Yield Analysis of Large-Scale Adiabatic-Quantum-Flux-Parametron Logic: The Effect of the Distribution of the Critical Current

D. Si, N. Takeuchi, K. Inoue, Y. Yamanashi, N. Yoshikawa

Department Electrical and Computer Engineering, Yokohama National University,

79-5 Tokiwadai, Hodogaya, Yokohama 240-8501, Japan

Corresponding author: N. Yoshikawa, +81-45-339-4259, e-mail: nyoshi@ynu.ac.jp

Abstract

We performed yield analysis of large-scale adiabatic-quantum-flux-parametron (AQFP) circuits using circuit simulations based on the Monte Carlo method assuming the distribution of the critical current of Josephson junctions. Based on the simulation results, we also made an analytical model for the circuit yield of large-scale AQFP circuits. The yield analysis indicates that AQFP integrated circuits containing 1 million gates can be realized when the interconnect inductance is about 40 pH and the normalized standard deviation of the critical current is less than 1%.

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1. Introduction

We are studying adiabatic-quantum-flux-parametron (AQFP) logic for realizing ultimately energy-efficient integrated circuits. In AQFP logic, the shape of the potential energy is adiabatically or gradually changed from a single-well potential to a double-well potential, by which the energy consumption of the logic gate can be reduced proportional to the operating frequency. We have investigated the parameter conditions for the AQFP logic gate to operate in the adiabatic mode [1, 2], and verified its low-energy logic operation with 10 zJ at 5 GHz experimentally [3]. The functional operation of a 1-bit AQFP full adder has been also demonstrated [4].

The current gain of the AQFP logic gate is substantially large and infinite in the ideal case where there is no distribution in the circuit parameters and no thermal fluctuation. However the non-uniformity of the critical current of the junction pair composing the AQFP logic gate brings about the offset in the input current, resulting in the malfunction of the logic gate in an extreme case. Besides, a long interconnection between AQFP gates causes the reduction of the input current for the next AQFP gate and enlarges the possibility of the malfunction. Both these effects would be obstacles when realizing large-scale AQFP integrated circuits.

In this study, we evaluate the relationship between the circuit yield and the distribution of the critical current I_c of AQFP circuits using circuit simulations based on the Monte Carlo method [5-8]. Furthermore, we also make an analytical model for the circuit yield of large-scale AQFP circuits based on the simulation results, taking account of the distribution of the critical current and the interconnect inductance.

Fig. 1 11 stages of AQFP gates used in the Monte Carlo simulations. Critical current of the junctions of the first stage AQFP gate: $I_{c01} = I_{c02} = 50 \mu\text{A}$. Critical current of the junctions of 10 stages of AQFP gates: $I_{c1} = I_{c2} = \dots = I_{c20}$ (Scattered according to the Gaussian distribution function whose average value, I_{c0} is $50 \mu\text{A}$). Coupling factors: $k_{x01} = k_{x02} = k_{x1} = k_{x2} = \dots = k_{x20} = k_1 = k_2 = \dots = k_{10} = 0.3$. Interconnect inductances between AQFP gates: $L_{imp1} = L_{imp2} = \dots = L_{imp10}$. Loop inductances of AQFP gates: $L_{01} = L_{02} = L_1 = L_2 = \dots = L_{20} = 1.32 \text{ pH}$. Output inductances of AQFP gates: $L_{out1} = L_{out2} = \dots = L_{out9} = 10.5 \text{ pH}$. Input inductances of AQFP gates: $L_{in} = L_{in1} = L_{in2} = \dots = L_{in10} = 10.5 \text{ pH}$. Input current for the first stage AQFP gate: $I_{in} = 5 \mu\text{A}$. Excitation current of AQFP gates: $I_{x1} = I_{x2} = I_{x3} = 1343 \mu\text{A}$.

Fig. 2 The dependence of the correct operation probability of 11 stages of AQFP gates on the normalized standard deviation σ/I_{c0} of the critical current for the different values of the interconnect inductances L_{imp} .

2. Evaluation of circuit yield using Monte Carlo method

The circuit yields of multiple stages of AQFP gates were calculated by using the Monte Carlo method, where each circuit parameter was scattered according to the Gaussian distribution function. We prepared many sets of circuit parameters for multiple stages of AQFP gates, and repeated circuit simulations many times to evaluate the circuit yields.

11 stages of AQFP gates used in the Monte Carlo simulation are shown in Fig. 1. In the simulation, the critical currents of 10 stages of AQFP gates except the first stage were scattered according to the Gaussian distribution function with the standard deviation of σ . We have prepared circuit parameters for 25,000 sets of the AQFP circuits, whose critical currents were scattered, and conducted circuit simulations to evaluate the probability of the correct operation of the circuit for different interconnect inductance L_{inp} .

Fig. 2 shows the dependence of the correct operation probability of the 11 stages of AQFP gates on the normalized standard deviation σ/I_{c0} of the critical current for different values of the interconnect inductances L_{inp} . It can be seen from the figure that the correct operation probability decreases with the increase of the standard deviation σ . In addition, the probability also decreases when the interconnect inductance becomes large. Therefore, an acceptable interconnect inductance L_{inp} exists for the given standard deviation σ , which is determined by the fabrication process.

Fig. 3. Comparisons of the correct operation probabilities of the 11 stages of AQFP gates derived from the analytical model and the Monte Carlo simulations. (a) $L_{\text{inp}} = 50 \text{ pH}$ and $\Delta I_{c\text{RL}} = 3.4 \text{ } \mu\text{A}$, (b) $L_{\text{inp}} = 30 \text{ pH}$ and $\Delta I_{c\text{RL}} = 4.93 \text{ } \mu\text{A}$, (c) $L_{\text{inp}} = 70 \text{ pH}$ and $\Delta I_{c\text{RL}} = 2.59 \text{ } \mu\text{A}$.

