DOCTORAL DISSERTATION

A STUDY ON DYNAMIC RESPONSE OF SPAR-type FLOATING WIND TURBINE IN WAVES USING 3-D GREEN FUNCTION METHOD CONSIDERING ROTATION OF WINDMILL BLADES

SPAR型浮体式洋上風力発電施設の波浪中動揺に関するブレードの回転を考慮した3次元グリーン関数法を用いた研究

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To My Parent, Wife and Family Members
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Abstract

Nowadays, the energy problem is one of the fundamental issues facing the world. Since 1960’s, people have begun to realize that our current dependence on fossil fuels needs to change, but no significant steps have been taken to cut consumption until very recently. We are nearing what has been deemed a critical point in the consumption of fossil fuels; as we have nearly used half of the fossil fuels resources available throughout the world. Therefore, the development of renewable energy has become imperative. In this study, the wind energy has been focused.

The offshore expansion of wind turbines is becoming increasingly common, especially in Europe. It is widely known that the vast marine space is one of the main sources of renewable energy. So, the floating offshore wind turbines (FOWTs) are suitable in deep sea. Recently, a number of research groups have paid much attention to the study of FOWTs.

Most of the researches have been carried out concerning the SPAR-type FOWT in an upright condition. However, there is still unknown part, when the horizontal force is applied on FOWTs, they incline and the motion might be changed.

Another important physical effect on FOWT is gyroscopic effect of rotating rotor blades. Therefore, to investigate the effect of the gyro moment and the effect of angle of inclination on the motion of the actual FOWT, two types of experiment were carried out in a water tank using a 1/360 scale model of a prototype FOWT.

First, the interaction between the change of rotational speed as well as moment of inertia of the blade and the motion of the FOWT has been studied in regular waves.
Second, the interaction between the rotary motion of the wind turbine blade and the dynamic motion of the SPAR-type FOWT has been studied at small angles of inclination in regular waves. Finally, compare the experimental results with the results of numerical simulation and discuss the findings. Numerical calculations have been carried out using potential theory based on the 3-D panel method.
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1. Introduction

Recently, the annual energy consumption of the world keeps increasing more and more. These situations are mainly supported by consumption of the fossil fuel. But as global warming and exhaustion of fossil fuels become a serious issue, the development of alternative energy sources such as hydro energy, solar energy and wind energy has become imperative. The offshore wind energy is one of the most important renewable energy resources which can cover worldwide energy demands. This form of energy can practically substitute to replace the fossil fuels. The wind energy is observed as an essential and powerful energy resource for the socio-economic development and economic growth which helps in reducing the dependency on fossil fuels and provides clean energy. It has been estimated that about 10 million MW of energy are continuously available in the earth’s wind and it provides security at a time when decreasing global reserves of fossil fuels threatens the long-term sustainability of global economy.

Wind energy is the fastest growing renewable energy sources, increasing at an annual rate of 25% with a worldwide installed capacity of 239 GW in 2011[1]. Thus, wind energy
emerged as a promising technology for the utilization of offshore wind resources for the large scale generation of electricity.

The generation of power from wind can be obtained from wind turbines which convert wind energy to electrical energy. The wind turbines can produce large quantities of electricity as compared to other energy sources which are generally placed onshore and offshore. It has been observed that relatively low surface roughness of the ocean results in higher wind speeds. So the floating offshore wind turbines (FOWTs) are the best possible options for generating electricity.

1.1 History of Wind Turbine

The wind turbines have been used by mankind since at least 200 BC. The first accepted establishment of the use of wind turbines in the tenth century in Persia [2]. With the advent of the industrial era, windmills were practically relegated to pump water for agriculture use. In the year 1885, wind energy was first used for the production of electrical energy by Poula Cour in Askov, Denmark [3]. He converted an old wooden windmill into the first wind turbine, which covered the energy demands of Askov high school. Thus, from 1885, the use of wind energy for the production of electricity progressed with the increase in energy demands.

The concept of FOWTs in offshore region came after 1930 and it was suggested that the wind turbines to be placed on pylons, but the suggestion was never used. It was approximately 42 years after the original idea; the vision for large-scale FOWTs was introduced by Professor William E. Heronemus at the University of Massachusetts in 1972.
[4], but it was not until the mid 1990’s, after the commercial wind industry was well established, that the topic was taken up again by the mainstream research community.

Concepts of how the vast amount of wind energy available offshore may be harnessed have varied since this initial proposal. In 1998, a conceptual design for ‘FLOAT – an offshore floating wind turbine’ was proposed. This design represented a marriage of the wind power and offshore oil and gas technology. The objective of the FLOAT project was to develop a FOWT system enabling the economic generation of electricity from wind power in offshore locations typically at 100 – 300 feet water depth [5]. Figure 1 below shows a perspective view of the proposed FLOAT design.

![Figure 1](image)

**Fig. 1.1:** Floater design: Perspective view [5]

In 2003, the development of analytical tools emerged for modeling the turbine loads and fatigue damage due to platform motion. The effect that the motion would have on the wind turbine was found by calculating the aerodynamic and inertia loads on the blades in a
two-dimensional state domain representing the blade and vessel motion respectively [6].

Numerous organizations and governmental agencies have accelerated the pace of FOWT development in recent years.

Blue H has built and installed a proof-of-concept FOWT at a 75% scale size in 113.0 [m] deep water 21.3 [km] off the coast of southern Italy. This model was installed in 2008 and decommissioned in early 2009. Blue H plans to launch a second proof-of-concept tension legged platform for a 2MW floating wind turbine in 2012[7].

The world’s first operational deep-water floating large-capacity wind turbine is the Hywind, in the north sea of Norway. The Hywind was towed out to sea in early June 2009. Afterwards, many countries took part in the construction of FOWTs but among them Denmark, Netherlands, Germany, Spain, Japan, and United States are currently the world leaders in wind energy technology.

**1.2 Components of Floating Offshore Wind Turbines**

The wind turbine has seven major subsystems such as blades, nacelle, controller, generator, rotor, tower, and floating body. The detail description of these components is as follows-

**1.2.1 Blades**

The generation of power increases with the increase in the number of blades. Most of the wind turbines have three blades, though there are some with two blades. The blades are generally 30.0 [m] to 50.0 [m] long, with the most common sizes around 40.0 [m]. Blade
weights vary, depending on the design and materials. A 40.0 [m] LM Glass fiber blade for a 1.5 MW turbine weighs 5,780.0 [kg] (6.4 tons) and one a 2.0 MW turbine weighs 6,290.0 [kg] (6.9 tons).

1.2.2 Nacelle

The nacelle houses the main components of the wind turbine, such as the controller, gearbox, generator, and shafts. This part protects the wind turbine equipment.

1.2.2.1 Controller

The controller monitors the condition of the turbine and controls the turbine movement. The control system changes the blade pitch, nacelle yaw, and generator loading of a wind turbine. The control system can also change the pitch of the blades to alter the amount of torque produced by the rotor. The purpose of the control system is to maximize power output.

1.2.2.2 Gearbox

The gearbox present in the turbine helps in increasing the rotational speed of the shaft. A low-speed shaft feeds into the gearbox and a high-speed shaft feeds from the gearbox into the generator. Some turbines use direct drive generators that are capable of producing electricity at a lower rotational speed.

1.2.2.3 Generators

Wind turbines typically have a single AC generator that converts the mechanical energy
from the wind turbines rotation into electrical energy. Clipper wind power uses a different design that features four DC generators. Offshore wind turbines typically send power through cables.

1.2.3 Rotor

The rotor includes both the blades and the hub which consists of normally two or three blades attached to a hub. The system performance of the wind turbine is based on the selection of blade number, shape, and length. The rotor can be either upwind or downwind design. Most wind turbines are three bladed upwind designs.

1.2.4 Towers

Towers are usually tubular steel towers 60.0[m] to 80.0[m] high that consist of three sections of varying heights. There are some towers with heights around 100.0[m]. The tower supports the wind turbine nacelle and rotor.

1.2.5 Floating Body

The floating body supports all wind turbine elements in the ocean. The floating body is tied up by the mooring systems and has enough buoyancy to support the structure. Floating body that achieve stability by using ballast weights hung below a buoyancy part which creates righting moment and high inertial resistance to pitch and roll motion and usually enough draft to offset heave motion[8].
The placement of wind turbines in harsh offshore environments is an engineering challenge, which requires development of suitable platforms to support the floating turbines; therefore, floating wind turbine concepts are classified according to their floater.

### 1.3 Floating Offshore Wind Energy

Currently, there are a number of offshore wind turbines with floating foundation concepts in various stages of development. The main concern is to study the FOWTs in deep water depth where the generation of power can be improved. They are divided into four main categories:

- **SPAR-type**
- **Tension Leg Platform type**
- **Semi-submersible type**
- **Pontoon (Barge) type**

In general, the SPAR-types have a better heave performance than the Semi-submersible types due to their deep draft and reduced vertical wave exciting forces [9], but have more pitch motion, since the water plane area contribution to stability is reduced. Therefore, in this study, SPAR-type FOWT has been selected as the basis of research because of its simpler shape and expected cost-effectiveness. TLPs have very good heave and angular motions, but the complexity and cost of the mooring installation, the change in tendon tension due to tidal variations, and the structural frequency coupling between the mast and the mooring system, are three major hurdles for such systems. Semi-submersible concepts
with a shallow draft and good stability in operational and transit conditions are significantly cheaper to tow out, install and commission than SPAR-type, due to their draft. In the next subsection, we will discuss various offshore floating wind turbines in detail.

1.3.1 SPAR-type FOWT

The SPAR-type FOWT comprises the floating foundation which is referred as the floater, the tower and the rotor-nacelle assembly (RNA). The floater may be towed in the horizontal position to calm waters near the deployment site. It is then upended, stabilized, and the tower and the RNA mounted by a derrick crane Barge-type before finally being towed by escort tugs in the vertical position to the deployment site for connection to the mooring system (Fig. 1.2).

![Fig. 1.2: Statoil Hywind - computer model and full scale. The model shows the depth of the platform below the sea surface [10]](image-url)
The floating foundation consist a steel and/or concrete cylinder filled with a ballast of water and gravels to keep the center of gravity well below the center of buoyancy which ensures the wind turbine floats in the sea. The floater is ballasted by permanent solid iron ore ballast, concrete or gravel from a chute. Alternatively, the ballast tanks may be injected with grout. It should be remarked that the SPAR-type is difficult to capsize. The draft of the floating foundation is usually larger than or at least equal to the hub height above the mean sea level for stability and to minimize heave motion. Therefore, it is necessary to have deep water for deployment of this SPAR-type FOWT as adequate keel to seabed vertical clearance is required for the mooring system to be effective.

The SPAR-type FOWT is usually kept in position by a taut or a catenary spread mooring system using anchor-chains, steel cables and/or synthetic fiber ropes. Alternatively, it may be moored by a single vertical tendon held at the base by a swivel connection that allows the wind turbine to revolve as the wind changes direction (as proposed by the company SWAY). This free yawing effect is similar to the swinging mechanism found in a Floating Production Storage and Offloading vessel (FPSO) turret mooring in the offshore oil and gas industry. Although favorable because the wind turbine will always face the direction of incoming wind thus optimizing power generation and the single vertical tendon may not provide for a degree of redundancy in the event of failure, resulting in unrestrained drifting of the floater. The first full scale SPAR-type FOWT has been deployed off the south-west coast of Karmoy Island, Norway by Statoil in the Hywind demonstration project.
1.3.2 Tension Leg Platform type FOWT

The TLP type comprises a floating platform structure to carry the wind turbine as in Fig. 1.3. In the offshore oil and gas industry, the conventional TLP platform comprises a square pontoon with columns on which the topside deck rests. A smaller version of this conventional hull form is the mini-TLP which has been adopted by the TLP-type FOWT. Unlike the SPAR-type FOWT which needs to be assembled in water, the TLP wind turbine may be assembled and commissioned onshore thereby avoiding the logistic difficulties of offshore assembly.

![Fig. 1.3: TLP FOWT](image)

The fully fitted up platform is towed to the deployment site thus precluding the need to charter and mobilize expensive heavy-lift vessels or derrick crane Barge-types for offshore construction. The floating platform is held in position by vertical tendons (also called tethers) which are anchored either by a template foundation, suction caissons or by pile driven anchors. The pre-tensioned tethers provide the righting stability. This type of FOWT has a relatively less dynamic response to waves when compared to the SPAR-type, the
Semi-submersible type or the Pontoon type. A TLP wind turbine has since been installed off the coast of Puglia, southern Italy by Blue H Technologies. This large scale prototype is used to test the assembly, transportation and installation of the TLP-type wind energy converter as well as to serve as a metering platform with sensors to measure site specific data.

1.3.3 Pontoon (Barge) type FOWT

The Pontoon type FOWT (Fig.1.4) has a very large pontoon structure to carry a group of wind turbines. The large pontoon structure achieves stability via distributed buoyancy and by taking advantage of the weighted water plane area for righting moment. The Pontoon type may be moored by conventional catenary anchor chains. However, the setback of the Pontoon type wind turbine is that it is susceptible to the roll and pitch motions in waves experienced by ocean going ship shaped vessels and may only be sited in calm seas, like in a harbour, sheltered cove or lagoon.

![Fig. 1.4: Barge-type FOWT](image-url)
The National Maritime Research Institute (NMRI) in Tokyo has made some studies on such pontoon type FOWT. It should be remarked that there are clearly hybrid types of FOWTs, for example a combination of SPAR-type floater and tension leg mooring system. Also, there is an interesting concept of a sailing-type floating wind turbine that was studied at the National Institute for Environmental Studies, Japan. The floating wind power plant has no mooring system but navigates with sails and azimuth thrusters. The self-sailing and self-propelled mobility allows the wind farm to move to a location that maximizes the generation of wind power as well as to weather route from storms.

Based on the buoyancy stabilized concept, NREL and MIT collaborated in a Barge-type FOWT. The Barge-type is adopted because of it is simplicity in design, fabrication and installation.

