Spin structure function \( g_2(x,Q^2) \) and twist-3 operators in large-\( N_C \) QCD

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It is shown in the framework of operator product expansion and the renormalization group method that the twist-3 part of flavor nonsinglet spin structure function \( g_2(x,Q^2) \) obeys a simple Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation in the large \( N_C \) limit even in the case of massive quarks (\( N_C \) is the number of colors). There are four different types of twist-3 operators which contribute to \( g_2 \), including quark-mass-dependent operators and the ones proportional to the equation of motion. They are not all independent, but are constrained by one relation. A new choice of the independent operator bases leads to a simple form of the evolution equation for \( g_2 \) at large \( N_C \).

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I. INTRODUCTION

In experiments of polarized deep inelastic leptonproduction, we can obtain information on the spin structures of the nucleon, which are described by the two functions \( g_1(x,Q^2) \) and \( g_2(x,Q^2) \). The QCD effects on \( g_1 \) and \( g_2 \) have been extensively studied [1] since earlier papers [2–4]. Increasingly accurate measurements of \( g_1 \) have been performed at SLAC, CERN, and DESY [5], while the \( g_2 \) measurements still have limited statistical precision [6].

In the language of operator product expansion (OPE), the twist-2 operators contribute to \( g_1 \) in the leading order of \( 1/Q^2 \). As for the structure function \( g_2 \), on the other hand, both twist-2 and twist-3 operators participate in the leading order. Moreover, the number of participating twist-3 operators grows with spin (moment of \( g_2 \)). Because of the increase of the number of operators and the mixing among these operators, the analysis of the twist-3 part of \( g_2 \) turns out to be rather complicated [7–14]. In other words, the \( Q^2 \) evolution equation for the moments of the twist-3 part of \( g_2 \) cannot be written in a simple form, but in a sum of terms, the number of which increases with spin.

For the case of the twist-3 flavor nonsinglet \( g_2 \), it has been observed by Ali, Braun, and Hiller (ABH) [15] that in the large \( N_C \) limit, \( g_2 \) obeys a simple Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [16]. In their formalism of working directly with the nonlocal operator contributing to the twist-3 part of \( g_2 \), they showed that local operators involving gluons effectively decouple from evolution equation for large \( N_C \). In fact their analysis has been made with massless quarks.

In this paper, I reanalyze the \( Q^2 \) evolution of \( \tilde{g}_2 \), the flavor nonsinglet twist-3 part of \( g_2 \), in the framework of the standard OPE and the renormalization group (RG) with massive quarks. Actually, the OPE analysis of \( \tilde{g}_2 \) has been performed already and the anomalous dimensions of the relevant twist-3 operators have been calculated [8,9,11,17,18]. However, to the best of my knowledge, the large \( N_C \) limit of \( \tilde{g}_2 \) has not been thoroughly studied so far in OPE and RG.

II. THE OPE ANALYSIS OF \( \tilde{g}_2 \)

The spin structure function \( g_2 \) receives contributions from both twist-2 and twist-3 operators. However, the twist-2 part of \( g_2 \) can be extracted once \( g_1 \) is measured [19]:

\[
\tilde{g}_2^{\text{tw-2}}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(y,Q^2)}{y} dy. \tag{1}
\]

Thus the difference

\[
\tilde{g}_2(x,Q^2) = g_2(x,Q^2) - \tilde{g}_2^{\text{tw-2}}(x,Q^2) \tag{2}
\]

contains the twist-3 contributions only.

The twist-3 operators which enter the OPE for the flavor nonsinglet \( \tilde{g}_2 \) are the following (I follow the notation and conventions of Refs. [17,18] and omit the flavor matrices \( \lambda_i \)):
\[ R_F^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{i^{n-1}}{n} \sum_{l=1}^{n-1} \left( (n-1) \bar{\psi} \gamma_5 \gamma^\sigma D^{(\mu_1 \cdots \mu_{n-1})} \psi - \sum_{l=1}^{n-1} \bar{\psi} \gamma_5 \gamma_\mu D^{(\sigma \mu_1 \cdots \mu_{n-1})} D^{\mu_1 \cdots \mu_{n-1}} \psi \right) \] (traces),

