimen from the observation of the beach marks. The fatigue crack growth at the lower part is clearly affected by interaction of the plate free surface.

The proximity factor was determined by the beach mark shape just before penetration of the subsurface flaw. Figure 3-3 illustrates the determination of the proximity factor \( Y \). The depth and length of the last beach mark are \( 2a \) and \( \ell \), and the distance to the free surface is \( S \). Then the proximity factor \( Y \) is given by \( Y = S/a \), where aspect ratio of the last beach mark just before the penetration is \( a/\ell \).

The proximity factors \( Y \) obtained by experiments are shown in Figure 3-4. The factor is not a constant value, as provided by FFS Codes and Standards,
and $Y$ is increasing with decreasing aspect ratio $a/\ell$. This means that the interaction between subsurface flaw and plate free surface becomes large when the aspect ratio is small.

Therefore, the interaction due to stress intensity factor is analysed as described hereafter.

![Figure 3-4 Proximity factor $Y (= S/a)$ obtained by experiments [43]](image)

### 3.2.3. Stress Intensity Factor Interactions for Subsurface Flaws

#### 3.2.3.1. Calculations conditions

Stress intensity factors $K_1$ and $K_0$ for elliptical subsurface flaws were calculated by the X-FEM method, where $K_1$ is the stress intensity factor at the Point 1 (Flaw A in Figure 3-1), and $K_0$ is the stress intensity factor for subsurface flaw (Flaw B in Figure 3-1) at the location of the Point 0, where the flaw B is in the centre of the plate. Flaws A and B are the same sizes. Stress intensity factor at Point 2 in Figure 3-1 was not selected. This is because the stress intensity factor at Point 2 is also amplified by the free surface when the subsurface flaw is very close to the free surface [48].
The width of the plate is $W = 200$ mm and the thickness is $t = 100$ mm. The height of the plate is $L = 400$ mm. XFEM model is presented in Figure 3-5. The flaw depth is a constant value of $2a = 5$ mm, lengths $\ell$ are $\ell = 8$, 10, 13.3, 20, and 40 mm. The aspect ratios become $a/\ell = 0.125$, 0.25, 0.375, 0.5 and 0.625. The distances $S$ are parameters from 0.5 to 6 mm. The applied tensile stress for the plate is 127 MPa. Mesh cross-section of the subsurface flaw in the centre of the plate is shown in Figure 3-6.

![Figure 3-5 XFEM model used for SIF interaction calculations and location of flaw at mid-height](image-url)
3.2.3.2. Calculations results

The ratios of stress intensity factors $K_1/K_0$ were obtained by XFEM analysis as a function of distance $S$, where the ratios $K_1/K_0$ are interactions of stress intensity factors affected by the component free surfaces. Figure 3-7 shows the interactions as a parameter of aspect ratios $a/\ell$. As can be seen in Figure 3-7, the interaction of $K_1/K_0$ increases with decreasing the distance $S$. In addition, $K_1/K_0$ becomes large when the flaw aspect ratio $a/\ell$ is small.
In accordance to the fatigue crack growth rate of specimens material given in Eq. (3-4), fatigue crack growth rates for subsurface flaws at Points 1 and 0 are expressed by $\frac{da}{dN} = 2.10 \times 10^{-11} (\Delta K^I)^{4.83}$, replacing $\Delta K^I$ to $\Delta K_1$ and $\Delta K_0$, respectively. Ratios of fatigue crack growth rates at Points 1 and 0 are given by $(\Delta K^I/\Delta K^0)^{4.83}$. The ratios $(\Delta K^I/\Delta K^0)^{4.83}$ of fatigue crack growth rates at Points 1 and 0 are derived from Figure 3-7 and are presented in Figure 3-8. Fatigue crack growth rate for subsurface flaw at Point 1 increases rapidly with decreasing the ligament distance $S$. In addition, the growth rate significantly increases, when the flaw aspect ratio is small. That is, fatigue crack growth rate for subsurface flaw with small aspect ratio is faster when the flaw approaches the free surface and, consequently, the subsurface flaw easily penetrates the ligament.
Equivalent fatigue crack growth rate is obtained from Figure 3-8. When \((\Delta K_1/\Delta K_0)^{4.83}\) is a fixed value, the relationship of \(a/\ell\) and \(S\) for the same fatigue crack growth rates, which is called an equilibrium fatigue crack growth rate, are derived from Figure 3-8. Figure 3-9 shows the equivalent fatigue crack growth rates for subsurface flaws for the cases of \((\Delta K_1/\Delta K_0)^{4.83}\) = 1.2 to 2.0. The ligament distance \(S\) for the flaw small aspect ratio is large under constant equivalent fatigue crack growth rates. For example, when the ratio of fatigue crack growth rate is \((\Delta K_1/\Delta K_0)^{4.83}=1.4\), the distance is \(S = 4.5\) mm for \(a/\ell = 0.1\), and \(S = 1.5\) mm for \(a/\ell = 0.65\). A flaw with large aspect ratio must be close to the free surface for the same fatigue crack growth rate.
Figure 3-9 Distance from free surface under equivalent stress intensity factor interaction

The ordinate of Figure 3-9 is the distance between subsurface flaw and free surface. When the distance is divided by the flaw depth $a$, the ordinate becomes $S/a$, which corresponds to proximity factor $Y (= S/a)$. Figure 3-10 is the proximity factor $Y$ and the aspect ratio $a/\ell$. Experimental data determined by fatigue beach marks are plotted as open circles in Figure 3-10. Comparing the experimental data and the various curves of $(\Delta K_1/\Delta K_0)^{4.83}$, it is seen that the curve of $(\Delta K_1/\Delta K_0)^{4.83}=1.4$ is a fit to the experimental data. When converting $(\Delta K_1/\Delta K_0)^{4.83}=1.4$ into stress intensity factor interaction, $K_1/K_0=1.07$. 

\[
\left(\frac{K_1}{K_0}\right)^{4.83} \\
-1.2 \\
-1.3 \\
-1.4 \\
-1.5 \\
-1.7 \\
2 \\
\]  

Aspect ratio, $a/\ell$
3.2.4. Proposal of Proximity Factor for Rules

The proximity factor from subsurface to surface flaw is not a constant value as provided by FFS Codes and Standards. This is because the subsurface flaw near the free surface is affected by interaction of the free surface. Fatigue crack growth rate becomes fast when the aspect ratio is small, as mentioned before.

From the equivalent fatigue crack growth rates and the experimental data, proximity factor can be proposed for codification, herein. The curve of $(\Delta K_i/\Delta K_0)^{4.83} = 1.4$ in Figure 3-10 is close to the experimental data, as mentioned before. Figure 3-11 shows the curve of $(\Delta K_i/\Delta K_0)^{4.83} = 1.4$ and the experimental data. The slope of the experimental data is slightly higher than the curve. The proximity factors of the experimental data were obtained from the last beach marks just before ligament penetration, as illustrated in Figure 3-3. Therefore, the data obtained by experiments are affected by the timing of beach mark introductions. The experimental data are not exact proximity factors $Y$, although the experimental data are very valuable. On the other hand, the curve of $(\Delta K_i/\Delta K_0)^{4.83} = 1.4$ means the same fatigue crack growth
rates. Fatigue crack growth rate at Point 1 is always 1.4 times higher than that at Point 0 in Figure 3-1, regardless of the aspect ratio $a/l$.

When making a rule, simple expression is required for usability. Using the curve of $(\Delta K_1/\Delta K_0)^{4.83} = 1.4$ and the experimental data, proximity factor can be proposed as follows:

$$Y = 1.0 - 1.4(a/l) \quad \text{for } 0 \leq a/l \leq 0.5$$
$$Y = 0.3 \quad \text{for } 0.5 < a/l$$

This proposal of proximity factor is shown in Figure 3-11 (red line). The proximity factor $Y$ is expressed as a linear function of $a/l$ for $a/l \leq 0.5$ and it is a constant value of $Y = 0.3$ for $a/l > 0.5$. 
3.3. FATIGUE CRACK GROWTH CALCULATIONS FOR PIPING AND VESSELS CONSIDERING SUBSURFACE TO SURFACE FLAW PROXIMITY RULES

In order to compare and discuss the remaining fatigue lives for subsurface flaws led by Eq. (3-1) and by Eq. (3-5), fatigue crack growth calculations were carried out for thin wall components like piping and thick wall components like RPVs.

These calculations were performed considering subsurface flaw configurations with different aspect ratios $a/\ell$, different values of initial ligament $S$ and different flaw sizes $a$.

3.3.1. Fatigue crack growth calculations in piping

3.3.1.1. Calculations conditions

The piping sizes employed consist of 12-in. and 6-in. Schedule 80 diameters with various circumferential subsurface flaws. The material properties of the piping are used austenitic stainless steel. The wall thickness 12-in. and 6-in. diameter piping are 17.4 mm and 11.0 mm, respectively. The subsurface flaws are located near inner surface of the piping.

Cyclic loading is a membrane stress, where the maximum stress is $\sigma_{\text{max}} = 123$ MPa, corresponding to the allowable design stress for stainless steel, and minimum stress is $\sigma_{\text{min}} = 0$ MPa The stress ratio of $R = \sigma_{\text{min}}/\sigma_{\text{max}} = 0$. Piping sizes and applied tensile stresses are tabulated in Table 3-1.
Fatigue crack growth rates were employed by the ASME Code Section XI, Appendix C [21]. The fatigue crack growth rates for austenitic stainless steels in air environment are given by:

\[
\frac{da}{dN} = C \cdot S (\Delta K) ^ n
\]

where \( da/dN \) is the fatigue crack growth rates in mm/cycle and \( \Delta K \) is the stress intensity factor range in MPa√m, \( n \) is a material constant given by \( n = 3.3 \); \( C \) is a scaling parameter to account for temperature and is provided in the ASME Code Section XI by:

\[
C = 10^{[4.714 + 1.34 \cdot 10^{-3}T - 3.34 \cdot 10^{-6}T^2 + 5.95 \cdot 10^{-9}T^3]}
\]

where \( T \) is the metal temperature for \( T \leq 430 \) °C and used as room temperature and \( S \) is a scaling parameter to account for \( R \) ratio and is given by:

\[
S = \begin{cases} 
1.0 & \text{for } R \leq 0 \\
1.0 + 1.8R & \text{for } 0 < R \leq 0.79 \\
-43.35 + 57.97R & \text{for } 0.79 < R < 1.0
\end{cases}
\]

As the \( R \) ratio is 0, the scaling parameter is \( S = 1.0 \) in the calculations.

Table 3-2 shows the initial sizes \( (a_0 \text{ and } \ell_0) \) and initial ligament value \( (S_0) \) of the subsurface flaws considered for the calculations. These flaws are similar sizes of allowable flaws for piping of the Acceptance Standard in the ASME Code Section XI.
Stress intensity factors $K_I$ for subsurface flaws shown in Figure 3-1 were calculated using the equation proposed by Miyazaki et al. [49]. During the calculation, the subsurface flaw is assumed to be an elliptical shape, and the position at the long axis of the elliptical shape was moved to the inner surface of the piping, because crack growth amount in the thickness direction to the inner side is larger than that outer side. When the aspect ratio $a/\ell$ of the subsurface flaw is beyond 0.5, stress intensity factor coefficient was obtained by extrapolation from $a/\ell = 0.5$.

<table>
<thead>
<tr>
<th>$t$ mm</th>
<th>$2a_0$ mm</th>
<th>$\ell_0$ mm</th>
<th>$a_0/\ell_0$</th>
<th>$S_0$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.4</td>
<td>3.48</td>
<td>58.0</td>
<td>0.03</td>
<td>1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>3.48</td>
<td>34.8</td>
<td>0.05</td>
<td>1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>3.48</td>
<td>17.40</td>
<td>0.1</td>
<td>1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>5.22</td>
<td>10.44</td>
<td>0.25</td>
<td>1.25, 1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>5.22</td>
<td>6.525</td>
<td>0.4</td>
<td>1.25, 1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>5.22</td>
<td>5.22</td>
<td>0.5</td>
<td>1.25, 1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td>11.0</td>
<td>3.48</td>
<td>58.0</td>
<td>0.03</td>
<td>1.0, 1.25, 1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>3.48</td>
<td>34.8</td>
<td>0.05</td>
<td>1.0, 1.25, 1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>3.48</td>
<td>17.40</td>
<td>0.1</td>
<td>1.0, 1.25, 1.5, 1.75, 2.0, 3.0</td>
</tr>
<tr>
<td></td>
<td>5.22</td>
<td>10.44</td>
<td>0.25</td>
<td>1.25, 1.5, 1.75, 2.0, 2.5</td>
</tr>
<tr>
<td></td>
<td>5.22</td>
<td>6.525</td>
<td>0.4</td>
<td>1.25, 1.5, 1.75, 2.0, 2.5</td>
</tr>
<tr>
<td></td>
<td>5.22</td>
<td>5.22</td>
<td>0.5</td>
<td>1.25, 1.5, 1.75, 2.0, 2.5</td>
</tr>
</tbody>
</table>
When the ligament $S$ reaches the proximity criterion $Y = 0.4$ or $Y = 1.0 - 1.4(a/\ell)$, the subsurface flaw is transformed into surface flaw according Eq. (3-2) and Eq. (3-3).

After the transformation to the surface flaw, the stress intensity factors for surface flaws were calculated by using the ASME Code Section XI [21]. In accordance with Eq. (3-3), a surface flaw with the aspect ratio of $a/\ell > 0.5$ is replaced with a semi-circular flaw of $a/\ell = 0.5$. Therefore, stress intensity factor for $a/\ell = 0.5$ is used for surface flaws, when $a/\ell > 0.5$.

The stress intensity factors for subsurface and surface flaws for piping are almost the same as those for plates [50]. Therefore, the stress intensity factors for piping are applicable to use for flat plate equations in the calculations.

During the fatigue crack growth calculations, the subsurface flaw approaches to the inner surface of the piping, and the subsurface flaw is transformed to surface flaw, when the distance $S$ is satisfied Eq. (3-1) or Eq. (3-5). Crack depths are not allowed the depth beyond the 75% of nominal wall thickness, in compliance of the ASME Code. Therefore, fatigue crack growth calculations were performed until 75% of nominal wall thickness. Number of cycles at the crack depth of 75% of nominal wall thickness is defined as remaining fatigue life. During the fatigue crack growth calculations, the flaws might be critical depths to occur failures before the 75% of nominal wall thickness. For assessing the remaining fatigue lives, the critical depths for failures are not considered herein.

### 3.3.1.2. Calculations results

The results present the fatigue crack growth curves as crack depth vs. number of cycles ($N$). Not all fatigue crack growth curves for piping are presented hereafter. Only the most relevant fatigue crack growth calculations results among all piping configurations are showed. However, since the scope of the calculations is to assess and to quantify the differences in terms of fatigue life led Eq. (3-1) and Eq. (3-5), all results are then treated in term of remaining
fatigue lives ratios as it is explained in section 3.3.1.3 Comparison of remaining fatigue lives.

3.3.1.2.1. 11 MM THICKNESS PIPING

The wall thickness of 6-in. diameter schedule 80 was used for calculations. The nominal wall thickness of the 6-in. diameter pipe is 11.0 mm. The crack depth calculations were performed until 8.25 mm, which is equal to the 75% of nominal wall thickness ($a = 0.75t$).

Fatigue crack growths in the wall thickness direction as a function of number of cycles are shown in Figure 3-12 and Figure 3-13 under the conditions of $t = 11.0$ mm and $S_0 = 2.0$ mm.

Figure 3-12 shows the fatigue crack growth curves for the initial crack depth, length and aspect ratio are $2a_0 = 3.48$ mm, $\ell_0 = 58.0$ mm and $a_0/\ell_0 = 0.03$. The flaw transformation given by Eq. (3-5) occurs at $N = 23000$ cycles, where the aspect ratio is slightly greater than 0.03. The number of cycles at $a = 0.75t$ is $N = 37000$ cycles. The number of cycles at transformation given by Eq. (3-1) is $N = 78000$ cycles and the number at $a = 0.75t$ is $N = 88000$ cycles. The aspect ratio at $N = 78000$ cycles increases to $a/\ell = 0.05$. The number of cycles at the flaw transformation and at $a = 0.75t$ for both fatigue lives are considerably different between the two curves.

![Fatigue crack growth in the thickness direction](image_url)

Figure 3-12 Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.03$ at $S_0 = 2.0$ mm and $t = 11.0$ mm
Figure 3-13 shows the fatigue crack growth curves for the initial crack depth, length and aspect ratio are $2a_0 = 5.22$ mm, $\ell_0 = 5.22$ mm and $a_0/\ell_0 = 0.5$, respectively. The flaw transformation given by Eq. (3-5) is at $N = 260000$ cycles, where the aspect ratio is 0.5. The number of cycles at transformation given by Eq. (3-1) is $N = 208000$ cycles and the number at $a = 0.75t$ is $N = 227000$ cycles. The aspect ratio at $N = 260000$ and 208000 cycles is $a/\ell = 0.5$ in accordance with Eq. (3-3). Almost all lives are subsurface flaw growths for both curves. The number of cycles at the flaw transformation and at $a = 0.75t$ are not large different between the two curves.

![Figure 3-13 Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.5$ at $S_0 = 2.0$ mm and $t = 11.0$ mm](image)

**3.3.1.2.2. 17.4 MM THICKNESS PIPING**

The wall thickness of 12-in. diameter schedule 80 was used for calculations. The nominal wall thickness of the 12-in. diameter pipe is 17.4 mm. The crack depth calculations were performed until 13 mm, which is equal to the 75% of nominal wall thickness ($a = 0.75t$).

Fatigue crack growth depth $a$ vs number of cycles $N$ is shown in Figure 3-14 for the initial flaw depth with $2a_0 = 3.48$ mm and $\ell_0 = 58.0$ mm. The initial aspect ratio is $a_0/\ell_0 = 0.03$, and the location of the subsurface flaw is $S_0 = 1.5$ mm. When using Eq. (3-5), $Y = 1.0 -1.4(a/\ell) = 0.97$. The initial location of
the subsurface flaw $S_0/a_0$ is 0.86, which is less than 0.97. Therefore, fatigue crack growth calculation starts as a transformed surface flaw. The number of cycles at $a = 0.75t$ is $N = 59000$ cycles.

On the other hand, when using $Y = 0.4$ provided by the ASME Code Section XI, the initial location of the subsurface flaw is $S_0/a_0 = 0.86$, which is greater than 0.4, fatigue crack growth calculation starts as a subsurface flaw, and the location of $S/a$ for the subsurface flaw satisfies with Eq. (3-1) at $N = 51000$ cycles. The aspect ratio increased from $a_0/\ell_0 = 0.03$ to $a/\ell = 0.04$ at the $N = 51000$ cycles. The number of cycles at $a = 0.75t$ is $N = 104000$ cycles. The remaining lives at $a = 0.75t$ are different between the curves of $Y = 0.4$ and $Y = 1.0 - 1.4(a/\ell)$.

On the other hand, when using $Y = 0.4$ provided by the ASME Code Section XI, the initial location of the subsurface flaw is $S_0/a_0 = 0.86$, which is greater than 0.4, fatigue crack growth calculation starts as a subsurface flaw, and the location of $S/a$ for the subsurface flaw satisfies with Eq. (3-1) at $N = 51000$ cycles. The aspect ratio increased from $a_0/\ell_0 = 0.03$ to $a/\ell = 0.04$ at the $N = 51000$ cycles. The number of cycles at $a = 0.75t$ is $N = 104000$ cycles. The remaining lives at $a = 0.75t$ are different between the curves of $Y = 0.4$ and $Y = 1.0 - 1.4(a/\ell)$.

Figure 3-14 Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.03$ at $S_0 = 1.5$ mm and $t = 17.4$ mm.
When the initial distance is \( S_0 = 2.0 \) mm, the calculations for both proximity factors of \( Y = 1.0 - 1.4(a/\ell) \) and \( Y = 0.4 \) start at subsurface flaws. Figure 3-15 shows the fatigue crack growth depths for the initial flaw with \( 2a_0 = 3.48 \) mm and \( \ell_0 = 58.0 \) mm with \( a_0/\ell_0 = 0.03 \). The subsurface flaw is transformed to a surface flaw at \( N = 24000 \) cycles by the proposed proximity factor of \( Y = 1.0 - 1.4(a/\ell) \) and at \( N = 82000 \) cycles by \( Y = 0.4 \). The aspect ratios at the transformation are \( a/\ell = 0.03 \) and 0.04, respectively. After these numbers of cycles at transformations, both flaws grow to the thickness direction to the 0.75 \( t \), with accompanying the same gaps of number of cycles. The number of cycles at \( a = 0.75t \) is \( N = 74000 \) cycles for the proposed proximity factor and \( N = 125000 \) cycles for \( Y = 0.4 \), respectively.