1.3.4 Semi-submersible type FOWT

The Semi-submersible (Fig.1.5) type comprises a few large column tubes connected to each other by tubular members. A wind turbine may sit on one of the column tubes or there could be wind turbines sitting in all the columns. Alternatively, the wind turbine may be positioned at the geometric center of the column tubes and supported by lateral bracing members. The column tubes provide the ballast and they are partially filled with water. In the float condition, the water-plane area of the columns primarily provides floatation stability. This design is good in providing stability to the wind turbine and it is relatively shallow draft allows for site flexibility.
The semi-submersible FOWT is kept in position by mooring lines. This type of FOWT may be constructed onshore. Until now, there is no semi-submersible FOWT in operation. Principle Power Inc. is promoting the semi-submersible type which consists of three column tubes with patented horizontal water entrapment heave plates at the bases. The heave plates primarily serve to reduce heave and pitch motion without increasing floating platform size.
1.4 Objective of Present Study

Deep sea wind firms consisting of FOWTs are deemed an important featured in meeting the world’s energy needs. However, little work has been done to assess the technical challenge that must be overcome to provide stable, durable and cost-effectiveness FOWT. This thesis research analyzes the motion of SPAR-type FOWT by conducting physical experiments in existing facilities at Yokohama National University.

Most of the study have been carried out regarding the motion of the SPAR-type FOWT at upright condition. However, in real sea environmental conditions, the FOWTs face many forces which prevent them from floating in an upright condition; they incline (Fig. 1.6) as a result of the winds, currents, typhoons cyclones, storms etc. Therefore, the motion of the FOWT might be changed at angle of inclination.

Fig. 1.6: SPAR-type FOWT at angle of inclination
This research intends to study the motion of SPAR-type FOWT at angle of inclination through scale model experiments as well as numerical computation and it can be predicted the behavior of actual model at angle of inclination.

The gyroscopic effect of rotating rotor blades is a significant physical effect on FOWT. The gyroscopic force is induced by the rotation of blades. This force has an influence on the motion of FOWT. The motion of the FOWT might be changed by a change in gyroscopic effect which depends on the angular velocity as well as moment of the inertia of the rotating blades. Therefore, to investigate the effect of gyro moment on the motion of the FOWT, another medal experiment was carried out.
1. 5 Literature Review

Several researches have been carried out concerning the dynamic behavior of the SPAR-type FOWT at upright position under the hydrodynamic load as well as aerodynamic load by the way of experimental analysis and numerical analysis.

However, there is no works have been focused on the effect of motion of a SPAR-type FOWT at angle of inclination. There have been several studies of the response of full scale and scaled FOWT to wave and wind loadings at upright position.

Tong [11] analyzed the technical and economic aspect of wind farms. The conceptual design for FLOAT which is a SPAR-type FOWT was presented. Nielson et al. [12] discussed the integrated dynamic analysis of SPAR-type FOWTs and they developed simulation models for Hywind and compared their numerical results with model scale test results. Skaare et al. [13] presented the importance of control strategies on fatigue life of floating wind turbines. They considered various environmental conditions and wind turbine control schemes. They showed the importance of the effect of pitch-angle control of blades on the dynamic response of the FOWT for wind speeds above the rated wind speed. Suzuki and Sato [14] investigated the load on turbine blade induced by motion of floating platform and design requirement for the platform. Here, the effect of a stabilizing the fin attached at the base of the FOWT in reducing the pitch motion of the floating SPAR-type wind turbine was analyzed.

Matsukuma and Utsunomiya [15] performed a motion analysis of a SPAR-type FOWT under steady wind considering rotor rotation. The wind loads acting on the rotor blades are calculated using the blade element momentum theory. As a result, the motion of yaw, sway
and roll are generated due to the effect of the gyro moment for the rotor-rotation. Utsunomiya et al. [16] continued the experimental validation for motion of a SPAR-type FOWT. In this case, the motion of a prototype SPAR-type FOWT was determined under regular and irregular waves and a steady horizontal force that simulates the steady wind condition was analyzed. Karimirad and Moan [17] carried out structural dynamic response analyses of a SPAR-type FOWT in the extreme survival condition. Numerical simulation for coupled wave and wind induced motion and structural response in harsh conditions for a parked FOWT were undertaken. Sultania [18] investigated the motion performance of a SPAR-type FOWT system during extreme sea conditions by (wind-wave coupled) time domain numerical simulation.

Recently, a detailed review on offshore floating wind turbine and the dynamic analysis of SPAR-type FOWT is studied by Bagbanci et al. [19].
Chapter 2

Hydrodynamic Analysis and Motion of Floating Wind Turbine in Waves

2.1 Introduction

The proper estimation of overall motion of a FOWT in waves is the fundamental problem. In order to predict the FOWT motions in waves, the FOWT is usually regarded as a rigid floating body having six degree of freedom, and the fluid force is estimated from linearized potential flow. Even after such linearized potential have been introduced, the solution of the resulting equation is still not easy to obtain. One of the major difficulties arises from the complicated free surface condition. Further difficulty is associated with fact that for practical FOWT its shape is usually described by the co-ordinates of discreet points rather than by a simple mathematical function. As a result, the solution can be only obtained numerically.

In this study, 3-D singular point distribution method is used to calculate the fluid force acting on a floating structure. In 3-D singular point distribution method, singular points are
distributed on the wetted surface of floating body. The basis of this method was introduced by W. D. Kim [20] and Garrison et al. [21]. After that Faltinsen et al. [22], Oortmersen et al. [23] and many other researchers have shown the effectiveness of 3-D singular point distribution method by applying a variety of three dimensional floating bodies.

This chapter is primarily concerned with mathematical formulations regarding 3-D singular point distribution technique. Various assumptions and boundary conditions are described. The expressions for green function are also mentioned here. Then the derivations of main parts of source density and velocity potential formulations are given. A numerical procedure is explained for calculating the source densities and velocities potential. As soon as the velocity potential is obtained, it is then a straightforward job to compute the added mass coefficient, damping coefficient, and first order wave exciting forces. The motion responses can be obtained by solving the equations of motions in the frequency domain by using the computed hydrodynamic coefficients and wave exciting forces.
2.2 Formulation of 3-D Singular Point Distribution Method

2.2.1 Governing Equation

Let \( \theta \)-xyz be the right-hand Cartesian co-ordinate system with xy plane on the mean free surface and z-axis directed vertically upwards through the center of the floating body as shown in Fig. 2.1. Assume that \( h \) is constant water depth, \( \beta \) is incident wave angle, \( P(p_1, p_2, p_3) \) is any point in the fluid, and \( Q(q_1, q_2, q_3) \) is the point on the wetted surface. The amplitude of the motion as well as the incident waves are supposed to be small, whereas the fluid is assumed to be incompressible, inviscid and irrotational.

![Co-ordinate system](image)

**Fig. 2.1:** Co-ordinate system
According to linear potential theory, the potential of a floating body is a superposition of the potentials. The potential due to undisturbed incident wave is $\phi_0$, the potential due to the diffraction of the undisturbed incident wave on the fixed body is $\phi_7$ and the radiation potentials due to the six body motions is $\phi_j$ ($j=1, 2...6$). The $j$-th ($j=1, 2...6$) mode motions are surge, sway, heave, roll, pitch, and yaw respectively. For the steady state condition, the total velocity potential, $\Phi$, can be written as:

$$\Phi = \text{Re}[\phi(x, y, z)e^{-i\omega t}]$$

(2.1)

where $\phi$ is a complex time independent quantity and $\omega$ is the wave circular frequency and it can be written as,

$$\omega = \frac{2\pi}{T}$$

(2.2)

where $T$ is the wave period. The potential function $\phi$ can be separated as follows:

$$\phi = -i\omega[(\phi_0 + \phi_7)\xi_a + \sum_{j=1}^{6} (\xi_j \phi_j)]$$

(2.3)

where $\xi_a$ and $\xi_j$ are incident wave amplitude and motion of the body in $j$-th mode respectively.

In the case of long-crested regular waves, the incident wave potential can be expressed as,

$$\phi_0 = \frac{g}{\omega} \frac{\xi_a}{\cosh(k(z+h))} e^{ik(x\cos\beta + y\sin\beta) + ikh}$$

(2.4)
where

\( g = \) acceleration of gravity

\( k = \) wave number

\( \beta = \) angle between incident waves and \( x \)-axis

The differential equation governing the fluid motion follows from the application of the continuity equation that yields the Laplace equation. The individual potentials are the solutions of this Laplace equation.

\[
\nabla^2 \phi = 0
\]  

(2.5)

### 2.2.2 Boundary Conditions

The potential functions should satisfy different boundary conditions. These conditions can be listed as follows:

1) The linearized free surface condition:

\[
\frac{\partial \phi}{\partial z} - \frac{\omega^2}{g} \phi = 0 \quad \text{on} \quad z=0
\]  

(2.6)

2) The boundary condition on the sea bottom:

\[
\frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z=-h
\]  

(2.7)
3) The boundary condition on the wetted surface of the floating body:

\[
\begin{align*}
\frac{\partial \phi_j}{\partial n} &= n_j \\
\frac{\partial \phi_0}{\partial n} + \frac{\partial \phi_j}{\partial n} &= 0
\end{align*}
\]

on the body surface (for \( j = 1, 2 \ldots 6 \)) \quad (2.8)

In which \( n_j \) is the outward unit normal vector of the wetted surface of the floating body in the \( j \)-th mode of motion, and given by following expression:

\[
n = (n_1, n_2, n_3), \quad (\mathbf{r} - \mathbf{r}_G) \times \mathbf{n} = (n_4, n_5, n_6)
\]

with \( \mathbf{r} \) is the position vector with respect to the co-ordinate of the point on wetted surface of floating body, \( \mathbf{r}_G \) the position vector with respect to the center of the gravity of the floating body.

4) The radiation condition of the potentials \( \phi_j \), which in polar co-ordinate state:

\[
\lim_{R \to \infty} \left[ \sqrt{R} \left( \frac{\partial \phi_j}{\partial R} - i k \phi_j \right) \right] = 0
\]

where, \( R = \sqrt{x^2 + y^2} \)

A simple body form of the floating body, solving the problem in analytical way is quit complex. Therefore, a proper numerical technique should be employed.
2.2.3 Numerical Procedure

According 3-D singular point distribution method, the potential function \( \phi_j \) \((j=1, 2\ldots 7)\) can be obtained over the wetted boundary surface of the floating body. If \( \sigma_j(Q) \) is considered as the strength of a source distributed over the wetted boundary surface at point \( Q(q_1, q_2, q_3) \), then the potential \( \phi_j(p) \) at any point \( P(x, y, z) \) in the fluid region can be expressed by using Green function as follows:

\[
\phi_j(P) = \int_S \sigma_j(Q) \cdot G(P : Q) ds \quad \text{for } j=1, 2\ldots 7 \tag{2.11}
\]

where \( G(P : Q) \) is well-known Green function satisfying the equation of continuity, the linearized boundary condition on the free surface, the boundary condition on sea bed, and the radiation condition. The series expression for Green function, given by John [24] is as follows:

\[
-4\pi G(p_1, p_2, p_3 : q_1, q_2, q_3) = \frac{2\pi(K^2 - k^2)}{k^2h - K^2h + K} \cosh k(p_3 + h) \cosh k(q_3 + h) \times \{Y_0(kR_{pq}) - iU_0(kR_{pq})\} \\
+ 4\sum_{n=1}^{\infty} \frac{(K^2 - k_n^2)}{k_n^2h - K^2h + K} \cos k_n(p_3 + h) \cos k_n(q_3 + h) K_0(k_nR_{pq})
\]

\tag{2.12}

and the integral form of Green function, given by Wehausen and Laitone [25] is as follows:
\(-4\pi G(p_1, p_2, p_3; q_1, q_2, q_3) = \frac{1}{r} + \frac{1}{r_1} + 2PV\int_0^\infty \frac{(\mu + K)e^{-\mu h} \cosh \mu (p_3 + h) \cosh(q_3 + h)}{\mu \sinh(\mu h) - K \cosh(\mu h)} d\mu \\
+ 2\pi (k^2 - K^2) \cosh k(p_3 + h) \cosh k(q_3 + h) J_0(kR_{pq}) \\
+ i \frac{k^2 - K^2}{k^2 - K^2 h + K} \cos \left(\frac{2\pi(k^2 - K^2)}{k^2 - K^2 h + K}\right) J_0(kR_{pq}) \right)

\tag{2.13}

\(J_0\) is the Bessel function of the first kind of zero order; \(Y_0\) is the Bessel function of second kind of zero order; \(K_0\) is modified Bessel function of the second kind of zero order; and

\[ K = \frac{\omega^2}{g} \tag{2.14} \]

\[ r = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2} \tag{2.15} \]

\[ r_1 = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + (p_3 - q_3)^2} \tag{2.16} \]

\[ R_{pq} = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2} \tag{2.17} \]

\(k\) and \(k_n\) satisfy the following dispersion relation.

\[ \frac{\omega^2}{g} = k \tanh kh \tag{2.18} \]

\[ \frac{\omega^2}{g} = -k_n \tanh k_n h \tag{2.19} \]
In Eqn. (2.13) $PV$ indicates a principal value of integral. As it is stated by Faltinsen et al. [21], for practical purpose Eqn. (2.12) should be used when $kR \geq 0.1$ and Eqn. (2.13) when $kR \leq 0.1$.

Taking the normal derivative of Eqn. (2.11) and let $P$ approach to $Q$, then the Eqn. (2.11) becomes

$$\frac{\partial \phi_j(P)}{\partial n} = \frac{1}{2} \sigma_j(P) + \int_s \sigma_j(Q) \frac{\partial G(P : Q)}{\partial n} ds$$  \hspace{1cm} (2.20)

The Eqn. (2.20) satisfies the kinematic boundary condition Eqns. (2.8) and (2.9) on the wetted surface of the floating body. Using the kinematic boundary conditions, the unknown source strength can be determined. Once the source strength is found, the velocity potential $\phi_j$ can be obtained from the Eqn. (2.11).

A numerical approach is required to solve the integral Eqn. (2.20), as it does not permit any solution in closed form. The wetted surface of the body is divided into $N$ number of rectangular panels of area $\Delta s$, as shown in Fig. 2.2.
The discreet form of the Eqn. (2.20) for the point $Q_m$ (m=1~N) and $Q_l$ (l=1~N) can be written as

$$\frac{1}{2} \sigma_j(Q_m) + \sum_{l=1}^{N} \int \sigma_j(Q_l) \frac{\partial G(Q_m : Q_l)}{\partial n} dS_l = \begin{cases} n_j(Q_m) & ; j = 1 \sim 6 \\ \frac{\partial \phi}{\partial n} & ; j = 7 \end{cases}$$

(2.21)

$G(Q_m, Q_l)$ is the induced velocity Potential for the elements $l$ from $m$ elements. The each panel can be expressed by single point source $\sigma_j(Q_m)$ (j=1, 2...6) the equation can be written as follows:

$$\frac{1}{2} \sigma_j(Q_m) + \sum_{l=1}^{N} \sigma_j(Q_l) \frac{\partial G(Q_m : Q_l)}{\partial n} dS = n_j(Q_m)$$

(2.22)

Above Eqn. (2.22) can be rewrite as the following matrix form:
The Eqn. (2.23) is a linear system with $N$ unknowns for radiation and diffraction problem and can be represented in the following simple form,

$$[A][x]=[B] \quad (2.24)$$

where $[A]$ is the coefficient matrix formed by the derivatives of the Green function $\frac{\partial G}{\partial n}$, $[x]$ is the matrix formed by the source strength $\sigma$ and $[B]$ is the coefficient matrix formed by the known conditions.