\[ R_I^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{1}{2n} \left( V_{i-1} - V_{n-1-i} + U_{i-1} + U_{n-1-i} \right), \quad (l=1, \ldots, n-2), \]

\[ R_m^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-2} m \left[ \bar{\psi} \gamma_5 \gamma^\sigma D^{(\mu_1 \cdots \mu_{n-2})} \gamma^\mu_{n-1} \psi \right] \] (traces),

\[ R_E^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-2} \frac{n-1}{2n} \left[ \bar{\psi} \gamma_5 \gamma^\sigma D^{(\mu_1 \cdots \mu_{n-2})} \gamma^\mu_{n-1} \psi \right] \] (traces),

\[ \left( n-1 \right) \bar{\psi} \gamma_5 \gamma^\sigma D^{(\mu_1 \cdots \mu_{n-1})} \psi - \sum_{l=1}^{n-1} \bar{\psi} \gamma_5 \gamma_\mu D^{(\sigma \mu_1 \cdots \mu_{n-1})} D^{\mu_1 \cdots \mu_{n-1}} \psi \right) \] (traces),

where \( \{ \} \) means complete symmetrization over the Lorentz indices and \( m \) represents the quark mass. The symbol \( S' \) denotes symmetrization on the indices \( \mu_1, \mu_2, \cdots, \mu_{n-1} \) and antisymmetrization on \( \sigma \mu_l \). The operators in Eq. (4) contain the gluon field strength \( G_{\mu\nu} \) and its dual tensor \( G_{\mu\nu} \) and they are given by

\[ V_i = -i g S' \bar{\psi} \gamma_5 D^{(\mu_1 \cdots \mu_{n-2})} \gamma^\mu_{n-1} \psi \] (traces),

\[ U_i = i^{n-1} g S' \bar{\psi} D^{(\sigma \mu_1 \cdots \mu_{n-2})} \gamma^\mu_{n-1} \psi \] (traces),

where \( g \) is the QCD coupling constant. The operator \( R_F^\sigma \) in Eq. (6) is proportional to the equation of motion (EOM) operator. The above twist-3 operators are not all independent, but they are constrained by the following relation [7,12]:

\[ R_F^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{1}{n} R_m^{\sigma\mu_1\cdots\mu_{n-1}} + \sum_{l=1}^{n-2} (n-1-l) R_I^{\sigma\mu_1\cdots\mu_{n-1}} + R_E^{\sigma\mu_1\cdots\mu_{n-1}}. \]

Thus in total there are \( n \) independent operators contributing to the \((n-1)\)th moment of \( g_2 \). But we will see later that in the \( N_C \to \infty \) limit, the \((n-1)\)th moment is expressed in terms of one operator \( R_F^{\sigma\mu_1\cdots\mu_{n-1}} \).

In all the analyses of \( g_2 \) performed so far in the framework of OPE and RG, operators \( R_F, R_I, R_m, R_E \) of Eqs. (4)-(6) have been taken as independent bases. In this paper I choose \( R_F, R_I, R_E \) as independent operators, replacing \( R_m \) with \( R_F \) of Eq. (3). The advantage of this choice of operator basis is that the coefficient functions take simple forms at the tree level. In fact we have [17]

\[ E_F^I(\text{tree}) = 1, \quad E_I^I(\text{tree}) = 0 \quad \text{for} \quad l = 1, \ldots, n-2, \] (10)

since the antisymmetric part of the short distance expansion for the product of two electromagnetic currents can be written at the tree level as

\[ \int d^4 x e^{i q \cdot x} \langle T [J_\mu(x) J_\nu(0)] \rangle \] (antisymmetric)

\[ = -i e \gamma_\mu \gamma_\lambda q^\lambda \sum_{n=1}^N \left( \frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_{n-1}} \times \{ R_q^{\sigma\mu_1\cdots\mu_{n-1}} + R_F^{\sigma\mu_1\cdots\mu_{n-1}} \} + \cdots, \]

where dots \( \cdots \) stands for nonleading terms and

\[ R_q^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-1} \bar{\psi} \gamma_5 \gamma^\sigma D^{(\mu_1 \cdots \mu_{n-2})} \gamma^\mu_{n-1} \psi \] (traces),

are twist-2 operators which contribute to \( g_1 \) and \( g_2^{\gamma^2} \). It is true that due to the relation, Eq. (9), \( R_F^{\sigma\mu_1\cdots\mu_{n-1}} \) can be expressed in terms of other operators. When eliminating \( R_F^\sigma \), we obtain a different set of coefficient functions. In other words, the (tree-level) coefficient functions are dependent upon the choice of the independent operators [17].

