LIMITING BEHAVIOR OF GENERALIZED QUADRATIC FORMS GENERATED BY REGULAR SEQUENCES III

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Abstract. In this paper, we consider the limit distributions of sums of
\[ \sum \sum W_n(\xi_i, \xi_j) \] when \( \{\xi_i\} \) is strongly mixing and for each \( n, W_n(x, y) \) admits
the eigenvalue expansion.

1. Main results

Let \( \{\xi_i\} \) be a strictly stationary sequence of random variables which are
defined on a probability space \( (\Omega, \mathcal{F}, P) \) and take values on a measurable space
\( (X, \mathcal{A}) \). We say that \( \{\xi_i\} \) satisfies the strongly mixing condition if
\begin{equation}
\alpha(n) = \sup_{A \in \mathcal{M}^a, B \in \mathcal{M}^b_n} |P(AB) - P(A)P(B)| \rightarrow 0 \quad \text{as } n \rightarrow \infty
\end{equation}
and that \( \{\xi_i\} \) satisfies the absolute regularity condition if
\begin{equation}
\beta(n) = E \left\{ \sup_{B \in \mathcal{M}^a_n} |P(B|\mathcal{M}^b_\omega) - P(B)| \right\} \rightarrow 0 \quad \text{as } n \rightarrow \infty.
\end{equation}
\( \mathcal{M}^a_n \) denotes the \( \sigma \)-algebra generated by \( \xi_1, \ldots, \xi_n \). It is obvious that if
\( \{\xi_i\} \) is absolutely regular, then it is strongly mixing since \( \beta(n) \leq \alpha(n) \).

In this paper, we consider only the strongly mixing case. Of course, the
results obtained remain valid when \( \{\xi_i\} \) is absolutely regular.

Next, let \( F \) be the distribution of \( \xi_1 \). Let \( L_2 \) be the space of all Borel
measurable functions which are defined on \( X \) and are square-integrable with
respect to \( F \). For each \( n(\geq 1) \) let \( W_n(x, y) \) be a symmetric kernel, i.e., a symmetric
square-integrable function with respect to \( F \times F \). Suppose that for all
\( n(\geq 1) \) and for all \( x \in X \)
\begin{equation}
E W_n(\xi_1, x) = 0.
\end{equation}
For each \( n(\geq 1) \), let \( A_n \) be a linear operator mapping from \( L_2 \) into \( L_2 \) such that
\begin{equation}
A_n : h \mapsto E W_n(\xi_1, \cdot)h(\xi_1).
\end{equation}
Let \( \{h_{n,i}\} \) (with \( h_{n,0}(x) = 1 \)) and \( \{\lambda_{n,i}\} \) be eigenvectors and eigenvalues of this operator, respectively. Then, it is obvious that for each \( n \) and each \( i \) \((h_{n,i}(\xi_{j}) : j \geq 1) \) satisfies the same mixing conditions as that of \( \{\xi_{j}\} \).

We assume that for all \( n(\geq 1) \) and for all \( j(\geq 0) \) the following relations hold:

(1.5) \[ |\lambda_{n,j}| \geq |\lambda_{n,j+1}|, \]

(1.6) \[ Eh_{n,j}(\xi_{1}) = 0, \quad Eh_{n,j}(\xi_{1}) = n^{-1}, \]

(1.7) \[ Eh_{n,j}(\xi_{1})h_{n,j}(\xi_{1}) = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j, \end{cases} \]

(1.8) \[ EW_{n}(\xi_{1}, x)h_{n,j}(\xi_{1}) = \lambda_{n,j}h_{n,j}(x). \]

Then, we have

(1.9) \[ \Sigma_{j=1}^{\infty} \lambda_{n,j}^{2} < \infty \]

and

(1.10) \[ W_{n}(x, y) = \Sigma_{j=1}^{\infty} \lambda_{n,j}h_{n,j}(x)h_{n,j}(y). \]

Define processes \( U_{n} = \{U_{n}(t) : 0 \leq t \leq 1\} \) by

(1.11) \[ U_{n}(t) = \sum_{1 \leq \ell \triangleleft S[nt]} W_{n}(\xi_{i}, \xi_{f}) \quad (0 \leq t \leq 1). \]

The following theorem is a generalization of a result in [2].

