Phase Space Evaluation of Distributed-Index Branching Waveguides

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Abstract—In this paper, we propose a new evaluation concept for characterizing multimode optical branching waveguides. In a ray tracing to simulate multimode branching waveguides, we introduce a phase space expression with the ray position and slope in order to describe the modal distribution of guided light. We have applied this technique for branching waveguides, which have a parabolic distributed-index, and clarified branching characteristics. Experimental results obtained for diffused branching waveguides are compared with theory.

I. INTRODUCTION

MULTIMODE optical branching circuits are important in optical communication network systems such as subscriber systems and local area networks. Thus far, several types of optical branching devices were reported [1]–[5]. In particular, a distributed-index (DI) waveguide is suitable for low loss coupling with distributed-index multimode fibers.

There has been a problem, however, that the branching characteristic strongly depends on the excited modal distribution. In order to solve this problem, we have proposed an integrated waveguide mode scrambler which has a built-in zigzag path [6], and analyzed branching characteristics by using a ray tracing [7]. On the other hand, to realize the star coupler which has multiports and a short device length, we have proposed a mesh waveguide optical star coupler with a mesh shape mixing region [8].

The above-mentioned study was aiming at stabilization of the branching ratio of only one branch. But in the case of multimode waveguides, there has been a problem that the branching ratio is unstable when some branches are connected in tandem, since the modal distribution of divided light at a branch is not in the steady state. In order to solve this, the evaluation of the modal distribution beyond the branch becomes necessary.

In this paper, in addition to a ray tracing method [7], we propose a phase space expression in terms of a ray position and slope to describe the modal distribution. We have applied this technique for some parabolic distributed-index branching waveguides. An experimental result

of a diffused waveguide is qualitatively compared with theory.

II. PHASE SPACE EXPRESSION OF RAYS

A. Phase Space

Let us suppose a slab waveguide with core width $2a$ and an arbitrary index distribution. A ray incident on this waveguide with an incident position x_i and slope $\dot{x}_i (= dx/dz)_{x=x_i}$ travels along a trajectory as shown in Fig. 1(a). We express the ray positions and slopes by a phase space to evaluate the modal distribution. The ray with position x and slope $\dot{x}$ in Fig. 1(a) moves in the phase space as shown in Fig. 1(b) while propagating in the waveguide. A critical ray, which is incident with $x_i = a$ and $\dot{x}_i = 0$, travels in the phase space along a contour as shown in Fig. 1(b). Thus the inside of a shaded area in Fig. 1(b) corresponds to guided modes, and the outside expresses the rays which are not bound in the waveguide. It is assumed that the modes are continuously distributed in the phase space since a large number of modes propagate with close spacing.

B. Phase Space of Multimode Waveguides

Let us consider a two-dimensional distributed-index waveguide as shown in Fig. 2, whose refractive index distribution is given by

$$\begin{aligned} n^2(x) &= n^2(0)[1 - (gx)^2], & |x| \leq a \\ &= n_2^2, & |x| > a \end{aligned} \quad (1)$$

where x is the distance from the core center. The position x and the slope $\dot{x}$ of a ray at a distance z from the input end can be expressed by

$$x = x_i \cos(gz) + (\dot{x}_i/g) \sin(gz) \quad (2a)$$

$$\dot{x} = -gx_i \sin(gz) + \dot{x}_i \cos(gz) \quad (2b)$$

where x_i is the position and $\dot{x}_i = \tan \theta_i$ is the slope of the ray at the input end. We normalize x by the half core width a and $\dot{x}$ by the slope of the maximum angle $\theta_{\max}$ as

$$X = x/a, \quad \dot{X} = \dot{x}/\tan \theta_{\max} \quad (3a)$$

$$X_i = x_i/a, \quad \dot{X}_i = \dot{x}_i/\tan \theta_{\max} \quad (3b)$$

Then (2a) and (2b) are reduced to

$$X = X_i \cos(gz) + \dot{X}_i \sin(gz) \quad (4a)$$

$$\dot{X} = -X_i \sin(gz) + \dot{X}_i \cos(gz). \quad (4b)$$

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Fig. 1. General concept of phase space: (a) real space, (b) phase space.

