Bessel–Thompson Filter Using Double-Series-Coupled Microring Resonator

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Abstract—We analyzed the dispersion characteristics of double-series-coupled microring resonator filter and found that the coupling efficiency between resonators for the zero third-order dispersion (Bessel–Thompson) filter is one-third of that of a flat passband (Butterworth) filter. By controlling the coupling efficiencies between waveguides, the flat-top group delay response (Bessel–Thompson filter response) was realized. However, under the symmetric arrangement of coupling efficiency, the Bessel–Thompson filter lost transmittance by 1.25 dB. Thus we found the optimum condition of coupling efficiencies for achieving zero third-order dispersion (flat-group delay) with unity transmittance is an asymmetric arrangement of coupling efficiencies.

Index Terms—Microresonators, optical waveguide filters, transfer function matrices, wavelength-division multiplexing.

I. INTRODUCTION

MICRORING resonators can create arbitrary spectrum response shape by synthesizing them [1]–[5]. The series-coupled microring resonator filter is advantageous to add/drop multiplexers since it can achieve a box-like spectrum response by optimizing the coupling efficiencies between the busline and the resonator and between the resonators. By controlling the resonant wavelengths of individual resonators independently, a hit-less wavelength-selective switch can be realized [6], [7]. Since the group delay dispersion (GDD) of resonator type filters, such as ring and Fabry–Perot resonators, is zero [8], [9], the influence of dispersion is small in the microring resonator [10]. Although the GDD of microring resonators of center wavelength is zero, the dispersion compensators by the superposition of delay characteristics of microring resonators are proposed and demonstrated [11]–[14]. However, the third-order dispersion (TOD) of microring resonators has not been discovered.

Generally, in an ultra-high-speed optical time-division multiplexing (OTDM) transmission system, GDD resulting from the group velocity dispersion (GVD) of transmission lines must be compensated. Although the GDD of microring resonators

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Fig. 1. Schematic of double-series-coupled microring resonator.

is zero, the TOD influences the transmission characteristics. The interaction between the TOD and the self-phase modulation causes the signal distortion [15]. Thus a TOD-free transmission and a TOD compensation are required. However, the conditions for the zero third-order dispersion of series-coupled microring resonator have not been made clear up to now. Thus, in this paper, we investigate the dispersion characteristic of double-series-coupled microring resonator and derive the optimum condition for zero third-order dispersion, which is generally known as the Bessel–Thompson filter. In particular, since the loss of zero third-order dispersion design under the symmetric arrangement of coupling efficiencies gets worse than the non-zero third-order dispersion design, we discovered a lossless third-order dispersion condition using an asymmetric arrangement of coupling efficiencies.

II. TRANSFER FUNCTION AND DISPERSION CHARACTERISTICS OF DOUBLE-SERIES-COUPLED MICRORING RESONATOR

The transmission characteristics of microring resonator are determined by the coupling efficiency between waveguides κ . The transfer functions of drop-port response D and through-port response T for a double-series-coupled microring resonator as shown in Fig. 1 are obtained by using the matrix formation [16] as follows:

$$D(\phi) = \frac{\sqrt{a(1-t_{b1})(1-t_r)(1-t_{b2})}}{e^{j\phi} - \sqrt{at_r}(\sqrt{t_{b1}} + \sqrt{t_{b2}}) + a\sqrt{t_{b1}t_{b2}}e^{-j\phi}} \quad (1)$$

$$T(\phi) = \frac{\sqrt{t_{b1}}e^{j\phi} - \sqrt{at_r}(1 + \sqrt{t_{b1}t_{b2}}) + a\sqrt{t_{b2}}e^{-j\phi}}{e^{j\phi} - \sqrt{at_r}(\sqrt{t_{b1}} + \sqrt{t_{b2}}) + a\sqrt{t_{b1}t_{b2}}e^{-j\phi}} \quad (2)$$

where a is the one round-trip transmittance in the resonator, t_{b1} is the transmittance at the coupling region between the input-through busline and the resonator, t_r is the transmittance at the

Fig. 2. Transmittance of double-series-coupled microring resonator.

coupling region between resonators, and t_{b2} is the transmittance at the coupling region between the add-drop busline and the resonator. Resonators are identical and phase shift in the resonators is $\phi = (\omega/c)n_{eq}L$. For the ease of formalism, the transfer function is expressed in terms of the transmittance at the coupling region $t = 1 - \kappa$.