From the inverse matrix of the derivatives of the Green function, source density can be calculated in the following way:

$$\left[ \begin{array}{c}
\sigma_{j1} \\
\sigma_{j2} \\
\vdots \\
\sigma_{jN}
\end{array} \right] = \left[ \begin{array}{cccc}
\frac{1}{2} + \frac{\partial G_{11}}{\partial n} S_1 & \frac{\partial G_{12}}{\partial n} S_2 & \cdots & \frac{\partial G_{1N}}{\partial n} S_N \\
\frac{\partial G_{21}}{\partial n} S_1 & \frac{1}{2} + \frac{\partial G_{22}}{\partial n} S_2 & \cdots & \frac{\partial G_{2N}}{\partial n} S_N \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial G_{N1}}{\partial n} S_1 & \frac{\partial G_{N2}}{\partial n} S_2 & \cdots & \frac{1}{2} + \frac{\partial G_{NN}}{\partial n} S_N
\end{array} \right]^{-1} \left[ \begin{array}{c}
n_{j1} \\
n_{j2} \\
\vdots \\
n_{jN}
\end{array} \right] \quad (2.25)$$
When the source density $\sigma_j$ on each panel is obtained the velocity potential can be calculated by using Eqn. (2.11) and expressed as follows:

$$\phi_j = \sum_{l=1}^{N} \sigma_{jl} \cdot G(Q_m, Q_l) \cdot S_l$$  \hspace{1cm} (2.26)

The Eqn. (2.26) gives the values of radiation potential $\phi_j (j=1\sim6)$ at the $l$ panel and the diffraction potential $\phi_j (j=7)$ is obtained in Eqn. (2.21).

### 2.2.4 Hydrodynamic Coefficients and Exciting Forces

After getting the velocity potential $\phi_{jl} (j=0\sim7, \ l=1\sim N, \ N$ is total number of element) on each element by the wetted surface boundary conditions, the hydrodynamic pressure of each point on the wetted surface of the element is given by,

$$P_{jm} = i\omega \rho \sum_{l=1}^{N} \sigma_{jl} \cdot G(Q_m, Q_l) \cdot S_l$$  \hspace{1cm} (2.27)

where $P_{jm}$ is the pressure of $m$ element in the direction $j$-th, $\rho$ is the fluid density. From radiation forces and moments added mass and damping coefficients are obtained as

$$m_{ij} = -\rho \text{Re}\{\sum_{l=1}^{N} \phi_{il} \cdot n_{ji} \cdot S_l\} \hspace{1cm} (i, \ j=1\sim6)$$  \hspace{1cm} (2.28)
\[ N_{ij} = -\rho \omega \text{Im}\left[ \sum_{l=1}^{N} \phi_{il} \cdot n_{jl} \cdot S_l \right] \quad (i, j=1 \sim 6) \quad (2.29) \]

where subscript \( ij \) of \( m_{ij} \), \( N_{ij} \) shows the coefficient of \( i \)-th motion by \( j \)-th motion, \( n_{jl} \) is the normal vector of \( j \)-th mode of each element \( l \) and \( S_l \) is the area of \( l \)-th element.

Wave exciting forces and moments \( F_i \) can be obtained by,

\[ F_i = i\omega \rho \sum_{l=1}^{N} (\phi_{bl} + \phi_{bl}) n_l S_l \quad (2.30) \]
2.3 Verification of Numerical Calculation

In order to validate the accuracy of the calculated value of 3-D singular point distribution method, a comparison with results of other theoretical value like Eigen function expansion [26, 27] is presented here. For computation of hydrodynamic forces, a simple shape of floating body such as cylinder is used and the configuration of cylinder is shown in Fig. 2.3. Therefore, surge and heave forces have been analyzed in the range of wave period $T=0.5\sim 10.5\,[\text{s}]$ and the angle of incident wave is $\beta=0^\circ$.

Figures 2.4~2.5 show the results of surge and heave mode of non dimensional forces. Both results show a good agreement for both calculations. In these figures, the horizontal axis is wave period and the vertical axis is dimensionless wave force. Therefore, the 3-D source distribution method shows a good prediction.

![Computational model](image)

**Fig. 2.3:** Computational model
**Fig. 2.4:** Comparison of directional surge force

**Fig. 2.5:** Comparison of directional heave force
2.4 Equation of Motion of Six Degrees Freedom

Let \((x, y, z)\) be the right-handed Cartesian co-ordinate system with the \(xy\) plane on the mean free surface and \(z\)-axis directed vertically upwards through the center of the FOWT as shown in Fig. 2.6. The motion of the FOWT in waves can be represented by equation of motion with six degrees freedom. The six degrees of freedom are composed of three translational motions and three angular motions. The translational displacements in the \(x, y, z\) directions are \(\xi_1, \xi_2, \xi_3\), respectively, where \(\xi_1\) is the surge, \(\xi_2\) is the sway, and \(\xi_3\) is the heave motion. Furthermore, the angular displacement of the motions about the \(x, y, z\) axes are \(\xi_4, \xi_5, \xi_6\) respectively, where \(\xi_4\) is the roll, \(\xi_5\) is the pitch and \(\xi_6\) is the yaw angle.

Fig. 2.6: Co-ordinate system of the FOWT with respect to center of water plane of FOWT
The dynamic equation of motion in the frequency domain of a FOWT in waves can be written as follows:

\[
\{(M + m)\ddot{\xi} + N\dot{\xi} + C\xi\} = F
\]  

(2.31)

where

- **M** = inertia matrix
- **m** = added mass matrix
- **N** = damping matrix
- **C** = restoring force matrix
- **F** = wave exciting force vector
- **\xi** = motion response vector

2.5 The Equation of Motion in the Frequency Domain

The motions of a FOWT in waves are determined from the solution of the motion equation (Eqn. (2.31)) and the six degrees of freedom motion can be written as follows:

\[
\xi = A_j e^{-i\omega t} \quad (j=1, 2\ldots 6)
\]  

(2.32)

where **A** is the amplitude of **j-th** mode, **\omega** is the wave circular frequency and **t** is time.
Differentiating Eqn. (2.32) with respect to time \( t \), the following equation is obtained.

\[
\dot{\xi} = -i\omega A_je^{-i\alpha} \quad \ddot{\xi} = -\omega^2 A_je^{-i\alpha}
\]  

(2.33)

where, \[ \frac{d}{dt} = \cdot \quad \frac{d}{dt^2} = \cdot \n\]

In order to obtain the dynamic equation of motion of FOWT in frequency domain, the Eqn. (2.33) is substituted into the Eqn. (2.31), and are shown as,

\[
\{-\omega^2(M + m) - i\omega N + C\}\xi = F
\]  

(2.34)

The \((6\times6)\) mass matrix \( M \) and the \(6\times6\) restoring force matrix \( C \) can be written respectively as follows:

\[
M = \begin{bmatrix}
M & 0 & 0 & 0 & Mz_G & 0 \\
0 & M & 0 & -Mz_G & 0 & 0 \\
0 & 0 & M & 0 & 0 & 0 \\
0 & -Mz_G & 0 & I_{1x} & 0 & 0 \\
Mz_G & 0 & 0 & 0 & I_{2y} & 0 \\
0 & 0 & 0 & 0 & 0 & I_{3z}
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
C_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & C_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

where, \( M \) is the mass of the FOWT, \( z_G \) is the vertical co-ordinate of the center of gravity in the body fixed co-ordinate system and \( I_{1x} \), \( I_{2y} \) and \( I_{3z} \) are the moments of
inertia of the FOWT with respect to the $x$, $y$ and $z$ axes respectively. $C$ is the coefficient of restoring force can be written as follows:

$$ C_{33} = \rho g A_w $$

(2.35)

$$ C_{44} = C_{55} = \rho g VGM $$

(2.36)

where, $A_w$ is the water-plane area of draft, $V$ is the volume and $GM$ is the metacentric height of the FOWT. The restoring force coefficient $C_{11}$ and $C_{22}$ are some percent (value seems to be reasonable by the model) of mooring force which is added directly to $C_{33}$. This values ($C_{11}$ and $C_{22}$) was referred by Ishikawa who used in numerical calculation [28].
2.6 Inertial Force Due to Rotational Motion

Since the rotor gyroscopic effect on the motion of FOWT is one of the objectives of the present work, it is appropriate to discuss the physics of this effects when applied to a FOWT.

When the rotor of the FOWT is turn, there is an induced gyroscopic force which has an influence on the motion of the FOWT. Therefore, the equation of gyro moment is added to the dynamic equation of motion of the FOWT. So, It is formulated the equation of moment occurring by the rotation of the blades of the FOWT as follows:

The center of the blades which rotates with angular velocity \( \omega_r (=\text{const.}) \) is at \( O_r \). The rotary axis of the blades is defined as the \( x \)-axis as shown in Fig. 2.7. Let \((x_r, y_r, z_r)\) be the right hand Cartesian co-ordinate system of the rotating blades with \( x_r \) and \( y_r \) are in the direction of horizontal plane perpendicular to each other and the \( z_r \)-axis is directed vertically upwards (see Fig. 2.7). Assume that the vibrational angle of the blades around the \( x_r \)-axis is \( \xi_{r4} \), around the \( y_r \)-axis is \( \xi_{r5} \) and around the \( z_r \)-axis is \( \xi_{r6} \). The angular velocity of those angles depends on the angular velocity \( \omega \) of the periodic external force. These angles are defined as follows:

\[
\begin{align*}
\xi_{r4} &= \xi_{r4}^0 e^{-i\omega t} \\
\xi_{r5} &= \xi_{r5}^0 e^{-i\omega t} \\
\xi_{r6} &= \xi_{r6}^0 e^{-i\omega t}
\end{align*}
\]  

(2.37)

Here, \( \xi_{r4}^0, \xi_{r5}^0, \xi_{r6}^0 \) on the right side of Eqn. (2.37) show the amplitudes of the rotational angle of the motion. Therefore, the angular velocities \( \omega_{r4}, \omega_{r5}, \omega_{r6} \) around the respective axis can be written as,
Differentiating this equation with respect to time, the following equation is obtained.

\[
\begin{align*}
\dot{\omega}_r &= -\omega^2 \xi_{r4} e^{-i\omega t} \\
\dot{\omega}_5 &= -\omega^2 \xi_{r5} e^{-i\omega t} \\
\dot{\omega}_6 &= -\omega^2 \xi_{r6} e^{-i\omega t}
\end{align*}
\]

(2.39)

In order to obtain the moments for each axis, these equations are substituted into
Euler’s equations of motion. Euler’s equations are represented by using the moments $M_{r4}$, $M_{r5}$ and $M_{r6}$, and are shown as,

$$
M_{r4} = I_{r4}\dot{\omega}_{r4} - (I_{r2} - I_{r3})\omega_{r5}\omega_{r6} \\
M_{r5} = I_{r2}\dot{\omega}_{r5} - (I_{r3} - I_{r1})\omega_{r6}\omega_{r4} \\
M_{r6} = I_{r3}\dot{\omega}_{r6} - (I_{r1} - I_{r2})\omega_{r4}\omega_{r5}
$$

(2.40)

where $I_{r1}$, $I_{r2}$ and $I_{r3}$ are the moments of inertia of the blades around the each axis.

The following equation can be obtained after substituting Eqns. (2.38) and (2.39) into Eqn. (2.40).

$$
M_{r4} = -I_{r1}\omega_{r4}^2 e^{-i\omega t} + (I_{r2} - I_{r3})\omega_{r5}^2 e^{-i\omega t} \\
M_{r5} = -I_{r2}\omega_{r5}^2 e^{-i\omega t} + (I_{r3} - I_{r1})\omega_{r6}^2 e^{-i\omega t} (\omega_r - i\omega_{r4}^0 e^{-i\omega t}) \\
M_{r6} = -I_{r3}\omega_{r6}^2 e^{-i\omega t} + (I_{r1} - I_{r2})\omega_{r4}^2 e^{-i\omega t} (\omega_r - i\omega_{r5}^0 e^{-i\omega t})
$$

(2.41)

By considering the vibrational amplitude of Eqn. (2.41) to be small, the product of vibrational amplitude can be ignored, and Eqn. (2.41) can be written as Eqn. (2.42)

$$
M_{r4} = -I_{r1}\omega_{r4}^2 e^{-i\omega t} \\
M_{r5} = -I_{r2}\omega_{r5}^2 e^{-i\omega t} + (I_{r3} - I_{r1})\omega_{r6}^2 e^{-i\omega t} \\
M_{r6} = -I_{r3}\omega_{r6}^2 e^{-i\omega t} + (I_{r1} - I_{r2})\omega_{r4}^2 e^{-i\omega t}
$$

(2.42)

The 2nd terms on the right hand side of Eqn. (2.42) for $M_{r5}$ and $M_{r6}$, represent the gyro moment of rotation of the blades. Therefore, the gyro moment $M_{gyro}$ is as shown in the following dynamic equation of motion.

$$
\{-\omega^2 (M + m) - i\omega (N - M_{gyro}) + C]\ddot{\xi} = F
$$

(2.43)
where $M_{\text{gyro}}$ is written as follows:

$$
M_{\text{gyro}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (I_{r3} - I_r)\omega_r \\
0 & 0 & 0 & (I_{r1} - I_{r2})\omega_r & 0
\end{bmatrix}
$$

(2.44)

Finally, when the inverse matrix of the matrix of the left hand side of Eqn. (2.43) is multiplied by both sides of equation Eqn. (2.43), the dynamic equation is obtained as follows:

$$
\xi = \{-\omega^2 (M + m) - i\omega(N - M_{\text{gyro}}) + C\}^{-1} F
$$

(2.45)
2.6.1 Application of Wind Turbine Rotor

The gyroscopic effect of rotating rotor blades is an important physical effect on FOWTs that must be modeled in scaled modeled experiments. A flat blade is chosen as rotor for its simplicity. The moment of inertia of rotating blade is changed with time. Hence, the moment of inertia of rotating blade of two axes which are perpendicular to the axis of rotation is a function of time and angular velocity of rotor.