Fig. 2. Ray trajectory in a parabolic distributed-index waveguide.

TABLE I
PHASE SPACE EXPRESSION OF MODAL DISTRIBUTION

		Distributed Index	Step Index
Region of X and $\dot{X}$		$X^2 + \dot{X}^2 \leq 1$	$ X \leq 1, \dot{X} \leq 1$
Expression of Modal Distribution in Phase Space (Uniform Mode Excitation)			
Corresponding Value to Mode Order		$\sqrt{X^2 + \dot{X}^2}$	$ \dot{X} $
Lower Mode		$\sqrt{X^2 + \dot{X}^2} \rightarrow 0$	$ \dot{X} \rightarrow 0$
Higher Mode		$\sqrt{X^2 + \dot{X}^2} \rightarrow 1$	$ \dot{X} \rightarrow 1$
Propagating Condition of Ray			
Mode Conversion	Higher Mode ↓ Lower Mode		
	Lower Mode ↓ Higher Mode		

From (4a) and (4b), bound rays in the waveguide are obtained from the condition

$$X^2 + \dot{X}^2 \leq 1. \quad (5)$$

Therefore, in the case of a parabolic distributed-index waveguide, propagation rays are in the inside of a unit circle, whereas the outside expresses the radiation rays as shown in Table I. Since it is considered that the distance from the origin corresponds to the mode order, a ray which is close to the origin is regarded as a low order mode. All rays rotate clockwise with a same radial velocity as they propagate in the phase space. The mode conversion is expressed by the change of the rotation radius as shown in Table I.

On the other hand, in the case of a step index waveguide as shown in Fig. 3, we normalize x by the half core width a and $\dot{x}$ by the slope of the critical angle θ_c as

$$X = x/a, \quad \dot{X} = \dot{x}/\tan \theta_c \quad (6a)$$

$$X_i = x_i/a, \quad \dot{X}_i = \dot{x}_i/\tan \theta_c. \quad (6b)$$

Fig. 3. Ray trajectory in a step-index waveguide.

In this case, rays propagate in the waveguide by repeating the total internal reflection. Thus propagation rays in the phase space are inside a square and the outside expresses the radiation rays. Since it is considered that the slope of a ray $|\dot{X}|$ corresponds to the mode order, a ray which is close to the X axis is regarded as a low order mode. Bound rays propagate in the phase space so that the absolute value of the ray slope doesn't change.

As stated above, using the phase space expression in terms of the ray position and slope, the modal distribution can be evaluated intuitively by the distribution of ray plots.

III. RESULTS OF RAY TRACING FOR DISTRIBUTED-INDEX MULTIMODE BRANCHING WAVEGUIDES

A. Mixing and Branching at Y-Branch

At first, we simulated the ray tracing for a mixing and/or a branching waveguide as shown in Fig. 4, which is a fundamental structure in the optical branching waveguide. The refractive index profile of the tapered part is assumed that two waveguides join gradually as shown in Fig. 4. This is considered as the case that the waveguide is formed through a thin mask much narrower than the final waveguide width as in the case of diffused waveguides.

Fig. 5 expresses the insertion loss versus the branching angle of the 2×1 mixing waveguide, under the condition that incident rays are distributed uniformly in a waveguide. The loss was obtained by counting the ratio of radiated rays to the incident rays. The solid line corresponds to the model in which there is a bend before the mixing and the dotted line represents the model without a bend as shown in the insert. It is found that the insertion loss is about 3 dB and is almost independent of the branching angle. This can be explained by the fact that the total area of phase space of two input ports shrinks into half of the initial state.

Thus in order to reduce the mixing loss, we assumed the incident condition of lower order modes as $(X_i^2 + \dot{X}_i^2)^{1/2} \leq \frac{1}{2}$. This result is shown in Fig. 6. It is found that the insertion loss decreased markedly as expected, and the rapid increase for the solid line is considered to be caused by the bend before the mixing. Next, we calculated the change of modal distribution in the mixing part for $\theta_B = 2^\circ$, and the result is shown in Fig. 7. In this figure, the vertical axis is enlarged 2.5 times larger than the horizontal axis for a good visualization at Y-branch. It is found that the incident rays which are lower order modes coming from the right input port are converted to higher order modes little by little, joining the left side waveguide at

Fig. 4. Model of refractive index profile at Y-branch.