![Figure 3-15 Fatigue crack growth in the thickness direction for \( a_0/\ell_0 = 0.03 \) at \( S_0 = 2.0 \) mm and \( t = 17.4 \) mm](image)

In case of initial distance of \( S_0 = 3.0 \) mm, the number of cycles for subsurface flaws becomes long. Figure 3-16 shows the relationship between fatigue crack growth depths and number of cycles for the initial flaw with \( 2a_0 = 3.48 \) mm, \( \ell_0 = 58.0 \) mm and \( a_0/\ell_0 = 0.03 \). The subsurface flaw is transformed to a surface flaw at \( N = 87000 \) cycles by the proposed proximity factor and the aspect ratio is \( a/\ell = 0.04 \), and the transformation by \( Y = 0.4 \) is at \( N = 136000 \) cycles and the aspect ratio at transformation is \( a/\ell = 0.06 \). The difference of
the remaining lives at $a = 0.75t$ between the curves of $Y = 1.0 - 1.4(a/\ell)$ and $Y = 0.4$ are relatively small.

![Fatigue crack growth](image)

Figure 3-16 Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.03$ at $S_0 = 3.0$ mm and $t = 17.4$ mm

When looking through Figure 3-14 to Figure 3-16 for the case of initial aspect ratio of $a_0/\ell_0 = 0.03$, the number of cycles at the transformation increase with increasing the initial distance of $S_0$. During the fatigue crack growth, the aspect ratio for subsurface flaw changes from $a_0/\ell_0 = 0.03$ to larger number. For example, when the subsurface flaw is located $S_0 = 3.0$ mm, the aspect ratios at transformation become $a/\ell = 0.04$ for the new proximity factor and $a/\ell = 0.06$ for $Y = 0.4$, as mentioned above. The number of cycles at the transformation used by the proposed proximity factor is always smaller than that by the Code proximity factor $Y = 0.4$. In addition, the remaining lives given by the proposed proximity factor is always shorter than that given by the Code proximity factor.

Figure 3-17 shows the fatigue crack growth with large aspect flaw. Initial subsurface flaw depth and length are $2a_0 = 5.22$ mm and $\ell_0 = 5.22$ mm. The aspect ratio is $a_0/\ell_0 = 0.5$, which is a circular flaw. The flaw is located at $S_0 = 3.0$ mm. The fatigue life given by the proximity factor of $Y = 1.0 - 1.4(a/\ell)$ is longer than that by $Y = 0.4$. The transformation from subsurface to surface
flaw given by the proposed proximity factor is at \(N = 432000\) cycles, the aspect ratio at the cycles is \(a/\ell = 0.5\), and the number of cycles at \(a = 0.75t\) is \(N = 509000\) cycles. On the other hand, transformation given by \(Y = 0.4\) was occurred at \(N = 385000\), the aspect ratio at the cycles is \(a/\ell = 0.5\), and the number of cycles at \(a = 0.75t\) is \(N = 469000\) cycles. The aspect ratio at transformation is always \(a/\ell = 0.5\), because of the circle flaw of \(a_0/\ell_0 = 0.5\), in accordance with Eq. (3-3). The difference of the remaining lives for \(a_0/\ell_0 = 0.5\) at \(a = 0.75t\) is small compared with that for \(a_0/\ell_0 = 0.03\). It is notable that fatigue life given by \(Y = 1.0 - 1.4(a/\ell)\) is longer than that by \(Y = 0.4\).

![Fatigue crack growth in the thickness direction for a subsurface flaw located near component free surface](image)

*Figure 3-17 Fatigue crack growth in the thickness direction for \(a_0/\ell_0 = 0.5\) at \(S_0 = 3.0\) mm and \(t = 17.4\) mm*

From Figure 3-12 and Figure 3-15 it can also be deduced that thin wall thickness could increase the relative difference of remaining fatigue lives between \(Y = 1.0 - 1.4(a/\ell)\) and \(Y = 0.4\), because the remaining ligament thickness \((t-a)\) becomes short after the flaw transformation. This is discussed hereafter.

### 3.3.1.3. Comparison of remaining fatigue lives

As seen from the curves presented in Figure 3-12 to Figure 3-17, the remaining fatigue lives for subsurface flaws located near component free surface are influenced by the initial aspect ratios, locations, component
thickness and the applied proximity factors. The proposed proximity factor given by Eq. (3-5) was developed by experimental data and equivalent fatigue crack growth rates, as mentioned before. It is therefore considered as reference case. Based on the proposed proximity factor, the current Code provided by \( Y = 0.4 \) estimates long lives when the initial aspect ratio is small. This means that the current ASME Code might lead to an un-conservative estimation for small aspect ratios.

In order to better quantify the differences in terms of remaining fatigue lives, the ratios of remaining fatigue lives defined as:

\[
\frac{N_{1.0-1.4(a/\ell)}}{N_{0.4}}
\]  

are calculated for all the analyzed flaw configuration at \( a = 0.75t \), where \( N_{1.0-1.4(a/\ell)} \) and \( N_{0.4} \) are the numbers of cycles estimated by the proposed proximity rule of \( Y = 1.0 - 1.4(a/\ell) \) and the current one \( Y = 0.4 \) respectively.

Since the proposed proximity rule \( Y = 1.0 - 1.4(a/\ell) \) is considered as the reference case, when \( N_{1.0-1.4(a/\ell)}/N_{0.4} > 1 \), the current proximity rule leads to conservative prediction i.e. to lower fatigue lives. On the contrary, when \( N_{1.0-1.4(a/\ell)}/N_{0.4} < 1 \), the current proximity rule leads to un-conservative predictions i.e. to higher fatigue lives. Therefore, the suitability of the current ASME Code proximity rule \( Y = 0.4 \) can be assessed by the magnitude of the range of \( N_{1.0-1.4(a/\ell)}/N_{0.4} \) around 1.

Figure 3-18 demonstrates the ratio of \( N_{1.0-1.4(a/\ell)}/N_{0.4} \) for most thin wall thickness piping i.e. with \( t = 11.0 \) mm.

The number of cycles \( N_{1.0-1.4(a/\ell)} \) given by the proposed proximity factor is 30% against \( N_{0.4} \) for \( a_\rho/\ell_0 = 0.03 \) and \( S_0 = 1.5 \) mm. The estimated fatigue life using \( Y = 0.4 \) gives long life. This is un-conservative estimation. For case of \( a_\rho/\ell_0 = 0.5 \) the remaining lives \( N_{1.0-1.4(a/\ell)} \) are always longer than \( N_{0.4} \). This is overly conservative, particularly, 43 % long at \( S_0 = 1.0 \) mm.
The ratios of the fatigue remaining lives defined by Eq. (3-9) at \( a = 0.75t \) are presented in Figure 3-19 for piping with \( t = 17.4 \) mm.

The remaining life of \( N_{1.0-1.4(a/\ell_0)} \) for \( a_0/\ell_0 = 0.03 \) and \( S_0 = 1.75 \) mm is one half live of the \( N_{0.4} \). When \( a_0/\ell_0 = 0.5 \), \( N_{1.0-1.4(a/\ell_0)} \) is always larger than \( N_{0.4} \).
For both the analyses i.e. piping with $t = 11$ mm and with 17.4mm, it appears that the ratio of remaining fatigue lives $N_{1.0-1.4(a/\ell)/N_{0.4}}$ depends on the initial aspect ratio $a_0/\ell_0$ and flaw location $S_0$. Furthermore, the relative differences of remaining fatigue lives led by $Y = 1.0 - 1.4(a/\ell)$ and $Y = 0.4$ cannot be considered as negligible since they are beyond 30% which is a maximum value for the scatter of fatigue analyses. In particular, for small aspect ratios, the current proximity rule $Y = 0.4$ leads to un-conservative predictions (higher remaining lives). Therefore, for thin wall components like piping, the current proximity rule $Y = 0.4$ of the ASME Code should be updated by using $Y = 1.0 - 1.4(a/\ell)$.

### 3.3.2. Fatigue crack growth in vessels

#### 3.3.2.1. Calculations conditions

The wall thickness of vessel type component considered for these fatigue crack growth analyses is $t = 200$ mm. It has to be noted that with $t = 200$ mm, the component may be considered as “thick wall” component. The subsurface flaws are located close to the inner surface of the vessel.

The cyclic loading is a membrane stress, where the maximum stress is $\sigma_{\text{max}} = 123$ MPa and minimum stress is $\sigma_{\text{min}} = 0$ MPa The stress ratio of $R$ is $R = \sigma_{\text{min}}/\sigma_{\text{max}} = 0$.

The material is assumed being common vessel ferritic steel SA533B. The crack growth rate used for the calculations is the one given by the ASME Code Section XI, Appendix A [21] for ferritic steels in air environment:

$$\frac{da}{dN} = CS(\Delta K_1)^n$$

(3-10)

where $\frac{da}{dN}$ is the fatigue crack growth rates in mm/cycle and $\Delta K_1$ is the stress intensity factor range in MPa$\sqrt{m}$, $n$ is a material constant given by $n = 3.07$ ; $C$ is a scaling constant given by $C = 3.78\times10^{-9}$ and $S$ is a scaling parameter to account for $R$ ratio and is given by:
\[ S = 25.72 \times (2.88 - R)^{3.07} \quad (3-11) \]

As the \( R \) ratio is 0, the scaling parameter is \( S = 1.0 \) in the calculations.

For the fatigue crack growth calculations in thick wall component like vessels, 9 subsurface flaw configurations have been considered.

Table 3-3 shows the initial sizes \((a_0, \ell_0)\) and initial ligament value \((S_0)\) of the subsurface flaws considered for the calculations. The initial dimensions are around the dimensions given in the Table IWB-3510-1 (Allowable Planar Flaws in Ferritic steels) of the ASME Code Section XI Acceptance Standards.

<table>
<thead>
<tr>
<th>( t ) mm</th>
<th>( 2a_0 ) mm</th>
<th>( \ell_0 ) mm</th>
<th>( a_0/\ell_0 )</th>
<th>( S_0 ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>8.8</td>
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<td>0.05</td>
<td>2.0</td>
</tr>
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<td></td>
<td>8.8</td>
<td>88</td>
<td>0.05</td>
<td>4.0</td>
</tr>
<tr>
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<td>8.8</td>
<td>88</td>
<td>0.05</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>8.8</td>
<td>88</td>
<td>0.05</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>50</td>
<td>0.1</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>50</td>
<td>0.1</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>15.2</td>
<td>30.4</td>
<td>0.25</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>23.2</td>
<td>29</td>
<td>0.4</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>30.4</td>
<td>30.4</td>
<td>0.5</td>
<td>8.0</td>
</tr>
</tbody>
</table>

The fatigue crack growth calculations are performed with XFEM method. This method allows computing the values of the stress intensity factor \( K_1 \) all along the crack front. Therefore, using the fatigue crack growth rate defined in Eq. (3-10) and Eq. (3-11), the increment \( da \) for a number of cycle \( dN \) can
be calculated for each point of the crack front. The flaw shape is then updated accounting for the actual size increase and shape evolution of the crack front.

Consequently, by using XFEM a flaw can freely propagates in a stress field without any constraint on its shape. In other words, a flaw initially elliptical will not necessary propagate by keeping its elliptical shape. Propagating using XFEM will lead to more realistic results.

The width of the model plate is $W = 400$ mm, the height of the plate is $L = 400$ mm and the thickness is $t = 200$ mm. The flaws are located in the mid-height plane (with refined mesh around the flaw location). The cyclic loading is applied on the top of the model. The bottom is fully constraint. The XFEM model is presented in Figure 3-20. The flaws are model by the level-set method. Figure 3-21 presents a cross section with a subsurface flaw ($a_0/l_0 = 0.4$ and $S_0 = 8$ mm) such as modelled by level-set method.

![Figure 3-20 XFEM model (mm) used for fatigue crack growth calculations in vessels and location of flaw at mid-height](image)

![Figure 3-21 Cross section (mm) in the plane of the subsurface flaw (example for $a_0/l_0 = 0.4$ and $S_0 = 8$ mm)](image)
The main steps for the fatigue crack growth calculation are illustrated in Figure 3-22 with a subsurface flaw such as modeled by level-set method. They are as follows: during growth of the initial subsurface flaw under fatigue cyclic loading, the ligament $S$ is calculated during the growth of the flaw. When the ligament $S$ reaches the proximity criterion $Y = 0.4$ or $Y = 1.0 - 1.4(a/l)$, the subsurface flaw is transformed into surface flaw according to Eq. (3-2) and Eq. (3-3). It has to be noted that using this methodology, when the proximity criterion is reached, the flaw shape is no longer elliptical because of the interaction with the free surface. This is accounted through XFEM calculations, as explained above. However, when the subsurface flaw is transformed into surface flaw, the flaw shape is elliptical again. This does not necessary remains true during the propagation of this new surface flaw.

![Figure 3-22 Steps of fatigue crack growth according to subsurface to surface proximity rules](image)

### 3.3.2.2. Calculations results

For each configuration of Table 3-3, fatigue live results for subsurface and transformed surface flaws are shown in Figure 3-23 to Figure 3-31. Although the calculations of crack growths were performed to 75% of the wall thickness of the plate, crack depths against number of cycles are shown to be 40 to 60 mm to make clear the differences of the transformation of subsurface to surface flaws. It is reminded that the fatigue life calculated using the proposed proximity rule $Y = 1.0 - 1.4(a/l)$ is considered as the reference case since this $Y$ is based on experiments.
In the case of initial aspect ratio of $a/\ell = 0.05$, the subsurface flaws with the distance $S_0 = 2$ and $4$ mm start to calculate as surface flaws for the proposing proximity rule $Y = 1.0 - 1.4(a/\ell)$, and for the ASME proximity rule $Y = 0.4$, fatigue crack growth calculations start as subsurface flaws, as shown in Figure 3-23 and Figure 3-24. The fatigue lives for $S_0 = 2$ mm are almost the same between the both rules.

![Figure 3-23 Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.05$ at $S_0 = 2.0$ mm ($t = 200$ mm)](#)

![Figure 3-24 Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.05$ at $S_0 = 4.0$ mm ($t = 200$ mm)](#)
When the ligament distance of $S_0 = 6$ and 8 mm, fatigue crack calculations start as subsurface flaws for both $Y = 1.0-1.4(a/l)$ and $Y = 0.4$. The differences of the number of cycles at the transformation are significantly large between $Y = 1.0-1.4(a/l)$ and $Y = 0.4$, as shown in Figure 3-25 and Figure 3-26.

![Figure 3-25 Fatigue crack growth in the thickness direction for $a_0/l_0 = 0.05$ at $S_0 = 6.0$ mm ($t = 200$ mm)](image1)

![Figure 3-26 Fatigue crack growth in the thickness direction for $a_0/l_0 = 0.05$ at $S_0 = 8.0$ mm ($t = 200$ mm)](image2)
In the case of initial aspect ratio of $a/\ell = 0.1$, the subsurface flaw located at $S_0 = 4$ mm is transformed to a surface flaw before calculation for $Y = 1.0-1.4(a/\ell)$, and it is transformed at around $N = 6.02 \times 10^5$ cycles for $Y = 0.4$, as shown in Figure 3-27. When $S_0 = 8$ mm, fatigue crack calculations start as subsurface flaws and transformations occur at around $N = 1.05 \times 10^5$ cycles for $Y = 1.0-1.4(a/\ell)$ and $N = 1.65 \times 10^5$ cycles for $Y = 0.4$, as shown in Figure 3-28. It is suggested that the ligament distance $S_0$ affects fatigue lives.

---

**Figure 3-27** Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.1$ at $S_0 = 4.0$ mm ($t = 200$ mm)

![Figure 3-27](image1)

---

**Figure 3-28** Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.1$ at $S_0 = 8.0$ mm ($t = 200$ mm)

![Figure 3-28](image2)
In the case of initial distance $S_0 = 8$ mm for $a/\ell = 0.1$, 0.25, 0.4 and 0.5, crack depths versus number of cycles $N$ are shown in Figure 3-28 to Figure 3-31. It can be seen that, at the boundary of $a/\ell = 0.4$, fatigue life tendencies for the same subsurface flaws calculated by $Y = 0.4$ and $Y = 1.0 - 1.4(a/\ell)$ are reversed.

**Figure 3-29** Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.25$ at $S_0 = 8.0$ mm ($t = 200$ mm)

**Figure 3-30** Fatigue crack growth in the thickness direction for $a_0/\ell_0 = 0.4$ at $S_0 = 8.0$ mm ($t = 200$ mm)
3.3.2.3. Comparison of remaining fatigue lives

A first analysis of the results in thick wall components exhibits that in case of small aspect ratio \( a_0 / \ell_0 < 0.4 \), the use of the current ASME Code proximity rule \( Y = 0.4 \) might lead to un-conservative predictions (higher remaining fatigue lives). However, in case of large aspect ratio \( a_0 / \ell_0 > 0.4 \), the use of the current ASME proximity rule \( Y = 0.4 \) leads to conservative predictions (lesser remaining fatigue lives).

In order to better quantify the differences in terms of remaining fatigue lives, the ratios of remaining fatigue lives defined in Eq. (3-9) are calculated. The flaw depths \( a \) for the determination of the \( N_{1.0-1.4(a/l)} / N_{0.4} \) have been selected in order to highlight the tendencies of the comparison exercise, they are presented in Table 3-4. These flaw depths are lower than 0.75\( t \) but are all larger than the size corresponding to the transformation from subsurface to surface flaw.
For the flaw configuration with $a_0/\ell_0 = 0.05$, four values of the ligament $S_0$ have been analyzed. Therefore, the results of $N_{1.0-1.4(a_0/\ell_0)}/N_{0.4}$ are presented according to $S$ in Figure 3-32. For the other flaw configurations, the results of $N_{1.0-1.4(a_0/\ell_0)}/N_{0.4}$ are presented according to $a_0/\ell_0$ in Figure 3-33.
These results highlight that for small aspect ratios \((a_0/l_0 < 0.4)\), the remaining lives \(N_{0.4}\) calculated by \(Y = 0.4\) is about 20 to 30 % longer than \(N_{1.0-1.4(a/l)}\) calculated by \(Y = 1.0 - 1.4(a/l)\) whereas that for large aspect ratio \((a_0/l_0 > 0.4)\), the remaining fatigue life calculated by \(Y = 0.4\) is about 30 % shorter than that calculated by \(Y = 1.0 - 1.4(a/l)\). However, these differences of
maximum 30% in terms of number of cycles can be considered as being within the scatter of fatigue analyses. Therefore, for thick wall components like vessel, the current proximity rule \( Y = 0.4 \) of the ASME Code appears not to be updated.

### 3.4. FATIGUE CRACK GROWTH CALCULATIONS IN ADDITIONAL CONFIGURATIONS

Through the last calculations it was shown that for thin wall components like piping, the current ASME Code Section XI proximity rule can provide non conservative fatigue lives. As a consequence, the current proximity rule given in Eq. (3-1) should be updated by the proposed new proximity rule given in Eq. (3-5) for piping geometry. On the other hand, for thick wall components like vessels, the current proximity rule and the proposed one lead to relatively similar fatigue lives. Therefore, the current proximity rule does not need to be updated for thick wall components.

Both those conclusions highlight that the subsurface-to-surface proximity rule should be updated according to the thickness of the component or according to the type of component i.e. piping or vessel.

In order to define at best the limit for the thickness-dependence of the use of Eq. (3-1) or Eq. (3-5) for the subsurface-to-surface proximity rule, additional fatigue crack growth calculations are carried out on numerous flaw configurations in thick wall pipes and thin wall vessels. The scope is to cover the range of the thickness values for classical B&PV components and to propose an improved subsurface-to-surface proximity rule depending also on the thickness of the component.
3.4.1. **Range of thickness for additional configurations**

As mentioned before, in order to assess the suitability of the current ASME Code Section XI proximity rule \( Y = 0.4 \), fatigue crack growth calculations have been performed using Eq. (3-1) and Eq. (3-5) on subsurface flaw configuration, for a large range of aspect ratio and ligament distances in thick wall components (thickness \( t = 200 \) mm) and thin wall components \((t = 11 \text{ mm and } t = 17.4 \text{ mm})\).

Furthermore, it has to be recalled that the experiments which have led to the proposed proximity factor limit of \( Y = 1.0 - 1.4(a/l) \) have also to be considered as configurations for which the current proximity factor limit given in Eq. (3-1) is not suitable. These experiments were led on 30mm thickness specimens.

Therefore, calculations results on piping and vessels reminded in the introduction and conclusions of experimental results can be summarized as follows: for thickness around 200mm and beyond, proximity rule \( Y = 0.4 \) provided in current Code appears not to be updated; for thickness around the range of 11 mm to 30 mm, the proximity rule provide in current Code should be updated by \( Y = 1.0 - 1.4(a/l) \).

As the thickness clearly appears playing a role in the suitability of the subsurface-to-surface proximity rule provided in the ASME Code Section XI, it is relevant to present the aforementioned conclusions in a graph using the remaining ligament \( S_2 = t - S_0 - 2a_0 \) defined in Figure 3-34.