The transmittance of double-series-coupled microring resonator varies depending on the coupling efficiencies, as shown in Fig. 2, where κ_{rB} is the coupling efficiency between resonators for the Butterworth response (maximally flat passband condition [1]).

The phase term of the drop-port response Φ_d is obtained by

$$\Phi_d = \tan^{-1} \left[\frac{\sqrt{k_d} - A_d t_r \sin(\phi)}{A_d t_r - \sqrt{k_d} \cos(\phi)} \right] \quad (3)$$

where

$$A_d = \frac{(\sqrt{t_{b1}} + \sqrt{t_{b2}})^2}{4\sqrt{t_{b1}t_{b2}}} t_r \quad (4)$$

$$k_d = \frac{(\sqrt{t_{b1}} + \sqrt{t_{b2}})^2 (1 + a\sqrt{t_{b1}t_{b2}})^2}{16at_{b1}t_{b2}} t_r. \quad (5)$$

A_d denotes the symmetric property between t_{b1} and t_{b2} . When the coupling configuration is symmetric, $A_d = t_r$. k_d denotes the deviation from the Butterworth condition [16]. The coupling efficiency between resonators at the Butterworth filter condition is defined by $t_{rB} = t_r/k_d$. When $k_d = 1$, the boxlike spectrum, i.e., the Butterworth spectrum, is obtained. When $k_d < 1$, the transmission spectrum has double peaks. When $k_d > 1$, the transmission spectrum has a reduced single peak.

Group delay τ_d is obtained by differentiating (3) with respect to the angular frequency ω

$$\begin{aligned} \tau_d &= \frac{\partial \Phi_d}{\partial \omega} = \frac{\partial \phi}{\partial \omega} \frac{\partial \Phi_d}{\partial \phi} \\ &= \frac{n_{eq}L}{c} \frac{\sqrt{k_d} - A_d (\sqrt{k_d} - A_d \cos(\phi))}{[(1 - A_d)(k_d - A_d) + A_d (\sqrt{k_d} - \cos(\phi))^2]}. \end{aligned} \quad (6)$$

Fig. 3 shows the group delay spectrum responses of the drop-port for various coupling conditions. Although the group delay has a single peak in the single microring resonator, the flat-top double-series coupled microring resonator has two peaks in the group delay characteristics as shown by the solid line in Fig. 3.

Fig. 3. Group delay characteristics.

Fig. 4. Group delay dispersion.

The group delay at the center wavelength is determined by the normalized integrated transmittance I_T

$$I_T = \int_{-\pi}^{\pi} |D(\phi)/D(0)|^2 d\phi. \quad (7)$$

The group delay dispersion of the drop-port response GDD_d is obtained by differentiating (6) with respect to the angular frequency ω and is given by

$$\begin{aligned} GDD_d &= \frac{\partial^2 \Phi_d}{\partial \omega^2} = \left(\frac{\partial \phi}{\partial \omega} \right)^2 \frac{\partial^2 \Phi_d}{\partial \phi^2} \\ &= \left(\frac{n_{eq}L}{c} \right)^2 \frac{\sqrt{k_d} - A_d \sin(\phi)}{[(1 - A_d)(k_d - A_d) + A_d (\sqrt{k_d} - \cos(\phi))^2]}. \end{aligned} \quad (8)$$

Fig. 4 shows the group delay dispersion of the drop-port response. The slope of the group delay dispersion at the center wavelength varies depending on the coupling efficiencies. The slope of the single microring resonator is opposite to that of the Butterworth double-series-coupled microring resonator. When the coupling efficiency between resonators is weak, the slope is close to horizontal.

