RECENT DEVELOPMENT OF STUDIES ON SCALE EFFECT

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1. Introduction

In 1986, a sub-committee was organized in Japan Society of Mechanical Engineers (JSME) to standardize a new scale effect formula by studying the outstanding problems of scale effect such as dependence of V (loss distribution coefficient) on specific speed, roughness effect and so on. Then the sub-committee established a new performance conversion method and published it as JSME S 008 in 1989 [1]. After its publication, the second sub-committee has been organized to revise the conversion method by collecting the results of further study.

The most difficult problem in the study of scale effect is that the accuracy of the measurement of prototype efficiency is incompetent to verify the scale effect. Therefore, most of the study was conducted by using both theoretical analyses on numerical models of hydraulic machines and its verification by laboratory model tests. For this study, various numerical models which may simulate the performance of a whole machine were developed independently at institutes and industries in Japan. Most of them employed 3D flow analysis for major parts of the machines. In order to assess the wall friction, they used boundary layer calculation or wall friction loss calculation using flat plate analogy. These numerical models were refined and improved by comparing their simulation results with performance characteristics obtained by laboratory model tests. By using these models, the hydraulic loss in the machines were separated into scalable loss and non-scalable loss and the scale effect on the scalable loss was assessed quantitatively.

This paper describes the summary of the results of these studies and the essential part of the newly proposed performance conversion method which will be published soon as a new version of JSME S 008.

Fig. 1 Flux diagrams of hydraulic machines

2. Structure of the New Scale Effect Formula

The energy flux through hydraulic machines is conceptually represented by the diagrams set out in Fig. 1 a) and b). As indicated in these diagrams, the hydraulic efficiency of hydraulic machine η_h is given by the product of the following three internal efficiencies.

$$\eta_h = \eta_E \eta_v \eta_r \quad (1)$$

where, η_E is specific energy efficiency concerning all specific hydraulic energy loss in main water passages, η_v is volumetric efficiency concerning leakage loss and η_r is power efficiency of runner/impeller concerning disk friction.

Scale effect observed in performance characteristics of geometrically homologous hydraulic machines is caused by the change of wall friction loss due to the difference of Reynolds number and the relative roughness of the wall surface. Since the wall friction mostly causes dissipation of the specific hydraulic energy of the main flow, scale effect appears primarily in the specific hydraulic energy of the machine E . Scale effect also appears in leakage flow loss and in disk friction loss of the runner/impeller due to the difference of Reynolds number.

In the new performance conversion method, conversion factors for each internal efficiencies proposed by T. Ida are newly introduced. [5] They are specific energy efficiency scale factor $F_E = \eta_{EP} / \eta_{EM}$, volumetric efficiency scale factor $F_v = \eta_{vP} / \eta_{vM}$ and power efficiency scale factor $F_r = \eta_{rP} / \eta_{rM}$, where subscript P indicates the values of the prototype and M for those of the model. By using these scale factors, the hydraulic efficiency of a model can be converted to the prototype

efficiency as set out below.

$$\eta_{hP} = F_E F_v F_r \eta_{hM} \quad (2)$$

These scale factors are standardized in the new conversion method as explained hereinafter.

Scale effect causes also shifting of performance. These scale factors are also applicable to the conversion of E , Q and P , respectively. [2]

<u>Turbine</u>	<u>Pump</u>
$E_P = F_E^{-1} \left(\frac{n_P}{n_M} \right)^2 \left(\frac{D_P}{D_M} \right)^2 E_M$	$E_P = F_E \left(\frac{n_P}{n_M} \right)^2 \left(\frac{D_P}{D_M} \right)^2 E_M \quad (3)$

$Q_P = F_v^{-1} \left(\frac{n_P}{n_M} \right) \left(\frac{D_P}{D_M} \right)^3 Q_M$	$Q_P = F_v \left(\frac{n_P}{n_M} \right) \left(\frac{D_P}{D_M} \right)^3 Q_M \quad (4)$
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$P_P = F_r \left(\frac{n_P}{n_M} \right)^3 \left(\frac{D_P}{D_M} \right)^5 P_M$	$P_P = F_r^{-1} \left(\frac{n_P}{n_M} \right)^3 \left(\frac{D_P}{D_M} \right)^5 P_M \quad (5)$
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3. New Formula for Conversion of Specific Energy Efficiency

One of the major changes from the existing method to the new method is the use of relative scalable loss δ_E instead of loss distribution coefficient V . The definition of δ_E is given by the following equation.

$$\delta_E = E_{Lf} / E \quad (6)$$

where, E_{Lf} is the scalable specific energy loss due to wall friction.