The graph in Figure 3-35 presents the analyzed flaw configurations (calculations and experiments) as points in the coordinate axes \( t \) and \( S_2/a_0 \).
This way of presenting the results allows highlighting two areas in Figure 3-35: the so-called “Y = 0.4 zone” where Y = 0.4 does not need to be updated and the so-called “Y = 1.0 – 1.4(a/l) zone” where Y = 0.4 should be replaced by Y = 1.0 – 1.4(a/l).
However, as it can be seen in Figure 3-36 no flaw configuration has been calculated in a large thickness range area, the so called “Intermediate area”. Therefore, in order to confirm the limit value of $10a_0$ for $S_2$ as limit between both $Y$ expressions, additional flaw configurations need thus to be calculated in the “Intermediate area”. This purpose is addressed hereafter.

### 3.4.2. Additional subsurface flaw configurations

In order to complete the analyses in the “Intermediate area”, additional fatigue crack growth calculations are carried out considering thick wall piping and thin wall vessel components.

#### 3.4.2.1. Thick wall piping

Figure 3-37 presents the initial dimensions ($a_0$ and $\ell_0$) and initial ligament value ($S_0$) of the subsurface flaw configurations considered in a 75 mm thickness piping component (typical thickness of PWR vessel outlet nozzle). The initial dimensions are around the dimensions given in the Table IWB-3514-2 (Allowable Planar Flaws in Austenitic steels) of ASME Code Section XI Acceptance Standards [21].
The calculations conditions are the same as those used for previous fatigue crack growth calculations in piping: the fatigue crack growth rate is given by Eq. (3-6), the stress intensity factors $K_i$ are analytically calculated in using the equation proposed by Miyazaki et al. [49] and the solutions of the ASME Code Section XI [21]. Fatigue crack growth calculations were performed until 75% of nominal wall thickness $t$. All the calculation conditions are fully described in section 3.3.1.1 Calculations conditions.

![Figure 3-37 Additional subsurface flaw configurations considered for fatigue crack growth analyses in thick wall piping [unit=mm]](image)

3.4.2.2. Thin wall vessel

Figure 3-38 presents the initial dimensions ($a_0$ and $\ell_0$) and initial ligament value ($S_0$) of the subsurface flaw configurations considered in a 140 mm thickness component (typical thickness of BWR vessel). The initial dimensions are around the dimensions given in the Table IWB-3510-1 (Allowable Planar Flaws in Ferritic steels) of the ASME Code Section XI Acceptance Standards.

The calculations conditions are the same as those used for previous fatigue crack growth calculations in vessels: the fatigue crack growth rate is given by Eq. (3-10), the fatigue crack growth calculations are performed with XFEM method with a 140 mm thickness model. Fatigue crack growth calculations were performed until 50% of nominal wall thickness $t$. All the calculation conditions are fully described in section 3.3.2.1 Calculations conditions.
3.4.2.3. Summary of subsurface flaw configurations

Figure 3-39 summarizes all the subsurface flaw configurations considered in the framework of this Chapter. It appears that the “Intermediate area” is now completed by the flaw configurations in thick wall piping (t = 75 mm) and in thin wall vessels (t = 140 mm).

Therefore, if the additional fatigue crack growth calculations demonstrate that the “Y = 0.4 zone” can be extended up to the 140 mm thickness configurations and that the “Y = 1.0 – 1.4(a/l) zone” can be extended up to the 75 mm thickness configurations, the limit value of 10a_0 for S_2 as limit between both Y expressions is confirmed.
3.4.3. Calculations results

Not all the fatigue crack growth curves as crack depth vs. \( N \) are presented hereafter. Only the most relevant fatigue crack growth calculations results among all additional configurations are showed. However, since the scope of the calculations is to assess and to quantify the differences in terms of fatigue life led Eq. (3-1) and Eq. (3-5), all results are presented in term of remaining fatigue lives ratios defined in Eq. (3-9). It has to be reminded that the calculations are performed until the crack depth reaches 0.75\( t \) for thick wall piping geometry (\( t = 75 \text{ mm} \)) and 0.5\( t \) for thin wall vessel geometry (\( t = 140 \text{ mm} \)).

3.4.3.1. Thick wall piping

Figure 3-40 to Figure 3-42 show the fatigue crack growth curves for the initial aspect ratio \( a_0/\ell_0 = 0.05, 0.4 \) and 0.5 and the initial value of the ligament \( S_0 = 8 \text{ mm} \).

Figure 3-40 exhibits that, for the small aspect ratio \( a/\ell = 0.05 \), the differences of the number of cycles at 75% of the thickness are significantly large between \( Y = 1.0 - 1.4(a/\ell) \) and \( Y = 0.4 \). The fatigue life led by \( Y = 0.4 \) are higher than those led by \( Y = 1.0-1.4(a/\ell) \). This highlights a non-conservatism of the current ASME Code proximity rule.

From Figure 3-41 it appears that, for \( a/\ell = 0.4 \) and for \( S_0 = 8 \text{ mm} \), the fatigue lives are almost the same between the both rules. For other value of \( S_0 \) this conclusion is still valid as it is demonstrated through the remaining fatigue lives ratios calculations in section 3.4.4 Comparison of remaining fatigue lives.
Figure 3-40 Fatigue crack growth in the thickness direction for $a_0/l_0 = 0.05$ at $S_0 = 8.0$ mm ($t = 75$ mm)

Figure 3-41 Fatigue crack growth in the thickness direction for $a_0/l_0 = 0.4$ at $S_0 = 8.0$ mm ($t = 75$ mm)

Figure 3-42 shows that, for $a/l = 0.5$, the fatigue life tendencies for the same value of $S_0$ calculated by $Y = 0.4$ and $Y = 1.0 - 1.4(a/l)$ are reversed in comparison to small aspect ratios $a/l$. The relative difference between both fatigue lives is small at 75% of the thickness.
3.4.3.2. Thin wall vessel

Figure 3-43 and Figure 3-44 show the fatigue crack growth curves for the initial aspect ratio $a_0/\ell_0 = 0.05$, 0.4 and the initial value of the ligament $S_0 = 5$ mm. Figure 3-45 shows the fatigue crack growth curves for the initial aspect ratio $a_0/\ell_0 = 0.5$ and the initial value of the ligament $S_0 = 4$ mm.

Figure 3-43 exhibits that for the small aspect ratio $a/\ell = 0.05$, the fatigue lives led by $Y=0.4$ are higher than those led by $Y = 1.0-1.4(a/\ell)$. However these differences are relatively small in comparison to the number of cycle at 50% of the thickness.

For $a/\ell = 0.4$, Figure 3-44 shows that the fatigue lives for $S = 4$ mm are almost the same between the both rules For other value of $S$ this conclusion is still valid as it is demonstrated through the remaining fatigue lives ratios calculations in section 3.4.4 Comparison of remaining fatigue lives.
It appears from Figure 3-45 that, for \( a/l = 0.5 \), the fatigue life tendencies for similar value of \( S \) calculated by \( Y = 0.4 \) and \( Y = 1.0 - 1.4(a/l) \) are reversed in comparison to small aspect ratios \( a/l \). The relative difference between both fatigue lives remains relatively small at 50% of the thickness.
3.4.4. Comparison of remaining fatigue lives

In order to better quantify the differences in terms of remaining fatigue lives, the ratios of remaining fatigue lives defined in Eq. (3-9) are calculated for all the additional flaw configurations at \( a = 0.75t \) for piping and \( a = 0.5t \) for vessel.

3.4.4.1. Thick wall piping

Figure 3-46 presents the value of the ratio of fatigue lives, defined in Eq. (3-9), for the additional flaw configurations in thick wall piping \( (t = 75\text{mm}) \). This ratio is presented according to the initial distance from the free surface and for the initial aspect ratios from 0.05 to 0.5.

It appears that for configurations with an aspect ratio \( a/\ell = 0.5 \), the current proximity factor \( Y = 0.4 \) leads to conservative prediction i.e. lower fatigue lives than the proposed proximity factor \( Y = 1.0 - 1.4(a/\ell) \).

For configurations with an aspect ratio \( a/\ell = 0.4 \), as the ratio \( N_{1.0-1.4(a/\ell)}/N_{0.4} \) is close to 1, the current proximity factor leads thus to similar predictions to the ones led by the proposed proximity factor \( Y = 1.0 - 1.4(a/\ell) \).
For configurations with small aspect ratio \(a/\ell = 0.05\) to 0.25, the current proximity factor \(Y = 0.4\) leads to un-conservative predictions i.e. higher fatigue lives than the proposed proximity factor \(Y = 1.0 - 1.4(a/\ell)\). This non-conservatism increases with the initial distance to the free surface \(S_0\), reaches a maximum before slowly decreasing.

For the smallest aspect ratio \(a/\ell = 0.05\) the difference of the remaining lives is greater than 30%. This non-conservative difference is outside the scatter of fatigue crack growth analyses. Therefore, for subsurface flaws in piping up to 75 mm thickness, the current surface-to-surface proximity rule should be updated using \(Y = 1.0 - 1.4(a/\ell)\).

![Figure 3-46 Ratio of remaining lives \(N_{1.0-1.4(a/\ell)} / N_{0.4}\) for \(t = 75\) mm](image)

### 3.4.4.2. Thin wall vessel

Figure 3-47 presents the value of the ratio of fatigue lives \(N_{1.0-1.4(a/\ell)} / N_{0.4}\) for the analysed configurations in thin wall vessels \((t = 140\) mm). This ratio is presented according to the initial distance from the free surface and for initial aspect ratios from 0.05 to 0.5.

For configurations with an aspect ratio \(a/\ell = 0.5\), the current proximity factor \(Y = 0.4\) leads to conservative prediction i.e. lower fatigue lives than the proposed proximity factor \(Y = 1.0 - 1.4(a/\ell)\).
For configurations with an aspect ratio $a/\ell = 0.4$, as the ratio $N_{1.0-1.4(a/\ell)} / N_{0.4}$ is close to or higher than 1, the current proximity factor leads to suitable or conservative predictions.

For configurations with small aspect ratio ($a/\ell = 0.05$ to 0.25), the current proximity factor $Y = 0.4$ leads to non-conservative predictions i.e. higher fatigue lives than the proposed proximity factor $Y = 1.0 - 1.4(a/\ell)$. However, the difference of the remaining lives is not greater than 15%. This value is within the scatter of fatigue crack growth analyses. Therefore, for subsurface flaws in vessels from 140 mm thickness, the current ASME Code proximity rule $Y = 0.4$ appears not to be updated.

![Figure 3-47 Ratio of remaining lives $N_{1.0-1.4(a/\ell)} / N_{0.4}$ for $t = 140$ mm](image)

**3.5. PROPOSAL OF IMPROVED PROXIMITY FACTOR FOR RULES**

Through the fatigue crack growth calculations and the assessment of the fatigue life ratios, it has been demonstrated that for subsurface flaws in piping up to 75 mm thickness, the current surface-to-surface proximity rule should be updated using $Y = 1.0 - 1.4(a/\ell)$ and that for subsurface flaws in vessels from 140 mm thickness, the current ASME Code proximity rule $Y =$
0.4 is suitable. In other words, it is shown from Figure 3-39 that the “\(Y = 0.4\) zone” can be extended up to the 140mm thickness configurations and that the “\(Y = 1.0 - 1.4(a/\ell)\) zone” can be extended up to the 75mm thickness configurations. Figure 3-39 is therefore updated in Figure 3-48. This demonstrates that the limit value of \(10a_0\) for \(S_2\) as limit between both \(Y\) expressions can be confirmed.

![Figure 3-48 Extension of \(Y= 1.0-1.4(a/\ell)\) and \(Y=0.4\) zones in graph \(S_2/a_0\) vs. \(t\)](image)

Through these calculations, it can be concluded that one improved proposal for the subsurface-to-surface proximity rules is:

\[
Y = 0.4 \quad \text{for vessels} \tag{3-13}
\]
\[
Y = 1.0 - 1.4(a/\ell) \quad \text{for piping}
\]

or comparing the remaining ligament \(S_2\) and the initial flaw depth \(2a_0\):

\[
Y = 0.4 \quad \text{for } S_2 > 10a_0 \tag{3-14}
\]
\[
Y = 1.0 - 1.4(a/\ell) \quad \text{for } S_2 \leq 10a_0
\]
3.6. SUMMARY AND CONCLUSIONS

Fitness-for Service Codes and Standards provide proximity rules for re-characterization of subsurface flaws located near component free surfaces, where the limit values for the proximity factor $Y$ are constant values. However, it was highlighted through experiments that the limit value of the proximity factors shall be a function of the flaw aspect ratios, and that this limit value increases with decreasing aspect ratios. Based on stress intensity factor interaction and experimental results, equivalent fatigue crack growth rates were obtained for the subsurface flaws a new subsurface-to-surface proximity rule was proposed; it is given in Eq. (3-5).

Fatigue crack growths in thin wall components like piping (thickness = 11 mm and 17.4 mm) were then calculated for subsurface flaws using on one hand the proximity rule of Eq. (3-1) provided by the ASME Code Section XI and on the other hand the proposed proximity rule of Eq. (3-5). It was found that the current Codes gives un-conservative remaining lives for subsurface flaws with small aspect ratios. The differences of the fatigue remaining lives are significantly affected by flaw aspect ratios. In case of un-conservative predictions, the relative differences between remaining fatigue lives led by Eq. (3-1) and Eq. (3-5) can reach more than 50%. This cannot be considered as negligible since they are beyond 30% which is a maximum value for the scatter of fatigue analyses. It is concluded that in piping the current proximity rule provided by the ASME Code appears to be updated by the new proposed one.

In parallel, similar fatigue crack growth analyses were performed in thick wall component like vessel (thickness = 200 mm) for subsurface flaws using on one hand the proximity rule of Eq. (3-1) provided by the ASME Code Section XI and on the other hand the proposed proximity rule of Eq. (3-5). As a results of the comparison of the two proximity rules, the current Code leads to longer remaining lives for subsurface flaws with small aspect ratios and to shorter remaining lives for subsurface flaws with large aspect ratios. The
difference of the remaining lives in thick wall component is not greater than 30%. This value is within the scatter of fatigue crack growth analyses. Therefore, for thick wall components, the current ASME Code proximity rule appears not to be updated.

Both those conclusions highlighted that the subsurface-to-surface proximity rule should be updated according to the thickness of the component or according to the type of component i.e. piping or vessel. Therefore, additional fatigue crack growth calculations were carried out in order to determine at best the limit value of the thickness for updating the subsurface-to-surface proximity rule. These additional fatigue crack growth analyses were performed for subsurface flaws in thick wall piping (thickness = 75 mm) and thin wall vessel (thickness = 140 mm) using the proximity rules provided by ASME Code Section XI and the proposed proximity rule.

Through all these calculations covering the range of the thickness value for the classical B&PV components, an improved proposal for the subsurface-to-surface proximity rule can be proposed as follows:

\[
Y = 0.4 \quad \text{for vessels}
\]
\[
Y = 1.0 - 1.4\left(\frac{a}{\ell}\right) \quad \text{for piping}
\]

or comparing the remaining ligament \(S_2\) and the initial flaw depth \(2a_0\):

\[
Y = 0.4 \quad \text{for } S_2 > 10a_0
\]
\[
Y = 1.0 - 1.4\left(\frac{a}{\ell}\right) \quad \text{for } S_2 \leq 10a_0
\]
4.1. INTRODUCTION

Multiple flaws are often detected in structural components in petrochemical, pipeline, offshore and power plants during operation. In addition, multiple flaws can also be found in other problems such as the components subjected to thermal fatigue [51]. It is not easy to assess structural integrity of these components because multiple flaws interact with each other. Both numerical and experimental investigations have been carried out for such flaws in the past. For example, in [52] SIFs were obtained for multiple two-dimensional flaws in plates subjected to tensile stresses. SIF solutions for multiple three-dimensional surface flaws under both tension and bending loads were analysed in [53] and [54]. In [55] and [56] results based on Japanese experimental programs on planar surface flaws were reported and investigated.

If multiple flaws are found in a structural component, FFS Codes such as ASME Code Section XI [21], British Standards (BS) 7910 [29], European Project FITNET [28], etc. provide combination rules. Although the concepts of the combination rules are the same, specific criteria are different among FFS Codes. Therefore, it is easily expected that, when fatigue crack growths are analysed, the remaining lives for the cracked components after flaw combination could be different. However, in any case, the conservatism of the results provided by different combination rules has to be guaranteed and demonstrated [57], [58].
This Chapter firstly presents the combination rules provided by the FFS Codes in the frame of fatigue crack growth analyses.

The next part of this Chapter shows the fatigue crack growth results i.e., the remaining lives led by these different combination rules considering two adjacent surface flaws in a pipe.

In order to assess the conservatisms of these aforementioned analyses, the Chapter also addresses fatigue crack growth calculations conducted using XFEM accounting for actual flaw interactions between the two flaws.
4.2. COMBINATION RULES FOR MULTIPLE FLAWS IN CODES

4.2.1. Combination rules of ASME Code Section XI

The ASME Code Section XI [21] provides combination rules for surface and subsurface flaws. Figure 4-1 shows two surface flaws characterized as semi-elliptical surface flaws in accordance with the ASME Code. The flaw depths are \( a_1 \) and \( a_2 \), flaw lengths are \( \ell_1 \) and \( \ell_2 \), and the distance (also called “ligament”) between the two flaws is \( S \). When \( S \) is small, the net-stress between two flaws increases and mechanical interaction of each flaw increases as well.

In accordance with the ASME Code, flaw combination rules are applied for assessments at inspection, subcritical crack growth calculations such as fatigue and stress corrosion crack (SCC) growth and fracture estimations. The combination rules are expressed by:

\[
S = \begin{cases} 
0.5 \max(a_1, a_2) & \text{for fracture} \\
0 & \text{for fatigue and SCC} 
\end{cases}
\]  

(4-1)

In case of subcritical crack growths such as fatigue and SCC growths, the distance \( S = 0 \) means that two flaws are not combined before touching each other. The fatigue crack growth rates at the surface points between two flaws are accelerated by each flaw interaction and the number of cycles becomes short. However, from the view point of the total fatigue life, number of cycles at the distance \( S \) is small. In addition, experimental data also showed that it was not necessary to consider interaction for two flaws [55].

Equation (4-1) means that the combination rule should be considered for fracture with \( S \leq 0.5 \max(a_1, a_2) \), while it is not required for fatigue or SCC assessments.
4.2.2. Combination rules of BS 7910 and FITNET

The BS 7910 [29] and FITNET [28] Codes also provide a combination rule as given by Eq. (4-2), where the rules depend on initial flaw aspect ratio. Hereafter, these two codes are referred to simply as BS 7910:

\[
S = \begin{cases} 
\min(\ell_1, \ell_2) & \text{for } \frac{a_1}{\ell_1} \text{ or } \frac{a_2}{\ell_2} > 0.5 \\
0.5 \max(a_1, a_2) & \text{for } \frac{a_1}{\ell_1} \text{ and } \frac{a_2}{\ell_2} \leq 0.5 
\end{cases} \tag{4-2}
\]

Note that the BS 7910 Code does not mention any combination rule for fatigue assessments whereas the FITNET Code states “It is not necessary to apply the flaw interaction criteria in a fatigue assessment. However, if there is any doubt, multiple flaws should be combined”. From the standpoint of a conservative evaluation, the combination rule in Eq. (4-2) is investigated in the fatigue growth calculations.

When flaw shapes are deep \((a_1/\ell_1 \text{ or } a_2/\ell_2 > 0.5)\), the distance \(S\) between two flaws is compared with the flaw length. On the other hand, when the flaws are elongated, \((a_1/\ell_1 \text{ and } a_2/\ell_2 \leq 0.5)\), the distance \(S\) is compared with half flaw depth, whichever is greater.
4.2.3. Combination Rule in the FKM, SSM and GB/T19624 Codes

In the FKM [30], SSM [25] and GB/T19624 [32] Codes, the same combination rule for surface flaws is given as seen in Eq. (4-3). Hereafter, these three codes are referred to simply as FKM:

\[ S = \min(\ell_1, \ell_2) \] (4-3)

Equation (4-3) shows that the distance \( S \) is always compared with the flaw lengths of multiple flaws. It is found that Eq. (4-3) give the same rule as Eq. (4-2) for the case of \( a_1/\ell_1 \) or \( a_2/\ell_2 > 0.5 \), whereas it is different from Eq. (4-2) when \( a_1/\ell_1 \) and \( a_2/\ell_2 \leq 0.5 \).

4.2.4. Combination Rule in the API 579, HPI S and A16 Codes

The API 579 [31], HPI S [34] and A16 [24] Codes provide the same combination rule as expressed in Eq. (4-4). Hereafter, these three codes are referred to simply as API 579.

\[ S = 0.5 \times (\ell_1 + \ell_2) \] (4-4)

Equation (4-4) shows that the distance \( S \) is compared with half of the sum of flaw lengths for multiple flaws. It should be noted that, for two similar flaws with the identical flaw length, the combination rule deduced from Eq. (4-4) is equivalent to that deduced from Eq. (4-3).