The angular velocity of the blade is \( \omega_r \), and the axis of rotation of the blade is \( x \)-axis, the moment of inertia around the axis of rotation is \( I_1 \) and around the two axes \( y \) and \( z \) which are perpendicular to the axis of rotation is \( I_2 \) and \( I_3 \), respectively.

Considering the length of the flat blade is \( a \) [m], width of the blade is \( b \) [m] and the initial phase of the flat blade is \( \delta = 0 \). After \( t \) second the position of the flat blade will look like the Fig. 2.8. Let \( z'_1 \) be the axis which passes through the center of gravity of the flat blade and parallel to the \( z_r \)-axis .The moment of inertia around the \( z'_1 \)-axis is \( I'_3 \) (see Fig 2.9, \( \theta = \omega_r t + \delta \)) can be written as.

\[
I'_3 = \rho \int_{\frac{a}{2}}^{\frac{b}{2}} \int_{\frac{b}{2}}^{\frac{a}{2}} r^2 \, dx \, dy
\]  
(2.46)

where \( r \) is the distance from the \( z'_1 \)-axis. Using \( x, y \) and \( \theta \), \( r \) can be written as follows:

\[
r = \frac{|x \tan \theta - y|}{\sqrt{\tan^2 \theta + 1}}
\]  
(2.47)
Fig. 2.8: Moment of inertia of the blade around the $z_r$-axis

Fig. 2.9: Moment of inertia of the blade around the $z_r'$-axis
In order to obtain the moment of inertia around $r'$, substitute the value of $r$ into the Eqn. (2.46) and obtain the following equation.

\[
\frac{I_3'}{\rho} = \frac{1}{\tan^2 \theta + 1} \left( \frac{1}{12} a^3 b \tan^2 \theta + \frac{1}{12} ab^3 \right) \tag{2.48}
\]

Now, substituting $\tan \theta = \frac{\sin \theta}{\cos \theta}$

\[
\frac{I_3'}{\rho} = \frac{1}{12} a^3 b \sin^2 \theta + \frac{1}{12} ab^3 \cos^2 \theta \tag{2.49}
\]

\[
\frac{I_3'}{\rho} = \frac{1}{24} a^3 b (1 - \cos 2\theta) + \frac{1}{24} ab^3 (1 + \cos 2\theta) \tag{2.50}
\]

\[
\frac{I_3'}{\rho} = \frac{1}{24} a^3 b (1 - \cos(2\omega, t + 2\delta)) + \frac{1}{24} ab^3 (1 + \cos(2\omega, t + \delta)) \tag{2.51}
\]

Therefore, using parallel axis theorem; the moment of inertia $I_3$ around the $z_r$-axis can be written as follows:

\[
I_3 = I_3' + Ml^2 \tag{2.52}
\]

where $M$ is the mass of the blade and $l$ is the distance from the center of gravity of the blade to the $z_r$-axis. Therefore, $l$ can be written as,

\[
l = \frac{a}{2} \sin \theta \tag{2.53}
\]

So, the moment of inertia $I_3$ around the $z_r$-axis is shown as follows:
\[ I_3 = \frac{1}{24} (1 - \cos(2\omega_t + 2\delta))Ma^2 + \frac{1}{24} (1 + \cos(2\omega_t + \delta))Mb^2 + \frac{1}{8} (1 - \cos(2\omega_t + 2\delta)Ma^2 \]

(2.54)

Similarly, the moment of inertia \( I_2 \) is obtained around the \( y_r \)-axis by adding \( \pi/2 \) with \( \theta \), is shown as follows:

\[ I_2 = \frac{1}{24} (1 + \cos(2\omega_t + 2\delta))Ma^2 + \frac{1}{24} (1 - \cos(2\omega_t + \delta))Mb^2 + \frac{1}{8} (1 + \cos(2\omega_t + 2\delta)Ma^2 \]

(2.55)

The following equations can be obtained after simplifying the Eqns. (2.54) and (2.55) respectively.

\[ I_2 = \frac{1}{24} M(4a^2 + b^2) + \frac{1}{24} M(4a^2 - b^2)\cos(2\omega_t + 2\delta) \]

(2.56)

\[ I_3 = \frac{1}{24} M(4a^2 + b^2) + \frac{1}{24} M(b^2 - 4a^2)\cos(2\omega_t + 2\delta) \]

(2.57)

The second term of the right-hand side of the above equations is a function of \( \omega_r \) and it is regardless of the incident wave frequency \( \omega \) which is a component of harmonic vibration to change over time. Therefore, in numerical calculation, when the equations of motion are solved in the frequency domain, \( \omega_r \) is taken as a constant.
2.7 Co-ordinate Transformation of Moment of Inertia

The inclination effect on the motion of the FOWT is the main objective of the present study. In this study, it is considered that the moment of inertia of the FOWT depends on co-ordinate transformation. Therefore, it is necessary to determine the moment of inertia around the center of the FOWT at the angle of inclination.

Assume that, the values of the moment of inertia $I_{x}$, $I_{y}$ and $I_{z}$ with respect to the $x$, $y$, and $z$ axes of FOWT to be known. Determine the values of the moment of inertia $I_{u}$, $I_{v}$ and $I_{w}$ with respect to the $u$, $v$ and $w$ axes, which are inclined at an angle $\theta$ to the $x$ and $z$-axis, as shown in Fig. 2.10.

![Diagram](image)

**Fig. 2.10:** Moment of inertia with respect to the inclined axes
Consider the co-ordinates for a typical differential area $dA$ (with respect to Fig. 2.10) are given by $x$ and $z$ with respect to $x$ and $z$ axes, and by $u$ and $w$ relative to the $u$ and $w$ axes. The relations between these co-ordinates can be obtained from the Fig. 2.11.

**Fig. 2.11:** Co-ordinate transformation of moment of inertia

\[ x = r \cos \alpha, \quad z = r \sin \alpha \quad (2.58) \]

\[ u = r \cos(\alpha - \theta) \quad w = r \sin(\alpha - \theta) \quad (2.59) \]

\[ u = x \cos \theta + z \sin \theta, \quad w = z \cos \theta - x \sin \theta \quad (2.60) \]

By definition, the values of $I_{1u}$ and $I_{3w}$ are as follows:

\[ I_{1u} = \int w^2 dA, \quad I_{3w} = \int u^2 dA \quad (2.61) \]

\[ I_{1u} = \int (x^2 \sin^2 \theta + z^2 \cos^2 \theta + 2xz \sin \theta \cos \theta) dA \quad (2.62) \]

\[ I_{lu} = I_{1x} \cos^2 \theta + I_{3z} \sin^2 \theta + P_{xz} \sin 2\theta \quad (2.63) \]
where, $P_{xz}$ is product of moment of inertia.

\[ I_{1u} = \frac{I_x + I_{3z} + I_{1x} - I_{3z}}{2} \cos 2\theta + P_{xz} \sin 2\theta \quad (2.64) \]

Similarly,

\[ I_{3w} = \frac{I_{1x} + I_{3z} - I_{1x} - I_{3z}}{2} \cos 2\theta - P_{xz} \sin 2\theta \quad (2.65) \]

If an area has an axis of symmetry or if either one or both reference axes are axes of symmetry, the product of the inertia is zero. Therefore, the Eqns. (2.64) and (2.65) can be written respectively as follows:

\[ I_{1u} = \frac{I_x + I_{3z} + I_{1x} - I_{3z}}{2} \cos 2\theta \quad (2.66) \]

\[ I_{3w} = \frac{I_{1x} + I_{3z} - I_{1x} - I_{3z}}{2} \cos 2\theta \quad (2.67) \]

Since the moment of inertia does not change with respect to $y$-axis. Therefore, the moment of inertia at angle of inclination can be written as follows:

\[ I_{1u} = \frac{I_x + I_{3z} + I_{1x} - I_{3z}}{2} \cos 2\theta \quad (2.68) \]

\[ I_{2v} = I_{2y} \quad (2.69) \]

\[ I_{3w} = \frac{I_{1x} + I_{3z} - I_{1x} - I_{3z}}{2} \cos 2\theta \quad (2.70) \]
Considering Eqns. (2.68)–(2.670), the generalized mass matrix of the FOWT at the angle of inclination can be written as:

\[
M = \begin{bmatrix}
M & 0 & 0 & 0 & M_{zG} & 0 \\
0 & M & 0 & -M_{zG} & 0 & 0 \\
0 & 0 & M & 0 & 0 & 0 \\
0 & -m_{zG} & 0 & I_{1w} & 0 & 0 \\
M_{zG} & 0 & 0 & 0 & I_{2v} & 0 \\
0 & 0 & 0 & 0 & 0 & I_{3w}
\end{bmatrix}
\]  

(2.71)

Now substituting Eqn. (2.71) into Eqn. (2.45), the dynamic equation of motion of the FOWT is obtained at the angle of inclination.
Chapter 3

Validation Check of the Numerical Calculation

3.1 Introduction

Assessment of hydrodynamic behavior for a FOWT or other floating structure has become an integral part of design process and it is increasingly becoming common nowadays. In the past decade great strides have been made in the application of numerical methods to marine hydrodynamics problems. This has been made possibly by ever improving numerical methods and the continuously increasing speed of computers. Accuracy of the numerical solution must be balanced against the computational efforts, time and cost. According to Beck et al. [29] all numerical codes should go through three stages before they can be used routinely in design. The first stage is VERIFICATION-the demonstration that the code is bug free and the output is numerically correct. Usually the numerical codes are verified by convergence checks, computing known solutions for simple test geometries and testing other codes. The second stage is called VALIDATION stage. It is actually the comparison of the numerical prediction with physical results. Validation is usually done by comparing the numerical results with the detailed experimental results. The Final stage is
ACCREDATION- the certification that a specific code is acceptable for certain type of design problem.

In this chapter the numerical accuracy of the computer code is validate by comparing the present numerical results with the existing numerical results calculate by other researchers with different numerical techniques. In addition, the motion responses of platform are investigated with change of depth-diameter of buoyancy part of FOWT and expected heave motion is reduced.

3.2 Validation Check

In order to evaluate the accuracy and applicability of the present calculation results of SPAR-type FOWT in the frequency domain, a comparative study is made with Suzuki’s et al. [30] published results. Suzuki et al. calculated wave exciting force by using Morison’s equation. The assumed FOWT model was proposed by Suzuki et al. [30]. The sketch of the assumed (prototype) model is shown in Fig. 3.1. The principal particulars of the model are listed in Table 3.1. The responses of surge, heave, pitch and yaw motions are shown in Figs. 3.2~3.5. In this figures, the horizontal axis represents the incident wave period 0.0~40.0[s] and the vertical axis represents the non dimensional RAO (response amplitude operator).

In this study, the RAO of heave and yaw shows a good agreement with Suzuki’s results in the case of blade no rotation. But discrepancies are observed in surge and pitch motion. It is found that surge and pitch motion have maximum value in the vicinity of wave periods 18.0~19.0[s]. The difference of resonant period is about 2.0[s] and the response of surge and pitch motion is large in present calculation. The cause of this difference of resonant period
is that the difference of the fluid force due to the external force, such as a different calculation method has been used. Overall a qualitative agreement is found in the comparison.

Fig. 3.1: Rough sketch of concepts of SPAR-type FOWT (type 1)
### Table 3.1: Model parameters (type 1)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FOWT tower</strong></td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>80.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>7.0 [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>415.0 [ton]</td>
</tr>
<tr>
<td><strong>Buoyancy part</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>20.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>20.5 [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>1022.0 [ton]</td>
</tr>
<tr>
<td><strong>Water filling column</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>60.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>8.0 [m]</td>
</tr>
<tr>
<td>Mass (before water filling)</td>
<td>360.0 [ton]</td>
</tr>
<tr>
<td>Mass (after water filling)</td>
<td>3693.0 [ton]</td>
</tr>
<tr>
<td><strong>Ballast</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>15.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>8.0 [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>5280.0 [ton]</td>
</tr>
<tr>
<td><strong>All</strong></td>
<td></td>
</tr>
<tr>
<td>Displaced volume</td>
<td>10371.0 [m³]</td>
</tr>
<tr>
<td>Gross mass</td>
<td>10380.0 [ton]</td>
</tr>
<tr>
<td>Center of the buoyancy location $KB$</td>
<td>68.0 [m]</td>
</tr>
<tr>
<td>Center of gravity location $KG$</td>
<td>37.0 [m]</td>
</tr>
<tr>
<td>Metacentric height $GM$</td>
<td>32.0 [m]</td>
</tr>
<tr>
<td>Water plane area</td>
<td>330.0 [m²]</td>
</tr>
<tr>
<td>Radius of gyration [about x-axis]</td>
<td>41.0 [m]</td>
</tr>
<tr>
<td>Radius of gyration [about y-axis]</td>
<td>41.0 [m]</td>
</tr>
<tr>
<td>Radius of gyration [about z-axis]</td>
<td>5.0 [m]</td>
</tr>
</tbody>
</table>
**Fig. 3.2**: Comparison of RAOs of surge motion

**Fig. 3.3**: Comparison of RAOs of heave motion
Fig. 3.4: Comparison of RAOs of pitch motion

Fig. 3.5: Comparison of RAOs of yaw motion
3.3 Motion Response According to Platform Shape

In the process of SPAR-type FOWT design; an accurate assessment of motion of FOWT in waves is very important because the motion is related to overall safety in real sea condition. Large motion of FOWT can be occurred due to winds and waves. It has significant effects on generating efficiency. Under extreme waves and winds; power generation may be impossible or platform can be capsized. In addition, when considering to set up a FOWT in waves, the important thing is that, how to avoid the resonant phenomena of the motion. So, it is imperative to estimate motions of platform in design stage, and to optimize the shape of platform for the stable operation. In this research, the motion responses of platform are investigated with change of depth-diameter of buoyancy part of FOWT.