The renormalization constants for this new set of independent operators are written in the matrix form as

\[ \left( \begin{array}{c} R_F^\sigma \\ R_I^\sigma \\ R_E^\sigma \end{array} \right) = \left( \begin{array}{ccc} \bar{Z}_{FE} & \bar{Z}_{Fj} & \bar{Z}_{FE} \\ \bar{Z}_{IF} & \bar{Z}_{ij} & \bar{Z}_{IE} \\ 0 & 0 & \bar{Z}_{EE} \end{array} \right) \left( \begin{array}{c} R_F^n \\ R_I^n \\ R_E^n \end{array} \right) \] (11)

where the suffix \( R(B) \) denotes renormalized (bare) quantities.

Now we proceed to the moment sum rule for \( g_2 \). Define the matrix elements of composite operators between nucleon states with momentum \( p \) and spin \( s \) by
\[ \langle p,s \mid R_{\mu_1}^{\cdots \mu_{n-1}}(p,s) \rangle = -\frac{n-1}{n} d_n(s^\alpha p_{\mu_1} - s_{\mu_1} p^\alpha) p_{\mu_2} \cdots p_{\mu_{n-1}}, \]  
\[ \langle p,s \mid R_{\mu_1}^{\cdots \mu_{n-1}}(p,s) \rangle = -f_n^l(s^\alpha p_{\mu_1} - s_{\mu_1} p^\alpha) p_{\mu_2} \cdots p_{\mu_{n-1}}, \]  
\[ \langle p,s \mid R_{E}^{\cdots \mu_{n-1}}(p,s) \rangle = 0. \]  

Normalization is such that for free quark target, we have \( d_n = 1 \) and \( f_n = O(g^2) \). It is recalled that physical matrix elements of the EOM operators vanish [20]. Using Eqs. (14)–(16), we can write down the moment sum rule for \( g_2 \) as

\[ M_n = \int_0^1 dx x^{n-1} \frac{1}{g_2(x,Q^2)} = \frac{n-1}{2n} d_n E_n^x(Q^2) + \frac{1}{2} \sum_{i=1}^{n-2} f_n^l E_i^x(Q^2). \]  

The coefficient functions \( E_n^x(Q^2) \) and \( E_i^x(Q^2) \) satisfy the following renormalization group equation:

\[ \left( \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m(g) \frac{\partial}{\partial m} \right) E_i = \gamma_{ij} E_j \]

for \( i,j = F,1,\ldots,n-2 \), (18)

where \( \beta(g) \) and \( \gamma_m(g) \) are the QCD \( \beta \) function and the anomalous dimension of mass operator, respectively. The anomalous dimension matrix \( \gamma_{ij} \) of the composite operators \( R_{\mu_1}^{\cdots \mu_{n-1}} \) and \( R_{\mu_1}^{\cdots \mu_{n-1}} \) with \( l = 1, \ldots, n-2 \) is defined as

\[ \gamma_{ij} = Z_{ij} - Z_{ij} E \]  

for \( i,j = F,1,\ldots,n-2 \). (19)

Note that the anomalous dimension matrix which appears in Eq. (18) is a transposed one. This comes from our convention of defining renormalization constants and anomalous dimensions of the operators in Eqs. (13) and (19).