As it can be deduced from this section, the remaining lives obtained using the aforementioned combination rules could be different. It is therefore relevant to assess the discrepancies between the fatigue crack growth results led by Eq. (4-1) to Eq. (4-4) and, on the other hand, to compare them with realistic remaining lives as provided by XFEM analyses.
4.3. CALCULATIONS CONDITIONS OF FATIGUE CRACK GROWTH

In order to obtain remaining lives for pipes containing two adjacent circumferential surface flaws, similar flaws with aspect ratio less than or equal to 0.5 were employed. Figure 4-2 illustrates the two circumferential semi-elliptical surface flaws with a similar size in a pipe. The pipe employed is 6-inch (165.2 mm) diameter Schedule 80 pipe, where the wall thickness is 11.0 mm. The material of the pipe is an austenitic stainless steel. The two adjacent coplanar flaws are set to locate on the inner surface of the pipe.

![Figure 4-2 Two semi-elliptical surface flaws with similar size](image)

The pipe is subjected to a uniform tensile stress. A cyclic loading is applied with the maximum stress $\sigma_{\text{max}} = 123$ MPa which corresponds to the allowable design stress for a stainless steel, and the minimum stress $\sigma_{\text{min}} = 0$ MPa [44]. The stress ratio is $R = \sigma_{\text{min}}/\sigma_{\text{max}} = 0$. The calculation conditions are summarized in Table 4-1.

<table>
<thead>
<tr>
<th>Table 4-1 Calculation conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>Pipe</td>
</tr>
<tr>
<td>Cyclic load</td>
</tr>
<tr>
<td>Cyclic load</td>
</tr>
<tr>
<td>Crack growth</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

123
The reference fatigue crack growth rate curves provided by the ASME Code Section XI are used, although the other FFS Codes also provide fatigue crack growth rates. The fatigue crack growth for austenitic stainless steels is given by:

\[
d a/dN = C_0 (\Delta K_I)^n
\]  

(4-5)

where \( da/dN \) is the fatigue crack growth rate in mm/cycle, \( \Delta K_I \) means the range of stress intensity factor in MPa\(\sqrt{m} \), and \( C_0 \) and \( n \) are parameters depending on the material and environment. Based on the ASME Code, \( n \) is given as \( n = 3.3 \) and \( C_0 \) can be determined using Eq. (4-6):

\[
C_0 = CS
\]  

(4-6)

where \( C \) is a scaling parameter to account for temperature and is provided by:

\[
C = 10^{(-8.714+1.34\times10^{-5}T-3.34\times10^{-6}T^2+5.95\times10^{-9}T^3)}
\]  

(4-7)

where \( T \) is the metal temperature for \( T \leq 430 \, ^\circ C \), and \( S \) is a scaling parameter to account for \( R \) ratio \( (R = \sigma_{\text{min}}/\sigma_{\text{max}}) \) and is given by:

\[
S = \begin{cases} 
1.0 & \text{for } R \leq 0 \\
1.0 + 1.8R & \text{for } 0 < R \leq 0.79 \\
-43.35 + 57.97R & \text{for } 0.79 < R < 1.0 
\end{cases}
\]  

(4-8)

As \( R = 0 \) in the calculations, the scaling parameter becomes \( S = 1.0 \).

Table 4-2 shows initial sizes and locations of the two adjacent flaws. These flaw sizes are around the allowable sizes for 6-inch diameter pipes in accordance with the Acceptance Standard in the ASME Code. Two initial distances \( S_0 \) are considered: \( S_0 = 0.5 \) and \( 1.0 \) mm.

<table>
<thead>
<tr>
<th>Subject piping</th>
<th>( a_1 = a_2 )</th>
<th>( \ell_1 = \ell_2 )</th>
<th>( a_1/\ell_1, \ a_2/\ell_2 )</th>
<th>( S_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-in. pipe ( t = 11.0 )</td>
<td>1.74</td>
<td>34.8</td>
<td>0.05</td>
<td>0.5, 1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.6</td>
<td>0.15</td>
<td>0.5, 1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.48</td>
<td>0.5</td>
<td>0.5, 1.0</td>
</tr>
</tbody>
</table>
SIFs are necessary to carry out fatigue crack growth calculations. The current FFS Codes provide stress intensity factor tables for only a single flaw in a flat plate. These Codes do not include stress intensity factors for two neighboring surface flaws. When performing fatigue crack growth calculations, Code users calculate fatigue crack growths for each flaw separately, updating the distance $S$ during the crack growths. When the distance $S$ satisfies the combination rules, the two flaws are combined to a single semi-elliptical flaw. Therefore, interaction effects between two flaws are not considered for fatigue crack growth calculations.

Although the FFS Codes provide their own SIF tables, the SIFs for flat plates with surface flaws provided by the ASME Code were used for fatigue crack growth calculations based on ASME Code combination rules i.e., Eq. (4-1) as well as for the fatigue crack growth calculations based on other combination rules i.e., Eq. (4-2) to Eq. (4-4). In this way, the differences of remaining lives are only based on the combination rules. Indeed, if using SIFs given by other FFS Codes for the corresponding combination rules, differences in remaining lives make it difficult to establish whether it is a consequence of combination rules or of SIFs.

On the other hand, fatigue crack growth calculations by XFEM were conducted by the same conditions of surface flaw size and cyclic stresses. However, note that the XFEM calculations consider two surface flaws concurrently, taking hence into account the actual interaction of two neighboring flaws.

The width of the model plate is 200 mm, the height of the plate is 200 mm and the thickness is 11 mm. The flaws are located in the mid-height plane (with refined mesh around the flaw location). The cyclic loading is applied on the top of the model. The bottom is fully constraint. The XFEM model is presented in Figure 4-3. The flaws are model by the level-set method. Figure 4-4 presents a cross section with a two adjacent surface flaws ($a_1/\ell_1 = a_2/\ell_2 = 0.15$ and $S_0 = 1$mm) such as modelled by the level-set method.
Owing to XFEM, the stress intensity factors $K_1$ are computed all along the crack fronts of the two flaws. Therefore, using the fatigue crack growth rate defined in Eq. (4-5) to Eq. (4-8), the increment $da$ for a number of cycle $dN$ can be calculated for each point of the crack front. The flaw shape is then updated accounting for the actual size increase and shape evolution of the crack fronts i.e. the flaws freely propagate in a stress field without any constraint on their shape leading to realistic results.
4.4. FATIGUE CRACK GROWTH CALCULATIONS OF TWO SURFACE FLAWS

4.4.1. Two Surface Flaws without Interaction

In calculating fatigue crack growths for two flaws, Code users perform calculations for a single flaw independently, and always check the geometries of the flaws of depths, lengths and the distance of two flaws. Figure 4-5 illustrates the combination behaviors of fatigue crack growths for two similar semi-elliptical flaws with initial ligament $S_0$. Flaw #1 and flaw #2 grow symmetrically, independently. That is, crack growth amounts at the surface points are the same in the circumferential direction of the outer side and the inner side. Therefore, the mechanical flaw interaction is implicitly not considered.

The timing of the combination by the ASME Code is shown in Figure 4-5 (a), where flaw #1 and flaw #2 touch at the inner sides of the surface points. After the combination, flaw length $\ell$ becomes $\ell' = \ell_1' + \ell_2'$. Figure 4-5 (b) depicts the combination behavior for the BS7910, FKM and API 579 Codes. Hereafter, FKM and API 579 Codes are referred to simply as FKM since, as mentioned before, these two combination rules become identical for the two similar flaws. When the distance $S$ between two flaws meets the combination rules, the two flaws are combined to a single flaw, where the length of the combined flaw is expressed by $\ell' = \ell_1' + \ell_2' + S$. After combination of the two flaws, the single flaw length conducted by the BS7910 or FKM Code is always larger than that by the ASME Code. It can be easily deduced that remaining lives provided by the BS7910 or FKM Code will be shorter than those provided by the ASME Code.
4.4.2. Two Surface Flaws with Interaction

Combination rules are not necessary when fatigue crack growth calculations by XFEM are performed. This is because the XFEM method allows calculating two surface flaws concurrently, taking into account their actual interaction, as mentioned above. The geometry of the growing surface flaw is not symmetric, as shown in Figure 4-6. The fatigue crack growth at the surface point close to the adjacent surface point is larger than that at the surface point of the opposite side, based on stress intensity factor interaction [59]. After touching the surface points of the inner sides of the two flaws, the flaw is calculated as a single flaw with the flaw depth $a'$ and flaw length $\ell' = \ell_1' + \ell_2'$ as shown in Figure 4-6. The growth behavior of two surface flaws is similar to the experimental behavior observed at the fractured surfaces of flat plate specimens with two surface flaws [60].
4.5. CALCULATION RESULTS OF FATIGUE CRACK GROWTHS

Fatigue crack growths for two similar circumferential semi-elliptical surface flaws in pipes were calculated using stress intensity factors provided by the ASME Code and using the XFEM method as explained above. Flaw in the thickness direction of the pipe is important, because flaw depth is strongly related to coolant leakage. The fatigue crack growth calculations were terminated when the single combined flaw depth reaches 75% of the nominal wall thickness, i.e., $a/t = 0.75$.

The relationships between the flaw depth $a/t$ and the number of cycles $N$ are shown in Figure 4-7 to Figure 4-12, obtained by the ASME Code, the BS7910 Code, the FKM Code and the XFEM method.

Figure 4-7 is the fatigue crack growth curves $a/t$ vs. $N$ for the initial flaw sizes with $a_1 = a_2 = 1.74$ mm and $\ell_1 = \ell_2 = 34.8$ mm. The initial aspect ratios are $a_1/\ell_1 = a_2/\ell_2 = 0.05$ and the initial distance of the two surface flaws is $S_0 = 0.5$ mm. In accordance with the combination rule given by the ASME Code i.e., $S = 0$ mm, two flaws were combined at $N = 126200$. The fatigue crack
growth curve shows a discontinuous increase at \( N = 126200 \). On the other hand, in the case of the BS 7910 and FKM Code, the two initial flaws are already combined before the fatigue crack growth calculation, because \( S_0 = 0.5 \text{ mm} \) is less than \( S = 0.5 \max (a_1, a_2) = 0.5 \times 1.74 = 0.87 \text{ mm} \) for BS 7910 Code or less than \( S = \min (\ell_1, \ell_2) = 34.8 \text{ mm} \) for the FKM Code. In the case of the XFEM, two initial flaws grow with interaction and contact at \( N = 101000 \). The number of cycles at the flaw combination by the XFEM is reasonably smaller than that by the ASME Code. It is asserted that the difference of the numbers comes from the flaw interactions.

Fatigue crack growths in the thickness direction occur gradually as shown in Figure 4-7. The number of cycles at \( a/t = 0.75 \) for the ASME Code, the BS 7910 Code, the FKM Code and the XFEM is tabulated in Table 4-4. As can be seen that the number of cycles at \( a/t = 0.75 \) for the ASME Code is nearer to that for the XFEM than that for the BS 7910 and the FKM Codes.

![Fatigue crack growth results for \( a/t = 0.05 \) and \( S_0 = 0.5 \text{ mm} \)](image)

Fatigue crack growth curves \( a/t \) vs. \( N \) for the initial flaw sizes with \( a_1 = a_2 = 1.74 \text{ mm} \), \( \ell_1 = \ell_2 = 34.8 \text{ mm} \) and \( S_0 = 1.0 \text{ mm} \) are shown in Figure 4-8. The distance between two initial flaws is slightly larger compared with the flaw location shown in Figure 4-7. In the case of the FKM Code, the two initial flaws are already combined before calculations, because \( S_0 < S \) (i.e., 1.0 mm < 34.8 mm). In the case of the BS 7910 Code, the two initial flaws grow
independently, because \( S_0 > S \) (i.e., 1.0 mm > 0.87 mm). Two flaws are combined at \( N = 14580 \) for the ASME Code and \( N = 77200 \) for the BS 7910 Code. Needless to say, the number of cycles at combination for the ASME Code is larger than that for the BS 7910 Code. Total lives at \( a/t = 0.75 \) are shown in Table 4-4. The remaining lives at \( a/t = 0.75 \) are greater when the distances between two flaws are large.

![Fatigue crack growth results for \( a/\ell = 0.05 \) and \( S_0 = 1.0 \) mm](image)

Figure 4-8 Fatigue crack growth results for \( a/\ell = 0.05 \) and \( S_0 = 1.0 \) mm

Figure 4-9 and Figure 4-10 show fatigue crack growth curves for flaws with \( a_1 = a_2 = 1.74 \) mm, \( \ell_1 = \ell_2 = 11.6 \) mm and \( a_1/\ell_1 = a_2/\ell_2 = 0.15 \), where \( S_0 = 0.5 \) mm in Figure 4-9 and \( S_0 = 1.0 \) mm in Figure 4-10. In the case of the FKM Code, the two initial flaws are already combined before calculations for both \( S_0 = 0.5 \) and 1.0 mm. In the case of the BS 7910 Code, the initial two flaws are combined before calculation, as \( S_0 = 0.5 \) mm is less than \( S = 0.87 \) mm, and they are not combined when \( S_0 = 1.0 \) mm. The fatigue crack growth curves for the BS 7910 and FKM Codes are relatively close to those for the ASME Code, compared with Figure 4-7 and Figure 4-8. The remaining lives at \( a/t = 0.75 \) are shown in Table 4-4.
Figure 4-9 Fatigue crack growth results for $a/\ell = 0.15$ and $S_0 = 0.5$ mm

Figure 4-10 Fatigue crack growth results for $a/\ell = 0.15$ and $S_0 = 1.0$ mm

Figure 4-11 and Figure 4-12 show fatigue crack growth curves for large aspect flaws with $a_1/\ell_1 = a_2/\ell_2 = 0.5$, where $a_1 = a_2 = 1.74$ mm and $\ell_1 = \ell_2 = 3.48$ mm. The distance between two flaws are $S_0 = 0.5$ mm for Figure 4-11 and $S_0 = 1.0$ mm for Figure 4-12. For the FKM Code, the two initial flaws are combined before calculations when $S_0 = 0.5$ and $1.0$ mm. For the BS 7910 Code, the initial two flaws for $S_0 = 0.5$ mm are combined before calculations while they are not combined for $S_0 = 1.0$ mm. The fatigue crack growth curves for the BS 7910 and FKM Codes are more close to those for the ASME Code, compared with Figure 4-9 and Figure 4-10. Fatigue crack
growth curves for the BS 7910 and FKM Codes approach to those for the ASME Code, when the initial flaw aspect ratio increases. It can be observed that the fatigue crack growth curves calculated by the XFEM are almost close to the curves for the ASME Code. The remaining lives at $a/t = 0.75$ are shown in Table 4-4. It can also be seen that the number of cycles at $a/t = 0.75$ for $S = 0$ is close to that by the XFEM method.

![Fatigue crack growth results for $a/t = 0.5$ and $S_0 = 0.5$ mm](image1)

![Fatigue crack growth results for $a/t = 0.5$ and $S_0 = 1.0$ mm](image2)
4.6. COMPARISON OF REMAINING LIVES

The number of cycles at combination from the fatigue crack growth calculations is summarized in Table 4-3. The asterisk mark ‘*’ means that the two surface flaws are already combined before the fatigue crack growth calculations. It is found that the number of cycles at combination for the BS 7910 Code shows a significant difference with that from XFEM. On the other hand, the difference in the number of cycles at combination for the ASME Code and XFEM is not so large for a small aspect ratio while it is significant for a large aspect ratio. In particular, the maximum difference is obtained as 53% for the case of $a/\ell = 0.5$ and $S_0 = 0.5$ mm.

Table 4-3 Fatigue crack growth results at combination

<table>
<thead>
<tr>
<th>$a/\ell$</th>
<th>$S_0$ mm</th>
<th>Number of cycles $N$ at combination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FKM</td>
</tr>
<tr>
<td>0.05</td>
<td>0.5</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>*</td>
</tr>
<tr>
<td>0.15</td>
<td>0.5</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>*</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>*</td>
</tr>
</tbody>
</table>

*Combined before the fatigue crack growth calculations

In addition, remaining fatigue lives for two discrete surface flaws were obtained by using numerical calculations with three combination rules and the XFEM method.

When the distance $S_0$ of the two flaws is 0.5 mm, remaining fatigue lives provided by the BS 7910 and FKM Codes are the same, regardless of the initial flaw aspect ratio. When $S = 1.0$ mm, difference in remaining fatigue lives provided by the BS 7910 and FKM Codes is relatively small with the maximum difference of 15% for the case of $a/\ell = 0.05$. It is observed that fatigue lives calculated by the combination rules provided by the BS 7910
Code and the FKM Code are shorter, compared with the fatigue life calculated by the rule of \( S = 0 \) provided by the ASME Code. Particularly, in the case of \( S_0 = 0.5\text{mm} \), fatigue life for \( a/\ell = 0.05 \) by the BS 7910 Code is 36% smaller than that by the ASME Code, whereas fatigue life for \( a/\ell = 0.05 \) by the FKM Code is 48% smaller than that by the ASME Code. It means that the combination rules of the BS 7910 Code and the FKM Code always give conservative remaining lives than the rule of the ASME Code. The remaining lives at \( a/t = 0.75 \) is tabulated in Table 4-4.

<table>
<thead>
<tr>
<th>( a/\ell )</th>
<th>( S_0 ) mm</th>
<th>Number of cycles ( N ) at ( a/t = 0.75 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FKM</td>
<td>BS 7910</td>
</tr>
<tr>
<td>0.05</td>
<td>0.5</td>
<td>104500</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>104100</td>
</tr>
<tr>
<td>0.15</td>
<td>0.5</td>
<td>238900</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>234600</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>599100</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>571800</td>
</tr>
</tbody>
</table>

It is found that the fatigue life given by combination rule of \( S = 0 \) provided by the ASME Code is fairly close to that by the XFEM results. Finally, it can be seen that the combination rule of \( S = 0 \) in the ASME Code is better than the combination rules provided by the BS 7910 Code and the FKM Code.
4.7. SUMMARY AND CONCLUSIONS

When multiple flaws are detected in structural components, remaining lives of the components are estimated by fatigue crack growth calculations using combination rules in FFS Codes. These FFS Codes were found to provide different combination rules. As a consequence, the remaining lives led by those combination rules are also different.

In order to investigate the impact of the different combination rules on remaining lives, fatigue crack growth analyses for two adjacent surface flaws in a pipe subjected to cyclic tensile stress were performed using these different combination rules.

In parallel, fatigue lives taking into account interaction effect between the two flaws were conducted by XFEM.

As the results of the calculations, it is concluded that the fatigue lives based on the combination rule provided by other FFS Codes are always conservative, compared with the lives based on the combination rule provided by the ASME Code.

Moreover, the difference between the fatigue lives given by other FFS Codes and by the ASME Code is significantly large, when the initial distance between two flaws and flaw aspect ratios are small.

Finally, the fatigue lives calculated by XFEM are close to those by the ASME Code. It can be stated that the combination rule provided by the ASME Code is appropriate for fatigue crack growth calculations considering two similar flaws.
CHAPTER 5

CHARACTERIZATION RULES FOR QUASI-LAMINAR FLAWS

5.1. INTRODUCTION

During the 2012 outage at the Belgian Nuclear Power Plan (NPP) of Doel 3, specific ultrasonic in-service inspections were performed to check for underclad cracking in the reactor pressure vessel. No underclad defects were found but a large number of quasi-laminar indications were detected mainly in the lower and upper shells of the RPV. A second inspection was performed with UT probes able to inspect the whole thickness of the vessel and identified the same type of indications deeper in the material. The same inspection performed in few months later at Tihange 2 NPP showed similar indications but to a lesser extent. As a consequence, the Doel 3 and Tihange 2 NPPs had to stay in cold shutdown until it has been proved that they can be safely operated.

With the support of internal and external experts, the licensee started an investigation of the precise nature and origin of these indications, and built its analysis to determine whether or not the reactor units could safely resume operation in spite of the detected flaws. This extensive demonstration of the licensee, confirming the fitness-for-service of both RPVs, was recorded in two Safety Case Reports which were submitted for review to the Federal Agency for Nuclear Control (FANC) in October 2015, one for Doel 3 [61] and one for Tihange 2 [62]. The restart of both units was allowed by FANC on November 2015.
A large part of the Safety Case demonstration consisted of the Flaw Acceptability Assessment based on the ASME Code which Belgian NPPs have to refer to for Design and In-service Inspections.

In particular, from the principles of article IWA-3300 “Flaw Characterization” of the ASME Code Section XI, it is required combining closely spaced flaws in order to account for their mechanical interactions according to grouping rules.