**Theorem.** Let \( \{\xi_{i}\} \) be a strongly mixing strictly stationary sequence of random variables taking values in \((X, \mathcal{A})\). Suppose there exist two positive numbers \( \rho \) and \( \delta(0 < \delta < 1) \) such that

(1.12) \[ K_{0} = \sup_{n \geq 1} \sup_{j \geq 1} E|n^{1/a}h_{n,j}(\xi_{1})|^{4+\rho+\delta} < \infty, \]

(1.13) \[ \sum_{n=1}^{\infty} n^{1+\rho/2} \alpha^{\delta/(4+\rho+\delta)}(n) < \infty. \]

Further, suppose the following relations hold:

(1.14) \[ \lim_{n \to \infty} \lambda_{n,j} = \lambda_{j} \quad \text{uniformly in } j, \]

(1.15) \[ \sup_{n} \Sigma_{j=1}^{\infty} |\lambda_{n,j}| < \infty \quad \text{and} \quad \Sigma_{j=1}^{\infty} |\lambda_{j}| < \infty, \]

(1.16) \[ \lim_{n \to \infty} n \Sigma_{j=1}^{\infty} Eh_{n,j}(\xi_{0})h_{n,j}(\xi_{1}) = a_{j} \quad \text{uniformly in } j. \]

Finally, suppose that for each \( j(\geq 1) \)

(1.17) \[ \{\sum_{\ell=1}^{n} h_{n,j}(\xi_{\ell}) : 0 \leq t \leq 1\} \quad D \longrightarrow B_{j} = \{B_{j}(t) : 0 \leq t \leq 1\} \quad \text{in } D[0, 1] \quad (n \to \infty), \]
where $B_i$'s are Wiener processes such that

\begin{equation}
EB_{j}(s)B_{j}(t)=\lim_{n\to\infty}\sum_{1}^{n} \sum_{1}^{n} Eh_{n,i}(\xi_{\ell_{1}})h_{n,j}(\xi_{\ell_{2}}).
\end{equation}

Then, we have

\begin{equation}
U_{n} \xrightarrow{D} U = \left\{ \sum_{j=1}^{n} \lambda_{j} \left( \int_{0}^{t} B_{j}(s) dB_{j}(s) + a_{j} t \right) : 0 \leq t \leq 1 \right\}
\end{equation}
in $D[0, 1]$ ($n \to \infty$).

As a special case, we consider the case where $W_{n}(x, y)$ is irrelevant to $n$, i.e.,

\begin{equation}
W_{n}(x, y) = G(x, y) \quad \text{for all } n(\geq 1).
\end{equation}

Define $\mu_{j}$ and $g_{j}(x)$ by

\begin{equation}
\lambda_{n,j} = \mu_{j}, \quad h_{n,j}(x) = g_{j}(x)
\end{equation}
and put

\begin{equation}
\sigma_{j}^{2} = 1 + 2 \sum_{i=1}^{\infty} Eg_{i}(\xi_{1})g_{j}(\xi_{i}).
\end{equation}

Assume that

\begin{equation}
\inf_{j=1}^{n} \sigma_{j} > 0.
\end{equation}

Let $V_{n} = \{ V_{n}(t) : 0 \leq t \leq 1 \}$ be the element of $D[0, 1]$ defined by

\begin{equation}
V_{n}(t) = \frac{1}{n} \sum_{1 \leq j \leq nt} G(\xi_{\ell}, \xi_{j}) \quad (t \in [0, 1]).
\end{equation}

