Index profiling of distributed-index lenses by a shearing interference method

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It is demonstrated that the transverse differential interference method is used as a reliable profiling technique for distributed-index rod lenses, and its accuracy and spatial resolution have been made clear. By the use of this technique the index profile of a distributed-index planar microlens which has 3-D index distribution has been measured.

I. Introduction

The refractive-index profiling of distributed-index lenses is required to characterize imaging and focusing properties of lenses which are related to fourth- and sixth-order coefficients of the series expansion expression of the index profile. The transverse interference method has been used for nondestructive index profiling of thin optical fibers.^{1,2,3} It was difficult, however, to apply this method to thick samples such as distributed-index rod lenses and preform rods of fibers, since the fringe shift in the interference pattern is so large that it overflows from the field of the interference microscope. To overcome this difficulty we propose the transverse differential interference (TDI) method.^{4,5}

In this paper we demonstrate the application of the TDI method^{4,5} to thick distributed-index lenses and preform rods. The optimum values of parameters which optimize the accuracy and spatial resolution are discussed. The application to a 3-D distributed-index microlens is also demonstrated.

II. Transverse Differential Interferometry

Figure 1 shows the optical setup of the TDI method. The sample is immersed in the index-matching oil whose refractive index is close to that of the surface or cladding of the sample. The sample is illuminated in its transverse direction. One of the beams divided by a Mach-Zehnder interferometer is shifted by s , which is small compared with the size of the sample, by means of a shearing prism. The fringe shift corresponds to the

difference of the optical path length between the original image and the shifted one, and it is approximated by the differential of the ordinary interference fringe shift. When we assume that the index profile is cylindrically symmetric, the index profile is calculated by¹

$$n(u) = n_2 \cdot \exp \left[-\frac{1}{\pi} \int_{u/n_2}^a \frac{\lambda}{D \cdot S} R_d(y) \frac{dy}{\sqrt{n_2^2 y^2 - u^2}} \right], \quad (1)$$

where n_2 is the refractive index of the outer surface or the cladding, a is the radius of the sample, D is the fringe interval, λ is the wavelength, and $R_d(y)$ is the observed fringe shift. The transverse distance r is given by

$$r = u/n(u).$$

Figure 2 shows an example of the measured profile of a distributed-index (or GRIN) rod lens. From this profile, the parabolic fourth- and sixth-order coefficients g , h_4 , and h_6 are evaluated to be 0.34, 4.6, and -44.5, respectively.

Figure 3 shows an example of the measured profile of a preform rod. This sample was observed to have ~9.0% ellipticity by rotating the preform rod.⁵ Figure 4 shows the change of the diameter and radii from different sides vs the rotation angle. Therefore, this ellipticity was corrected for in Fig. 3 by using an analysis discussed in Appendix A, and the index profiles measured in the directions of the major and minor axes almost coincide with each other.

III. Accuracy and Resolution

The accuracy of the TDI method is mainly determined by the reading error resulting from the data reading and the principal error resulting from the approximation of the difference by the differential of optical path length. The former corresponds to the standard deviation of the error, and from a computer simulation it was found that this is evaluated by⁵

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Fig. 1. Measuring system of transverse differential interferometry.

Fig. 2. Refractive-index profile of a distributed-index (or GRIN) rod lens.

Fig. 3. Measured index profile of a preform rod.

$$\frac{\epsilon}{\Delta n} = 0.19 \frac{\lambda}{2S \cdot \Delta n} \cdot \frac{\delta R}{D}, \quad (2)$$

where Δn is the index difference between the maximum index and the refractive index of the cladding, and δR is the reading error of data. The latter corresponds to the mean value of the error $\langle \delta n \rangle$ and varies with the ratio of the shearing distance to the size of the sample as shown in Fig. 5, because when the shearing distance becomes large the fringe shift cannot be approximated by the differential of the optical path length. The total error is given by the sum of errors given by Eq. (2) and

Fig. 4. Deposited cladding diameter and radius variation vs the rotation angle.

Fig. 5. Mean error vs the core radius normalized by S .

Fig. 6. Core radius dependence of the error resulting from the data reading in the TDI method.

Fig. 7. Lateral thin plate.