However, it early appeared that the characterization rules of ASME Code Section XI were adapted neither to quasi-laminar flaws nor to such densities of flaws as found in Doel 3 and Tihange 2 RPVs: the strict application of the current grouping rules to the actual quasi-laminar flaw populations led to unrealistic results and conclusions.

Therefore, an alternative methodology to derive suitable characterization rules for quasi-laminar flaws was developed, implemented and validated [36], [37].

It is the first time that specific grouping criteria are developed and used to successfully assess the acceptability of such flaws.

From this first-of-a-kind development, a Code Case was drawn up and proposed to the ASME in February 2014 for improving the Code regarding the characterization rules for quasi-laminar flaws. The proposed Code Case (Code Case N-848) was voted and approved by the ASME Section XI Committee on Nuclear In-service Inspection during the April 2015 Code meeting and will be part of the next editions of the Code [35].

In the first part of this Chapter the context and the motivations having led to the development of these specific characterization rules are fully detailed. Some relevant definitions, figures, dimensions… are also defined in order to clearly set the framework of the calculations which are presented further in the Chapter.
The next part of this Chapter presents the methodology used to derive the flaw proximity limits for two flaws close to each other suitable for the quasi-laminar flaws based on two-dimensional (2D) XFEM analyses. This methodology is in line with recent developments made (or being made) for the ASME XI and with recently published articles [59], [63], [64]. Based on the proximity criteria so calculated, the alternative characterization rules for quasi-laminar flaws that are put into practice when multiple flaws have the potential to interact is then depicted as implemented in [35].

As it is afterwards highlighted in the Chapter, these new proximity rules, based on 2D calculations, although better suited to quasi-laminar flaws are however highly conservative. Indeed, it appeared after performing some detailed 3D XFEM analyses that a three-dimensional (3D) approach could lead to more realistic, but still conservative, proximity rules for quasi-laminar flaws [36].

Therefore, the next part of this Chapter presents the updated methodology used to develop the flaw proximity rules through 3D XFEM analyses as well as the demonstration of the multiple levels of conservatism of the methodology by numerous sensitivity analyses [37]. The result of this more realistic approach has led to a revision of the Code Case N-848 which has been presented to the ASME Committee in November 2015.

Finally, in order to highlight the conservatism of these revised proximity rules for quasi-laminar flaws, a practical case of combined flaw will be analysed. This analysis compares the SIF of the single combined flaw to the SIF of each individual flaw constituting the combined flaw but simultaneously modelled in order to account for their actual interaction [42]. This is done through a so-called “XFEM multi-flaw analysis”.

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5.2. CONTEXT AND MOTIVATIONS

5.2.1. Definitions

5.2.1.1. Laminar flaw

In the sense of the ASME Code Section XI, the definition of the laminar flaw is as follows [21]:

“Planar indications oriented within 10 degrees of a plane parallel to the surface of the component shall be considered laminar flaws”.

5.2.1.2. Quasi-laminar flaw

As defined in the ASME Code Section XI Code Case N-848, the definition of the quasi-laminar flaw is as follows [35]:

Planar indications oriented within 20 degrees of a plane parallel to the surface of the component, including any laminar indications, shall be considered quasi-laminar flaws.

It has to be noted that all laminar flaws in the sense of the ASME XI Code are also quasi-laminar according to this definition.

For information, it has been demonstrated in the Safety Cases [61], [62] that the flaws in the Doel 3 and Tihange 2 RPVs are oriented within 16° of a plane parallel to the pressure retaining surface i.e. the surface of the component. They are therefore well quasi-laminar.

5.2.2. ASME Code Section XI

The proximity rules of the ASME Code Section XI address the laminar, the axial and the circumferential flaws, depicted in Figure 5-1, but are not adapted for quasi-laminar flaws as it is discussed hereafter.

In practice, on one hand, if a flaw is oriented with a tilt lower than or equal to 10 degrees relative to the pressure retaining surface (i.e. laminar flaw in the
sense of the ASME Code), it has to be projected in the plane of the pressure retaining surface in order to assess its potential interaction with other flaws according to the proximity rules of article IWA-3360 of Section XI as shown in Figure 5-2.

On the other hand, if a flaw is oriented with a tilt higher than 10 degrees (= planar flaw in the sense of the ASME Code) relative to the pressure retaining surface, it has to be projected in the axial and circumferential plane, as illustrated in Figure 5-3, in order to assess its potential interaction with other flaws according to the proximity rules of article IWA-3330 of Section XI.
Figure 5-2 Proximity criteria for laminar flaws as defined in figure IWA-3360-1 of [21]

Figure 5-3 Axial and circumferential projections of the flaw according to ASME Code section XI
It implies that laminar flaws (inclination less than 10 degrees) and planar flaws should be separately considered even if they are physically close to each other. However in Doel 3 and Tihange 2 RPVs, the flaw densities are such that both kinds of flaws coexist, have interactions and have thus to be treated simultaneously. This demonstrates that the proximity rules of the ASME Code Section XI are not adapted for such flaw densities of quasi-laminar flaws.

To even more emphasize the need of developing alternative proximity rules, let us focus only on laminar flaws. Indeed, for laminar flaws in the sense of the ASME Code Section XI, the proximity criteria defined by article IWA-3360, shown on Figure 5-2, refer to a maximum distance $S=1$ inch (25.4 mm) under which flaws have to be combined into one equivalent flaw with a larger size than the individual flaws in the group. This distance is thus an absolute distance independent of the size of the flaws. As explained in the document EPRI-NP-1406 [65], the 1 inch distance corresponds in fact to the capabilities of UT at the time of establishment of these criteria (~1970). UT techniques have strongly evolved since then and the 1 inch distance is no more adapted to the present-day UT capabilities. In addition, in the IWA-3360 criterion, there is no distinction between flaws lying in different planes. This implies the projection of all the flaws from parallel planes onto one single surface, i.e. the cylindrical inner surface of the vessel, whatever their actual distance through the wall of the RPV. Through this investigation, it clearly appears for laminar flaws that there is no mechanical interaction consideration behind the proximity criteria in the ASME Code Section XI because of the use 1 inch criterion and that the existing criteria for laminar flaws are overly conservative because of the projection of all the flaws onto one plane.

As a consequence, alternative proximity rules better suited to the quasi-laminar flaws were developed for assessing the acceptability of flaws in Doel 3 and Tihange 2 RPVs by fracture mechanics, firstly based on 2D calculations then updated using a 3D approach.
5.2.3. Flaw characterization and relevant dimensions

Each quasi-laminar flaw is bounded by the minimum bounding box (rectangular cuboid) that fully contains the area of the flaw as shown in Figure 5-4. This way of proceeding is consistent with article IWA-3300 “Flaw Characterization” of the ASME Code Section XI.

The diagonals of the faces of the box normal to the principal stresses \( \sigma_1 \) and \( \sigma_2 \) are denoted \( 2D_{11} \) and \( 2D_{21} \) for the first flaw and \( 2D_{12} \) and \( 2D_{22} \) for the second flaw as shown in Figure 5-4. As an example, in the RPV wall, \( \sigma_1 \) and \( \sigma_2 \) are the axial and the hoop stresses.

The distances between the boxes bounding the flaws are denoted \( S_1 \), \( S_2 \) (along the direction of \( \sigma_1 \) and \( \sigma_2 \)), and \( H \) (along the through wall direction), as shown in Figure 5-4.

---

Figure 5-4 Multiple Quasi-Laminar Flaws: Illustrative Flaw Configuration and Determination of Relevant Dimensions
The proximity rules for quasi-laminar flaws aims therefore at determining the limit value for $H$, $S_1$ and $S_2$ below which the flaws have to be combined and considered as one single flaw.

5.3. ALTERNATIVE CHARACTERIZATION RULES FOR QUASI-LAMINAR FLAWS BASED ON 2D XFEM CALCULATIONS

The objective of this section is to present the 2D approach used to derive the proximity rules for flaws close to each other which are suitable for quasi-laminar flaws. Then, the generic way of proceeding with flaws to be combined (i.e. the characterization rules) is explained. Finally the conservatisms of this 2D approach are demonstrated.

5.3.1. Flaw interaction

5.3.1.1. Stress Intensity Factor

The proximity rules are elaborated on the basis of the analysis of the Stress Intensity Factor (SIF) interaction between two flaws.

For fracture mechanics calculations, ASME XI Code focuses on the stress intensity factor $K_i$ corresponding to the fracture mode I (crack opening mode). Figure 5-5 illustrates a crack solicited in pure mode I.

In reality, since the quasi-laminar flaws are tilted as illustrated in Figure 5-1 and Figure 5-4, distinction is made between three particular fracture modes (relative displacement of crack lips), according to the direction of the load relative to that of the crack (see Figure 5-6).
The combination of these three fracture modes is taken into account using the following formula by defining an equivalent SIF as proposed in the French Code RSE-M [23]:

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2 + \frac{1}{1-\nu} K_{III}^2}$$  \hspace{1cm} (5-1)

As the flaw length in the out-of-plane dimension is implicitly considered as infinite, the proximity criteria are established on the basis of 2D XFEM calculations in plane strain. The mode III SIF $K_{III}$ is therefore not used and the equivalent SIF becomes:

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2}$$  \hspace{1cm} (5-2)
5.3.1.2. Interaction Factor
A literature survey of the proximity rules between two flaws other than laminar flaws (i.e. planar flaws) shows that the proximity limits are based on the analysis of the SIF in mode I i.e., $K_I$, of the two interacting flaws [59], [63], [64]. When distance between two flaws is decreasing, the mechanical interaction between them is increasing and consequently their SIF. This interaction can lead to the fracture of the ligament between the flaws as shown in Figure 5-7 and Figure 5-8.

Figure 5-7 Flaws sufficiently distant: no mechanical interaction between them

Figure 5-8 Flaws close enough: mechanical interaction leading to group
This mode I SIF increase is quantified by the so-called interaction factor defined as the ratio:

\[ \bar{\xi} = \frac{K_1}{K_0} \]  

(5-3)

between the mode I SIF of the flaw when subjected to the influence of the second flaw, \( K_1 \), and the mode I SIF of the same flaw considered as isolated, \( K_0 \), i.e., without any influence from other flaws in its vicinity, as illustrated in Figure 5-9.

Calculations and experimental tests [63] have been carried out on nonaligned flaws having an inclination of 90 degrees in a flat plate loaded with a tension perpendicular to the axes of the flaws (situation of Figure 5-7 and Figure 5-8). The results of these works have highlighted that the fracture of the ligament between flaws occurs from an increase of 6% in the SIF. In other words, the mechanical interaction between the flaws leads to combine the flaws when the interaction factor \( K_1/K_0 \) reaches 1.06.

This criterion of 6% increase in SIF has been applied for determining the proximity criteria with quasi-laminar flaws.
Further in this section, when we deal with interaction between quasi-laminar flaws, we use the following generic definition of the interaction factor $\xi$, based on the equivalent SIF, $K_{eq}$, as defined in Eq. (5-2):

$$\xi = \frac{K_{eq,1}}{K_{eq,0}}$$  \hspace{1cm} (5-4)

Figure 5-10 illustrates a typical example of the configurations that are considered in this 2D approach and defines the equivalent SIFs, $K_{eq,0}$ and $K_{eq,1}$, as used in Eq. (5-4).

![Figure 5-10 Schematic illustration of $K_{eq,0}$ and $K_{eq,1}$ definition](image)

All the parameters of the 2D configurations describing unequivocally the cases to solve are presented in details hereafter.

### 5.3.2. Hypotheses

#### 5.3.2.1. Influence of the flaw size

The experiments and the calculations of [63] have been carried out by analyzing the interaction between two flaws of same and different size.
The resulting 6% SIF increase criterion for the flaws combination has been determined by focusing on the flaw with highest $K_0$ i.e. the flaw with the largest size, let $2D_1$. Indeed, the flaws the most susceptible to initiate fracture are those with highest SIF.

Let $H$ and $S$ denote respectively the distance between the plane of the flaws and the distance between the flaws when projected in a same plane; let $2D_1$ and $2D_2$ denote the size of the flaws, as depicted in Figure 5-11.

![Figure 5-11 Definition of parameters for two non-aligned flaws](image)

Another key point of the results of [63] highlights that when the isovalue curves of $K_1/K_0$ are plotted into the axes $S/D_1$ $H/D_1$, the area of interaction seems to decrease proportionally to the ratio $D_1/D_2$. This is physically obvious that the smaller the size of the second flaw is, the lower its influence on the first one is, for given values of $H$ and $S$. Consequently, for a given interaction, the areas of interaction are similar in the $S/\min(D_1,D_2)$ $H/\min(D_1,D_2)$ axes regardless of the size of the second flaw relative to the size of the first one. This point is schematically outlined in Figure 5-12 with two configurations of two flaws (a) and (b): $2D_2=2D_1$ and $2D_2=0.5\cdot2D_1$
Therefore, for configurations with two nonaligned flaws having an inclination of 90 degrees perpendicular to the direction of the tensile stress, as illustrated at the top of Figure 5-12, the criteria in order to reach 6%
interaction can be calculated with two flaws of the same size and generalized on the basis of the minimum flaw size.

This important result has been used and applied for the determination of proximity rules of quasi-laminar flaws.

Figure 5-13 shows the isovalue curves $K_1/K_0=1.06$ in the $S/\min(D_1,D_2)$ $H/\min(D_1,D_2)$ axes, calculated for three combinations of two 20 degrees tilted flaws, as shown in Figure 5-15: $2D_2 = 2D_1$, $2D_2 = 0.5 \cdot 2D_1$ and $2D_2 = 0.25 \cdot 2D_1$. The reference flaw size is $2D_1 = 20\text{mm}$.

The same statement can be done as for the 90 degrees tilted flaws about the similarity of the areas of interaction in the $S/\min(D_1,D_2)$ $H/\min(D_1,D_2)$ axes.

Consequently, Figure 5-13 demonstrates that the generalization of the criteria on the basis of the minimum flaw size is also relevant for quasi-laminar flaws.

Moreover, it has to be noticed that the area of interaction in the $S/\min(D_1,D_2)$ $H/\min(D_1,D_2)$ axes slightly decreases when the second flaw is smaller than ...
the reference one. Therefore, considering two same flaw sizes for the determination of the proximity criteria is also conservative.

For the 2D approach, those last results validate the use for calculations of the two following hypotheses:

**Hypothesis No. 1:**

*The interaction between flaws can be calculated with an arbitrary reference flaw size (2D=20mm) if the distance between flaws is made dimensionless by dividing by the flaw size.*

**Hypothesis No. 2:**

*The criteria in order to reach 6% interaction between quasi-laminar flaws are calculated with two flaws of the same size and generalized on the basis of the minimum flaw size.*

5.3.2.2. Influence of the flaw tilt

As mentioned above the flaws most susceptible to initiate fracture are those with highest SIF i.e. for a given flaw size, the most tilted flaws are the most susceptible to initiate fracture. Indeed, the more tilted the flaw is, the higher the SIF. By definition, the largest tilt for quasi-laminar flaws is 20 degrees relative to the pressure retaining surface. The most relevant choice for the reference flaw ($K_{eq0}$) is thus a 20 degrees tilted flaw for the determination of the proximity rules.

Moreover, a sensitivity study has been performed to determine the influence of relative inclination between the flaws: the tilt of the first flaw (reference flaw) is equal to 20 degrees and the tilt of the second one varies from -20 degrees to 20 degrees. The results of this analysis (the configuration according to Figure 5-15 is $H=0, S=2D$) are presented in Figure 5-14.

Figure 5-14 shows that the maximum interaction is reached when the tilts of the flaws are the same. The proximity rules are therefore computed by considering two flaws with a tilt of 20 degrees.
For the 2D approach, those last results validate the use for calculations of the two following hypotheses:

**Hypothesis No. 3:**

*The flaw interaction between quasi-laminar flaws is calculated considering the reference flaw tilted with $\alpha_1 = 20^\circ$*

**Hypothesis No. 4:**

*The inclination $\alpha_2$ of the second flaw is considered to be the same as the inclination $\alpha_1$ of the first flaw*

### 5.3.2.3. Extension of the criteria in the third dimension

In this first assessment of the proximity rules for quasi-laminar flaws, the proximity limits are calculated on the basis of 2D XFEM analyses. Therefore, the 3D biaxially loaded problem of Figure 5-4 degenerates, in 2D, into the uniaxially loaded problem shown in Figure 5-15.
Figure 5-15 Multiple Quasi-Laminar Flaws: 2D distance parameters between flaws and relevant dimensions

Figure 5-4 shows that, in 2D, among the two distances $S_1$ and $S_2$ along the principal stresses $\sigma_1$ and $\sigma_2$ only one is still defined, let $S_1$ (= $S$ in Figure 5-15). Therefore, the 2D calculations will lead to proximity criteria on $H$ and $S$ but not on the second distance parallel to the second principal stress ($S_2$). In order to define this third criterion, the criterion on $S_2$ is assumed to be the same as the one on $S$ (= $S_1$).

This assumption is consistent with the ASME Code Section XI approach which does not specify difference between $\sigma_1$ and $\sigma_2$ in terms of direction of load as illustrated in Figure 5-2 and in Figure 5-4.

The criterion related to $H$ is called the alignment criterion and the criterion related to $S$ (or $S_1$ and $S_2$ in 3D) is called the combination criterion.

That justifies the following hypothesis:
Hypothesis No. 5:

The combination criterion on $S_2$ is assumed to be the same as the one on $S_1$ ($= S$).

5.3.3. Computations and Results

5.3.3.1. Model description

The 2D configuration considered for the calculations as well as the material properties and the boundary conditions are presented in Figure 5-16. A highly refined 2D XFEM model has been used for calculations. It is presented in Figure 5-17.

![Diagram](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
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</tr>
<tr>
<td>Tensile stress $\sigma$ (MPa)</td>
<td>100</td>
</tr>
<tr>
<td>$2D$ (mm)</td>
<td>20</td>
</tr>
<tr>
<td>$LX=LY$ (mm)</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 5-16 2D configuration considered for computations

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5.3.3.2. Interaction factor calculations

The calculations of the interaction factor, $\xi$ defined in Eq. (5-4), have been performed by considering first a computational step of 0.5 and a range from 0 to 2 for $H/D$ and $S/D$. Afterwards, in order to better catch the limit value on $H/D$ and $S/D$ corresponding to $\xi = 1.06$, a more refined computational step of 0.2, a range from 0 to 1 for $H/D$ and a range from 0 to 1.8 for $S/D$.

The results are presented in Table 5-1 to Table 5-3.

Table 5-1 Results matrix of the interaction factor $K_{eq,1}/K_{eq,0}$
(STEP 0.5 on $S/D$ and $H/D$)

<table>
<thead>
<tr>
<th>$H/D$</th>
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<th>1.5</th>
<th>2</th>
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<tbody>
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<td>1.1104</td>
<td>1.0659</td>
<td>1.0436</td>
<td></td>
</tr>
<tr>
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<td>1.0465</td>
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</tr>
<tr>
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<td>1.0262</td>
<td>1.0352</td>
<td>1.0334</td>
<td></td>
</tr>
<tr>
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<td>1.0106</td>
<td>1.0169</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.0063</td>
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<td>0.9906</td>
<td>0.9978</td>
<td>1.0046</td>
<td></td>
</tr>
</tbody>
</table>
Table 5-2 Results matrix of the interaction factor $K_{eq,1}/K_{eq,0}$
(STEP 0.2 on $S/D$ and $H/D$) 1/2

<table>
<thead>
<tr>
<th>$K_{eq,1}/K_{eq,0}$</th>
<th>$S/D$</th>
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<tbody>
<tr>
<td></td>
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<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
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<tr>
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<td>0.9999</td>
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<td>1.0184</td>
</tr>
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<td>1.0065</td>
<td>1.1129</td>
</tr>
</tbody>
</table>

Table 5-3 Results matrix of the interaction factor $K_{eq,1}/K_{eq,0}$
(STEP 0.2 on $S/D$ and $H/D$) 2/2

<table>
<thead>
<tr>
<th>$K_{eq,1}/K_{eq,0}$</th>
<th>$S/D$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
<td>1.8</td>
</tr>
<tr>
<td>$H/D$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
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</tr>
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<tr>
<td>0.4</td>
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<td>1.0860</td>
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<td>1.0362</td>
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</tbody>
</table>

On the basis of the results of these tables the alignment ($H$) and the combination ($S$) criteria can be derived.
5.3.3.3. Alignment criterion \((H)\)

To determine the alignment criterion specifying the limit value of \(H\) below which the two flaws have to be considered as aligned, the configurations of Table 5-1 to Table 5-3 are investigated by varying \(H/D\) while \(S/D\) remains constant. This process is repeated for all the \(S/D\) values of Table 5-1. By comparing the interaction factor with the limit of 1.06, a general rule for \(H\) limit is derived. These results are presented in Figure 5-18 and Figure 5-19.