Fig. 5. Mixing loss versus branching angle (exciting all guided modes uniformly).

Fig. 6. Mixing loss versus branching angle (exciting lower order modes).

Fig. 7. Modal distributions of rays in a mixing waveguide.

the mixing part like a taper. This result shows the modal conversion effect at the mixing waveguide, which converts the modes from lower order modes to higher ones.

Fig. 8. Modal distributions of rays in a branching waveguide (distorted modal distribution before branching).

Fig. 9. Model of the ray tracing for a 2×2 coupler with a conventional straight mixing waveguide.

The final modal distribution after the mixing is not distributed uniformly but distorted.

Next, we considered the modal distribution of divided light in which the modal distribution is initially distorted before the branching. The model of the branching is the reversed mixing as shown in Fig. 4. The input and output ports are interchanged in Fig. 4. The result is shown in Fig. 8. It is seen that rays are divided into both sides at a branching like a taper and modes are converted to lower order modes. This shows the modal conversion effect of the branching waveguide, which is an inverted process of mixing. The modal distribution of output light after the branching is not uniform. This is caused by the partial modal distribution before the branching.

Therefore, in order to stabilize both the branching ratio and the modal distribution of divided light, it becomes necessary to introduce a kind of mode mixer which distribute the modes uniformly in the phase space before the branching.

B. A 2×2 Star Coupler with a Straight Mixing Waveguide

For the purpose of reducing the distortion of the modal distribution after the mixing, we traced the rays in a 2×2 star coupler with a conventional straight mixing region connected to DI fiber as shown in Fig. 9. The branching ratio is calculated by totaling output rays at the left and the right output ports. Fig. 10 shows the branching ratio and the loss versus the normalized mixing length, where we have assumed $\Delta = 1.9$ percent. The branching ratio and the radiation loss depend on waveguide parameters such as the mixing length and the branching angle. The

Fig. 10. Branching characteristic versus straight mixing length.

Fig. 11. Modal distribution in the phase space for a 2×2 coupler with a conventional straight mixing waveguide. Fig. 12. Model of the ray tracing for a 2×2 branching waveguide with a zigzag path mode scrambler.

optimum points where the branching ratio is unity appears periodically at every half pitch of the sinusoidal ray trajectory, which is $l/a = 16$ in our case. For example, in the case of $\theta_B = 2$ deg, one of the optimum points is the normalized mixing length $l/a = 68.5$. For this optimum point from the viewpoint of the branching ratio, the phase space expression of the output rays is shown in Fig. 11. Even if the branching ratio is unity, the modal distributions of two output ports are not similar to each other, and it is seen that the right port contains partially lower order modes and the left does higher order modes.

Therefore, in designing a branching waveguide, we must consider not only the branching ratio but also the modal distribution.

C. A Waveguide Mode Scrambler With Zigzag Path

Next, we simulated a 2×2 branching waveguide with zigzag path mode scrambler [6] by the ray tracing. The calculated model is shown in Fig. 12, and the basic function is to make the distorted modal distribution of mixed rays uniform by the repeat of zigzag bends. The branching ratio and the loss versus the bend number are shown in Fig. 13, where we have assumed $\Delta = 1.9$ percent. Fig. 14 shows the modal distribution of the output rays in the

Fig. 13. Branching characteristic versus bend number.

Fig. 14. Modal distribution in the phase space for a 2×2 coupler with a zigzag path mode scrambler.

Fig. 15. (a) (b) Motion of a modal distribution in the phase space (parabolic distributed-index).

case that the bend number is fifteen, where the branching ratio is nearly equal to unity. In spite of many bends, the distortion of the modal distribution remained without any change, compared with the result of a straight mixing model as shown in Fig. 11.