3.3.1 Variation of Depth-Diameter Ratio of Buoyancy Part

3.3.1.1 Models

The prototype model (type 2 model is shown in Fig. 4.4 and main particulars is shown in Table 4.4 in 4th chapter) is considered as the basic model, and depth-diameter ratio of buoyancy part of basic model is varied as case-1, case-2, case-3, and case-4 models. Without changing the water plane area, displacement and platform depth, the diameter of buoyancy part of case-1 model is changed to 18.2 [m] and depth of buoyancy part is changed to 25.0 [m]. The diameter of case-2, case-3 and case-4 models are changed to 22.6, 25.0, and 27.1[m] and depth of buoyancy part is changed to 15.0, 12.0, and 10.0[m]
respectively. Conceptual sketch of changes of buoyancy part are shown in Figs. 4.6 (a) ~ 4.6 (b) and their properties are listed in Table 3.2.

**Fig. 3.6 (a):** Conceptual sketch of basic model, case-1 and case-3 model
**Fig. 3.6 (b):** Conceptual sketch of basic model, case-3 and case-4 model

**Table 3.2:** Main properties of basic model, case-1, case-2, case-3, and case-4 model

<table>
<thead>
<tr>
<th></th>
<th>Basic model</th>
<th>Case-1</th>
<th>Case-2</th>
<th>Case-3</th>
<th>Case-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of buoyancy part</td>
<td>20.0 [m]</td>
<td>18.2 [m]</td>
<td>22.6 [m]</td>
<td>25.0 [m]</td>
<td>27.1 [m]</td>
</tr>
<tr>
<td>Change of buoy. part diameter</td>
<td>-</td>
<td>-9.0%</td>
<td>13.0%</td>
<td>25.0%</td>
<td>35.5%</td>
</tr>
<tr>
<td>Depth of buoyancy part</td>
<td>20.0 [m]</td>
<td>25.0 [m]</td>
<td>15.0 [m]</td>
<td>12.0 [m]</td>
<td>10.0 [m]</td>
</tr>
<tr>
<td>Change of buoy. part depth</td>
<td>-</td>
<td>25.0%</td>
<td>25.0%</td>
<td>-40.0%</td>
<td>-50.0%</td>
</tr>
<tr>
<td>Displacement volume</td>
<td>9801.8 [m³]</td>
<td>9771.1 [m³]</td>
<td>9787.2 [m³]</td>
<td>9811.2 [m³]</td>
<td>9789.3 [m³]</td>
</tr>
<tr>
<td>Change of displacement</td>
<td>-</td>
<td>-0.31%</td>
<td>-0.15%</td>
<td>0.096%</td>
<td>-0.13%</td>
</tr>
<tr>
<td>Pitch inertia ($I_{ss}$)</td>
<td>$4.81 \times 10^8$ [kg⋅m²]</td>
<td>$4.86 \times 10^8$ [kg⋅m²]</td>
<td>$4.75 \times 10^8$ [kg⋅m²]</td>
<td>$4.72 \times 10^8$ [kg⋅m²]</td>
<td>$4.68 \times 10^8$ [kg⋅m²]</td>
</tr>
</tbody>
</table>
3.3.2 Results and Discussion

In Figs. 3.7~3.10, surge, heave, pitch and yaw motion responses are shown with variation of diameter of buoyancy part of basic model. The horizontal axis represents the incident wave period 0.5~40.0[s] and the vertical axis represents the non dimensional RAO. The rotor rotation of the models was chosen as based on the Vestas (V-120) 4.5 MW wind turbine has a maximum designed rotor speed of 14.9 RPM.

In surge RAOs, it is observed that the peak responses are varied with the variation of diameter of buoyancy part of the basic model. In the case-1, the peak response and the resonant period are increased 6.0% and 2.65% respectively comparing the basic model. But in the case-2, case-3 and case-4 the peak response is reduced almost -34.0% and the natural period is changed -3.31%,-0.66% and -0.66% respectively.

In heave RAOs, resonant period is increased and the peak response decreased with the increasing of the diameter of buoyancy part. But, when diameter of buoyancy part is decreased the peak response increase and resonant period decrease.

The resonant period of pitch motion in the case-1 is increased 2.65% but in the case-2, case-3 and case-4 the resonant period is reduced -3.3%, -0.66% and -0.66% respectively.

The peak response of yaw motion is almost same in case-1 comparing the basic model but the resonant period is enhanced 2.65%. On the other hand, the peak response and resonance period is reduced in case-2, case-3 and case-4.

As a result, the variation of diameter of buoyancy part of the model has effect on the motion. As the diameter of the buoyancy part has been increased 25.0%, the peak response
of heave motion of the model is reduced and the resonant period is enhanced. It can be attributed that the large diameter of buoyancy part creates huge damping effect.

**Fig. 3.7**: The RAO of surge motion with the variation of diameter of buoyancy part

**Fig. 3.8**: The RAO of heave motion with the variation of diameter of buoyancy part
Fig. 3.9: The RAO of pitch motion with the variation of diameter of buoyancy part

Fig. 3.10: The RAO of yaw motion with the variation of diameter of buoyancy part
Chapter 4

Model Experiment in Water Tank

The experiments were carried out at the Laboratory of Sea and Air Control System, located in the Marine Building no. B of the Graduate School of Engineering at the main campus of Yokohama National University.

A couple of small scale experiments were carried out in a water tank in regular waves. The first experiment was to understand the change of gyroscopic effect on the motion of FOWT. The tower of a ground type wind turbine is fixed; the change in the rotor surface on revolution of the blade is extremely small. But in the case of FOWT, the whole floating structure is affected by sea waves and the rotor surface of the FOWT always vibrates during revolution. Therefore, the gyroscopic effect is present on the motion of FOWT.

In marine environment, the FOWTs face many forces which prevent them from floating in upright condition. It can be inclined by wind. Hence, in order to estimate its motion characteristics of actual model at angle of inclination, second model test in regular waves was performed.
4.1 Experimental Apparatus

4.1.1 Water Tank

In order to clarify the effects of change of gyro moment and the motion of the FOWT at the angle of inclination, a small scale experiments were carried out in a water tank in regular waves. The water tank was filled with fresh water. The water tank is composed with 16.0 [m] long, 1.0 [m] wide and 2.0 [m] deep. This water tank is long in the longitudinal direction (x-direction) of the model. Considering this water tank as a real sea, a 1/360 scale model which satisfies the Froude law of similarity (see Table 4.2) and the scale ratio is shown by Table 4.1. In addition, the appearance of the water tank is shown in Fig. 4.1.

<table>
<thead>
<tr>
<th></th>
<th>The water tank</th>
<th>The actual sea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>16.0 [m]</td>
<td>5400.0 [m]</td>
</tr>
<tr>
<td>Width</td>
<td>1.0 [m]</td>
<td>360.0 [m]</td>
</tr>
<tr>
<td>Water depth</td>
<td>1.5 [m]</td>
<td>540.0 [m]</td>
</tr>
<tr>
<td>Wave period</td>
<td>0.50~1.50 [s]</td>
<td>9.49~28.5 [s]</td>
</tr>
<tr>
<td>Wave height</td>
<td>0.006 [m]</td>
<td>2.16 [m]</td>
</tr>
</tbody>
</table>

Table 4.1: Similarity rule in the test tank
Fig. 4.1: The water tank

4.1.2 Wave Generating Device

A plunger type wave maker was located at one end of the tank and a wave absorption beach at the other end. Regular waves are generated by the wave maker in 1.5[m] water deep. The appearance of the plunger type wave maker is shown in Fig. 4.2. The wave generator of this water tank is capable of producing waves of periods 0.3~2.0 [s] and maximum wave height 0.3 [m].
Fig. 4.2: Plunger type wave generator
4.2. The Data Analysis

In order to normalize the experimental responses by the amplitude of incident waves, the propagation of incident wave was measured by ultrasonic displacement meter. The incident wave experimental data are obtained for wave height measurement, it may come out with other components including harmonic vibration of the wave period, and we only take out the harmonic components, called the amplitude. Also, the amplitude and phase are calculated for each experimental response of FOWT by Fourier analysis.

The derivation of amplitudes and phases; let the time history of a signal recorded be given by the periodic function \( f(t) \). Then, the Fourier representation of the signal \( f(t) \) is given by

\[
f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right)
\]  

(4.1)

To extract the amplitude and the phase of the harmonic components from the experimental data the periodic function \( f(t) \) can be written as:

\[
f(t) = a \cos \frac{2\pi t}{T} + b \sin \frac{2\pi t}{T}
\]  

(4.2)

where \( T \) and \( t \) is the period and time in second respectively and \( a \) and \( b \) are the Fourier coefficients which are evaluated from the integral as follows:

\[
a = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos \frac{2\pi t}{T} \, dt
\]  

(4.3)
\[ b = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin \left( \frac{2\pi t}{T} \right) dt \]  \hspace{1cm} (4.4)

In order to increase the accuracy of actual data analysis, it is considered \( N \) number of cycles rather than one cycle. Therefore, the Eqns. (4.3) and (4.4) can be written as,

\[ a = \frac{1}{N} \cdot 2 \int_{-\frac{T}{2}}^{\frac{N \cdot T}{2}} f(t) \cos \left( \frac{2\pi t}{T} \right) dt \]  \hspace{1cm} (4.5)

\[ b = \frac{1}{N} \cdot 2 \int_{-\frac{T}{2}}^{\frac{N \cdot T}{2}} f(t) \sin \left( \frac{2\pi t}{T} \right) dt \]  \hspace{1cm} (4.6)

After calculation \( a \) and \( b \) from the Eqns. (4.5) & (4.6), the amplitude \( A \) and phase \( \theta \) of the harmonic components are computed from,

\[ A = \sqrt{a^2 + b^2} \]  \hspace{1cm} (4.7)

\[ \theta = \tan^{-1} \frac{b}{a} \]  \hspace{1cm} (4.8)

The above analysis allows one to study the harmonic components of the incident wave and the corresponding response of FOWT.
4.3 Froude Scaling of Model

A model is a representation of a physical system that may be used to predict the behavior of the system in some desired respect. The physical system for which the predictions are to be made is called the prototype. The laboratory systems are usually thought of as models and are used to study the phenomenon of interest under carefully controlled conditions. From these model studies, specific predictions of one or more characteristics of some other similar system can be made. To do this, it is necessary to establish the relationship between the laboratory model and the prototype model.

Froude scaling was used for establishing scaling factors between the model and the full scale prototype model. A geometric scaling ratio used for this scaling is first defined by:

\[ L_p = \lambda \cdot L_m \]

(4.9)

where the subscripts \( p \) and \( m \) stands for prototype and model, respectively and \( \lambda \) is the scale factor of the model. Correspondingly, we get for areas and volume:

\[ A_p = \lambda^2 \cdot A_m \]

\[ V_p = \lambda^3 \cdot V_m \]

The Froude number in waves can be expressed by:

\[ Fr = \frac{U}{\sqrt{gL}} \]

(4.10)

where \( U \) is the characteristics velocity and \( L \) is a corresponding characteristics length. The characteristics velocity in the model, \( U_m \) is then calculated by matching \( Fr \) in the model and full scale, i.e. \( Fr_p = Fr_m \). Thus,
\[ U_p = \sqrt{\lambda} \cdot U_m \quad (4.11) \]

The scale ratio for any important physical parameter in this problem can be determined in a similar way. Table 4.2 describes the scaling of common parameters for the FOWT design.

**Table 4.2:** Scale ratios for common variables using Froude scaling

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimensions</th>
<th>Units</th>
<th>Scale ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>L</td>
<td>m</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Mass</td>
<td>M</td>
<td>kg</td>
<td>( \lambda^3 )</td>
</tr>
<tr>
<td>Area</td>
<td>L^2</td>
<td>m^2</td>
<td>( \lambda^2 )</td>
</tr>
<tr>
<td>Volume</td>
<td>L^3</td>
<td>m^3</td>
<td>( \lambda^3 )</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>1/T</td>
<td>1/s</td>
<td>1/( \sqrt{\lambda} )</td>
</tr>
<tr>
<td>Force</td>
<td>(M*L)/T^2</td>
<td>(kg*m)/s^2</td>
<td>( \lambda^3 )</td>
</tr>
<tr>
<td>Spring constant</td>
<td>MT^{-2}</td>
<td>kgs^{-2}</td>
<td>( \lambda^2 )</td>
</tr>
<tr>
<td>Wave height</td>
<td>L</td>
<td>m</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Wave period</td>
<td>T</td>
<td>s</td>
<td>( \sqrt{\lambda} )</td>
</tr>
<tr>
<td>Wave length</td>
<td>L</td>
<td>m</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>M*L^2</td>
<td>kg*m^2</td>
<td>( \lambda^3 )</td>
</tr>
</tbody>
</table>

It should be noted that when Froude scaling is applied, Reynolds number scaling is not granted.
4.4 Experimental Model

The assumed SPAR-type FOWT model was proposed by Suzuki et al. [30]. A conceptual sketch of the rotor with nacelle of FOWT is shown in Fig. 4.3 and the main parts of the model are listed in Table 4.3. In addition, a sketch diagram of FOWT full model is shown in Fig. 4.4 and the parameters of the model are listed in Table 4.4. The experimental model was build using 1/360 scale model of a prototype FOWT. The model was build following Froude’s law, and geometric similitude is almost invariably maintained wherever possible. The size was generally maintained by the dimensions of the water tank and its simulation capability of the environment of the water tank, for example; wave period.