On the other hand, the loss distribution coefficient V used in IEC Publication 995, which is denoted as V_{IEC} or that in the existing version of JSME S 008, which is denoted as V_{JSME} , is defined as follows;

$$V_{IEC} = \frac{(\text{Total scalable losses including disk friction, etc.})}{[(\text{Total scalable loss}) + (\text{Total non-scalable loss})]} \quad (7a)$$

$$V_{JSME} = E_{Lf} / E_L = E_{Lf} / (E_{Lf} + E_{Lns}) \quad (7b)$$

Whichever V is used, the problem to use these V values is that they include non-scalable loss E_{Lns} and others in their denominators which could be significantly affected by the difference of specific speed or individual design and, consequently, they could vary considerably from their standardized values depending on individual machine design. Also, it has been pointed out that, since these V values were standardized for average machines, the prototype efficiency stepped up from poor efficiency model by using these values would be unreasonably too high.

It is expected that relative scalable loss δ_E is less affected than V by the difference of specific speed or individual design since it does not include non-scalable loss like E_{Lns} . The values of δ_E for Francis turbines and pump turbines, which are obtained

Fig. 2 Scalable loss δ_E versus specific speed Fig. 3 V_{JSME} versus specific speed

from the investigation using various numerical models are shown in Fig. 2. Apparently, their scattering is less than that of V_{JSME} [2], which is shown in Fig. 3, for example.

Hence, δ_E is selected instead of V as the standardized reference of the scalable loss.

When the flow condition in both prototype and model is homologous, the ratio of their relative scalable loss denoted as Λ is given as follows;

$$\Lambda = (\delta_{EP} / \delta_{EM}) = (C_{fP} / C_{fM}) \quad (8)$$

where, C_f is the friction loss coefficient for main water passages.

Then new scale effect formulae for the conversion of the specific energy efficiency are proposed by T. Ida.[3] They are written as follows;

$$\text{For turbine: } F_E = \frac{1}{1 - (1 - \Lambda)\delta_E} \quad \text{or} \quad \Delta\eta_E = \eta_{EM} \left[\frac{1}{1 - (1 - \Lambda)\delta_E} - 1 \right] \quad (9)$$

$$\text{For pump: } F_E = 1 + (1 - \Lambda)\delta_E \quad \text{or} \quad \Delta\eta_E = \eta_{EM} (1 - \Lambda)\delta_E \quad (10)$$

For simplification, all the model values in this paper are assumed to be those measured at the reference Reynolds number.

Fig. 4 Efficiency step up $\Delta\eta_E$ versus model efficiency η_{EM}

The values of δ_E and V , which were collected from the results of the numerical model analysis, show some scattering as shown in Figs. 2 and 3. The uncertainty of $\Delta\eta_E$ caused by such scattering is examined and shown in Fig. 4. It is observed that the scattering of $\Delta\eta_E$ caused by the scattering of δ_E is less than that caused by V . Thus, by using the new formulae based on δ_E , the accuracy of $\Delta\eta_E$ can be improved considerably and the unreasonably large efficiency step up for poor efficiency turbines can be avoided.

4. Friction Coefficient Ratio Considering Roughness Effect

The effects of surface roughness on the wall friction loss are studied by T. Ida using the analogy of the friction loss of flat plate. Basic equations of the friction loss coefficient for flat plate which are used as a basis for the discussions hereinafter are shown below.

$$\text{For smooth surface: } C_{fS} = 0.455(\log_{10} Re_L)^{-2.58} \quad (11)$$

$$\text{For rough surface: } C_{fR} = [1.89 - 1.62 \log_{10}(k_s/L)]^{-2.5} \quad (12)$$

where, k_s is the equivalent sand roughness and L is the length of the plate.