Fig. 8. Shearing interference pattern of a sliced sample cut from the surface of a planar microlens.

shown in Fig. 5, as shown in Fig. 6. Therefore, the optimum value of the shearing distance is obtained by the condition that the total error takes the minimum value as follows:

$$S_{\text{opt}} \approx \left(0.41 \frac{\lambda}{\Delta n} \cdot \frac{\delta R}{D} \right)^{0.4} a^{0.6}, \quad (3)$$

where $\alpha = 2$ has been assumed. For example, when we assume that $\Delta n = 0.05$, $a = 1$ mm, $\lambda = 0.65$ μm , and $\delta R/D = 0.02$, the optimum is 19.5 μm and we can measure the index profile within the error of 0.4% .

The spatial resolution is determined by the shearing distance and the sampling interval of data. Therefore, the preferred sampling interval is close to the shearing distance (Appendix B).

IV. Index Profiling of Planar Microlens: Sliced Sample Method

The shearing interference method can be applied to sliced samples which have a circularly symmetric index profile such as sliced rod lenses and sliced preform rods. As an example we sliced the surface of a distributed index planar microlens⁶⁻⁸ made of glass into a lateral thin plate 50 μm thick as shown in Fig. 7. The sample was illuminated in the direction normal to the surface, and the shearing interference pattern was observed as shown in Fig. 8 by using the same interference microscope. When we assume that the sample is much thinner than the diffusion length of dopants and the index profile is radially symmetric, the index profile near the surface of the planar microlens can be obtained from the observed fringe shift as shown in Fig. 9. The index profile was calculated by solving a difference equation instead of approximating the difference by the differential of the optical path length. The radius of the mask which was used to prevent the diffusion of dopants is ~ 0.55 mm.⁷ The discrepancy between measured and theoretical profiles seems to result from an assumption that the diffusion constant is independent of the concentration of dopants. The diffusion constant may be dependent on the concentration of dopants, or the interaction of dopants seems to be a possible cause of this discrepancy.

The index profile along the optical axis is obtained by using a sliced sample containing the optical axis perpendicular to the surface as shown in Fig. 10. In this case the shearing distance must be larger than the size

Fig. 9. Measured index distribution of the surface of a planar microlens.

Fig. 10. Longitudinal thin plate.

Fig. 11. Interference pattern of a longitudinal thin plate.

Fig. 12. Refractive-index profile of a planar microlens in the depth direction.

of the sample. Figure 11 shows the total shearing interference pattern of this longitudinal thin plate. The thickness was $159 \mu\text{m}$. The interference fringes correspond to the cross section of equi-index surfaces. Therefore, an index profile in an arbitrary direction can be obtained by scanning in that direction. The index profile in the depth direction was obtained from this pattern as shown in Fig. 12. The large discrepancy between measured and theoretical curves seems to also be caused by the concentration dependent diffusion or the interaction of dopants. Figure 13 shows the 2-D index profile of this planar microlens. (The planar microlens has a 3-D index profile, but the profile can be represented by a 2-D function of the transverse distance and the depth because the profile is symmetric around the optical axis.) Dots represent the points sampled from the equi-index curves in Fig. 11.

V. Conclusions

The transverse differential interference method is suitable for thick samples such as distributed-index lenses and preform rods. The error can be reduced to $<1\%$ of the index difference when the shearing distance is chosen to be the optimum value. The same optical

setup is also applied to the longitudinal shearing interference method. When the sample is cut into sliced thin plates, a 3-D index profile such as a distributed-index planar microlens can be measured.

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Appendix A: Correction Formula of Elliptic Deformation in Determining the Index Profile

As mentioned in Sec. II, actual preform rods have some amount of ellipticity. Therefore, it becomes necessary to correct for the effect of elliptic deformation in determining the index profile precisely. Thus far, Chu⁹ has proposed a formula which corrects for the effect of elliptic deformation on a transverse interference fringe shift. But this formula includes an integral from zero to infinity at the point of $r = 0$, and so it is not suited for the numerical calculation.

Now, we assume a fiber or a preform rod sample which has an elliptic cross section as shown in Fig. 14. The refractive-index profile is assumed to be represented by $n(r')$ where

$$r'^2 = \left(\frac{x}{e}\right)^2 + y^2. \quad (\text{A1})$$

Fig. 13. Three-dimensional index distribution of a planar microlens.

Fig. 14. Elliptical deformation of core and ray trajectory (e : ellipticity).