In the leading-logarithmic approximation, the solutions of the RG equations in Eq. (18) are given as follows [21]:

\[ E_i^x(Q^2) = \exp \left[ \frac{\gamma_{0i}^0}{2 \beta_0 \ln \left( \alpha(Q^2) / \alpha(\mu^2) \right) / \beta_i } \right] \text{for } i = F,1,\ldots,n-2, \]  

where \( \alpha(Q^2) \) is the QCD running coupling constant, \( \beta_0 \) and \( \gamma_{0i}^0 \) are, respectively, one-loop coefficients of the \( \beta \) function and anomalous dimension matrix,

\[ \beta(g) = -\beta_0 g^3 + O(g^5), \quad \beta_0 = \frac{1}{(4\pi)^2} \frac{n_f - 2n_f}{3}, \]  

where \( n_f \) is the number of flavors, and we have used the fact that \( E_i^x(\mu^2) = 1 \) and \( E_i^x(\mu^2) = 0 \) (for \( i = 1,\ldots,n-2 \) ) at the lowest-order.

III. MOMENT SUM RULE FOR \( g_2 \) IN THE LARGE \( N_c \) LIMIT

Now we need the information on the anomalous dimensions \( \gamma_{ij}^0 \) for \( i = F,1,\ldots,n-2 \). We can get it without embarking on a new calculation of the relevant Feynman diagrams. We utilize the existing results on the anomalous dimension matrix for the operators \( R_1, R_m \) and \( R_E \). In the case of the conventional choice of \( R_1, R_m \) and \( R_E \) independent operators, the renormalization constant matrix takes a triangular form

\[ \begin{pmatrix} R_1^a \\ R_m^a \\ R_E^a \end{pmatrix} = \begin{pmatrix} Z_{ij} & Z_{im} & Z_{IE} \\ 0 & Z_{mm} & 0 \\ 0 & 0 & Z_{EE} \end{pmatrix} \]  

(23)

In the minimal subtraction (MS) renormalization scheme, \( Z_{ij} \) is expressed as

\[ Z_{ij} = \delta_{ij} - \frac{\mu^2}{16\pi^2} X_{ij} \]  

(24)

where \( \epsilon = (4-d)/2 \) with \( d \) the space-time dimension, and the components \( X_{ij} \) have been calculated [8,9,11,18]. The following is the result on \( X_{ij} \) taken from Ref. [18]:

\[ X_{ij} = C_G \left( \frac{(j+1)(j+2)}{(l+1)(l+2)(l-j)} + 2C_F - C_G \right) \left( (-1)^{i+j} \frac{n-2i-j+1}{n-2i-j+1} \right) \left( \frac{2(-1)^{i+j} + (-1)^{i+j}}{l(l+1)(l+2)} \right) \]  

(25)

\[ X_{H} = C_G \left[ 1 - \frac{1}{l+1} - \frac{1}{l+2} - \frac{1}{n-l} - S_{l-1} - S_{n-l-1} \right] + (2C_F - C_G) \left[ \frac{1}{n-l} + \frac{2(-1)^l}{l(l+1)(l+2)} - \frac{(-1)^l}{n-l} \right] \]  

(26)
If we impose that the renormalized and bare operators, respectively, satisfy the constraint Eq. (9), we find from Eqs. (13) and (23) that Z's are related to the conventional Z's as follows:

\[
\bar{Z}_{FF} = \frac{n}{n-1} \sum_{l=1}^{n-2} (n-1-l)Z_{lm},
\]

\[
\bar{Z}_{Fj} = -(n-1-j)\bar{Z}_{FF} + \sum_{l=1}^{n-2} (n-1-l)Z_{lj},
\]

\[
\bar{Z}_{lj} = Z_{lj} - \frac{n}{n-1} (n-1-j)Z_{lm},
\]

where \( l, j = 1, \ldots, n-2 \). Using MS scheme rule, \( 1/e \to \ln \mu^2 \), we obtain, from Eqs. (19) and (22),

\[
-8\pi^2 \gamma_{FF}^{(0)n} = X_{mm} + \frac{n}{n-1} \sum_{l=1}^{n-2} (n-1-l)X_{lm},
\]

\[
-8\pi^2 \gamma_{lj}^{(0)n} = X_{lj} - \frac{n}{n-1} (n-1-j)X_{lm},
\]