The curves of Figure 5-18 and Figure 5-19 show that the interaction between the flaws is reached for \(H/D \leq 0.7\). An additional margin of 20% has been conservatively considered such that the alignment criterion can be written as follows, after generalization based on the minimum flaw size as justified in hypothesis No. 2:

\[
H \leq 0.85 \min(D_1, D_2) \quad (5-5)
\]

![Interaction factor \(K_{eq,1}/K_{eq,0}\) vs. \(H/D\)](image)

Figure 5-18 Interaction factor \(K_{eq,1}/K_{eq,0}\) vs. \(H/D\) (step 0.5 on \(S/D\) and \(H/D\))
5.3.3.4. Combination criterion (S)

To determine the combination criterion specifying the limit value of S below which the two flaws have to be considered as aligned, the configurations of Table 5-1 to Table 5-3 are investigated by varying S/D while H/D remains constant. This process is repeated for all the H/D values of Table 5-1. By comparing the interaction factor with the limit of 1.06, a general rule for S limit is derived. These results are presented in Figure 5-20 and Figure 5-21.

The curves of Figure 5-20 and Figure 5-21 show that the interaction between the flaws is reached for S/D ≤ 1.65. An additional margin of 20% has been conservatively considered such that the alignment criterion can be written as follows, after generalization based on the minimum flaw size as justified in Hypothesis No. 2:

\[ S \leq 2 \min(D_1, D_2) \]  \hspace{1cm} (5-6)
Figure 5-20 Interaction factor $K_{eq,1}/K_{eq,0}$ vs. $S/D$ (step 0.5 on $S/D$ and $H/D$)

Figure 5-21 Interaction factor $K_{eq,1}/K_{eq,0}$ vs. $S/D$ (step 0.2 on $S/D$ and $H/D$)
5.3.4. Multiple quasi-laminar flaws: generic expression of proximity rules and characterization

As explained in section 5.2.3 Flaw characterization and relevant dimensions and shown in Figure 5-4, in 3D, each flaw is associated to two diagonals (instead of one diagonal in 2D): the diagonals of the faces of the box normal to the principal stresses $\sigma_1$ and $\sigma_2$.

Since the 2D calculations leading to Eq. (5-5) and Eq. (5-6) have been performed in plane strain, the flaw length in the out-of-plane dimension is implicitly considered as infinite and thus the second diagonals as well. Therefore, when dealing with finite 3D boxes, the reference diagonal to consider in conjunction with Eq. (5-5) and Eq. (5-6) is always the smallest one for each box.

In addition, as justified in Hypothesis No. 5, the combination criterion on $S_2$ is assumed to be the same as the one on $S_1$ ($= S$).

As a consequence, when dealing with two 3D boxes as shown in Figure 5-4, Eq. (5-5) and Eq. (5-6) can be extended as follows:

$$H \leq 0.85 \min(\min(D_{11}, D_{21}), \min(D_{12}, D_{22}))$$  \hspace{0.5cm} (5-7)

$$S_1 \leq 2 \min(\min(D_{11}, D_{21}), \min(D_{12}, D_{22}))$$ \hspace{0.5cm} (5-8)

$$S_2 \leq 2 \min(\min(D_{11}, D_{21}), \min(D_{12}, D_{22}))$$ \hspace{0.5cm} (5-9)

If multiple discontinuous quasi-laminar flaws exist, each flaw shall be evaluated for its interaction with each adjacent flaw on an individual flaw basis, using the dimensions of the 3D box bounding the original flaw.

Finally, the generic process of the 3D extension of the proximity criteria can be expressed as follows.

Multiple discontinuous quasi-laminar flaws shall be combined into a single flaw if all three following proximity criteria are met, as specified in Figure 5-22:
\[ H \leq 0.85 \min(D_{11}, D_{12}, D_{21}, D_{22}) \]  
\[ S_1 \leq 2 \min(D_{11}, D_{12}, D_{21}, D_{22}) \]  
\[ S_2 \leq 2 \min(D_{11}, D_{12}, D_{21}, D_{22}) \]

where \(2D_{ij}\) denotes the diagonal of the face of box \(j\) normal to the principal stress \(\sigma_i\), as depicted in Figure 5-4, and the distances between the boxes bounding the flaws are denoted \(S_1\), \(S_2\) (along the direction of \(\sigma_1\) and \(\sigma_2\), respectively), and \(H\) (along the through wall direction), as shown in Figure 5-4.

![Diagram of proximity rules for multiple quasi-laminar flaws](image)

If the boxes are partially or totally overlapping in a direction, the proximity criterion in that direction is considered as met, as shown in Figure 5-23.
Flaw interaction within a group containing a greater number of individual quasi-laminar flaws than shown in Figure 5-4 shall be governed by the same criteria as above on $S_1$, $S_2$ or $H$. However, in all cases, the initial characterization of flaw interactions shall not require a recharacterization even if the bounding box of the combined flaw increases $D_{ij}$ values or reduces the separation distances $S_1$, $S_2$ or $H$.

5.3.4.1. Combined flaw: characterization and relevant dimensions

The combined flaw is sized by the minimum bounding box that contains the individual boxes meeting the proximity rules defined above in Eq. (5-10) to Eq. (5-12), as shown in Figure 5-24.
5.3.4.2. Acceptance Standards

For the purpose of defining the characteristics of the flaw to be used in conjunction with the allowable flaw size tables of the acceptance standards of Articles IWB-3500 and IWC-3500 of the ASME Code Section XI, the bounding box defined in Figure 5-24 for a combined flaw or in Figure 5-25 for a separate flaw has to be considered.

The bounding box has to be resolved into two rectangular planar flaws corresponding to the faces of the box normal to the principal stresses. The dimensions of these planar flaws are defined in Figure 5-24 for a combined flaw and in Figure 5-25 for a separate flaw. In these figures, $S$ is the distance from the box to the nearest surface of the component, $\ell$ is the side of the rectangle parallel to the inside pressure retaining surface of the component ($\ell_1$ or $\ell_2$) and $2d$ is the side of the rectangle normal to the inside pressure retaining surface of the component.
These characterization rules are presently implemented in the Code Case N-848 which is now part of the ASME Code Section XI [35].

![Diagram](image)

**Figure 5-25 Bounding box for separate flaw: relevant dimensions for IWB&IWC-3500 Acceptance Standards of ASME Code Section XI [21]**

### 5.3.5. Demonstration of the conservatism of the 2D approach

The development of the proximity criteria is based on 2D calculations. Since they are in reality applied to 3D flaws, the conservatism of this 2D approach has to be demonstrated.

In order to quantify this conservatism, the interaction factors have been computed considering 3D circular flaws, illustrated in Figure 5-26 and Figure 5-27, instead of 2D linear flaws in plane strain.

As a consequence, it has to be noted that, since we deal with circular flaws, the point of the reference circle where $K_{eq,0}$ and $K_{eq,1}$ are taken along the circumference for the calculation of the interaction factor $\xi$ is less obvious than for linear flaws as considered in 2D. Nevertheless, it can be said that, consistently with the aforementioned discussions led in the 2D approach, the
point taken for the calculation of $\xi$ is the one with the highest SIF as it is the most susceptible to initiate brittle fracture. The selection of this point is fully detailed in the section 5.4.1.2 Interaction factor.

5.3.5.1. Model description

The 2D XFEM model has been extruded to a 3D model with a thickness of $8D$ ($2D$ = diameter of the circles). The centers of the two circular flaws are in the middle plane of this model. This 3D configuration considered for the calculations as well as the material properties and the boundary conditions are presented in Figure 5-26.

A 3D XFEM model has been carried out and is presented in Figure 5-27.

![Figure 5-26 3D configuration considered for computations](image)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus $E$ (MPa)</td>
<td>193100</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Tensile stress $\sigma$ (MPa)</td>
<td>100</td>
</tr>
<tr>
<td>$2D$ (mm)</td>
<td>20</td>
</tr>
<tr>
<td>LX=LY (mm)</td>
<td>200</td>
</tr>
<tr>
<td>LZ (mm)</td>
<td>80</td>
</tr>
</tbody>
</table>

Figure 5-26 3D configuration considered for computations
5.3.5.2. Interaction factor calculations

The different values considered for $H/D$ and $S/D$ for the computations of the interaction factor $K_{eq,1}/K_{eq,0}$ are 0, 0.5, 1, 1.5. For all the combinations of $H/D$ and $S/D$, the interaction factors have been calculated (except, $H/D = S/D = 0$). The results are presented in Table 5-4.

<table>
<thead>
<tr>
<th>$K_{eq,1}/K_{eq,0}$</th>
<th>$S/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>$H/D$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>/</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0048</td>
</tr>
<tr>
<td>1.0</td>
<td>0.9891</td>
</tr>
<tr>
<td>1.5</td>
<td>0.9930</td>
</tr>
</tbody>
</table>

On the basis of the results of Table 5-4 the proximity criteria according to $H/D$ and $S/D$ can be determined and compared relative to those ones of the 2D computations. As a reminder, the proximity criteria correspond to an interaction factor $\xi$ of 1.06 or more.
5.3.5.3. Alignment criterion \((H)\)

The results of the computations of the interaction factor according to \(H/D\) for different values of \(S/D\) are given in Figure 5-28.

The curves of Figure 5-28 show that the interaction between the flaws is reached in 3D for \(H \leq 0.25D\). Since in 2D the area of interaction is reached for \(H \leq 0.7D\), these results show that the 2D approach is overly conservative for the alignment criterion \(H\).

5.3.5.4. Combination criterion \((S)\)

The results of the computations of the interaction factor according to \(S/D\) for different values of \(H/D\) are given in Figure 5-29.

The curves of Figure 5-29 show that the interaction between the flaws is reached in 3D for \(S \leq 0.7D\). Since in 2D the area of interaction is reached for \(S \leq 1.65D\), these results show that the 2D approach is overly conservative for the combination criterion \(S\).
5.3.5.5. Findings of 3D analyses on circles

Through the 3D XFEM calculations led on circles, it appears that an interaction of 6% is reached when:

\[ H \leq 0.25D \quad \text{and} \quad S \leq 0.7D \]  \hspace{1cm} (5-13)

whereas the 2D XFEM calculations led on linear flaws in plane strain have led to the following limits to reach an interaction of 6%:

\[ H \leq 0.7D \quad \text{and} \quad S \leq 1.65D \]  \hspace{1cm} (5-14)

That clearly demonstrates the high conservatism of the proximity rules derived from 2D approach.
5.4. ALTERNATIVE CHARACTERIZATION RULES FOR QUASI-LAMINAR FLAWS BASED ON 3D XFEM CALCULATIONS

As highlighted hereinabove through the 3D interaction analyses led on circles, the 2D approach, although better suited to quasi-laminar flaws, is highly conservative. The basic calculations performed on circles, tend therefore to demonstrate that a 3D approach would lead to more realistic proximity rules.

However, in order to make the results more generic and hence more robust, not only circles may be considered. Therefore, the 3D flaw interaction calculations are led on ellipses which are the generic expression of planar flaws.

The objective of this section is to present the 3D approach used to derive more realistic, but still conservative, proximity rules for flaws close to each other which are suitable for quasi-laminar flaws. Then, the generic way of proceeding with flaws to be combined (i.e. the characterization rules) is explained.

The conservatisms are demonstrated through the demonstration of each hypothesis done in this approach.

5.4.1. Flaw interaction

5.4.1.1. Stress Intensity Factor

As mentioned above, the proximity rules are based on the analysis of the Stress Intensity Factor (SIF) interaction between two flaws. For the 3D calculations of the flaw interaction between quasi-laminar flaws, the SIF to consider is given by Eq. (5-15) combining the three fracture modes:

\[ K_{eq} = \sqrt{K_I^2 + K_{II}^2 + \frac{1}{1-\nu} K_{III}^2} \]  

(5-15)
5.4.1.2. Interaction factor of elliptical flaws

When dealing with elliptical flaws, the SIF $K_{eq}$ is calculated at each point $t$ of the crack front (see Figure 5-30). Therefore, the interaction factor $\xi$ due to the presence of a second flaw can be assessed at each point $t$ of the crack front:

$$\xi(t) = \frac{K_{eq,1}(t)}{K_{eq,0}(t)}$$

(5-16)

However, the most relevant point is the one with the highest SIF as it is the most susceptible to initiate brittle fracture. The interaction factor thus becomes:

$$\xi = \xi(t^*) \quad where \quad t^* | K_{eq,1}(t^*) = \max_t \left( K_{eq,1}(t) \right)$$

(5-17)

![Figure 5-30 Stress Intensity Factor of reference flaw alone (left) and in presence of a second flaw (right)](image)

As for the 2D calculations, the criterion of 6% increase in SIF is applied for determining the proximity criteria with quasi-laminar flaws using 3D calculations.

In other words, two quasi-laminar flaws have to be combined if:

$$\xi(t) = \frac{K_{eq,1}(t)}{K_{eq,0}(t)} \geq 1.06$$

(5-18)
5.4.2. Hypotheses

Since the calculations are carried out in 3D, numerous parameters are involved such as the flaw size, the flaw aspect ratio (elliptical shape of the flaw), the flaw inclinations (two possible inclinations), the relative position of the flaws, the loading (uniaxial or biaxial).

Therefore a reference case, fixing and linking these parameters, has to be defined. This reference case is established through several hypotheses. The conservatisms related to the use of this reference case for proximity rules calculations has to be demonstrated. This is done in section 5.4.5 Justification of hypotheses.

5.4.2.1. Influence of the flaw size

As far as the influence of the flaw size on the flaw interaction is concerned, the principles mentioned in the 2D calculations section 5.3.2.1 Influence of the flaw size are independent of the dimension. Therefore both hypotheses related to this section remain valid and are used for the 3D calculations:

**Hypothesis No. 1:**

*The interaction between flaws can be calculated with an arbitrary reference flaw size (2D=20mm) if the distance between flaws is made dimensionless by dividing by the flaw size.*

**Hypothesis No. 2:**

*The criteria in order to reach 6% interaction between quasi-laminar flaws are calculated with two flaws of the same size and generalized on the basis of the minimum flaw size.*

The conservatism of these hypotheses will be demonstrated further in this Chapter, as well as all the conservatisms related to the following hypotheses.
5.4.2.2. Influence of the flaw aspect ratio

When dealing with 3D elliptical flaws, another degree of freedom that has to be covered is the flaw aspect ratio, quantified by $D_{21}/D_{11}$, as presented in Figure 5-31. While $D_{11}$ is set to 10mm (Hypothesis No. 1), the ratio $D_{21}/D_{11}$ is considered as a parameter for the calculation of interaction between flaws.

![Figure 5-31 Illustration of diagonals $D_{11}$, $D_{12}$, $D_{21}$ and $D_{22}$ for hypotheses on flaw aspect ratio](image)

**Hypothesis No. 3:**

The criteria in order to reach 6% interaction between quasi-laminar elliptical flaws are calculated considering that $D_{21} = D_{11}$ and generalized on the basis of the maximum flaw diagonals.

5.4.2.3. Influence of the tilt

As mentioned before the flaws most susceptible to initiate fracture are those with highest SIF i.e. for a given flaw size, the most tilted flaws are the most susceptible to initiate fracture. Indeed, the more tilted the flaw is, the higher the SIF. By definition, the largest tilt for quasi-laminar flaws is 20 degrees relative to the pressure retaining surface. The most relevant choice for the reference flaw ($K_{eq,0}$) is thus a 20 degrees tilted flaw for the determination of...
the proximity rules since, the largest tilt for quasi-laminar flaws is 20 degrees relative to the retaining surface of the component.

However, the elliptical flaw included in the box can have two different inclinations as shown in Figure 5-32: one around Y-axis (β) and one around X-axis (θ). While the main inclination of the reference flaw β₁ is set to 20°, the angles β₂, θ₁ and θ₂ are considered as parameters (up to 20° by definition of quasi-laminar flaws) for the calculation of interaction between flaws.

**Hypothesis No. 4:**
*The flaw interaction between quasi-laminar elliptical flaws is calculated considering the reference flaw tilted with β₁ = 20° and θ₁ = 0°.*

![Inclination of β* around Y-axis and Inclination of θ* around X-axis](image)

**Figure 5-32 Definition of inclinations of elliptical flaws**

**Hypothesis No. 5:**
*The inclinations β₂ and θ₂ of the second flaw are considered to be the same as the inclinations β₁ and θ₁ of the first flaw.*
5.4.2.4. Influence of the loading

As calculations are performed in linear elastic conditions, the SIF is directly proportional to the stress field in the vicinity of the flaw so that the value of the interaction factor \( \xi \) is independent from the absolute value of the loading. Therefore, an arbitrary value of 100MPa has been considered.

On the other hand, as illustrated in Figure 5-4, the flaws can be subject to a bi-axial loading composed by the principal stresses \( \sigma_1 \) and \( \sigma_2 \). While \( \sigma_1 \) is set at 100MPa, the load \( \sigma_2 \) is considered as a parameter for the calculation of interaction between flaws.

**Hypothesis No. 6:**

*The flaw interaction between quasi-laminar elliptical flaws is calculated considering a uniaxial loading \( \sigma_1 \) (i.e. \( \sigma_2 = 0 \)).*

5.4.2.5. Influence of the distance \( S_2 \)

As illustrated in Figure 5-4, the distance between two flaws is represented by the three parameters \( H, S_1 \) and \( S_2 \). These parameters are varying to cover the whole interaction domain.

**Hypothesis No. 7**

*The flaw interaction between quasi-laminar elliptical flaws is calculated considering that the centers of flaws are aligned in \( S_2 \).*

Justification of Hypotheses No. 1 to No. 7 and their related conservatisms are demonstrated in section 5.4.5 *Justification of hypotheses.*

5.4.3. Computation and Results

The objective of this paragraph is to explain the methodology used to derive the proximity rules for quasi-laminar flaws.
5.4.3.1. Model description

5.4.3.1.1. MODEL GEOMETRY

The XFEM calculations are performed using 3D models.

The flaws are considered to be in an infinite domain so that the model consists of a cube illustrated in Figure 5-33 with dimensions \( L_x = L_y = L_z = 10 \times 2D \) which is large compared to the size of the flaws (\( 2D = 20 \text{mm} \)). Consequently, the model is large enough in such a way that the stress distribution in the area of interest is not influenced by applying the boundary conditions at the extremities of the model.

![Figure 5-33 Infinite domain modelled by a cube of dimensions \( L_x, L_y \) and \( L_z \)](image_url)

5.4.3.1.2. FLAW DEFINITION

Following Hypotheses No. 1 to No. 7, the flaws modelled to calculate the interaction factors are characterized as follows (see Figure 5-34):

- Flaw 1 (reference flaw)
  - \( D_{11} = D_{21} = D = 10 \text{mm} \)
  - \( \beta_1 = 20^\circ \)
  - \( \theta_1 = 0^\circ \)
- Flaw 2
  - $D_{12} = D_{11}$
  - $D_{22} = D_{21}$
  - $\beta_2 = \beta_1$
  - $\theta_2 = \theta_1$

- $H$ and $S_1$ vary in the interaction domain

- $S_2$ is such that the centres of the flaws are aligned in $Y$

---

Figure 5-34 Two flaws tilted with an angle $\beta$ around $Y$ axis (XZ view - left) and with centres aligned in $Y$ (XY view – right)
5.4.3.1.3. **BOUNDARY CONDITIONS**

As mentioned in Hypothesis No. 6, the flaws are subjected to a uniaxial loading $\sigma_1$ in the X-direction. Therefore, a constant tensile stress is applied on the upper face of the cube.

The other boundary conditions are illustrated in Figure 5-35.

![Boundary conditions of the 3D XFEM reference model](image)

**Figure 5-35** Boundary conditions of the 3D XFEM reference model

5.4.3.1.4. **MESH FOR XFEM CALCULATIONS**

The mesh has been refined in the vicinity of the crack front, following best practices for XFEM calculations i.e., with a mesh size proportional to the flaw size and decreasing with distance to the crack front. The mesh is illustrated in Figure 5-36 and Figure 5-37 for a flaw configuration corresponding to $(H/D, S_1/D) = (0.21, 0.56)$. 
5.4.3.2. Methodology

5.4.3.2.1. INTERACTION DOMAIN

The first step to determine the proximity rules is to calculate the interaction domain with respect to the distances between flaws $H$ and $S_1$. The interaction domain corresponds to the values $(H, S_1)$ for which the interaction factor $\xi$ is higher than 1.06:

$$(H, S_1) \text{ such that } \xi(H, S_1) = \frac{K_{eq1}(H,S_1)}{K_{eq,0}} \geq 1.06 \quad (5-19)$$

The variables $H$ and $S_1$ are made dimensionless by dividing by the flaw size $D$, as mentioned in Hypothesis No. 1.
In order to draw the boundary of the interaction domain which corresponds to a 6% SIF interaction between flaws, two series of calculations are foreseen, as illustrated in Figure 5-38.