The transverse interference pattern is assumed to be observed in the direction of the major or the minor axis. Although the probing beam is actually refracted through the core, in this analysis the straight ray trajectory is assumed for simplicity. Then, the transverse interference fringe shift $R(y)$ is given by

$$\begin{aligned} \frac{\lambda}{D} R(y) &= 2 \int_0^{e\sqrt{a^2-y^2}} [n(r') - n_2] dx \\ &= 2e \int_y^a \frac{[n(r') - n_2] r' dr'}{r'^2 - y^2}. \end{aligned} \quad (\text{A2})$$

This is the Abelian transform of $n(r') - n_2$, and the refractive-index profile is given by

$$n(r') = n_2 - \frac{1}{\pi} \int_{r'}^a \times \frac{\lambda}{D} \left(\frac{1}{e} \cdot \frac{dR(y)}{dy} \right) \frac{dy}{\sqrt{y^2 - r'^2}}. \quad (\text{A3})$$

Since this analysis assumes the straight ray trajectory, Eq. (A3) involves the principal error of $\sim 0.8\%$ of the index difference. However, if Eq. (1) which corrects for the ray refraction is used to calculate the index profile including an elliptical deformation, the error due to the elliptical deformation is about $(1 - e)$ of the index difference. Therefore, when the ellipticity is larger than 1%, Eq. (A3) should be used to correct for the elliptical deformation.

Appendix B: Spatial Resolution

The spatial resolution is determined by both the resolving power of the microscope and the maximum spatial frequency limited by the sampling interval of data. The resolving power d is approximately expressed by

$$d = 0.61 \frac{\lambda}{\text{N.A.}},$$

where λ is the wavelength of light source, and N.A. is the numerical aperture of the microscope object lens. The resolving power is of the same order of magnitude as the wavelength. The maximum spatial frequency limited by the sampling interval is calculated in the following way.

The Fourier transform of the data reduction formula Eq. (1) gives

$$\begin{aligned} N(\Omega) &= \int_0^\infty g(y) J_0(\Omega y) y dy \\ [g(y) &= 0, \quad a < y] \end{aligned} \quad (\text{B1})$$

where $N(\Omega)$ is the Fourier transform of $n(r) - n_2$, $g(y) = -\lambda/(2D) \cdot 1/y \times (dR)/(dy)$, and Ω is the spatial angular frequency. Equation (B1) states that $N(\Omega)$ is the Hankel transform of $g(y)$. When $N(\Omega) = 0$ for $\Omega > \Omega_m$, the Hankel inversion of Eq. (B1) gives

$$g(y) = \Omega_m^2 \int_0^1 N(k) J_0(\Omega_m k y) k dk. \quad (\text{B2})$$

On the other hand, $N(\Omega)$ is expressed by the Fourier-Bessel transform as follows:

$$N(k) = \sum_{\nu=1}^\infty a_\nu J_0(\alpha_{0\nu} k), \quad (\text{B3})$$

where $\alpha_{0\nu}$ is the ν th zero of $J_0(x)$, and

$$a_\nu = \frac{2}{|J_0(\alpha_{0\nu})|^2} \int_0^1 N(t) J_0(\alpha_{0\nu} t) t dt. \quad (\text{B4})$$

Substitution of Eq. (B2) into Eq. (B4) gives

$$a_\nu = \frac{2}{|J_0(\alpha_{0\nu})|^2} g\left(\frac{\alpha_{0\nu}}{\Omega_m}\right). \quad (\text{B5})$$

Further, substituting Eq. (B5) into Eq. (B3) and Eq. (B3) into Eq. (B2) in turn, the following sampling formula for $g(y)$ is obtained:

$$\begin{aligned} g(y) &= \sum_{\nu=1}^\infty \frac{1}{|J_0(\alpha_{0\nu})|^2} g\left(\frac{\alpha_{0\nu}}{\Omega_m}\right) \\ &\times \int_0^1 J_0(\alpha_{0\nu} k) \cdot J_0(\Omega_m k y) k dk. \end{aligned} \quad (\text{B6})$$

Since $g(y) = 0$, for $y > a$ the maximum value Ω_m is approximated by

$$\Omega_m = \alpha_{0N}/a, \quad (\text{B7})$$

where N is the number of sampled data. When N is much larger than unity, α_{0N} is approximated by

$$\alpha_{0N} \simeq \pi \frac{4N - 1}{4},$$

and so Ω_m is rewritten by

$$\Omega_m = \frac{\pi}{a} \cdot \frac{4N - 1}{4} \simeq \frac{\pi}{a} N. \quad (\text{B8})$$

From Eq. (B8) it is seen that the maximum spatial frequency is inverse to the sampling interval a/N . Since the number of sampled data is usually 50–100, the sampling interval is smaller than or of the same order of magnitude as the resolving power of the microscope in the case of optical fibers whose core radius is several tens of micrometers. On the other hand, in the case of preform rods the spatial resolution is nearly equal to the sampling interval.

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