Let $B_{j}' = \{ B_{j}'(t) : 0 \leq t \leq 1 \}$ ($j=1, 2, \cdots$) be Wiener processes such that

\begin{equation}
EB_{j}'(s)B_{j}'(t) = \lim_{n\to\infty}\frac{1}{n\sigma_{j}\sigma_{j}'} \sum_{1}^{n} \sum_{1}^{n} Eg_{i}(\xi_{1})g_{j}(\xi_{i}) \quad (s, t \in [0, 1]).
\end{equation}

Let $V = \{ V(t) : 0 \leq t \leq 1 \}$ be the element of $D[0, 1]$ defined by

\begin{equation}
V(t) = \sum_{n=0}^{\infty} \frac{1}{2} \lambda_{s} \sigma_{s} \{(B_{j}'(t))^{2} - t\sigma_{s}^{2}\} \quad (t \in [0, 1]).
\end{equation}

Then, Theorem can be written as follows.

**Corollary.** Let $\{ \xi_{j} \}$ be as in Theorem. Suppose (1.4)-(1.8) and (1.21) hold and there exist two positive numbers $\rho$ and $\delta$ ($0 < \delta < 1$) such that

\begin{equation}
\sup_{j=1}^{n} \| g_{j}(\xi_{1}) \|_{4+\rho+\delta} < \infty
\end{equation}
and (1.12) holds. Then

\begin{equation}
V_{n} \xrightarrow{D} V \quad \text{in } D[0, 1] \quad \text{as } n \to \infty.
\end{equation}
2. Auxiliary results

In what follows, $c$, with or without subscript, denotes an absolute constant, and put $||\xi||_{r}=E|\xi|^{r}(r\geq 2)$ when the expectation of $|\xi|^{r}$ is finite. For a given triangular array $\{\xi_{n,j}\}$, let $M^{(n)}_{h,k}$ be the $\sigma$-algebra generated by $\xi_{n,j}$ $(h\leq j\leq k)$ where $n\geq 1$ is an integer and $1\leq h<k\leq N_{n}$. Put

$$\alpha_{*}(k)=\sup_{n\geq 1,h\leq k}\sum_{h+1 \leq N_{n}}\alpha(M^{(n)}_{h,k}, M^{(n)}_{h+k,N_{n}}) \rightarrow 0 \quad (k \rightarrow \infty).$$

Lemma 2.1. Let $\{\eta_{n,i}: 1 \leq i \leq N_{n}, n \geq 1\}$ be a triangular array of strictly stationary strongly mixing sequences of zero-mean random variables. Suppose there exist two positive numbers $\rho(\geq 2)$ and $\delta(0<\delta<1)$ such that

(2.1) \( \sup_{n}E|\eta_{n,1}|^{\rho+\delta} < \infty \),
(2.2) \( \sum_{n=1}^{\infty}n^{\rho/2-1}\alpha_{*}^{\delta/(\rho+\delta)}(n) < \infty \).

Put

(2.3) \( K_{n}=\max\{n\Vert\eta_{n,1}\Vert_{\rho+\delta}, n^{\rho/2}\Vert\eta_{n,1}\Vert_{\rho+\delta}\} \).

Then, the following inequalities hold:

(2.4) \( E|\Sigma_{\ell=1}^{n}\eta_{n,\ell}|^{\rho} \leq cK_{n} \),
(2.5) \( E|\sum_{1\leq \ell<\ell'\leq n}\eta_{n,\ell}\eta_{n,\ell'}|^{\rho/2} \leq cK_{n} \).