It is straightforward to calculate the above \( \bar{\gamma}_{lj}^{(0)n} \) using the expressions \( X_{lj} \) in Eqs. (25)–(28). Especially, we obtain

\[
8\pi^2 \gamma_{lj}^{(0)n} = 4C_F \left( S_{n-1} - \frac{1}{4} + \frac{1}{2n} \right),
\]

\[
8\pi^2 \gamma_{lj}^{(0)n} = -(2C_F - C_G) \left( n-1-j \right) \left( 2S_{n-1} - S_j - S_{n-j-1} + 1 + \frac{1}{n} \right)
\]

\[
+ \sum_{l=1}^{j-1} (n-1-l) \left( \frac{1}{n-1} + \frac{2(-1)^j}{j(j+1)(j+2)} \right)
\]

\[
+ \sum_{l=j+1}^{n-2} (n-1-l) \left( \frac{1}{n-1} + \frac{2(-1)^j}{j(j+1)(j+2)} \right)
\]

\[
\left( -1 \right)^{l+j} \frac{n-2C_j (n-1-l+j)}{n-2C_{l-1} (n-1)(l-j)} + \frac{2(-1)^j}{l(l+1)(l+2)} \right) \right).
\]

for \( j = 1, \ldots, n-2 \).
Now we see that the mixing anomalous dimension $\gamma_{FJ}^{(0)n}$ turns out to be proportional to $(2C_F - C_G)$. Since

$$C_F = \frac{N_C^2 - 1}{2N_C}, \quad C_G = N_C,$$

we have $2C_F = C_G$ and thus $\gamma_{FJ}^{(0)n} = 0$ in the $N_C \to \infty$ limit. Then Eq. (20) gives

$$E_F^n(Q^2) = \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{\gamma_{FF}^{(0)n}/2\beta_0},$$

$$E_I^n(Q^2) = 0 \quad \text{for} \quad l = 1, \ldots, n-2.$$ (41)

Returning to Eq. (17), we find that, at $N_C$ going to infinity, the moment sum rule for $\bar{g}_2$ takes a simple form as follows:

$$\int_{0}^{1} dx x^{n-1} \bar{g}_2(x, Q^2) = \frac{n-1}{2n} d_n \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{\gamma_{FF}^{(0)n}/2\beta_0}$$ (42)

with

$$\gamma_{FF}^{(0)n} = 2N_C(S_{n-1} - 1/4 + 1/2n)$$

(43)

In other words, at large $N_C$, the operators $R^\mu_1 \cdots \mu_{n-1}$ involving the gluon fields decouple from the evolution equation of $\bar{g}_2$ and the whole contribution is represented by one type of operators $R^\mu_1 \cdots \mu_{n-1}$. With the substitution $C_F = N_C/2$ and $n = j + 1$, the anomalous dimension $8\pi^2\gamma_{FF}^{(0)n}$ coincides with Eq. (18) of Ref. [15]. This completes the reproduction, in the framework of OPE and RG, of the ABH result on $\bar{g}_2$.

IV. SUMMARY AND DISCUSSION

It should be emphasized that we have reproduced the ABH result without assuming massless quarks. A question expected to come up immediately is that the replacement of the mass-dependent operator $R^\mu_1 \cdots \mu_{n-1}$ with $R^\mu_1$ may be equivalent to working with massless quarks. The answer is no. Indeed it can be shown that even when we include the mass-dependent operator $R^\mu_1 \cdots \mu_{n-1}$ among the independent operator bases, we reach the same conclusion. Let us take, for an example, $R^\mu_1$, $R^\mu_1$ (with $l = 2, \ldots, n-2$), $R^\mu_1$ and $R^\mu_1$ as independent operators replacing one quark-gluon operator $R^{\mu_1 \cdots \mu_{n-1}}$. With this choice of new operator bases, the moment sum rule for $\bar{g}_2$ is written in terms of the coefficient functions $E_F^n(Q^2)$, $E_I^n(Q^2)$ with $l = 2, \ldots, n-2$, and $E_I^n(Q^2)$. The renormalization constants for these operators are written as

$$E_F^n(Q^2) = \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{\gamma_{FF}^{(0)n}/2\beta_0}$$

(44)