- The first one includes a coarse discretization of the \((H/D, S_1/D)\) domain in order to localize the area of 6% interaction. The step between two calculations is \(H/D = S_1/D = 0.2\) (left hand side of Figure 5-38).

- Afterwards, a refined discretization in the area of 6% interaction allows better describing the boundary of the interaction domain. The step between two calculations is \(H/D = S_1/D = 0.05\) in this area (right hand side of Figure 5-38).

![Figure 5-38 Determination of the boundary of the interaction domain (\(\xi = 1.06\) interaction curve)](image)

### 5.4.3.2.2. PROXIMITY CRITERIA

Once the boundary curve of the interaction domain is known, it has to be translated into a mathematical expression to be applied it to actual flaws. This translation includes two conservatisms:

- A rectangle based on the highest \(H/D\) and highest \(S_1/D\) values of the 6% interaction curve is drawn (dash green line in Figure 5-39);

- An additional margin of 20% is considered to determine the threshold parameters \(k_h\) and \(k_s\) (solid green line in Figure 5-39).
On the basis of these definitions of the threshold parameters, the proximity criteria determining if two flaws have to be grouped are expressed as one alignment criterion:

$$H \leq k_h D$$

and one combination criterion:

$$S_1 \leq k_s D$$

The final expression of proximity criteria based on the previously defined interaction domain requires some precisions to be applicable to actual flaws. Indeed, as explained in Hypothesis No. 2, the flaw size $D$ corresponds to the minimum flaw size i.e.:

$$D = \min (D_1, D_2)$$

As explained in Hypothesis No. 3, the size of a flaw is the maximum size of its diagonals i.e.:

$$D_1 = \max (D_{11}, D_{21})$$

$$D_2 = \max (D_{12}, D_{22})$$
Moreover, the interaction criteria have been defined considering $S_1$ as a varying parameter, a uniaxial load $\sigma_1$ and centres of flaws aligned in $S_2$. Considering $S_2$ as a varying parameter, a uniaxial load $\sigma_2$ and centres of flaws aligned in $S_1$ would have led, by symmetry, to the same results. Therefore, the interaction criterion on $S_1/D$ can be applied to $S_2/D$ provided that a uniaxial load maximizes the interaction (Hypothesis No. 6).

Finally, the proximity rules, based on dimensions of Figure 5-4, can be expressed as follows:

Two quasi-laminar flaws shall be combined if all three of the following proximity criteria are met:

\[
H \leq k_h \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \tag{5-25}
\]

\[
S_1 \leq k_s \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \tag{5-26}
\]

\[
S_2 \leq k_s \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \tag{5-27}
\]

5.4.3.3. Alignment and Combination criteria

Results in terms of interaction domain are presented in Figure 5-40, dots representing the calculation points. The coarse and refined discretizations provide the same results in terms of interaction curve.
The limit values of the domain correspond to \((H/D)_{\text{max}} = 0.28\) and \((S_1/D)_{\text{max}} = 0.61\). Therefore, adding a 20% margin on these results, the proximity criteria determining whether two flaws have to be grouped are expressed through the alignment criterion:

\[
H/D \leq 0.34
\]  \hspace{1cm} (5-28)

and the combination criterion:

\[
S_1/D \leq 0.73
\]  \hspace{1cm} (5-29)

As it has been done for the 2D calculations, another way to illustrate the results is to plot the interaction factor \(\xi\) as a function of \(H/D\) considering \(S_1/D\) as constant (see Figure 5-41) and as a function of \(S_1/D\) considering \(H/D\) as constant (see Figure 5-42).

In the following figures, it is shown that considering a 20% margin corresponds to a flaw interaction of only about 4.5% instead of 6%.

Figure 5-41 Interaction factor \(\xi\) with different constant values of \(H/D\)
Finally, two quasi-laminar flaws shall be grouped if all three of the following proximity criteria are met:

\[
H \leq 0.34 \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \quad \text{(5-30)}
\]

\[
S_1 \leq 0.73 \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \quad \text{(5-31)}
\]

\[
S_2 \leq 0.73 \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \quad \text{(5-32)}
\]

These results have been reached by computing interaction on 3D elliptical flaws. They can be compared to the very conservative 6% interaction curve obtained through calculations of 2D infinite flaws presented in Figure 5-13. This comparison is shown in Figure 5-43 and Table 5-5.
This comparison clearly highlights the conservatisms of the proximity rules obtained using 2D XFEM calculations, especially since the section 5.4.5 Justification of hypotheses will demonstrate that the 3D results, although more realistic, remains still conservative.

5.4.4. Application to multiple quasi-laminar flaws and characterization

The application of proximity rules to multiple quasi-laminar flaws follows the same aforementioned principles i.e., if multiple discontinuous quasi-laminar flaws exist, each flaw shall be evaluated for its interaction with each
adjacent flaw on an individual flaw basis, using the dimensions of the 3D box bounding the original flaw.

Finally, the generic process of the 3D proximity rules can be expressed as follows:

Multiple discontinuous quasi-laminar flaws shall be combined into a single flaw if Eq. (5-30) to Eq. (5-32) are met. Those proximity rules are specified in Figure 5-44 where $2D_{ij}$ denotes the diagonal of the face of box $j$ normal to the principal stress $\sigma_i$ and the distances between the boxes bounding the flaws are denoted $S_1$, $S_2$ (along the direction of $\sigma_1$ and $\sigma_2$, respectively), and $H$ (along the through wall direction), as shown in Figure 5-4.

![Figure 5-44 Proximity rules for multiple quasi laminar flaws (3D calculations) – Projection onto principal stress planes](image)
If the boxes are partially or totally overlapping in a direction, the proximity criterion in that direction is considered as met, as shown in Figure 5-23.

Flaw interaction within a group containing a greater number of individual quasi-laminar flaws than shown in Figure 5-4 shall be governed by the same criteria as above on $S_1$, $S_2$ or $H$. However, in all cases, the initial characterization of flaw interactions shall not require a recharacterization even if the bounding box of the combined flaw increases $D_{ij}$ values or reduces the separation distances $S_1$, $S_2$ or $H$.

For the purpose of defining the characteristics of the flaw to be used in conjunction with the allowable flaw size tables of the acceptance standards of Articles IWB-3500 and IWC-3500 of the ASME Code Section XI, the bounding box defined in Figure 5-24 for a combined flaw or in Figure 5-25 for a separate flaw has to be considered. The bounding box has then to be resolved into two rectangular planar flaws corresponding to the faces of the box normal to the principal stresses.

5.4.5. Justification of hypotheses

The scope of this section is to validate and justify the hypotheses listed in the section 5.4.2 Hypotheses. This section also highlights the conservative approach followed when formulating each of these hypotheses.

As it has been shown in Figure 5-40 that the coarse and refined discretization of $H$ and $S$ led to the same results, the calculations supporting the justifications will be performed with the coarse discretization.

5.4.5.1. Hypothesis No. 1

The interaction between flaws can be calculated with an arbitrary reference flaw size ($2D=20mm$) if the distance between flaws is made dimensionless by dividing by the flaw size.
Although this hypothesis is trivial, it is demonstrated through a sensitivity study on the flaw size $D$. This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 Model description except that the relative position between flaws has been fixed to $(H/D; S_1/D) = (0; 0.6)$ and that three different values for $D$ have been considered i.e., 5mm, 10mm and 20mm.

The objective of the sensitivity study is to compare the flaw interaction calculated for all cases.

Results provided in Table 5-6 show that the flaw interaction does not vary with $D$. Therefore, the hypothesis of choosing an arbitrary reference flaw size ($2D=20mm$) and dividing the distance between flaws by the flaw size is valid.

<table>
<thead>
<tr>
<th>$D$</th>
<th>$D = 5mm$</th>
<th>$D = 10mm$</th>
<th>$D = 20mm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction factor $\xi$</td>
<td>1.056</td>
<td>1.057</td>
<td>1.057</td>
</tr>
</tbody>
</table>

5.4.5.2. Hypothesis No. 2

The criteria in order to reach 6% interaction between quasi-laminar elliptical flaws are calculated with two flaws of the same size and generalized on the basis of the minimum flaw size.

In order to demonstrate this hypothesis, a sensitivity study on the ratio $D_2/D_1$ between flaw sizes is performed, $D_1$ and $D_2$ being defined in Eq. (5-23) and Eq. (5-24).

This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 Model description except that three flaw size ratios $D_2/D_1$ have been considered: 1, 0.5 and 0.25.
The objective of the sensitivity study is to compare the interaction domains related to the different ratios $D_2/D_1$ and expressed as a function of the minimum flaw size.

Results presented in Figure 5-45 show that the reference case i.e. $D_1 = D_2$ bounds the cases where the second flaw is smaller than the reference flaw. Therefore, the hypothesis of calculating the flaw interaction with two flaws of the same size and generalized on the basis of the minimum flaw size is verified.

Moreover, applying criteria calculated on the basis of this hypothesis on two flaws with different sizes will lead to combine some flaws whose interaction is lower than 6%, which is conservative.

5.4.5.3. Hypothesis No. 3

The criteria in order to reach 6% interaction between quasi-laminar elliptical flaws are calculated considering that $D_{21} = D_{11}$ and generalized on the basis of the maximum flaw diagonals.

In order to demonstrate this hypothesis, a sensitivity study on the flaw aspect ratio $b/a$ is performed as depicted in Figure 5-46. The flaw aspect ratio is directly related to the ratio $D_{11}/D_{21}$.
This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 Model description except that the distance $H$ between flaws has been fixed to 2mm and that several flaw aspect ratios $b/a$ have been considered from 0.75 to 2.

The objective of the sensitivity study is to calculate the distance $S_1$ for which the flaw interaction is 6%, and then to assess the ratio:

$$k_s = S_1 / \min(\max(D_{11}, D_{21}), \max(D_{12}, D_{22}))$$

(5-33)

Results provided in Table 5-7 show that the distance $S_1$ for which the flaw interaction is 6% increases with growing aspect ratio. However, when dividing by the maximum of $D_{11}, D_{21}$, the maximum value of the ratio $k_s$ i.e., 0.58, is reached for $D_{21}/D_{11} = 1$.

Therefore, the hypothesis of using flaw configuration with $D_{21} = D_{11}$ to determine the interaction criteria and generalizing the criteria on the basis of the maximum flaw diagonals is verified.
Moreover, applying criteria calculated on the basis of this hypothesis on flaws with \( D_{21} \neq D_{11} \) will lead to combine some flaws whose interaction is lower than 6%, which is conservative.

### 5.4.5.4. Hypothesis No. 4

The flaw interaction between quasi-laminar elliptical flaws is calculated considering the reference flaw tilted with \( \beta_1 = 20^\circ \) and \( \theta_1 = 0^\circ \).

In order to demonstrate this hypothesis, a sensitivity study on the flaw inclination \( \theta_1 = \theta_2 = \theta \), defined in Figure 5-32, is performed.

This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 Model description except that three values for \( \theta \) have been considered: 0°, 10° and 20°.

The objective of the sensitivity study is to compare the interaction domains related to the different values of \( \theta \).

The results presented in Figure 5-47 show that the reference case (with \( \theta = 0^\circ \)) bounds the cases where \( \theta > 0^\circ \) in terms of flaw interaction.

---

**Table 5-7 Results of sensitivity study on flaw aspect ratio**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Cases 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{21} ) (mm)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Aspect ratio ( b/a ) (-)</td>
<td>0.75</td>
<td>0.94</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>( D_{11}/D_{21} ) (-)</td>
<td>0.82</td>
<td>1</td>
<td>1.06</td>
<td>1.54</td>
<td>2.03</td>
</tr>
<tr>
<td>( D_{11} ) (mm)</td>
<td>8.2</td>
<td>10</td>
<td>10.6</td>
<td>15.4</td>
<td>20.3</td>
</tr>
<tr>
<td>6% interaction reached for ( S_1 = ) (mm)</td>
<td>4.3</td>
<td>5.8</td>
<td>6.0</td>
<td>8.5</td>
<td>9.9</td>
</tr>
<tr>
<td>6% interaction reached for ( k_s = ) (-)</td>
<td>0.43</td>
<td>0.58</td>
<td>0.57</td>
<td>0.55</td>
<td>0.49</td>
</tr>
</tbody>
</table>
Therefore, the hypothesis of considering the reference flaw tilted with $\theta = 0^\circ$ is verified.

Moreover, applying criteria calculated on the basis of this hypothesis on flaws with $\theta > 0^\circ$ will lead to combine some flaws whose interaction is lower than 6%, which is conservative.

5.4.5.5. Hypothesis No. 5

The inclinations $\beta_2$ and $\theta_2$ of the second flaw is considered to be the same as the inclinations $\beta_1$ and $\theta_1$ of the reference flaw.

In order to demonstrate this hypothesis, a sensitivity study on the inclinations $\beta_2$ and $\theta_2$ of the second flaw is performed.

5.4.5.5.1. Sensitivity Study on $\beta_2$

This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 Model description except that the relative position between flaws has been fixed to $(H/D ; S_1/D) = (0 ; 0.6)$ and that several flaw inclinations $\beta_2$ have been considered from $-20^\circ$ to $+20^\circ$, as illustrated in Figure 5-48.
The objective of the sensitivity study is to compare the flaw interaction calculated for all cases.

In the domain of definition of quasi-laminar flaws i.e., oriented within 20° of plane parallel to the surface of the component, the results presented in Figure 5-49 show that the flaw interaction is maximum for $\beta_2 = 20^\circ$. Therefore, the hypothesis of using flaw configurations with $\beta_2 = \beta_1$ to determine the interaction criteria is justified.

![Configuration of flaws for sensitivity study on the angle $\beta_2$](image1)

![Results of sensitivity study on the angle $\beta_2$](image2)
Moreover, applying criteria calculated on the basis of this hypothesis on flaws with \( \beta_2 \neq \beta_1 \) will lead to combine some flaws whose interaction is lower than 6%, which is conservative.

5.4.5.6. **Sensitivity study on \( \theta_2 \)**

This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 *Model description* except that the relative position between flaws has been fixed to \((H/D ; S_1/D) = (0 ; 0.6)\) and that several flaw inclinations \( \theta_2 \) have been considered from 0° to 20°, as illustrated in Figure 5-50

![Figure 5-50 Configuration of flaws for sensitivity study on the angle \( \theta_2 \)](image)

The objective of the sensitivity study is to compare the flaw interaction calculated for all cases.

Results presented in Table 5-8 show that the flaw interaction does not vary with \( \theta_2 \). Therefore, the hypothesis of using flaw configurations with \( \theta_2 = \theta_1 \) to determine the interaction criteria is valid.
Table 5-8 Results of sensitivity study on $\theta_2$

<table>
<thead>
<tr>
<th></th>
<th>$\theta_2 = 0^\circ$</th>
<th>$\theta_2 = 10^\circ$</th>
<th>$\theta_2 = 20^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction factor $\xi$</td>
<td>1.057</td>
<td>1.057</td>
<td>1.057</td>
</tr>
</tbody>
</table>

5.4.5.7. Hypothesis No. 6

The flaw interaction between quasi-laminar elliptical flaws is calculated considering a uniaxial loading $\sigma_1$ (i.e. $\sigma_2 = 0$).

In order to demonstrate this hypothesis, a sensitivity study on the load ratio $\sigma_2/\sigma_1$ is performed.

This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 Model description except that an inclination $\theta = 20^\circ$ is considered in such a way that the fracture modes are also solicited by principal stress $\sigma_2$ and that a bi-axial loading with several load ratios $\sigma_2/\sigma_1$ has been considered. It has also to be noted that the boundary conditions of the model have been adapted to account for the principal stress $\sigma_2$, as shown in Figure 5-51. The flaws are subjected to a bi-axial loading $\sigma_1$ in the X-direction and $\sigma_2$ in the Y-direction.

![Figure 5-51] Boundary conditions of the 3D X-FEM model in case of bi-axial loading

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The objective of the sensitivity study is to compare the interaction domains related to the different values of load ratios $\sigma_2/\sigma_1$.

Results of the bi-axial loading calculations are presented in Figure 5-52. They show that the maximum interaction is reached when the load ratio $\sigma_2/\sigma_1$ is minimum i.e., when the loading is uniaxial.

Therefore, the hypothesis of considering a uniaxial loading is verified.

This sensitivity study related to the load ratio has been completed by another one to assess the impact of variations of angle $\theta$ with 2 different load ratios.

As illustrated in Figure 5-53 for a position $(H/D ; S_1/D) = (0 ; 0.6)$, the uniaxial case with no inclination in $\theta$ is once again the one which leads to the highest flaw interaction, which confirms the conservatism of selecting it as the reference case.
Moreover, applying criteria calculated on the basis of this hypothesis on flaws solicited with a biaxial loading ($\sigma_2/\sigma_1 > 0$) will lead to combine some flaws whose interaction is lower than 6%, which is conservative.

**5.4.5.8. Hypothesis No. 7**

*The flaw interaction between quasi-laminar elliptical flaws is calculated considering that the centres of flaws are aligned in $S_2$.*

In order to demonstrate this hypothesis, a sensitivity study on the distance $S_2$ is performed.

This sensitivity study is conducted with the same model and flaw definition as the reference case described in the section 5.4.3.1 *Model description* except that the centre of the second flaw move from its original position (centres aligned in $S_2$) in the Y-direction, as depicted in Figure 5-54.

The objective of the sensitivity study is to compare the interaction domains related to the different level of overlap.
The results presented in Figure 5-55 and Figure 5-56 clearly show that as soon as one deviates from the complete overlap position, the interaction rapidly decreases. Moreover, if there is no overlap between flaws ($S_2 \geq 0$), the interaction is lower than $\sim 2\%$ whatever $H$ and $S_1$ values i.e., lower than the limit value of 6% considered for flaw combination.
However, the combination rules are determined based on cases with complete overlap. Therefore, applying such rules on flaws with no overlap ($S_2 \geq 0$) will lead to combine some flaws whose interaction is much lower than 6%, which is very conservative.
5.4.5.9. Findings of the justification of hypotheses

Through the justification of the hypotheses leading to the design of the reference flaw configuration, it is demonstrated that the reference flaw configuration used to determine the 3D proximity criteria has been designed to maximize the flaw interaction, enveloping all possible flaw configurations.

In particular, the conservative hypotheses linked to the definition of the reference flaw configuration are two flaws of the same size, of the same inclination, included in boxes with the same aspect ratio, both tilted in only one direction, subjected to uniaxial loading and with centre aligned.

Of course, the actual flaws to which the proximity rules are applied never correspond exactly to the reference flaw configuration. Therefore, as soon as one deviates from any of these hypotheses, the flaw interaction will be overestimated.

This is illustrated in the next section considering a real-life application of the 3D proximity rules.

5.4.6. Demonstration of the conservatism of the 3D approach

The scope of this section is to illustrate the conservatism of the 3D proximity rules presented in Eq. (5-30) to Eq. (5-32). This is done through a so-called multi flaws analysis which is a detailed XFEM calculation of a group of flaws meeting the proximity criteria. For that purpose, the proximity criteria given in Eq. (5-30) to Eq. (5-32) are thus applied to a material including quasi-laminar flaws. Among the resulting groups of flaws, one group of 9 flaws is arbitrary selected for XFEM calculation and illustration of the conservatisms. According to the grouping process, each flaw in this group is considered to interact with at least one other flaw of the group i.e., each flaw in this group satisfies Eq. (5-30) to Eq. (5-32) with at least one other flaw of the group.
5.4.6.1. Model description

It is considered that the group of flaws is located in a cylindrical vessel with inner radius $R_{in}$ and outer radius $R_{out}$. Only a $20^\circ$ sector of this cylinder is considered in the XFEM model as illustrated in Figure 5-57. This sector is submitted to a unit pressure $p = 1MPa$ and to an axial stress corresponding to the pressure end cap effect $\sigma_{ap} = p R_{in}^2 / (R_{out}^2 - R_{in}^2)$. This pressure leads thus to a biaxially loaded configuration.

Each quasi-laminar flaw is bounded by the minimum bounding box that fully contains the area of the flaw, according to Figure 5-4. For SIF calculation purpose, the flaws composing the selected group are geometrically described by means of an elliptical shape. This ellipse has the largest size and angle to remain included in the flaw box (see zoom in Figure 5-57). The equivalent SIF $K_{eq}$, given in Eq. (5-1), is calculated at each point of the crack front.

![Figure 5-57 XFEM model of the multiple-flaw analysis](image)

The mesh used for the XFEM analysis is more refined in the vicinity of the crack fronts, with a mesh size proportional to the flaw size and decreasing with distance to the crack front, as shown in Figure 5-58.
5.4.6.2. Computations and results

Figure 5-59 illustrates the three models that have been considered to highlight the conservatism of the proximity rules:

(a) The multi-flaws model includes all the flaws belonging to the selected group. Therefore, the interaction between flaws is taken into account in the SIF evaluation. The maximum SIF $K_{eq,1,MAX} = 13.74 \text{ MPa/mm}$ is found on the crack front of flaw “x”. According to the definition of the interaction factor $\xi$ given in section 5.4.1.2 Interaction factor of elliptical flaws, $\xi$ has therefore to be assessed at this point of this flaw “x”.