Fig. 4. The relationship between the normalized standard deviation σ/I_{c0} of critical current and the circuit yield for different L_{inp} in a million stages of AQFP circuits.

(a)

(b)

Fig. 5. The relationship between the normalized standard deviation σ/I_{c0} of critical current and the circuit yield for different circuit scale when L_{inp} is 40 pH. (a) $k = 0.3$, (b) $k = 0.5$.

3. Circuit yield model for large-scale AQFP integrated circuits

We will make an analytical model to estimate the circuit yields of large-scale AQFP circuits. Since the distribution of the critical current of AQFP circuits obeys the Gaussian distribution function, the model can be made based on the error function. We define the parameter ΔI_{cRL} , which represents the maximum allowable difference between the two critical currents of the AQFP gate for the correct operation. ΔI_{cRL} can be determined by circuit simulations.

By using the ΔI_{cRL} , the correct operation probability of N stages of AQFP gates can be derived by the error function. Assuming the standard deviation of the critical current of the junctions composing AQFP circuits is σ , the standard deviation of the difference between the two critical currents of the AQFP gate is given by

$$\sigma_{\Delta I_c} = \sqrt{2}\sigma. \quad (1)$$

As stated above, the AQFP gate can operate correctly when the difference between the two critical current of the AQFP gate is within $\pm\Delta I_{cRL}$. Therefore, the correct operation probability of the AQFP gate is given by

$$P_1 = \text{erf}\left[\frac{\Delta I_{cRL}}{\sqrt{2}(\sigma_{\Delta I_c})}\right] \quad (2)$$

by using the error function. The correct operation probability of the N stages of AQFP gates is, then, given by

$$P_N = (P_1)^N. \quad (3)$$

The relationship between interconnect inductance L_{imp} and ΔI_{cRL} was calculated and is listed in Table 1. The comparisons of the correct operation probabilities of the 11 stages of AQFP gates for different values of the interconnect inductances L_{imp} are shown in Fig. 3, where the curve represents the calculation results using the equations (1) – (3) and the dots are the simulation results using the Monte Carlo method. From the figure, we can find that the curve using the analytical model agrees well with the results calculated by the Monte Carlo method for the different valued of the interconnect inductance. Therefore, it can be considered that the analytical model is appropriate for calculating the circuit yields of large-scale AQFP circuits.

Table 1. The relationship between interconnect inductance L_{imp} and ΔI_{cRL} .

L_{imp} [pH]	ΔI_{cRL} [μA]
10	8.93
20	6.36
30	4.93
40	4.03
50	3.40
60	2.94
70	2.59
80	2.31

4. Evaluation of the circuit yield of large-scale AQFP integrated circuit

Utilizing the analysis method derived in the previous section, we will estimate the circuit yields of the large-scale AQFP circuits for the different values of the standard deviation of the critical current and the interconnect inductance. Fig. 4 shows the dependence of the circuit yields of a million stages of AQFP gates on the normalized standard deviation σ/I_{c0} of the critical current for different values of the

interconnect inductance L_{inp} . As can be seen in the figure, in order to achieve a million stages of AQFP gates with a 98% yield, the normalized standard deviation σ/I_{c0} of the critical current is required to be less than 2.3%, 1.6%, 1.0% and 0.58% for the interconnect inductance L_{inp} of 10 pH, 20 pH, 40 pH and 80 pH, respectively. In our current design of AQFP circuits, the interconnect inductance is limited to be less than 40 pH for insuring enough operating margin. When using the COU layer in the AIST standard process [9], this inductance value corresponds to the wire with the length and width of 400 μm and 8 μm , respectively. In this condition, we can estimate that a million stages of AQFP gates can be achieved, when the normalized standard deviation σ/I_{c0} of the critical current is less than 1.0%. It should be noted here that this analysis is performed in the pessimistic condition, where all AQFP gates are connected by wires of the same length with the largest inductance. In the actual circuit design, however, most gates are connected each other by using shorter wires with smaller inductance, resulting in larger circuit yields.

Fig. 5(a) shows dependence of the circuit yields of the AQFP circuits with the interconnect inductance of 40 pH on the circuit scale. It can be seen from the figure that the normalized standard deviation σ/I_{c0} of the critical current is required to be less than 1.3%, 1.1% and 0.94% to achieve AQFP circuits with the scale of 1000 stages, 100 thousand stages and 10 million stages, respectively.

The coupling coefficient k of the superconducting transformer between the AQFP gates is assumed to be 0.3 in the current design. It is possible to improve the coupling coefficient k up to about 0.5 by improving coupling structure. Fig. 5(b) shows the relationship between the circuit yields of AQFP circuits and the normalized standard deviation σ/I_{c0} of the critical current when the interconnect inductance L_{inp} is 40 pH and coupling coefficient k is 0.5 for different circuit scales. From these results, one can see that larger scale of AQFP circuits can be achieved if the coupling coefficient k is increased, because the input current to the next AQFP gate is increased.

5. Conclusion

We investigated the circuit yields of AQFP circuits by using the Monte Carlo method, when the critical current of the junction is scattered due to the fabrication process. Based on the Monte Carlo simulation results, we established an analytical model for estimating the circuit yield of large-scale AQFP integrated circuits using the error function. It was shown from the analytical model that AQFP integrated circuits containing 1 million gates can be realized when the wire length is less than 400 μm , and the normalized standard deviation of the distribution of the critical current is less than 1.0%.

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