The admissible sand roughness can be obtained as the solution of $C_{fS} = C_{fR}$. The result is;

$$(k_{Sadm}/L) = 10^{\left\{ (1.89/1.62) - (1/1.62) \left[(\log_{10} Re)^{2.58} / 0.455 \right]^{0.4} \right\}} \approx 24.0 Re_L^{-0.931} \quad (13)$$

The extreme right is the approximation for the middle equation. The admissible roughness thus obtained is shown in Fig. 5. The admissible roughness given by the Schlichting's equation is also plotted for comparison.

Fig. 5 Admissible sand roughness versus Reynolds number

Fig. 6 Friction loss coefficient of sand roughened flat plate

In general, actual surface roughness e , which is usually expressed in terms of R_a , is not the same as k_s . Therefore, when using the admissible sand roughness obtained above, the following conversion is required.

$$e_{adm} = k_{Sadm} / C_k \quad (14)$$

where, C_k is a coefficient to be obtained experimentally. Unfortunately, although a few results of the experimental studies on C_k were published, they did not sufficiently coincide with each other. Further study is required in this regard.

The variation of friction loss coefficient C_f for different Reynolds number and surface roughness is shown in Fig. 6. In this diagram, point M represents C_f for a rough model (strictly speaking, a model not perfectly smooth) and point P for a rough prototype (similarly, a prototype not perfectly smooth). Here, we consider a smooth model A which has the same Reynolds number with M and a smooth prototype B with the same Reynolds number with P. Then, the conversion of the friction loss coefficient from the rough model M to the rough prototype P is divided into three steps; namely, $M \rightarrow A \rightarrow B \rightarrow P$. Thus, the ratio of friction loss coefficient, $\Lambda_{M \rightarrow P}$, may be expressed by the following equation.

$$\Lambda_{M \rightarrow P} = \Lambda_{M \rightarrow A} \times \Lambda_{A \rightarrow B} \times \Lambda_{B \rightarrow P} = \Lambda_{A \rightarrow B} \times \Lambda_{B \rightarrow P} / \Lambda_{A \rightarrow M} \quad (15)$$

Firstly, by Eq. (11), $\Lambda_{A \rightarrow B}$ (smooth model to smooth prototype) is given as the first line below and it can be approximated with good accuracy by the equation at the second line.

$$\begin{aligned} \Lambda_{A \rightarrow B} &= (C_{fSB} / C_{fSA}) = \left[(\log_{10} Re_{LP}) / (\log_{10} Re_{LM}) \right]^{-2.58} \\ &\approx (Re_{LP} / Re_{LM})^{-0.155} \end{aligned} \quad (16)$$

Secondly, $\Lambda_{B \rightarrow P}$ (smooth machine to rough machine) is obtained. By combining Eqs. (11) and (12), it is given as the first line below and it can be closely approximated by the equation at the second line.

$$\begin{aligned} \Lambda_{B \rightarrow P} &= (C_{fRP} / C_{fSB}) = \left[1.89 - 1.62 \log_{10} (k_{SP} / L_P) \right]^{-2.5} / 0.455 (\log_{10} Re_{LP})^{-2.58} \\ &\approx 0.678 (k_{SP} / L_P)^{0.18} Re_{LP}^{0.155} \end{aligned} \quad (17)$$

Similarly $\Lambda_{A \rightarrow M}$ is obtained as follows;

$$\Lambda_{A \rightarrow M} = 0.678 (k_{SM} / L_M)^{0.18} Re_{LM}^{0.155} \quad (18)$$

By substituting Eqs. (16), (17) and (18) into Eq. (15), we obtain;

$$\begin{aligned} \Lambda_{M \rightarrow P} &= (Re_{LP} / Re_{LM})^{-0.155} \left[(k_{SP} / L_P) / (k_{SM} / L_M) \right]^{0.18} (Re_{LP} / Re_{LM})^{0.155} \\ &= \left[(k_{SP} / L_P) / (k_{SM} / L_M) \right]^{0.18} \end{aligned} \quad (19)$$