Again imposing that the renormalized and bare operators, respectively, satisfy the constraint Eq. (9), we find that $\tilde{Z}$'s are related to conventional $Z$'s as follows:

$$\tilde{Z}_{FF} = \frac{1}{n-2} \sum_{l=1}^{n-2} (n-1-l)Z_{lj},$$

$$\tilde{Z}_{Fj} = -(n-1-j)\tilde{Z}_{FF} + \sum_{l=1}^{n-2} (n-1-l)Z_{lj},$$

$$\tilde{Z}_{fm} = -\frac{n-1}{n} \tilde{Z}_{FF} + \frac{n-1}{n} Z_{mm} + \sum_{l=1}^{n-2} (n-1-l)Z_{lm}.$$ (47)

Then it is easy to obtain the following one-loop coefficients of the relevant anomalous dimensions

$$8\pi^2\gamma_{FF}^{(0)n} = 4C_F \left( S_{n-1} - \frac{1}{4} + \frac{1}{2n} \right)$$

(48)

$$8\pi^2\gamma_{Fj}^{(0)n} \times (2C_F - C_G) \quad \text{for} \quad j = 2, \ldots, n-2,$$

$$8\pi^2\gamma_{fm}^{(0)n} \times (2C_F - C_G).$$ (50)

Inserting these anomalous dimensions to the solutions of the RG equations for the coefficient functions $E_F^n(Q^2)$, $E_I^n(Q^2)$ ($l = 2, \ldots, n-2$) and $E_I^n(Q^2)$, we obtain in the large $N_C$ limit

$$E_F^n(Q^2) = \left( \frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{\gamma_{FF}^{(0)n}/2\beta_0}$$

(52)

$$E_I^n(Q^2) = 0 \quad \text{for} \quad l = 2, \ldots, n-2,$$

$$E_I^n(Q^2) = 0.$$ (54)
Thus we reach the same conclusion Eq. (42) even when we include the mass-dependent operators among the independent operator bases.

A few comments are in order. Firstly, the twist-3 quark-gluon operators $R^a_{\gamma J}$ decouple from the evolution equation for $g_2$ at large $N_c$. This might be explained by an argument on quark condensate [7]. A hint is that the mixing anomalous dimensions $\gamma_{f J}$ turn out to be proportional to $(2C_F - C_G)$. There are two types in the products of color matrices entering into the calculation of anomalous dimensions for the flavor nonsinglet $g_2$:

$$T^bT^aT^b = \left( C_F \frac{1}{2} C_G \right) \frac{1}{2N_c} T^a,$$

$$T^bT^bT^a = C_F \frac{1}{2} N_c \frac{1}{N_c} T^a.$$  

(55)

(56)

It is argued in Ref. [7] that the quark condensate contains all colors and at large $N_c$, the condensate polarization becomes small and that the combination $T^bT^aT^b$ is connected with condensate polarization effects.

Secondly, we have chosen particular sets of the independent operators and reached a simple form for the moments of $g_2$ in the large $N_c$ limit. However, arbitrariness in the choice of the operator bases should not enter into physical quantities [17]. A different choice of the operator bases leads to different forms for the anomalous dimension matrix and the coefficient functions. Recall that the constraint, Eq. (9), gives a relation among the tree-level coefficient functions and also a relation among the matrix elements of the operators. After diagonalizing the anomalous dimension matrix and using these relations, we can arrive at the same conclusion for the moments of $g_2$ in the $N_c \rightarrow \infty$ limit. What we did in this paper is that we chose particular sets of bases from the beginning which include an operator that represents the whole contribution to $g_2$ for large $N_c$.

Finally, the nucleon has other twist-3 distributions, namely, chiral-odd distributions $h_1(x, Q^2)$ and $e(x, Q^2)$ [22]. Just like the $g_2$ case, the $Q^2$ evolutions of flavor nonsinglet $h_1(x, Q^2)$ and $e(x, Q^2)$ turn out to be quite complicated due to mixing with quark-gluon operators, the number of which increases with spin. However, it has been proved [23] that in the large $N_c$ limit, these twist-3 distributions also obey a simple DGLAP equation. The proof holds true only when we work with massless quarks.

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