The third-order dispersion of drop-port response at the center frequency ($\phi = 2\pi n$, n is an integer) TOD_d is given by the third-

Fig. 5. Third-order dispersion at the center frequency.

order derivation of Φ_d with respect to the angular frequency, ω and is given by

$$\begin{aligned} \text{TOD}_d &= \left. \frac{\partial^3 \Phi_d}{\partial \omega^3} \right|_{\phi=2\pi n} = \left(\frac{\partial \phi}{\partial \omega} \right)^3 \left. \frac{\partial^3 \Phi_d}{\partial \phi^3} \right|_{\phi=2\pi n} \\ &= \left(\frac{n_{eq} L}{c} \right)^3 \frac{\sqrt{k_d} - A_d [2A_d \sqrt{k_d} - \sqrt{k_d} - A_d]}{(A_d - \sqrt{k_d})^3}. \quad (9) \end{aligned}$$

Fig. 5 shows the third-order dispersion at the center frequency. TOD takes maxima at the Butterworth condition. When the coupling efficiency between resonators is about one-third of the Butterworth condition, TOD is zero.

The coupling condition for the Bessel–Thompson filter, where the third-order dispersion is zero, is given by (10) shown at the bottom of the page. In the symmetric arrangement, $t_{b1} = t_{b2} = t_b$, (10) is rewritten as

$$t_r = \frac{1 + 4t_{rB} - \sqrt{1 + 8t_{rB}}}{2t_{rB}} \sim \frac{2 + t_{rB}}{3} \quad (11)$$

where t_{rB} is the transmittance at the coupling region between resonators—for the Butterworth filter, $t_{rB} = 1 - \kappa_{rB}$.

Fig. 6 shows the coupling efficiencies for the Bessel–Thompson filter response. The Bessel–Thompson filter is realized when κ_r is about one-third of that of the Butterworth filter condition.

As shown in Fig. 2, the bandwidth of the Bessel–Thompson response is narrower than that of the Butterworth response. The finesse of the Butterworth and the Bessel–Thompson response changes with the coupling efficiency between busline and resonator, as shown in Fig. 7. The 3 dB-bandwidth of the Bessel–Thompson response is 1.5 times narrower than that of the Butterworth response.

III. FABRICATION AND MEASUREMENT

We fabricated double-series-coupled microring resonators as shown in Fig. 1. The lower cladding material was thermally

Fig. 6. Coupling efficiency for Bessel–Thompson response.

Fig. 7. Finesse of double-series-coupled microring resonator with Butterworth and Bessel–Thompson response.

grown SiO_2 ($n = 1.44$ at $\lambda = 1.55 \mu\text{m}$). The core material was sputter deposited 17 mol% $\text{Ta}_2\text{O}_5 - \text{SiO}_2$ ($n = 1.65$ at $\lambda = 1.55 \mu\text{m}$) and the channel waveguides were formed by photolithography and reactive ion etching using C_2F_6 gas. The upper cladding material was sputter deposited SiO_2 ($n = 1.45$ at $\lambda = 1.55 \mu\text{m}$). The core size was $1.4 \times 1.2 \mu\text{m}^2$, which satisfied the single-mode condition. The resonators and the busline waveguides were formed in the same layer. The ring resonator was a racetrack shape, and the coupling strength was controlled by changing the length of the parallel region of the straight waveguides L_r . The radius of the racetrack resonator was $100 \mu\text{m}$ and the length of the straight region was $35 \mu\text{m}$. The core spacing between the busline and the resonator was $1.0 \mu\text{m}$, and that between resonators was $1.3 \mu\text{m}$.

Figs. 8 and 9 show the measured group delay of the symmetric double-series-coupled microring resonator with $L_r = 12$ and $10 \mu\text{m}$, respectively. These characteristics were measured using an optical network analyzer Q7761 made by ADVANTEST. The polarization of the measured results was the TM-mode. There was slight polarization dependence because the coupling efficiency had polarization dependence. When the transmittance is slightly flat, as shown in Fig. 8, the group delay has dual peaks.

$$t_r = \frac{\left[1 + 18a\sqrt{t_{b1}t_{b2}} + a^2t_{b1}t_{b2} - (1 + a\sqrt{t_{b1}t_{b2}})\sqrt{1 + 34a\sqrt{t_{b1}t_{b2}} + a^2t_{b1}t_{b2}} \right]}{2a(\sqrt{t_{b1}} + \sqrt{t_{b2}})^2} \quad (10)$$

Fig. 8. Measured group delay of double-series-coupled microring resonator ($L_r = 12 \mu\text{m}$).