Proof. (2.4) is easily proved by modifying the proof of Theorem in [3]. (2.5) is obtained from (2.3) since

$$E|\sum_{\ell=1}^{n} \sum_{\ell' \neq \ell}^{n} \eta_{n,\ell} \eta_{n,\ell'}|^{\rho/2} = 2^{-\rho/2}E|(\sum_{\ell=1}^{n} \eta_{n,\ell})^{2} - \sum_{\ell=1}^{n} \eta_{n,\ell}^{2}|^{\rho/2} \leq 2^{-\rho/2}E|\sum_{\ell=1}^{n} \eta_{n,\ell}|^{\rho} \leq 2^{-\rho/2}E|\sum_{\ell=1}^{n} \eta_{n,\ell}|^{\rho}. \quad \square$$

By the methods in [6] and [7] we can prove the following lemma.

Lemma 2.2. Let $\{\eta_{n,i}: 1 \leq i \leq N_{n}, n \geq 1\}$ be a triangular array of strictly stationary strongly mixing sequences of zero-mean random variables. Suppose there exist two positive numbers $\rho$ and $\delta$ $(0<\delta<1)$ such that

(2.6) \( \sup_{n\geq 1}E|N_{n}^{1/2}\eta_{n,1}|^{\rho+\delta} < \infty \),
(2.7) \( \sum_{n=1}^{\infty}n^{1+\rho/2}\{\alpha(n)\}^{\delta/(\rho+\delta)} < \infty \).

Suppose

(2.8) \( a=\lim_{n \rightarrow \infty}N_{n} \sum_{i=1}^{N_{n}}E\eta_{n,1}\eta_{n,i+1} \).
exists. Finally, suppose that

\[ \{ \sum_{i=1}^{N_{n}} \eta_{n,i} : 0 \leq t \leq 1 \} \overset{D}{\longrightarrow} \{ B(t) : 0 \leq t \leq 1 \} \quad \text{in } D[0, 1] \]

as \( n \to \infty \).

Then, we have that as \( n \to \infty \)

\[ \left\{ \sum_{1 \leq \ell_{1} < \ell_{2} \leq N_{n}} \eta_{n, \ell_{1}} \eta_{n, \ell_{2}} : 0 \leq t \leq 1 \right\} \overset{D}{\longrightarrow} \left\{ \int_{0}^{t} B(s) \, dB(s) + at : 0 \leq t \leq 1 \right\} \quad \text{in } D[0, 1]. \]

Lemma 2.2 can be easily extended to the multidimensional case as follows.

**Lemma 2.3.** Let \( \{ (\eta_{n,i}^{(1)}, \ldots, \eta_{n,i}^{(d)}) : 1 \leq j \leq N_{n}, n \geq 1 \} \) be a triangular array of strictly stationary strongly mixing sequence of \( d \)-dimensional zeromean random vectors. Suppose for each \( l \) (\( 1 \leq l \leq d \)) \( \{ \eta_{n,j}^{(l)} : 1 \leq j \leq N_{n}, n \geq 1 \} \) satisfies conditions (2.6)-(2.9). Suppose further

\[ a_{l} = \lim_{n \to \infty} N_{n} \sum_{j=1}^{N_{n}} \sum_{k=1}^{N_{n}} E \eta_{n,j}^{(l)} \eta_{n,k+1}^{(l)} \quad (n \to \infty) \]

exist. Then, we have that as \( n \to \infty \)

\[ \left\{ \sum_{t=1}^{T} \mu_{t} \sum_{i_{1} \leq i_{2} \leq N_{n}} \eta_{n,i_{1}}^{(l)} \eta_{n,i_{2}}^{(l)} : 1 \leq i_{1} < i_{2} \leq N_{0} 0 \leq t \leq 1 \right\} \overset{D}{\longrightarrow} \left\{ \sum_{j=1}^{d} \int_{0}^{t} B_{j}(s) \, dB_{j}(s) + a_{j}t : 0 \leq t \leq 1 \right\} \quad \text{in } D[0, 1] \]

where \( \mu_{1}, \ldots, \mu_{d} \) are arbitrary constants and \( \{ B_{j}(t) : 0 \leq t \leq 1 \} (j=1, \ldots, d) \) are a collection of Wiener processes such that

\[ EB_{j}(s)B_{j}(t) = \lim_{n \to \infty} \sum_{l=1}^{N_{n}} \sum_{k=1}^{N_{n}} E \eta_{n,j}^{(l)} \eta_{n,k}^{(l)} \quad \text{for all } s \leq t. \]

3. **Proof of Theorem**

We use the method in \([7]\).