The experimental model of SPAR-type FOWT is shown as a photo and sketch in Fig. 4.5 and Fig. 4.6 respectively. The parameters of the model are listed in Table 4.5. A flat blade with two small “weights A” was used as the rotor in the experimental model. No attempts were made to create the wind force. A motor was placed at the top of the tower to create the gyroscopic effect. This well-known mechanical force arises when a rotor rotating around a certain axis undergoes a rotation around a different axis. The tower part and the buoyancy part were made of balsa and the column which was under water was made of polycarbonate pipe. Then ballast was used inside the column of end part. In order to create steady angle of inclination, a moveable “weight B” was shifted away in the negative direction of x-axis (see Fig. 4.6) on an aluminum light rod which was connected to the tower of the model.
Fig. 4.3: Conceptual sketch of rotor and nacelle

Table 4.3: Principal particulars of FOWT

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>3.0 blades</td>
</tr>
<tr>
<td>Blade shape</td>
<td>Flat plate blade</td>
</tr>
<tr>
<td>Rotor mass</td>
<td>60.0 [ton]</td>
</tr>
<tr>
<td>Blade length</td>
<td>60.0 [m]</td>
</tr>
<tr>
<td>Blade width</td>
<td>3.0 [m]</td>
</tr>
<tr>
<td>Blade thickness</td>
<td>1.5 [m]</td>
</tr>
<tr>
<td>Nacelle configuration</td>
<td>Rectangular solid</td>
</tr>
<tr>
<td>Nacelle height</td>
<td>4.0 [m]</td>
</tr>
<tr>
<td>Nacelle width</td>
<td>4.0 [m]</td>
</tr>
<tr>
<td>Nacelle length</td>
<td>10.0 [m]</td>
</tr>
<tr>
<td>Nacelle mass</td>
<td>190.0 [ton]</td>
</tr>
<tr>
<td>Cut in wind speed</td>
<td>4.0 [m/s]</td>
</tr>
<tr>
<td>Cut out wind speed</td>
<td>25.0 [m/s]</td>
</tr>
<tr>
<td>Rotor rotational speed</td>
<td>9.9~14.9 [rpm]</td>
</tr>
<tr>
<td>Optimum number of rotation</td>
<td>12.4 [rpm]</td>
</tr>
<tr>
<td>Nominal output</td>
<td>4.5 [MW]</td>
</tr>
</tbody>
</table>
**Fig. 4.4:** Rough sketch of concepts of SPAR-type FOWT (type 2)
### Table 4.4: Model parameters (type 2)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FOWT tower</strong></td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>80.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>8.0 [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>415.0 [ton]</td>
</tr>
<tr>
<td><strong>Water filling column 1</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>10.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>8.0 [m]</td>
</tr>
<tr>
<td>Mass (before water filling)</td>
<td>60.0 [ton]</td>
</tr>
<tr>
<td>Mass (after water filling)</td>
<td>560.0 [ton]</td>
</tr>
<tr>
<td><strong>Buoyancy part</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>20.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>20.0 [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>1022.0 [ton]</td>
</tr>
<tr>
<td><strong>Water filling column 2</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>45.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>8.0 [m]</td>
</tr>
<tr>
<td>Mass (before water filling)</td>
<td>270.0 [ton]</td>
</tr>
<tr>
<td>Mass (after water filling)</td>
<td>2520.0 [ton]</td>
</tr>
<tr>
<td><strong>Ballast</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>15.0 [m]</td>
</tr>
<tr>
<td>Diameter</td>
<td>8.0 [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>5280.0 [ton]</td>
</tr>
<tr>
<td><strong>All</strong></td>
<td></td>
</tr>
<tr>
<td>Displaced volume</td>
<td>9802.0 [m(^3)]</td>
</tr>
<tr>
<td>Gross mass</td>
<td>10047.0 [ton]</td>
</tr>
<tr>
<td>Center of the buoyancy location ( KB )</td>
<td>58.0 [m]</td>
</tr>
<tr>
<td>Center of gravity location ( KG )</td>
<td>35.0 [m]</td>
</tr>
<tr>
<td>Metacentric height ( GM )</td>
<td>24.0 [m]</td>
</tr>
<tr>
<td>Water plane area</td>
<td>50.0 [m(^2)]</td>
</tr>
<tr>
<td>Radius of gyration [about ( x )-axis]</td>
<td>40.0 [m]</td>
</tr>
<tr>
<td>Radius of gyration [about ( y )-axis]</td>
<td>40.0 [m]</td>
</tr>
<tr>
<td>Radius of gyration [about ( z )-axis]</td>
<td>5.0 [m]</td>
</tr>
</tbody>
</table>
Fig. 4.5: Photo of the experimental model

Fig. 4.6: Sketch of the experimental model
### Table 4.5: Experimental model parameters

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FOWT tower</strong></td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>226.0 [mm]</td>
</tr>
<tr>
<td>Diameter</td>
<td>20.0 [mm]</td>
</tr>
<tr>
<td>Mass</td>
<td>29.7 [g]</td>
</tr>
<tr>
<td><strong>Aluminum bar</strong></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>300.0 [mm]</td>
</tr>
<tr>
<td>Mass</td>
<td>3.0 [g]</td>
</tr>
<tr>
<td><strong>Moveable “weight B” (2 pieces)</strong></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>7.0 [mm]</td>
</tr>
<tr>
<td>Diameter</td>
<td>20.0 [mm]</td>
</tr>
<tr>
<td>Mass</td>
<td>32.0 [g]</td>
</tr>
<tr>
<td><strong>Buoyancy part</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>54.0 [mm]</td>
</tr>
<tr>
<td>Diameter</td>
<td>70.0 [mm]</td>
</tr>
<tr>
<td>Mass</td>
<td>51.4 [g]</td>
</tr>
<tr>
<td><strong>Hollow column</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>141.0 [mm]</td>
</tr>
<tr>
<td>Diameter</td>
<td>22.0 [mm]</td>
</tr>
<tr>
<td>Mass</td>
<td>11.1 [g]</td>
</tr>
<tr>
<td><strong>Ballast</strong></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>60.0 [mm]</td>
</tr>
<tr>
<td>Diameter</td>
<td>22.0 [m]</td>
</tr>
<tr>
<td>Mass</td>
<td>161.2 [g]</td>
</tr>
<tr>
<td><strong>All</strong></td>
<td></td>
</tr>
<tr>
<td>Displaced volume</td>
<td>292077 [mm³]</td>
</tr>
<tr>
<td>Gross mass</td>
<td>290.6 [g]</td>
</tr>
<tr>
<td>Center of the buoyancy location $KB$</td>
<td>196.0 [mm]</td>
</tr>
<tr>
<td>Center of gravity location $KG$</td>
<td>153.0 [mm]</td>
</tr>
<tr>
<td>Metacentric height $GM$</td>
<td>45.0 [mm]</td>
</tr>
<tr>
<td>Water plane area</td>
<td>315.0 [mm²]</td>
</tr>
</tbody>
</table>
4.4.1 Blade and Nacelle

Gyroscopic effect of rotating rotor blades is an important physical effect on FOWT. The change of gyroscopic effect depends on the change of velocity and moment of inertia. The change of velocity of the rotor of actual model is impossible to achieve in practice. An alternative method using 1/360 scale, a flat blade with two “small weights A” was used in the experiment as a rotor of the FOWT, as shown in Fig. 4.7.

The dimensions of the flat blade are listed on the Table 4.6. The flat blade was divided with some holes from the center of the rotation. The holes are located every 25.0 [mm] from the center of rotation. To investigate the change of gyroscopic effect on the motion of a FOWT, a small “weights A” were positioned at different locations on the flat blade to change the velocity as well as moment of inertia. Here, bolts were used as small “weight A”.

![Small “weight A”](image-url)

**Fig. 4.7:** Rotating flat blade with weights
Table 4.6: Dimensions of flat blade

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>220.0 [mm]</td>
</tr>
<tr>
<td>Width</td>
<td>5.0 [mm]</td>
</tr>
<tr>
<td>Thickness</td>
<td>2.0 [mm]</td>
</tr>
<tr>
<td>Mass</td>
<td>0.9 [g]</td>
</tr>
<tr>
<td>Total mass (including small “weight A”)</td>
<td>2.0 [g]</td>
</tr>
</tbody>
</table>

In the experiment no attempts were made to create the wind force. A motor was used to create the rotor rotation and assume it as a nacelle, as shown in Fig. 4.8. The motor model was PNN13RE09HD from Minabea Motor Manufacturing Corporation. The dimensions of the motor are shown in Fig. 4.9 and motor performance is listed in Table 4.7. As a power source of the motor, two button batteries (model no.LR41 (1.5 [v]) were connected in parallel. In addition, a bearing unit (see Fig. 4.10) was used in the connection portion between the nacelle and tower top part.

Fig. 4.8: The nacelle
Fig. 4.9: Motor dimensions

Table 4.7: Motor performance

<table>
<thead>
<tr>
<th>Model number</th>
<th>PNN13RE09HD (From Minabea Motor Manufacturing Corporation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating voltage</td>
<td>1.0~3.5 [v]</td>
</tr>
<tr>
<td>Proper load</td>
<td>1.0 [g\cdot m]</td>
</tr>
<tr>
<td>Load current value</td>
<td>109 [mA]</td>
</tr>
<tr>
<td>No-load speed</td>
<td>16,426 [r/min]</td>
</tr>
<tr>
<td>Mass</td>
<td>3.8 [g]</td>
</tr>
<tr>
<td>Mass (Total mass of nacelle)</td>
<td>8.0 [g]</td>
</tr>
</tbody>
</table>
Fig. 4.10: Bearing unit
4.5 Angular Velocity of Rotating Flat Blade

To calculate the angular velocity of the rotation of flat blade, a CCD Laser Displacement Meter (see Table 4.8) was used. Cycle collection of CCD Laser Displacement Meter was measured 1 [kHz]. A schematic diagram of the experimental set-up is shown in Fig. 4.11. A wall plate was installed near the flat blade. The reference distance from the Meter to the flat blade was 30.0 [mm]. To measure the period of displacement by rotating flat blade was assumed to be 0.0 distance between the displacement and the wall. In order to calculate the change of the angular velocity as well as change of moment of inertia of the flat blade, the attached “weights A” were varied from 25.0 [mm] to 100.0 [mm] from the center of the flat blade. Three times measurements were carried out in each condition; the case 1, 2 and 3 indicate the results in 1st, 2nd and 3rd time respectively and the values are listed in Table 4.9, Table 4.10, Table 4.11 and Table 4.12.

From Eqn. (2.44), it is seen that the motion of the FOWT is affected by gyro moment. Gyro moment leading to FOWT motion in pitch, the rotating rotor create a new component of gyro moment yaw. This new component of gyro moment may either enhance or decrease the yaw motion of FOWT. The change of gyroscopic force depends on the change of velocity and moment of inertia of rotating flat blade. In order to calculate the change of angular velocity as well as change of moment of inertia of the flat blade, the attached “weights A” were varied from 25.0 [mm] to 100.0 [mm] from the center of the blade and results are listed in Table 4.12. From Table 4.13, it is concluded that, the angular velocity of the blade decreases with increasing the distance from the center. On the other hand, the moment of inertia increases with increasing the distance. The measured change of gyroscopic effect is larger at $d=25.0$ [mm] and decrease with increasing the distance.
**Fig. 4.11:** Experimental set-up for measurement of angular velocity of rotating flat blade

**Table 4.8:** Laser displacement meter

<table>
<thead>
<tr>
<th>Type</th>
<th>Sensor</th>
<th>LK-030</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplifier Unit</td>
<td>LK-2000</td>
<td></td>
</tr>
<tr>
<td>The number of channels</td>
<td>Chanel -2</td>
<td></td>
</tr>
<tr>
<td>Reference distance</td>
<td>30.0 [mm]</td>
<td></td>
</tr>
<tr>
<td>Measurement range</td>
<td>±5.0 [mm]</td>
<td></td>
</tr>
<tr>
<td>Source</td>
<td>Red semiconductor laser</td>
<td></td>
</tr>
<tr>
<td>Analog output</td>
<td>Voltage output</td>
<td>±5.0 [v]</td>
</tr>
<tr>
<td></td>
<td>Output impedance</td>
<td>100.0[Ω]</td>
</tr>
<tr>
<td></td>
<td>Current output</td>
<td>4.0~20.0 [mA]</td>
</tr>
<tr>
<td>Sub-function</td>
<td>Auto zero</td>
<td></td>
</tr>
</tbody>
</table>
### Table 4.9: Angular velocity of rotating flat blade: weight positioned $d=25.0$[mm]

<table>
<thead>
<tr>
<th></th>
<th>Period [s]</th>
<th>Angular velocity [rad/s]</th>
<th>Number of revolution [r/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.147</td>
<td>42.73</td>
<td>6.80</td>
</tr>
<tr>
<td>2nd</td>
<td>0.150</td>
<td>41.91</td>
<td>6.67</td>
</tr>
<tr>
<td>3rd</td>
<td>0.151</td>
<td>41.60</td>
<td>6.62</td>
</tr>
<tr>
<td>Average</td>
<td>0.149</td>
<td>42.1</td>
<td>6.7</td>
</tr>
</tbody>
</table>

### Table 4.10: Angular velocity of rotating flat blade: weight positioned $d=50.0$[mm]

<table>
<thead>
<tr>
<th></th>
<th>Period [s]</th>
<th>Angular velocity [rad/s]</th>
<th>Number of revolution [r/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.160</td>
<td>39.0</td>
<td>6.22</td>
</tr>
<tr>
<td>2nd</td>
<td>0.163</td>
<td>38.42</td>
<td>6.11</td>
</tr>
<tr>
<td>3rd</td>
<td>0.165</td>
<td>37.98</td>
<td>6.04</td>
</tr>
<tr>
<td>Average</td>
<td>0.163</td>
<td>38.5</td>
<td>6.12</td>
</tr>
</tbody>
</table>

### Table 4.11: Angular velocity of rotating flat blade: weight positioned $d=75.0$[mm]

<table>
<thead>
<tr>
<th></th>
<th>Period [s]</th>
<th>Angular velocity [rad/s]</th>
<th>Number of revolution [r/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.176</td>
<td>35.5</td>
<td>5.65</td>
</tr>
<tr>
<td>2nd</td>
<td>0.179</td>
<td>34.91</td>
<td>5.55</td>
</tr>
<tr>
<td>3rd</td>
<td>0.180</td>
<td>34.8</td>
<td>5.53</td>
</tr>
<tr>
<td>Average</td>
<td>0.179</td>
<td>35.07</td>
<td>5.58</td>
</tr>
</tbody>
</table>
Table 4.12: Angular velocity of rotating flat blade: weight positioned \(d=100.0\,[\text{mm}]\)

<table>
<thead>
<tr>
<th></th>
<th>Period [s]</th>
<th>Angular velocity [rad/s]</th>
<th>Number of revolution [r/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^{st})</td>
<td>0.188</td>
<td>33.4</td>
<td>5.32</td>
</tr>
<tr>
<td>2(^{nd})</td>
<td>0.202</td>
<td>31.1</td>
<td>4.95</td>
</tr>
<tr>
<td>3(^{rd})</td>
<td>0.197</td>
<td>31.9</td>
<td>5.08</td>
</tr>
<tr>
<td>Average</td>
<td>0.196</td>
<td>32.0</td>
<td>5.10</td>
</tr>
</tbody>
</table>

Table 4.13: Effect of change of position of “weight A” on the flat blade

<table>
<thead>
<tr>
<th>Distance from the center of blade to the “weight A”, (d) [mm]</th>
<th>Angular velocity (\omega_r) [rad/s]</th>
<th>Moment of inertia (I_{r1}) [kg\cdot\text{m}^2]</th>
<th>Moment of inertia (I_{r2}) [kg\cdot\text{m}^2]</th>
<th>Moment of inertia (I_{r3}) [kg\cdot\text{m}^2]</th>
<th>((I_{r1} - I_{r2})\omega_r)</th>
<th>((I_{r3} - I_{r1})\omega_r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
<td>42.0</td>
<td>4.43\times10^{-7}</td>
<td>2.38\times10^{-7}</td>
<td>2.38\times10^{-7}</td>
<td>8.64\times10^{-6}</td>
<td>-8.64\times10^{-6}</td>
</tr>
<tr>
<td>50.0</td>
<td>38.5</td>
<td>5.39\times10^{-7}</td>
<td>3.33\times10^{-7}</td>
<td>3.33\times10^{-7}</td>
<td>7.92\times10^{-6}</td>
<td>-7.92\times10^{-6}</td>
</tr>
<tr>
<td>75.0</td>
<td>35.0</td>
<td>6.98\times10^{-7}</td>
<td>4.93\times10^{-7}</td>
<td>4.93\times10^{-7}</td>
<td>7.19\times10^{-6}</td>
<td>-7.19\times10^{-6}</td>
</tr>
<tr>
<td>100.0</td>
<td>32.0</td>
<td>9.21\times10^{-7}</td>
<td>7.16\times10^{-7}</td>
<td>7.16\times10^{-7}</td>
<td>6.58\times10^{-6}</td>
<td>-6.58\times10^{-6}</td>
</tr>
</tbody>
</table>
4.6 First Experiment: The Influence of Gyro Moment

As stated earlier, when the rotor of the FOWT is turned, there is an induced gyroscopic force which has an influence on the motion of FOWT. Hence, to verify this gyroscopic effect on the motion of FOWT; a model test was carried out in a water tank. Since the change of gyroscopic effect depends on the change of angular velocity and moment of inertia of the rotating flat blade, two small “weight A” were varied at different location on the flat blade (see Fig 4.7). The location of the small “weights A” from the center of the blade was taken as $d = 25.0, 50.0, 75.0, \text{ and } 100.0 [\text{mm}]$. For each condition, a water tank experiment was carried out.