In the above equation, L represents the length of vanes or passages. Therefore, in case of homologous machines, $(L_P / L_M) = (D_P / D_M)$. Also, using actual roughness e , (k_{SP} / k_{SM}) can be substituted by (e_P / e_M) . After all, $\Lambda_{M \rightarrow P}$ is obtained as follows;

$$\Lambda_{M \rightarrow P} = (D_P / D_M)^{-0.18} (e_P / e_M)^{0.18} \quad (20)$$

If the surface roughness of the model is smooth i.e. $e_M \leq e_{admM}$, the following equation can be used for the conversion from a smooth model to a rough prototype.

$$\Lambda_{M \rightarrow P} = (D_P / D_M)^{-0.18} (e_P / e_{admM})^{0.18} \quad (21)$$

Since a prototype has generally larger Reynolds number and smaller admissible roughness, its surface roughness is mostly hydraulically rough. Especially, surfaces of runner/impeller and distributor are exposed to high velocity flow and subjected to significant roughness effect. Therefore, it is generally necessary to apply Eq. (20) or (21) to the wall friction loss on these areas taking into account the difference of the roughness of both model and prototype; namely, $(e_P / e_M) \neq 1$. Regarding the other areas, most of them are painted and their roughness is almost identical for both model and prototype. Therefore, for these areas, Eq. (20) or (21) assuming $(e_P / e_M) = 1$ can be applied.

Then, similar to the existing code JSME S 008, a coefficient β which is defined by the following equation is introduced.

$$\beta = \frac{(\text{Friction loss in runner / impeller and distributor})}{(\text{Total friction loss in main water passage})} \quad (22)$$

By using β , the friction loss coefficient ratio $\Lambda_{M \rightarrow P}$ applicable to actual machine is obtained as follows;

$$\begin{aligned} \Lambda_{M \rightarrow P} &= (1 - \beta)(D_P / D_M)^{-0.18} (1/1)^{0.18} + \beta(D_P / D_M)^{-0.18} (e_P / e_M)^{0.18} \\ &= (D_P / D_M)^{-0.18} \left[(1 - \beta) + \beta(e_P / e_M)^{0.18} \right] \end{aligned} \quad (23)$$

The value of β can be estimated by the simulation analysis using numerical models. Some examples of its value at optimum condition are shown in Fig. 7. It becomes larger for higher specific speed machines, however, mostly it remains within the range of 0.6~0.85.

Fig. 7 β versus specific speed

5. Influence of the Approximation of Principal Coefficients

5.1 INFLUENCE OF RELATIVE SCALABLE LOSS δ_E

As shown in Fig. 2, the scattering of δ_E ranges over 0.037 ± 0.004 . The result of an example to examine the influence of this scattering is shown in Fig. 8. As observed in this diagram, the difference of $\Delta\eta_E$ caused by the above scattering of δ_E is only $\pm 0.13\%$. It suggests that a single value of $\delta_E = 0.037$ can be used practically as a standardized value for all Francis turbines of various specific speed.

5.2 INFLUENCE OF DEPENDENCE COEFFICIENT OF ROUGHNESS RATIO β

As shown in Fig. 7, the value of β varies over a range of 0.60 to 0.85. Another examination to see the influence of the different values of β was conducted. An example of the results of the conversion using different values of β is shown in Fig. 9. It indicates that, if β is selected within a range of 0.8 ± 0.2 , the dispersion of the resultant prototype efficiency does not exceed $\pm 0.1\%$. Thus the influence of the deviation of β on $\Delta\eta_E$ is small. It suggests that a constant value $\beta = 0.7$ may be used for all Francis turbines and pump-turbines in practical application.

In this figure, K_m and K_p are the dimensionless flow coefficient and output power coefficient, respectively, which are defined as follows:

$$K_m = (Q_1 / A_{R1}) / (2E)^{1/2} \quad (24)$$

$$K_p = P / [\rho A_{R1} (2E)^{1/2} E] \quad (25)$$

Fig. 8 Influence of the difference of δ_E on the converted prototype performances (Francis turbine: $\text{NsQ}=62$, $(e_P/e_M) = 4.0$)

Fig. 9 Influence of the difference of β on the converted prototype performances (Francis turbine: $\text{NsQ}=62$, $(e_P/e_M) = 4.0$)

It should be noted that K_m and K_p of the prototype are significantly influenced by scale effect since they include E , which is most likely to be affected by scale effect, in their denominators.