**Lemma 3.1.** Suppose conditions of Theorem are satisfied. Then, for any \( \varepsilon > 0 \) and any \( t(0 < t \leq 1) \)

\[ \lim_{N \to \infty} \sum_{l \leq j \leq N} \sum_{1 \leq \ell \leq N} \lambda_{n,k} h_{n,k}(\xi_{l}) h_{n,k}(\xi_{j}) \| > \varepsilon \| = 0 \]

holds uniformly in \( n \).

**Proof.** We note that

\[ I_{N} = E \left| \sum_{1 \leq l \leq j \leq N} \sum_{1 \leq \ell \leq N} \lambda_{n,k} h_{n,k}(\xi_{l}) h_{n,k}(\xi_{j}) \right|^{2} \]
\( \leq \sum_{k=N}^{\infty} \lambda_{n,k} E \left| \sum_{1 \leq i \triangleleft j \leq \left\lceil nt \right\rceil} h_{n,k}(\xi_i) h_{n,k}(\xi_j) \right|^2 + 2 \sum_{k' > k \geq N} |\lambda_{n,k}| \cdot |\lambda_{n,k'}| \\
\times |E \left\{ \sum_{1 \leq i \triangleleft j \leq \left\lceil nt \right\rceil} h_{n,k}(\xi_i) h_{n,k}(\xi_j) \sum_{1 \leq i' < j' \leq \left\lceil nt \right\rceil} h_{n,k'}(\xi') h_{n,k'}(\xi_{j'}) \right\} |. \)

Since by Lemma 2.1 (with \( r = 4 \)), (1.12) and (2.5)
\( E \left| \sum_{1 \leq i \triangleleft j \leq \left\lceil nt \right\rceil} h_{n,k}(\xi_i) h_{n,k}(\xi_j) \right|^2 \leq c \)
for all \( t (0 < t \leq 1) \) and \( n \), we have
(3.2) \( I_N \leq c (\sum_{k=N}^{\infty} |\lambda_{n,k}|)^{r} \)
for all \( n \). Now (3.1) follows from (1.14) and (3.2). \( \square \)

**Lemma 3.2.** For any \( \varepsilon (> 0) \) and for any \( t (0 < t \leq 1) \)
(3.3) \( \lim_{N \to \infty} P \left( \left| \sum_{k=N}^{\infty} \lambda_{k} \left( \int_{0}^{t} B_{k}(s) dB_{k}(s) + a_{k} t \right) \right| > \varepsilon \right) = 0 \)
holds.

**Proof.** Since
\( E \left| \sum_{k=N}^{\infty} \lambda_{k} \left( \int_{0}^{t} B_{k}(s) dB_{k}(s) + a_{k} t \right) \right| \)
\( \leq \sum_{k=N}^{\infty} |\lambda_{k}| \left( E \left| \int_{0}^{t} B_{k}(s) dB_{k}(s) \right| + |a_{k}| \right) \leq c \sum_{k=N}^{\infty} |\lambda_{k}|, \)
(3.3) follows from (1.14). \( \square \)

**Lemma 3.3.** Suppose conditions of Theorem are satisfied. Let \( N \) be fixed arbitrary. Then, for each \( t (0 < t \leq 1) \)
(3.4) \( \sum_{1 \leq i \triangleleft j \leq \left\lceil nt \right\rceil} \sum_{k=1}^{N} \lambda_{n,k} h_{n,k}(\xi_i) h_{n,k}(\xi_j) \)
\( \rightarrow \sum_{k=1}^{N} \lambda_{k} \left( \int_{0}^{t} B_{k}(s) dB_{k}(s) + a_{k} t \right) \) \( (n \to \infty). \)

**Proof.** (3.4) is easily obtained from (1.13), (1.16) and Lemma 2.3. \( \square \)

**Lemma 3.4.** Suppose conditions of Theorem are satisfied. Then, for each \( t (0 < t \leq 1) \)
(3.4) \( U_n(t) \longrightarrow U(t) \) \( (n \to \infty) \).