4.6.1 Experimental Method

A schematic diagram of the water tank with experimental model is depicted in Fig. 4.12 (side view) and Fig. 4.13 (front view). In order to prevent the model from drifting, it was moored by two spring mooring lines. Top parts of the mooring lines were connected from the center of the gravity of the model and bottom parts of the mooring lines were connected to a box-shaped aluminum spindle (Fig. 4.14). This connection position was in the $y$ direction (see Fig. 4.39) of the model so that the pitch motion was not affected. The mooring stiffness was measured by experiment and the measured stiffness is 7.72 [N/m] (Table 4.14). The effect of mooring stiffness cannot be ignored. So, the value of the mooring stiffness is added directly to the restoring force coefficient in numerical computation which is presented in chapter 2.
The experiments were carried out with wave periods of 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, and 1.2 [s] for 0° wave heading (head waves). The 6.0 [mm] wave height was set in the wave machine. In each wave period, the characteristics wave amplitude was measured by Fourier analysis (see section 4.2).

Two red marker tapes were placed on the vertical tower part of the model (see Fig. 4.15 (a)) and also another two red tapes were placed on the horizontal light bar which was connected to the tower of the model (see Fig. 4.15 (c)).

In each wave period, a video of the surge, heave, pitch and yaw motion of the model was recorded by two digital cameras (300 frames per second) for 4.0 second of real time. Using camera-1, pictures of the motion of the red marks were taken in the $xz$ plane (see Fig. 4.15(a)) and analyzed their surge heave and pitch. On the other hand, camera-2 was used to take pictures of the motion of the red marks in the $xy$ plane (see Fig. 4.15 (c)) and analyzed their yaw motion.

**Fig. 4.12:** Experimental set-up (side view)
Fig. 4.13: Experimental set-up (front view)

Fig. 4.14: Mooring lines with aluminum spindle
Table 4.14: Dimensions of mooring lines

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1230.0 [mm]</td>
</tr>
<tr>
<td>Spring constant</td>
<td>7.72 [N/m]</td>
</tr>
</tbody>
</table>

Table 4.15: High speed camera

<table>
<thead>
<tr>
<th>Model number</th>
<th>Casio computer Co. Ltd EX-F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame rate</td>
<td>300.0 [fps]</td>
</tr>
<tr>
<td>Number of pixels</td>
<td>512×384 [pixels]</td>
</tr>
</tbody>
</table>
4.6.2 Experimental Analysis

A color tracking method is followed for video analysis. The animation is separated which has been recorded by digital cameras into still images (BMP file). The still images are 300.0 (512×384) frames per second and the total images are created 1200 images for each wave period. The components (Red>100, Green<90, Blue<90) of red tape’s color are extracted by analyzing program. The position of the red tapes is converted from pixel to millimeters generating the position time trajectory of each red tape. This time trajectory of the tape shows the time series response of the motion of the model. A sample result showing a photo of the model in the waves and the analysis data is shown in Fig. 4.15.

An example of time series data of pitch and yaw motion of the model is shown in Figs. 4.16, 4.17 in wave period 1.2 [s], respectively. Finally, using the time series data, the response of the motion is obtained by Fourier analysis (see section 4.2).
**Fig. 4.15 (a):** Picture from camera-1 ($xz$ plane)

**Fig. 4.15 (b):** Analyzed position of red tapes ($xz$ plane)
Fig. 4.15 (c): Picture from camera-1 (xy plane)

Fig. 4.15 (d): Analyzed position of red tapes (xy plane)

Fig. 4.15: Sample result showing photo of the model and the data; (a) camera-1, (b) camera-1, (c) camera-2 and (d) camera-2
**Fig. 4.16:** The time series response of pitch motion: rot., 0.0 [deg.], \( d=25.0 \) [mm], \( T=1.2 \) [s]

**Fig. 4.17:** The time series response of yaw motion: rot., 0.0 [deg.], \( d=25.0 \) [mm], \( T=1.2 \) [s]
4.6.3 Results and Discussion

Gyroscopic effect of the rotating rotor without wind is studied here. Although a rotating rotor that is extracting energy implies a thrust force on FOWT, this study is presented to isolate the gyroscopic effect on the FOWT motion.

Using the time series response of each motion, graphs of the frequency response of surge, heave, pitch, and yaw motion are obtained. The response amplitude operator (RAO) of the motion responses of surge, and heave is defined by the ratio of motion amplitude and incident wave amplitude (\(\xi_a\)). On the other hand, the RAO of pitch and yaw is defined by the ratio of motion amplitude and wave amplitude times wave number (\(k\)). The RAO is a function of normalized frequency, \(2r\omega^2/g\) where \(r\) is the radius of the buoyancy part of the experimental model, \(\omega\) is the wave frequency and \(g\) is the gravity acceleration. In the figures, symbols show the experimental results and lines represent numerical calculations.

From Figs. 4.18~4.33 shows the results for the RAOS of surge, heave, pitch and yaw motion for the “weight A” position \(d= 25.0, 50.0, 75.0, \) and \(100.0 \text{ [mm]}\) on the flat blade and the comparison on the numerical simulation using potential theory which is presented in chapter 2. A resonant period of have motion is seen near the normalized frequencies \(2r\omega^2/g=0.28\). At frequencies above 0.28, the RAOS of surge, heave, pitch and yaw show a good agreement with the experimental results for each condition. But discrepancies are observed in the experimental results for normalized frequencies lower than 0.28. It can be considered that the main reason for such discrepancies is that the small scale model is not effective at lower frequencies. Hence, the responses between the normalized frequencies \(0.28\) to \(1.13\) are discussed. Peak response is observed in surge and heave motion in
numerical computation. This response is maybe due to neglecting the damping effect in the numerical calculation.

The change of gyroscopic effect with the change of location of small “weights A” on the flat blade are shown in Figs. 4.34~4.37. The change of gyroscopic effect does not affect the surge, heave, and pitch motion, but its effect is observed on yaw motion. From Eqn. (2.44), it is seen that the yaw motion of the FOWT is affected by the gyro moment, which is clearly seen in numerical computations and experimental results above normalized frequencies of 0.28 (see Fig. 4.37).

Figure 4.38 shows the results of RAO of yaw motion at the normalized frequencies 0.44 and 0.57 with the change of gyro moment of the rotating flat blade. This linear relationship demonstrates that the yaw response is increased with increasing the change of gyro effect (see Table 4.13). As the change of gyro moment implies a higher response, the moment of inertia of the rotating blade plays an essential role in yaw response. Therefore, the change of gyro moment, which is presented theoretically in section 2.7 (chapter 2), has an effect on the numerical and experimental results for yaw motion of the FOWT. Infect, the gyro moment effect on the FOWT could obviously influence the performance of the wind turbine and might cause huge gyroscopic force acting on the top of the tower, so the gyro moment results are very useful for the design of the wind turbine and its tower structure.
**Fig. 4.18:** RAO of surge motion at $d= 25.0$ [mm]

**Fig. 4.19:** RAO of heave motion at $d= 25.0$ [mm]
Fig. 4.20: RAO of pitch motion at $d = 25.0$ [mm]

Fig. 4.21: RAO of yaw motion at $d = 25.0$ [mm]
Fig. 4.22: RAO of surge motion at $d=50.0$ [mm]

Fig. 4.23: RAO of heave motion at $d=50.0$ [mm]
Fig. 4.24: RAO of pitch motion at $d=50.0$ [mm]

Fig. 4.25: RAO of yaw motion at $d=50.0$ [mm]
Fig. 4.26: RAO of surge motion at $d = 75.0$ [mm]

Fig. 4.27: RAO of heave motion at $d = 75.0$ [mm]
Fig. 4.28: RAO of pitch motion at $d = 75.0$ [mm]

Fig. 4.29: RAO of yaw motion at $d = 75.0$ [mm]
Fig. 4.30: RAO of surge motion at $d = 100.0$ [mm]

Fig. 4.31: RAO of heave motion at $d = 100.0$ [mm]
Fig. 4.32: RAO of pitch motion at $d=100.0$ [mm]

Fig. 4.33: RAO of yaw motion at $d=100.0$ [mm]
Fig. 4.34: Comparison of RAOs of surge motion

Fig. 4.35: Comparison of RAOs of heave motion
**Fig. 4.36:** Comparison of RAOs of pitch motion

**Fig. 4.37:** Comparison of RAOs of yaw motion
Fig. 4.38: RAO of yaw motion with the change of gyro moment of the rotating blades for normalized frequencies $2\omega^2/g=0.44$ and $0.57$. 
4.6.4 Remarks

In this experiment, we could observe the effect of the gyro moment of rotating flat blade on the motion of the FOWT. The flat blade was rotated with small “weights A” to change the angular velocity as well as moment of inertia.

Throughout the motion analysis, it is possible to conclude that the yaw motion is affected by gyro moment. The change of gyro moment depends on the angular velocity and moment of inertia of rotating blades.

The effect of gyro moment could be reduced by enhancing the moment of inertia of the rotating flat blade (see Table 4.12). This experimental and numerical study indicates that for the actual prototype model, the yaw motion could be decrease with decreasing the gyro effect. Finally, the gyroscopic effect of the rotating flat blade on the motion of FOWT could be verified by this study. Further investigation shall be conducted, considering the combined effect of gyroscopic force and aerodynamics thrust force effect.
4.7 Second Experiment: Motion of a FOWT at Angle of Inclination

In marine environment, the FOWTs can be inclined by wind force (detailed description in section 1.4, in chapter 1). The inclination effect may affect the motion of the FOWTs. Since the purely numerical treatment of FOWT hydrodynamics has not reached a complete satisfactory stage, model test are still essential in design process and validation purpose. The model test must be performed such that model and full scale FOWTs exhibit similar behavior. Hence, in order to estimate its motion characteristics of actual model at angle of inclination, model test in regular waves was carried out in a water tank.

To determine the angle of inclination of the model, two small “weights B” were fastened to light aluminum bar which was connected to the tower of the model, as shown in Fig. 4.39. The dimensions of the “weights B” and light aluminum bar are concluded on the Table 4.5.
Fig. 4.39: Schematic of the experimental model with small “weight B”

### 4.7.1 Determination the Steady Angle of Inclination

In order to determine the steady angle of the model, the moveable “weights B” were shifted from the center of the tower on the bar. To create the positive angles (anti clockwise direction), the “weight B” was positioned every 20.0 [mm] from the center of the tower to 120.0 [mm] in the negative direction of x-axis (see Fig. 4.39). Similarly, to create the negative angles (clockwise direction), the “weight B” was positioned every 20.0 [mm] from the center of the tower to 120.0 [mm] in the positive direction of x-axis.
In each case, photograph was taken from the side of the model in the water tank and analyzed its images in the manner of previous sub-section 4.6.2. A sample photo of the model is shown in Fig. 4.40. The angles against the weight position are shown in Table 4.16. A relationship between the angle of the model and the position of the “weight B” is illustrated in Fig. 4.41.

![Sample photo of the model in water tank](image)

\[d=60[mm], \theta=-1.08[deg.]\]  
\[d=-60[mm], \theta=5.32[deg.]\]

**Fig. 4.40:** Sample photo of the model in water tank
Table 4.16: Moveable weight position and corresponding angle of inclination

<table>
<thead>
<tr>
<th>Position of the “weight B” [mm]</th>
<th>Angle of inclination of the model [deg.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0</td>
<td>8.82</td>
</tr>
<tr>
<td>-100.0</td>
<td>7.62</td>
</tr>
<tr>
<td>-80.0</td>
<td>6.37</td>
</tr>
<tr>
<td>-60.0</td>
<td>5.32</td>
</tr>
<tr>
<td>-40.0</td>
<td>2.15</td>
</tr>
<tr>
<td>-20.0</td>
<td>2.40</td>
</tr>
<tr>
<td>0.0</td>
<td>1.17</td>
</tr>
<tr>
<td>20.0</td>
<td>0.30</td>
</tr>
<tr>
<td>40.0</td>
<td>-0.65</td>
</tr>
<tr>
<td>60.0</td>
<td>-1.08</td>
</tr>
<tr>
<td>80.0</td>
<td>-3.91</td>
</tr>
<tr>
<td>100.0</td>
<td>-3.59</td>
</tr>
<tr>
<td>120.0</td>
<td>-4.23</td>
</tr>
</tbody>
</table>

Fig. 4.41: Approximation line of the weight position and the angle of inclination of the model
4.7.2 Results and Discussion

The second experiment was carried out at a number of variable angles $\theta = -4.0, -2.0, 0.0, 2.0, 4.0, 6.0,$ and $8.0$ degrees at the same wave periods and same wave height (as like as the first experiment) in regular waves. These angles were measured by the relationship of variable “weight B” position and the angle of inclination of the model which is shown in Fig. 4.41. In each angle, the experiment was performed in two cases and that was the blade rotation and no rotation.