Fig. 9. Measured flat-top group delay of double-series-coupled microring resonator ($L_r = 10 \mu\text{m}$).

On the other hand, when the coupling efficiency between resonators is smaller than Fig. 8, the flat-top group delay is successfully achieved.

IV. LOSSLESS BESSEL–THOMPSON FILTER BY ASYMMETRIC COUPLING ARRANGEMENT

In the Bessel–Thompson filter condition (10), the peak transmittance is reduced as shown in Fig. 2. The peak transmittance p is given by

$$p = \frac{a(1 - t_{b1})(1 - t_r)(1 - t_{b2})}{[1 + a\sqrt{t_{b1}t_{b2}} - \sqrt{a}(\sqrt{t_{b1}} + \sqrt{t_{b2}})\sqrt{t_r}]^2}. \quad (12)$$

Typically in the higher order series coupling, the symmetric arrangement $\kappa_{b1} = \kappa_{b2}$ is adopted for the higher transmittance. However, the peak transmittance is reduced when the coupling efficiency between resonators is weaker than the Butterworth filter condition. As shown in Figs. 8 and 9, the peak transmittance is slightly reduced in the flat-top group delay response. In the Bessel–Thompson condition, the peak transmittance is reduced as shown in Fig. 10. In the lossless case, the peak transmittance is 0.75. The remaining part of optical power transmits to the throughput.

When the coupling arrangement is asymmetric, the coupling condition for the Bessel–Thompson filter condition is given as shown in Fig. 11.

Fig. 12 shows the peak transmittance at the coupling condition for the Bessel–Thompson filter condition. When the ratio of coupling efficiencies between busline and resonator $\kappa_{b1} : \kappa_{b2}$ is

Fig. 10. Peak transmittance (symmetric coupling).

Fig. 11. Coupling condition for zero third-order dispersion.

Fig. 12. Peak transmittance at the asymmetric coupling arrangement and zero third-order dispersion.

1:3 or 3:1, the Bessel–Thompson filter response with the high peak transmittance is realized.

Fig. 13 shows the condition of coupling efficiencies κ_r and κ_{b2} as a function of κ_{b1} for the zero third-order dispersion with unity transmittance. When $\kappa_{b1} \ll 1$, κ_{b2} and κ_r can be simply expressed in terms of κ_{b1} by

$$\kappa_{b2} \sim \frac{\kappa_{b1}}{2(2 - \kappa_{b1})}, \quad (13)$$

$$\kappa_r \sim \kappa_{b2}^2 \sim \frac{\kappa_{b1}^2}{4(2 - \kappa_{b1})^2}. \quad (14)$$

V. CONCLUSION

The coupling efficiency condition for the Bessel–Thompson filter is derived. The coupling efficiency between resonators for

Fig. 13. Coupling efficiency for zero third-order dispersion and high peak transmittance.

the Bessel–Thompson filter is one-third of that of the Butterworth filter. The flat-top group delay response using double-series-coupled microring resonators was experimentally demonstrated.

However, the peak transmittance is reduced in the flat-top group delay response. Accordingly, we proposed an asymmetric coupling arrangement as the optimum condition to simultaneously realize the high peak transmittance and flat-top group delay.

By detuning from the Bessel–Thompson filter condition, both positive and negative TOD can be realized. Thus the double-series-coupled microring resonator has the possibility of providing TOD-free filter as well as either positive or negative TOD compensator.

This design may expand to the higher series-coupled microring resonator. However, the group delay response is corresponding to the transmission response with a Kramers–Kronig relation. The spectrum shape at the passband is the same regardless of the order in the Bessel–Thompson response. By increasing the order of series-coupling, the spectrum shape approaches to the Gaussian response.

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