**Proof.** Let \( t (0 < t \leq 1) \) be fixed. Let \( \varepsilon_1 \) and \( \varepsilon_2 \) be arbitrary small positive numbers. Then, by Lemmas 3.1 and 3.2 we can choose \( N \) so that
(3.5) \( P\left( \left| \sum_{1 \leq i \triangleleft j \leq \left\lceil nt \right\rceil} \sum_{k=N}^{\infty} \lambda_{n,k} h_{n,k}(\xi_i) h_{n,k}(\xi_j) \right| > \varepsilon_2 \right) < \varepsilon_1 / 2 \)
holds uniformly in $n$ and
\begin{equation}
P\left(\left|\sum_{k=N}^{\infty} \lambda_k \left\{\int_0^t B_k(s) dB_k(s) + a_k t\right\}\right| > \epsilon_2\right) < \epsilon_1/2.
\end{equation}
Therefore, we have
\begin{align}
P\left(\sum_{1 \leq k \leq \lceil nt \rceil} \sum_{k=1}^{N} \lambda_{n,k} \left[h_n, k(\xi_i) h_n, k(\xi_j)\right] < u - \epsilon_2\right) & - P\left(\sum_{k=1}^{N} \lambda_{k} \left\{\int_{0}^{t} B_{k}(s) dB_{k}(s) + a_{k} t\right\} < u + \epsilon_2\right) - \epsilon_1 \\
\leq P\left(U_n(t) < u\right) - P\left(U(t) < u\right)
\end{align}
(3.6)
\begin{equation}
P\left(\sum_{1 \leq k \leq \lceil nt \rceil} \sum_{k=1}^{N} \lambda_{n,k} \left[h_n, k(\xi_i) h_n, k(\xi_j)\right] < u + \epsilon_2\right) & - P\left(\sum_{k=1}^{N} \lambda_{k} \left\{\int_{0}^{t} B_{k}(s) dB_{k}(s) + a_{k} t\right\} > u - \epsilon_2\right) + \epsilon_1.
\end{equation}
(3.7)
So, using Lemma 3.3 (with $N$ fixed) we have
\begin{equation}
\lim_{n \rightarrow \infty} |P(U_n(t) < u) - P(U(t) < u)|
\end{equation}
\begin{equation}
\leq \epsilon_1 + P\left(u - \epsilon_2 \leq \sum_{k=1}^{N} \lambda_{n,k} \left\{\int_{0}^{t} B_{k}(s) dB_{k}(s) + a_{k} t\right\} \leq u + \epsilon_2\right).
\end{equation}
Now, the desired conclusion follows from (3.8), the continuity of the distribution of $\int_{0}^{t} B_{k}(s) dB_{k}(s) + a_{k} t$ and the arbitrariness of $\epsilon_1$ and $\epsilon_2$.