The experimental method and the data analysis that was computed for this experiment is similar to the methodology presented previously in the sub-sections 4.6.1 and 4.6.2. Using the same methodology, the experimental results are presented in this section and compared with the numerical results. The numerical simulations are calculated by using potential theory which is described in chapter 2.

At angle of inclination, the metacenter is displaced and the metacentric height is increased of the floating body. When the “weight B” of the model is shifted, the center of buoyancy shifts as the angle of inclination increasing and the water plane area increase. A relationship between the angle of inclination and the metacentric height (GM) of the FOWT model is illustrated in Fig. 4.42. As shown this figure, the metacentric height increases with enhancing the angle of inclination.
The moments of inertia of the FOWT at the angle of inclination are calculated with Eqns. (2.68) ~ (2.70) and the values are listed in Table 4.17. As can be seen, the moment of inertia around the \( w \)-axis increases with increasing the angle of inclination but the moment of inertia around the \( u \)-axis decreases with increasing angle of inclination. The moment of inertia around the \( v \)-axis is constant with the change of angle of inclination (see Eqn.(2.69)).
Table 4.17: Moment of inertia at the angle of inclination

<table>
<thead>
<tr>
<th>Angle of inclination, $\theta$ [deg.]</th>
<th>Moment of inertia, $I_{lu}$ [kg·m$^2$]</th>
<th>Moment of inertia, $I_{lv}$ [kg·m$^2$]</th>
<th>Moment of inertia, $I_{lw}$ [kg·m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.0</td>
<td>6.23×10$^{-4}$</td>
<td>6.26×10$^{-4}$</td>
<td>7.58×10$^{-5}$</td>
</tr>
<tr>
<td>-2.0</td>
<td>6.25×10$^{-4}$</td>
<td>6.26×10$^{-4}$</td>
<td>5.13×10$^{-5}$</td>
</tr>
<tr>
<td>0.0</td>
<td>6.26×10$^{-4}$</td>
<td>6.26×10$^{-4}$</td>
<td>3.53×10$^{-5}$</td>
</tr>
<tr>
<td>2.0</td>
<td>6.25×10$^{-4}$</td>
<td>6.26×10$^{-4}$</td>
<td>5.13×10$^{-5}$</td>
</tr>
<tr>
<td>4.0</td>
<td>6.23×10$^{-4}$</td>
<td>6.26×10$^{-4}$</td>
<td>7.58×10$^{-5}$</td>
</tr>
<tr>
<td>6.0</td>
<td>6.19×10$^{-4}$</td>
<td>6.26×10$^{-4}$</td>
<td>1.11×10$^{-3}$</td>
</tr>
<tr>
<td>8.0</td>
<td>6.14×10$^{-4}$</td>
<td>6.26×10$^{-4}$</td>
<td>1.62×10$^{-3}$</td>
</tr>
</tbody>
</table>

The experimental results are displayed below and a comparison is presented with numerical simulation in the case of blade rotation and no rotation at different angle of inclination, as shown in Figs. 4.43~4.70. In graphs, norot indicates the blade no rotation. On the other hand, rot indicates the blade rotation. In addition, cal. and exp. represent the computed results and the experimental results of FOWT respectively. The graphs of surge, heave, pitch and yaw are listed in order for each angle.

From these figures, it can be seen that the response of surge, heave and pitch motion of the FOWT are same in numerical calculation in the case of blade rotation and no rotation at each angles $\theta$= -4.0, -2.0, 0.0, 2.0, 4.0, 6.0, and 8.0 [deg]. On the other hand, the experimental results almost same beyond the normalized frequencies 0.28 (detail explanation in section 4.6.3). But still little discrepancies are observed in the experimental results. That is mainly because the influence of the difference of the two mooring forces due
to a subtle error in connection to the gravity of the model. The connection of mooring lines is important of small scale model.

Peak response is observed in surge and heave motion (near the resonant period 0.28) in numerical calculation in both cases and each angle. This peak response in the computed results is perhaps due to ignoring the damping effect in the numerical calculation. However, no significance difference is observed in surge, heave and pitch motion in the case of blade rotation and no rotation at each angle. Overall an agreement between the numerical and experimental results is good although.

As it has been described in section 2.6 (chapter 2) theoretically, the effect of the gyro moment (caused by rotation of blades) is seen only in yaw motion and it is confirmed in previous experiment. Therefore, no effect is observed in the case of blade rotation and no rotation in surge, heave and pitch motion at each steady angle but the effect is observed in yaw motion both numerically and experimentally. The yaw motion is smaller in the case of blade no rotation beyond the normalized frequencies 0.28. The computed numerical result of yaw motion is almost zero in the case of blade no rotation at each angle throughout the normalized frequencies. But the experimental results slightly larger than the numerical results in case of blade no rotation. It can be consider that the main reason of such influence is; the absolute value of have motion become large and directly affect to the rigid body motions.
Fig. 4.43: RAO of surge motion at -4.0 [deg.]

Fig. 4.44: RAO of heave motion at -4.0 [deg.]
**Fig. 4.45:** RAO of pitch motion at -4.0 [deg.]

**Fig. 4.46:** RAO of yaw motion at -4.0 [deg.]
**Fig. 4.47:** RAO of surge motion at -2.0 [deg.]

**Fig. 4.48:** RAO of heave motion at -2.0 [deg.]
Fig. 4.49: RAO of pitch motion at -2.0 [deg.]

Fig. 4.50: RAO of yaw motion at -2.0 [deg.]
Fig. 4.51: RAO of surge motion at 0.0 [deg.]

Fig. 4.52: RAO of heave motion at 0.0 [deg.]
**Fig. 4.53:** RAO of pitch motion at 0.0 [deg.]

**Fig. 4.54:** RAO of yaw motion at 0.0 [deg.]
Fig. 4.55: RAO of surge motion at 2.0 [deg.]

Fig. 4.56: RAO of heave motion at 2.0 [deg.]
Fig. 4.57: RAO of pitch motion at 2.0 [deg.]

Fig. 4.58: RAO of yaw motion at 2.0 [deg.]
**Fig. 4.59:** RAO of surge motion at 4.0 [deg.]

**Fig. 4.60:** RAO of heave motion at 4.0 [deg.]
**Fig. 4.61**: RAO of pitch motion at 4.0 [deg.]

**Fig. 4.62**: RAO of yaw motion at 4.0 [deg.]
**Fig. 4.63:** RAO of surge motion at 6.0 [deg.]

**Fig. 4.64:** RAO of heave motion at 6.0 [deg.]
Fig. 4.65: RAO of pitch motion at 6.0 [deg.]

Fig. 4.66: RAO of yaw motion at 6.0 [deg.]
Fig. 4.67: RAO of surge motion at 8.0 [deg.]

Fig. 4.68: RAO of heave motion at 8.0 [deg.]
Fig. 4.69: RAO of pitch motion at 8.0 [deg.]

Fig. 4.70: RAO of yaw motion at 8.0 [deg.]
Figures 4.71–4.74 show that the experimental results for the RAOs of surge, heave, pitch and yaw motion respectively and the comparisons with the numerical simulation using potential theory in the case of blades rotation. Results from multiple experiments at wave period are presented in order to show the variation of motion of FOWT with the change of angle of inclination. As can be seen, the RAOs of surge, heave and pitch are found to yield an identical response throughout the frequencies in the numerical calculation with respect to the change of angle of inclination. But it can be observed that the angle of inclination only has an effect on yaw motion. The RAO of yaw motion is smaller at angle of inclination, compared to the upright condition, which is confirmed numerically and experimentally. It can be attributed to that the moment of inertia around the w-axis increases with enhancing the angle of inclination (see Table 4.17), which obtained with Eqns. (2.68)–(2.70).

Figure 4.75 illustrates an important results of RAO of yaw motion with the change of angle of inclination at normalized frequencies, \(2r \omega^2/g=0.44, 0.57, 0.78\) and 1.13. This figure demonstrates an approximately linear relationship, where the RAO of yaw motion is reduced with increasing the angle of inclination. On the other hand, the yaw response is higher at lower frequencies. So, it is conclude that the effect of the angle of inclination is apparent at lower frequencies. However, such important information of angle of inclination of FOWT under forces would play a positive role in the future design of SPAR-type FOWT.
Fig. 4.71: RAOs of surge motion for different angle of inclination

Fig. 4.72: RAOs of heave motion for different angle of inclination
**Fig. 4.73:** RAOS of pitch motion for different angle of inclination

**Fig. 4.74:** RAOS of yaw motion for different angle of inclination
Fig. 4.75: Relationship between the RAO of yaw motion and the angle of inclination for normalized frequencies $\frac{2\omega^2}{g} = 0.44, 0.57, 0.78$, and 1.13
4.7.3 Remarks

In the above section, the effect of angle of inclination on the dynamic behavior of the FOWT was investigated by the way of experimental and numerical analysis.

In this experiment, we can observe the effect of the angle of inclination on the motion of the FOWT. Throughout the motion analysis; it is possible to conclude that the yaw motion is reduced at angle of inclination. The moment of inertia of FOWT has a significant effect on the yaw motion of the FOWT at angle of inclination. As shown by numerical model in section 2.7, the moment of inertia around the w-axis increase with increasing the angle of inclination.

Also it can be observed that there is good agreement in surge, heave, pitch and yaw RAOs for the numerical simulation and experiments above the normalized frequencies 0.28, confirming that the simulations are capturing the most important physical effects associated with the wave loading on the FOWT. These results could provide valuable information for the design of actual FOWT model to extract the power from open sea.
4.8 Numerical Simulation for Scale Effect

In order to study the scale effects on the motion of FOWT, two different sized models are set up for numerical simulations. The small-sized model stands for the experimental model and the large-sized model stands for the prototype. They are geometrically similar. The conditions are set up according to the Froude number similarity. Hence, they meet the requirements of the geometric similarity and the Froude number similarity.

The aim of the numerical simulation is to study the scale effects on the motion of FOWT, when the simulation results of the small-sized model are used to predict the situations of the large-sized model according to the similitude theory.

The full scale SPAR-type FOWT configurations were used to set the values for full-scale parameter. These configuration parameters are shown in Table 4.4. The scale ratios from Table 4.2 were then applied to determine the corresponding values for the scale model in order to set model design requirement. The rotor rotation for actual model was chosen as based on the Vestas (V-120) 4.5 MW wind turbine. The V-120, 4.5 MW wind turbine has a maximum designed rotor speed of 14.9 RPM [31], which translates into a 283.0 RPM rotor speed for our 360:1 scale model.

The results of numerical simulations of yaw motion of the FOWT for scale model and actual model are shown in Figs. 4.76 and 4.77, respectively. The horizontal axis represents wave period and the vertical axis represents the non dimensional RAO. The trend of these two simulations is the almost same but corresponding periods are different according to Froude law. It can be consider that the scale model does not meet the requirement of Reynolds number similarity.
**Fig. 4.76:** RAO of yaw motion of scale model at angle of inclination

**Fig. 4.77:** RAO of yaw motion of actual model at angle of inclination
Chapter 5

Conclusions

In the present thesis, two experiments were conducted in a water tank using Froude scaled of FOWT model. During the first testing, the gyroscopic effect was investigated on the motion of FOWT. At this stage, emphasis was placed to change the gyroscopic effect with change of angular velocity as well as change the moment of inertia of rotating blades. In order to change the angular velocity as well as moment of inertia of rotating blade small “weights A” (see Fig. 4.7) was positioned on the different location of rotating flat blade. Regular waves were imposed on the model to study surge, heave, pitch, and yaw motion. Motions analysis was conducted by using Fourier analysis.

It is observed that the gyroscopic effect caused by the rotation of flat blade has an effect on yaw motion of FOWT. The response of yaw motion increases with increasing the gyroscopic force. As shown by numerical model (section 2.6), the change of yaw motion depends on the change of angular velocity as well as moment of inertia. Hence, the computed response of yaw motion of FOWT using potential theory exhibits a good agreement with experimental results above the normalized frequency 0.28. These results indicate that the effect of gyro moment on the FOWT could obviously influence the
performance of the wind turbine in actual sea. Hence, the gyro moment results are very useful for the design of the wind turbine and its tower structure.

Second experiments were conducted at the angle of inclination of the model in the same water tank. The experiments were carried out concerning the dynamic behavior of SPAR-type FOWT at the angle of inclination. In marine environment, the FOWT faces many problems, e.g. it can be inclined by the wind. Therefore, to understand the dynamic behavior of actual model at angle of inclination, model test were performed. In order to create the angles, small “weight B” was shifted away from the tower of the model in the both direction of x-axis. In this experiment, the model was subjected in regular waves with known periods to study surge, heave, pitch, and yaw motion. Finally, the RAOs were obtained for each wave period conducting motion analysis by Fourier analysis.

It is observed that the variation of response of motion of a FOWT at angle of inclination is apparent in the yaw motion. The response of yaw motion is reduced at angle of inclination. As shown by theoretically in section 2.7 the moment of inertia around the w-axis increases with increasing the angle of inclination (see Table 4.17). Hence, the computed response of yaw motion of FOWT using potential theory decreases with increasing the angle of inclination and this result exhibits a good agreement with experimental results above the normalized frequency 0.28. This experimental and numerical study indicates that for the actual prototype model, the yaw motion could be reduced at angle of inclination. These results could provide valuable information for the design of FOWT to extract the power from open sea.
**Recommendation for Future Work**

In this study hydrodynamic analysis of FOWT has been carried out by numerically and experimentally. There are different problems which are related to the present research and need to further research on their nature.

In this study, the small scale model was used. However, the slight deviation of connection position of the center of gravity of the model has a significant impact on the experimental results. In addition, the small scale model is not effective at lower frequencies. Therefore, in order to obtain more accuracy of experimental data, a large scale model experiment is required.

Wind has not been generated in this research. In view of operating in marine environment, the FOWT experiences aerodynamic thrust load and gyroscopic effect. A clear understanding of this combined effect can lead to better design and operation of such system. The present study is restricted in regular waves; however, it can be extended in irregular waves.
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1971.


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