The reason why the zigzag mode scrambler does not work well for parabolic index waveguide can be explained in terms of the phase space expression in the following manner. First, we suppose an elliptical modal distribution as a model of distorted one as shown in Fig. 15(a). If these rays propagate some distance in a parabolic-index waveguide, the modal distribution moves as shown in Fig. 15(b) since all rays rotate with a same radial velocity in the phase space. Therefore even though rays propagate a long distance in a straight waveguide, the elliptical area of modal distribution is maintained. Moreover, if the co-ordinate of the phase space is slightly shifted by bends,

Fig. 16. A bellows waveguide mode scrambler.

Fig. 17. Branching characteristic versus number of bellows.

axial offsets, etc., the distortion isn't dissolved so much since the whole ellipse moves concentrically.

D. A Bellows Waveguide Mode Scrambler

As stated in the preceding sections, it becomes obvious that the conventional straight mixing waveguide and the zigzag path mode scrambler can't make the modal distribution uniform, in the case of a parabolic distributed-index waveguide. It is seen from Fig. 7 that only the tapered mixing region as shown in Fig. 4 can convert the order of modes.

Thus we propose a bellows waveguide mode scrambler as shown in Fig. 16, which utilizes the mode conversion at the tapered part. This mode scrambler dissolves the distorted modal distribution by the repeat of tapers and re-distributes the modes like a steady-state distribution. Fig. 17 shows the branching characteristics versus the number of bellows. It is seen that the branching ratio approaches unity and the loss increases with increasing the number of bellows. Showing the modal distribution of the output ports in Fig. 18, it is found that the modal distributions of the left and the right output ports are made uniform and similar to each other as the number of bellows increases. Even in the case of $M = 1$, the modal distribution of both output ports approaches a uniform one, and the insertion loss is about 0.6 dB. For $M = 8$, a sufficient stabilization of modal distribution for the both output ports is achieved with the insertion loss of about 3 dB. Fig. 19 illustrates the traces of rays in the bellows mode scrambler for $M = 1$. It is recognized that the modal conversion occurs in the bellows waveguide like a taper.

Next we calculated the branching characteristic against the position of the incident light, that is the change of the

Fig. 18. Modal distribution in the phase space for a 2×2 coupler with a bellows waveguide mode scrambler. Fig. 19. Ray trajectories in a 2×2 branching waveguide with a bellows waveguide mode scrambler ($M = 1$). Fig. 20. Branching characteristic versus position of incident light ($M = 1$ and $M = 6$).

excited mode distribution. The result is shown in Fig. 20. It is seen that the branching characteristic is very stable in the case of $M = 6$, and is stable comparatively for $M = 1$ if the input position lies within the range of $|x/a| \leq$

Fig. 21. Modal distribution versus position of incident light ($M = 1$).

0.3. The modal distribution of the output light for this condition is illustrated in Fig. 21. It is seen that the modal distribution remains stable against the change of the input position.

For practical uses, since it is regarded that the uniform modal distribution is achieved for $M = 1$, we can conclude that this new mode scrambler is useful to achieve the branching characteristics which are stable with respect to the branching ratio and the modal distribution of the output light with sufficient low loss.

IV. EXPERIMENTS

We fabricated experimental branching waveguides by an ion exchange technique [1], [6] for the purpose of comparing with calculated results.

A glass substrate was coated by a metal mask and a waveguide pattern was opened. This substrate was immersed into a molten salt which was kept at high temperature. A metal ion which brings about a high refractive index was migrated into a glass substrate through stripe window masks by applying electric field. After the first migration process, the metal mask was removed and a low index ion was migrated again through the surface under applied field in order to bury the waveguide. The cross section is a slightly distorted hemicycle with the width of 100 μm and the depth of 50 μm .

First, we measured the insertion loss of a Y-branching waveguide as shown in Fig. 22, which is dependent on the branching angle θ_B . A He-Ne laser light ($\lambda = 0.633 \mu\text{m}$) was coupled into a distributed-index fiber (GI-50), and this fiber was butt-jointed to the input end of the waveguide. The dotted line at 1.95 dB is the propagation loss of the waveguide. The open circles and squares show the mixing loss which is defined by the ratio of the output power to the input power coming from one of the branches. The closed circles exhibit the branching loss which is the ratio of the input power to the total power of

Fig. 22. Mixing loss and branching loss versus branching angle of Y-branching waveguide.