6. Conversion of Volumetric Efficiency

Volumetric efficiency η_v is determined by leakage flow loss and it is subjected to scale effect. In the new conversion method, it is converted to the prototype value by the following equation.

$$\eta_{vP} = F_v \eta_{vM} \quad (26)$$

The value of η_{vM} obtained by the numerical analysis is plotted against specific speed and shown in Fig. 10. Those for Francis turbines and pump-turbines are represented by a common single curve. [4]

Fig. 10 Volumetric efficiency η_{vM} versus specific speed

Detailed study to determine Λ_v and F_v is conducted by T. Ida. [5] The scale factor F_v can be expressed as follows;

$$F_v = 1 / [\eta_{vM} + \Lambda_v (1 - \eta_{vM})] \quad (\text{for turbine}) \quad (27)$$

$$F_v = [(1 - \Lambda_v) / \eta_{vM}] + \Lambda_v \quad (\text{for pump}) \quad (28)$$

As a result of the study, it is made clear that F_v may be regarded as 0.997~0.998 within the practical range of $(Re_P / Re_M) = 5 \sim 25$ and $nsQ = 25 \sim 100$, when the geometry of the seal clearances of both model and prototype is homologous.

7. Conversion of Power Efficiency of Runner/impeller

Similar to volumetric efficiency, power efficiency η_r concerning disk friction of runner/impeller can be converted by the following formula.

$$\eta_{rP} = F_r \eta_{rM} \quad (29)$$

The value of η_{rM} is investigated by using the numerical models and the results are shown in Fig. 11.

Fig. 11 Power efficiency of runner/impeller η_{rM} versus specific speed

The scale factor F_r is given by the following equation. [5]

$$F_r = \left[(1 - \Lambda_r) / \eta_{rM} \right] + \Lambda_r \quad (\text{for turbine}) \quad (30)$$

$$F_r = 1 / \left[(1 - \Lambda_r) \eta_{rM} + \Lambda_r \right] \quad (\text{for pump}) \quad (31)$$

where, Λ_r is the ratio of the disk friction loss coefficient. If it is assumed that the surface roughness relevant to the disk friction is hydraulically rough, the following approximate equation is obtained.

$$\Lambda_r = (D_P / D_M)^{-0.27} (e_P / e_M)^{0.12} \quad (32)$$

Finally, the following equations for F_r are derived.

$$F_r = (0.388 / \eta_{rM}) + 0.612 \quad (\text{for turbine}) \quad (33)$$

$$F_r = 1 / (0.995 - 8.2 nsQ^{-2}) \quad (\text{for pump}) \quad (34)$$

8. Conclusion

New performance conversion method with regard to scale effect is proposed for the new version of JSME S 008. Compared with the existing version, it is expected that the new method gives more reasonable and accurate conversion. In addition, introduction of scale factors makes the calculation of performance shifting due to scale effect much easier.

Reference

- 1 JSME S 008 (1989) Performance conversion method for hydraulic turbines and pumps
- 2 Ida, T. et al. (1990) Advanced scale-up formulae for performances of hydro-turbines and pump-turbines considering surface roughness, *Proc. IAHR Symp. in Belgrade* 1, B4
- 3 Ida, T. (1994) New formula for efficiency step-up of hydraulic turbines, *Proc. IAHR Symp. in Beijing* 2, 827-840
- 4 Kurokawa, J. et al. (1994) Accurate determination of volumetric and mechanical efficiencies and leakage behaviour of Francis turbine and Francis pump-turbine, *Proc. IAHR SYMP. in Beijing* 2, 889-900
- 5 Ida, T. (1995) New formulae for scaling-up hydraulic efficiency of hydraulic turbines, *Journal of Hydraulic Research* 33, 2, 147-162