**Lemma 3.5.** Suppose conditions of Theorem are satisfied. Then, $\{U_n\}$ is tight.

**Proof.** Since for any $s$ and $t \ (0 \leq s < t \leq 1)$
\begin{equation}
U_n(t) - U_n(s) = \sum_{k=1}^{N} \lambda_{n,k} \left\{\Sigma_{\ell=1}^{\lceil ns \rceil} h_n, k(\xi_i) \sum_{j=\lceil ns \rceil+1}^{\lceil nt \rceil} h_n, k(\xi_j)\right\} + \sum_{\lceil ns \rceil+1 \leq \ell < \lceil nt \rceil} h_n, k(\xi_i) h_n, k(\xi_j),
\end{equation}
to prove that $\{U_n\}$ is tight, it suffices to show that for all $n$ sufficiently large
\begin{equation}
E \left|\sum_{k=1}^{N} \lambda_{n,k} \left\{\Sigma_{\ell=1}^{\lceil ns \rceil} h_n, k(\xi_i) \sum_{j=\lceil ns \rceil+1}^{\lceil nt \rceil} h_n, k(\xi_j)\right\}\right|^{2+\rho/2} \leq c(t-s)^{1+\rho/4}
\end{equation}
and
\begin{equation}
E \left|\sum_{k=1}^{N} \lambda_{n,k} \left\{\sum_{\lceil ns \rceil+1 \leq \ell < \lceil nt \rceil} h_n, k(\xi_i) h_n, k(\xi_j)\right\}\right|^{2+\rho/2} \leq c(t-s)^{1+\rho/4}
\end{equation}
(3.9) (3.10)
(cf. [1]). Let $n$ be fixed and put $h_n, k(\xi_i) = \eta_{k,i} \ (1 \leq i \leq n, \ k \geq 1)$. Further, put $\sum_{k=1}^{N} |\lambda_{n,k}| = d, \ \lceil ns \rceil = m$ and $\lceil nt \rceil = l$. Then, by the Schwarz inequality, the Jensen inequality and Lemma 2.1 (with $r=4+\rho$) we have
LHS of \((3.9)\)
\[ \leq E\{ (\sum_{k=1}^{\infty} |\lambda_{n,k}| |\sum_{l=m+1}^{t} \eta_{k,l}|^{2} )^{1+\rho/4} \leq E\{ (\sum_{k=1}^{m} |\lambda_{n,k}| |\sum_{l=m+1}^{t} \eta_{k,l}|^{2} )^{1+\rho/4} \} \]
\[= d^{2+\delta/2} \left[ E\{ (\sum_{k=1}^{m} |\lambda_{n,k}| |\sum_{l=m+1}^{t} \eta_{k,l}|^{2} )^{1/2} \right] \]
\[(3.11)\]
\[ \leq d^{2+\delta/2} \left[ E\{ (\sum_{k=1}^{m} |\lambda_{n,k}| |\sum_{l=m+1}^{t} \eta_{k,l}|^{2} )^{1/2} \right] \]
\[ \leq d^{2+\delta/2} \left[ E\{ (\sum_{k=1}^{m} |\lambda_{n,k}| |\sum_{l=m+1}^{t} \eta_{k,l}|^{2} )^{1/2} \right] \]
\[ \leq c \left[ \sup_{k\geq 1} E\{ |\sum_{i=1}^{m} \eta_{k,i}|^{4+\rho} \} \right]^{1/2} \leq c(t-s)^{1+\rho/4} \].

Similarly, by the Jensen inequality and Lemma 2.2

LHS of \((3.10)\)
\[ \leq d^{2+\rho/2} E\{ \sum_{k=1}^{m} |\lambda_{n,k}| |\sum_{i=1}^{m} \eta_{k,i} \eta_{k,i} |^{2+\rho/2} \leq \}
\[ \leq d^{2+\rho/2} \sup_{k=1} E\{ |\sum_{i=1}^{m} \eta_{k,i} |^{4+\rho} \} \]
\[ \leq c \left[ \sup_{k\geq 1} E\{ |\sum_{i=1}^{m} \eta_{k,i}|^{4+\rho} \} \right]^{1/2} \leq c(t-s)^{1+\rho/4} \],

which implies \((3.10)\). Thus, we have the desired conclusion. \(\square\)

**Proof of Theorem.** By Lemma 3.4 we can easily show that each finite dimensional distribution of \(U_{n}\) converges weakly to that of \(U\). Hence, from Lemma 3.5 Theorem follows. \(\square\)

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