(b) The single flaw model is calculated to assess the SIF of flaw “x” alone, i.e. without considering the interaction with other flaws. At the same place on the crack front where $K_{eq,1,MAX}$ was found, the SIF is $K_{eq,0} = 13.59 \text{ MPa/mm}$. Consequently, one can assess the interaction factor $\xi = \frac{K_{eq,1,MAX}}{K_{eq,0}} = 1.011$.

According to the proximity rules definition, the interaction between the two reference flaws inside the interaction domain is higher than 6%. In the present example, it is shown that interaction between actual flaws is
around 1%. Therefore, although the group includes more than two flaws, the interaction is still conservatively considered by the proximity rules.

(c) The last model consists of the resulting combined flaw which is the elliptical flaw with largest size and angle included in the minimum bounding box that fully contains all the interacting flaws. The maximum SIF of the resulting flaw $K_{eq, group, MAX} = 30.32 \text{ MPa}\sqrt{\text{mm}}$ which is more than twice that of the most critical flaw inside the group calculated in (a). This result shows that replacing interacting flaws with a combined flaw induces a conservative assessment of the maximum SIF.

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![Diagram](image)

Figure 5-59 SIF from XFEM calculations for unit pressure load
5.5. SUMMARY AND CONCLUSIONS

In the frame of the fitness-for-service demonstration of both Doel 3 and Tihange 2 RPVs, alternative characterization rules for quasi-laminar flaws were developed.

Firstly, 2D XFEM calculations were used. They led to proximity rules given in Eq. (5-10) to Eq. (5-12) that are presently implemented in the Code Case N-848 [35] which is now part of the ASME Code Section XI.

However, those 2D XFEM proximity rules, although better suited to quasi-laminar flaws, were shown highly conservative.

As a consequence, 3D XFEM calculations were then used to develop more realistic proximity rules for quasi-laminar flaws. The result of this more realistic approach led to the proximity rules given in Eq. (5-30) to Eq. (5-32) as well as to a revision of the Code Case N-848 presented to the ASME Committee in November 2015.

For the 3D calculations, a reference case has been designed to calculate the flaw interaction as a function of the distance between flaws. Appropriate sensitivity analyses were performed to validate the hypotheses considered when defining the reference case. They cover the parameters that potentially affect interaction between elliptical flaws: flaw size, flaw aspect ratio, flaw inclination, load ratio and relative position between flaws. Moreover, these justifications also highlight the multiple levels of conservatism of this methodology.

For validation purpose, a 3D XFEM multi-flaws analysis involving multiple flaws has been performed. Results emphasized that flaw interaction was conservatively considered by the proximity rules, even in groups involving more than two flaws. Moreover, it has been shown that replacing interacting flaws with a combined flaw allows a conservative assessment of the maximum SIF.
6.1. INTRODUCTION

Delamination or laminar flaws are sometimes detected during manufacturing and/or plant operations. They occur by inclusions such as Manganese Sulphur (MnS), hydrogen flakes, etc., in steels at manufacturing. A laminar flaw is a subsurface flaw considered as parallel to the rolling direction of the component, where the applied stress is typically parallel to the rolling direction. The definition of the laminar flaw is provided only in the ASME Boiler & Pressure Vessel Code Section XI [21] and API 579/ASME FFS-1 Code [31]. Besides, these two Codes provide different characterization rules (grouping criteria and sizing) for multiple laminar flaws.

Nevertheless, unlike the combination criteria for non-laminar flaws which are based on mechanical interaction analyses, the combination criteria for laminar flaws provided in both the Codes [21] and [31] are based either on UT capabilities in the ’70s as in [21] or on engineering judgment as in [31]. There is a lack of consistence between the treatments of both kinds of flaws i.e., non-laminar vs. laminar.

Therefore a harmonization of the concepts, which the grouping criteria are based on, should be needed. In other word, as the mechanical interaction between the flaws is, by definition, the most relevant concept to combine the flaws, it should also be applied to laminar flaws.
In parallel, as mentioned in the previous Chapter, a large number of quasi-laminar flaw indications were detected in nuclear power reactor vessels [61], [66]. The observed indications were caused by hydrogen flaking induced during the manufacturing process. The maximum tilt of quasi-laminar flaws with respect to the surface of the component is slightly larger than the one of laminar flaws i.e., 20 degrees instead of 10 degrees for laminar flaws. For the fitness-for-service demonstration of these vessels, alternative characterization rules for quasi-laminar flaws have been developed. This development has been performed on the basis of flaw interaction analyses.

The current Chapter proposes a revision of the characterization rules for laminar flaws based on the characterization rules developed for quasi-laminar flaws [35], [37]. In fact, this revision is a particular application of the previous Chapter results and is thus also based on mechanical interaction concept. This Chapter aims therefore at harmonizing the grouping rules of the current Codes dealing with laminar flaws.
6.2. CURRENT QUASI-LAMINAR AND LAMINAR FLAWS COMBINATION RULES

As this dissertation essentially focuses on ASME Code Section XI, the revision of the proximity rules for laminar flaws is proposed from the current proximity rules of the ASME Code Section XI.

6.2.1. Quasi-laminar flaws combination rules

As a reminder, the proximity rules developed for quasi-laminar flaws can be expressed as follows:

Each quasi-laminar flaw shall be bounded by the minimum bounding box that fully contains the area of the flaw. Two quasi-laminar flaws shall be grouped if all three of the following proximity criteria are met:

\[ H \leq 0.34 \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \]  \hspace{1cm} (6-1)

\[ S_1 \leq 0.73 \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \]  \hspace{1cm} (6-2)

\[ S_2 \leq 0.73 \min (\max (D_{11}, D_{21}), \max (D_{12}, D_{22})) \]  \hspace{1cm} (6-3)

where the distances between the flaws \( H, S_1, S_2 \) and the diagonals of the bounding boxes \( D_{11}, D_{21}, D_{12}, D_{22} \) are defined in Figure 6-1.
6.2.2. Laminar flaws combination rules

According to the ASME Code, if there are two or more laminations, these laminations are projected to a single plane and if the separation distance of the projected laminations is lower than or equal to 1 in (= 25.4 mm), the laminations shall be combined into a single large rectangular flaw of dimensions $\ell$ and $w$ as shown in Figure 6-2.
This resulting laminar flaw of dimensions $\ell$ and $w$ is then used for the purpose of defining the characteristics of the flaw to be used in conjunction with the acceptance standards of IWB-3500 or IWC-3500.

For laminar flaws, the aforementioned acceptance standards of ASME Code provide tables of allowable areas. Therefore, it is necessary to give an area for the combined laminar flaw. In compliance to the subsurface flaw idealization procedure of the most FFS Codes and in particular the ASME Code Section XI, a subsurface flaw is bounded by rectangle and is defined as elliptical in shape, as illustrated in Figure 6-3.
With this in mind, the area of the combined laminar flaw is deemed to be 0.75 times the area of $w \times \ell$. This approximatively corresponds to the area of the ellipse $A_{\text{ellipse}}$ bounded by the rectangle $w \times \ell$:

$$A_{\text{ellipse}} = \pi \times (w/2 \times \ell/2) = \pi/4 \times w \times \ell \approx 0.75 \times (w \times \ell) \quad (6-4)$$

This equivalent area of $0.75 \times (w \times \ell)$ shall be then compared to the allowable area of the acceptance standard of the ASME Code Section XI.

### 6.3. REVISION OF LAMINAR FLAWS PROXIMITY RULES

As explained hereinbefore and depicted in Figure 6-2, the current ASME combination methodology for laminar flaws does not consider the offset distance of the laminations in the wall of the component i.e., laminar flaws could be combined to a single laminar flaw even if the flaw planes are by far spaced which makes no sense.

Moreover, the combination rules for laminar flaws are not intended to deal with a large number of laminar flaws as detected in nuclear power reactor vessels [61], [66].
Finally, as mentioned in the Introduction of this Chapter, the ASME proximity rules for laminar flaws are not based on mechanical interaction analyses as it is the case for all other kinds of flaws in all the FFS Codes but on UT considerations.

Those points argue for the present proposal of updated proximity rules for laminar flaws.

### 6.3.1. Preliminary statements

These updated proximity rules for laminar flaws are based on calculations performed for the quasi-laminar flaws. This approach is fully justified and pertinent since, by definition, the laminar flaws (tilt ≤ 10°) are a particular case of the quasi-laminar flaws (tilt ≤ 20°). In other words, the laminar flaws are a subset of the quasi-laminar flaws. Therefore, the proximity criteria in Eq. (6-1) to Eq. (6-3) could be directly applied to laminar flaws.

However, ultrasonic examinations qualified for laminar flaws detection provide signals in the form of rectangular boxes bounding the flaw projections on the surface of the component, as shown by the rectangle \( w \times \ell \) in Figure 6-4. UT measurements, which flaw assessment is based on, are thus the dimensions of rectangles instead of dimensions of 3D boxes as for quasi-laminar flaws. No diagonals are provided by UT for laminar flaws. As a consequence, combination rules for quasi-laminar flaws cannot be used as is for laminar flaws.

Nevertheless, if it is assumed that, from UT examinations, laminar flaws were also sized by 3D bounding boxes, as presented in Figure 6-4, the diagonals of the 3D boxes could be assimilated to the sides of the rectangle parallel to the surface of the component i.e.:

\[
2D_1 = w \quad \text{and} \quad 2D_2 = \ell \quad \text{(6-5)}
\]
Indeed, let us consider the side $w$:

$$w = 2D_1 \cos \alpha$$  \hspace{1cm} (6-6)

where $\alpha$ is the tilt of the diagonal $2D_1$ with respect to the surface of the component. By definition of laminar flaw, the maximum value $\alpha_{\text{max}}$ of this tilt is $10^\circ$. Therefore, for any laminar flaw, the maximum relative difference $\Delta_{\text{max}}$ between $2D_1$ and $w$ is:

$$\Delta_{\text{max}} = \frac{2D_1 - 2D_1 \cos 10^\circ}{2D_1} = 1.5\%$$  \hspace{1cm} (6-7)

The same reasoning can be done for the side $\ell$ and the diagonal $2D_2$.

This fully justifies the assumptions of Eq. (6-5) i.e., for any laminar flaw, the sides $w$ and $\ell$ of the bounding rectangle can be assimilated to the corresponding diagonals $2D_1$ and $2D_2$ of a hypothetical 3D bounding box as presented in Figure 6-4.
6.3.2. Updated proximity rules

Based on both assumptions of Eq. (6-5), the quasi-laminar flaws combination rules, given in Eq. (6-1) to Eq. (6-3), can be therefore adapted to the laminar flaws, sized by bounding rectangles, as follows:

Multiple laminar flaws as shown in Figure 6-5 shall be combined into a single flaw if all three of the following proximity criteria are met:

\[
H \leq 0.17 \min\{\max(w_1, \ell_1), \max(w_2, \ell_2)\} \quad (6-8)
\]

\[
S_1 \leq 0.37 \min\{\max(w_1, \ell_1), \max(w_2, \ell_2)\} \quad (6-9)
\]

\[
S_2 \leq 0.37 \min\{\max(w_1, \ell_1), \max(w_2, \ell_2)\} \quad (6-10)
\]

If the detected areas of those flaws are partially or totally overlapping in any one direction, the proximity criteria in that direction are met.

Figure 6-5 Configuration and determination of relevant dimensions of multiple laminar flaws
For the purpose of defining the characteristics of the flaw to be used in conjunction with the acceptance standards of the ASME Code Section XI, the dimensions $w$ and $\ell$, shown in Figure 6-5, of the resulting rectangle are given by:

\begin{align*}
  w &= w_1 + w_2 + S_2 \\
  \ell &= \ell_1 + \ell_2 + S_1
\end{align*}

(6-11) \quad (6-12)

In compliance to the current sizing used for laminar flaws in ASME Code, the area the combined laminar flaw shall be 0.75 times the area of the resulting rectangle $w \times \ell$ that contains the detected area of the flaws that are within the proximity limits defined in Eq. (6-8) to Eq. (6-10).
6.4. SUMMARY AND CONCLUSIONS

Combination of laminar flaws is at this time addressed only in the ASME Code Section XI [21] and API 579/ASME FFS-1 Code [31]. However, these combination criteria for laminar are not based on mechanical interaction analyses as it is the case for non-laminar flaws.

Moreover, the combination rules for laminar flaws are not intended to deal with a large number of laminar flaws as detected in nuclear power reactor vessels [61], [66].

To this end, since laminar flaws are a subset of quasi-laminar flaws, a revision of the characterization rules for laminar flaws has been proposed on the basis of the characterization rules developed for quasi-laminar flaws [35], [37].

These revised combination rules for laminar flaws, given in Eq. (6-8) to Eq. (6-10), are thus also based on mechanical interaction between the flaws as for non-laminar flaws, harmonizing in this way the combination rules for both laminar and non-laminar flaws.

The combination rules developed for multiple laminar flaws were contributed to Code changes, which were proposed to the ASME Committee in February 2016.
CHAPTER 7

CONCLUSIONS

When flaws are detected in a component, flaw assessments have to be done in order to demonstrate the fitness-for-service (FFS) of the component for continued operation. The first step of the flaw assessment is the flaw characterization which aims at determining the flaw geometry for analysis. The characterization of a flaw is thus highly important and essential and is done according to characterization rules provided in the FFS Codes.

In this frame, this dissertation presents the works carried out for the purpose of assessing and improving the flaw characterization rules of the FFS Codes. Furthermore, this work also proposed innovative flaw characterization rules in order to overcome some lacks in the current FFS rules. Most of the calculations presented in this dissertation were carried out using the XFEM software Morfeo Crack specifically dedicated to Fracture Mechanics.

In Chapter 1, the theoretical background of XFEM method was presented. This highlighted the main features and advantages of this computational method for Fracture Mechanics i.e., the ease of use (modelling of cracks by level-sets method), the accuracy (stress singularity captured with the near-tip field enrichment) and the flexibility (no restriction of crack geometry). In order to demonstrate the know-how in the use in XFEM as implemented in Morfeo Crack software as well as the robustness of the results obtained, a benchmark was conducted in collaboration with Engineering Mechanics Corporation of Columbus. This benchmark, including a wide range of 3D models with cracks, involved two complete different calculation methodologies i.e., XFEM implemented in Morfeo Crack software and FEAM implement in FRAC@ALT software. Both simulations agreed very well in terms of stress intensity factors and interaction factor as well. From
this benchmark it was stated that the XFEM Morfeo Crack software used to perform the XFEM calculations in this work was validated.

The Chapter 2 addressed a State of the Art of the flaw characterization rules of the worldwide available FFS Codes and Standards and presented the differences in the specific criteria in the flaw characterization procedures in among these FFS Codes. The flaw characterization principles are the same among the FFS Code: (i) determination of flaw size, shape and orientation, (ii) transformation of subsurface flaw to a surface flaw, (iii) alignment and combination of multiple flaws. However, this State of the Art highlighted that the specific criteria of these rules are significantly different among the different FFS Codes. It can be easily deduced that the remaining lives of the flawed components will be different depending on which Code is used, although the original flaws are the same. This will lead to different margins assessment to the structural integrity of flawed components from various Codes and Standards. Therefore, characterization rules should be developed, harmonized to make more uniform the flaw assessment in the components.

Different flaw combination rules were addressed in the next Chapters of the dissertation in order to assess them, to improve them or to harmonize them.

So, Chapter 3 addressed specific criteria for the rules on transforming subsurface flaws to surface flaws when a flaw is located near the free surface of a component. The proximity factors in the current FFS Codes rules are defined by constant values, regardless of the flaw aspect ratios $a/l$. However, it was highlighted through fatigue crack growth experiments that the re-characterization from subsurface to surface flaw highly depends on aspect ratio of the subsurface flaw. As a consequence, XFEM calculations of the interaction between subsurface flaw and free surface of the component were conducted in order to correlate equivalent fatigue crack growth rates to the experimental data. A more suitable flaw-to-surface proximity rule depending on the initial flaw aspect ratio was then deduced. The remaining fatigue lives for subsurface flaws were then compared using this new proximity rule and the current rule provided in the ASME Code Section XI. This comparison
was carried out on thin wall component like piping and on thick wall components like pressure vessels by considering different flaw aspect ratios, flaw sizes and distances from subsurface flaws to the free surface of the component. Results showed that in thin wall component the current ASME Code Section XI subsurface-to-surface proximity rule appears to be updated by the new one, whereas, for thick wall components the current ASME Code proximity rule appears not to be updated. Then, it was stated that the subsurface-to-surface proximity rule should be updated according to the thickness of the component or according to the type of component i.e. piping or vessel. In order to better define the limit of the thickness for updating the subsurface-to-surface proximity rule, therefore, additional fatigue crack growth calculations were done in intermediate thickness components. As a result, an improved subsurface-to-surface proximity rule depending on the aspect ratio of the flaw as well as on the type or on thickness of the component was developed and proposed.

In Chapter 4, the combination rules provided by the ASME, BS 7910 and FITNET Codes in the frame of fatigue crack growth analyses were investigated. In order to quantify the influence of the different combination rules on remaining lives, fatigue crack growth analyses for two adjacent surface flaws in a pipe subjected to cyclic tensile stress were performed. Since it was expected that these combination rules would lead to different remaining lives, fatigue crack growth calculations were conducted using XFEM. So the actual interaction between the flaws was accounted for as well as the natural flaw shape evolution. This allowed assessing the conservatisms of the remaining lives induced by the considered combination rules. It was found that the fatigue lives provided by the BS 7910 and the FITNET Codes are always conservative in comparison to those based on the ASME Code combination rule. Nevertheless, the fatigue lives calculated by XFEM are close to those led by the ASME Code. From these results, it was stated that the combination rule provided by the ASME Code is appropriate for fatigue crack growth calculations.
Chapter 5 presented the development and the validation of characterization rules addressing the quasi-laminar flaws. The quasi-laminar flaws are first-of-a-kind flaws, induced by hydrogen flaking phenomenon, detected in the Reactor Pressure Vessels of Doel 3 and Tihange 2. Due to their specific orientation and their high densities, FFS Codes were not suited to characterize such flaws. Therefore, specific proximity rules were developed using firstly 2D XFEM calculations. Although better suited to high densities of quasi-laminar flaws, these 2D XFEM rules appear very conservative and improvable. This was carried out in a second step using 3D XFEM calculations. Those 3D XFEM calculations led to more realistic but still conservative proximity rules for quasi-laminar flaws. The conservatisms of the proximity rules were demonstrated through a lot of sensitivity analyses dealing with all calculations parameters as well as through a real-life case, involving multiple flaws, which was explicitly calculated in a multi-flaws XFEM calculation. These specific proximity rules for quasi-laminar flaws have led to the Code Case N-848 which is now part of the ASME Code Section XI.

For sake of harmonization a proposal to update the proximity rules for laminar flaws was finally proposed in Chapter 6. Up to now, laminar flaws characterization rules are based on UT capabilities or on engineering judgment. Unlike planar flaws (flaws perpendicular to principal stresses), the Fracture Mechanics concepts are thus not considered for laminar flaws in FFS Codes. Therefore, since laminar flaws are a subset of quasi-laminar flaws, the proximity rules developed for quasi-laminar flaws were adapted to laminar flaws. The fundamentals of these alternative proximity rules are now in line with proximity rules of other kinds of flaws. The combination rules developed for multiple laminar flaws were contributed to Code changes, which were proposed to the ASME Committee in February 2016.

It appears through the works and the results presented in this dissertation that the continuous improvements of the flaw characterization rules are essential
and necessary. This dissertation addressed some of them. However, a number of points are still currently treated in the ASME Working Groups or are open for future works. As an example, the combination criteria for multiple planar flaws are currently based on the flaw depths regardless of their aspect ratio \( a/\ell \). It is easy to understand that, in the same way as for the subsurface-to-surface proximity rules, these criteria should be updated to account of the values of \( a/\ell \). Another example could be the consideration of the ductile failure of the ligament between flaws or between a flaw and the free surface of a component. These issues involve elastic-plastic fracture mechanics calculations or purely plastic analyses and could be more difficult to generalize and standardize because of the dependence in the material properties. Finally, very few FFS Codes clearly address the flaw assessment in mixed mode i.e., when a flaw is subjected to mode I, mode II and mode III. This point should be also improved because the flaws are never purely in mode I as very conservatively assumed in the FFS Codes.

On the other hand, although XFEM appears as a very powerful methodology for Fracture Mechanics analyses, some enhancements could still be made. Even if elastic-plastic Fracture Mechanics is manageable with XFEM, the elastic-plastic crack propagation is not yet in use in Morfeo Crack and is still in development.

These points may therefore constitute some topics for future works to improve the characterization rules of the FFS Codes as well as to push the limits of the XFEM.
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