Fig. 23. Near field pattern and far field pattern of output light.

both branches. These mixing and branching losses were measured with the same branching waveguide in the opposite direction. It is found that the mixing loss is larger than the branching loss by 2.5 dB for small branching angle, although there is the variation depending on the side of the input port because of the imperfection of the waveguide. Since the incident light was focused on a fiber, the lower order modes were predominantly excited. Considering this fact, the measured result seems to coincide with the calculated one, which is the mixing loss of 3 dB. In contrast with this, the branching loss is less than 1 dB. The rapid increase of losses with the increase of branching angle is considered to be caused by the discrepancy of the actual refractive index profile from a parabolic one.

Next we measured the modal distribution of the Y-branching waveguide. It can be regarded that the distribution of ray positions can be obtained from the near field pattern (NFP) and that of ray slopes can be obtained from the far field pattern (FFP), as shown in Fig. 23. An AlGaAs-LED light ($\lambda = 0.83 \mu\text{m}$) was used to reduce the speckle noise in the NFP and the FFP. Fig. 24 shows the NFP and the FFP of the mixing waveguide, and Fig. 25 shows those of the branching waveguide. In these pictures, the brightness distribution along a scanning line A-A' is displayed at the bottom of each picture. Comparing both NFP's, the NFP of the mixing waveguide is almost rectangular, but on the other hand that of the branching waveguide is almost triangular. Thus it can be seen that output rays of the mixing waveguide are distributed uniformly in the waveguide, and those of the branching waveguide lie in the central part of the waveguide. As for both FFP's, it is also seen that the FFP of the mixing waveguide is like mesa and those of the branching waveguide are almost triangular. Therefore, it can be con-

Fig. 24. Modal distribution of a fabricated 2×1 mixing waveguide. Fig. 25. Modal distribution of a fabricated 1×2 branching waveguide.

cluded that output rays of the mixing waveguide are higher order modes and those of the branching waveguide are lower order modes. These results are in good agreement with the calculated results.

Next we considered modal distributions of a zigzag path mode scrambler and a bellows waveguide mode scrambler. The sample with a zigzag mode scrambler has the number of bend $M = 18$. The observed NFP and FFP are shown in Fig. 26. From Fig. 26, it is seen that NFP's are different from each other. The FFP of the left output port is like a triangular shape and that of the right output port is like a mesa shape. Thus we can conclude that the light from the left output port includes lower order modes and that from the right output port includes higher order modes. This result coincides with the theoretical one, in which the modal distributions are distorted.

Last, we measured the NFP and FFP of the sample with bellows waveguide mode scrambler, which has the number of bellows $M = 1$. The branching ratio was almost

Fig. 26. Modal distribution of a fabricated 2×2 coupler with a zigzag mode scrambler. Fig. 27. Modal distribution of a fabricated 2×2 coupler with a bellows waveguide mode scrambler.

unity. The results are shown in Fig. 27. Comparing with the result of a zigzag path mode scrambler, it is seen that the distribution of ray positions and slopes of both output ports becomes similar to each other. Thus we can conclude that the modal distributions of both output ports become close to each other. This result shows that a bellows mode scrambler is effective to make the modal distribution uniform, which is consistent with the theoretical one.

In this way, we can estimate the modal distribution by the NFP and FFP.

V. CONCLUSION

In this paper, we have proposed an evaluation concept for multimode branching waveguides, i.e., a ray tracing

for the branching ratio and a phase space expression with ray positions and slopes to describe the modal distribution. Using this method, we made the characteristics of some branching waveguides clear and showed that we must consider not only the branching ratio but also its modal distribution. Based upon the calculated results, we proposed a bellows waveguide mode scrambler, which have very stable branching characteristics. The modal distribution of a diffused branching waveguide was measured from the NFP and FFP, and the theoretical results were compared with the measured ones.

We believe that this evaluation method can be applied for other microoptic components such as a planar microlens [3].

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