Photonic Crystal k -Vector Superprism

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Abstract—We theoretically investigate the resolution of the photonic crystal (PC) superprism as a narrow-band filter at 1.55 μm wavelength range. First, we defined the equi-incident-angle curve against the dispersion surface of each photonic band in the Brillouin zone and calculated the beam collimation, wavelength sensitivity, and resolution parameters for a PC. The result indicated that the conventional superprism which deflects the Poynting vector of light cannot achieve a high resolution and the miniaturization of the PC, simultaneously. So, we proposed a new superprism (k -vector prism), which deflects the k vector and enhances the refraction angle at an angled output end of the PC. We estimated that the resolution is the same as or higher than that of the conventional prism and the PC can be significantly miniaturized. Through the finite-difference time-domain simulation of light propagation, we observed a correspondence and a characteristic difference against the above analysis.

Index Terms—Filter, finite-difference time-domain (FDTD), photonic crystal (PC), superprism, wavelength-division multiplexing (WDM).

I. INTRODUCTION

THE photonic crystal (PC) is a multidimensional diffraction grating designed by solid-state physics theory. It is expected to realize various novel microoptic devices. Thus far, an ultrasmall laser [1], an ultrasmall waveguide [2], and a resonant-type narrow-band filter [3], all of which utilize the photonic bandgap (PBG), have been demonstrated. On the other hand, an ultralow group velocity and an anomalous dispersion occur at the frequency range higher than the PBG. Especially, the superprism phenomenon has attracted much attention, because it allows wide-angle deflection of a light beam by a slight change of wavelength and incident angle. It has been theoretically discussed and experimentally demonstrated by Russel [4] and Kosaka *et al.* [5]. As applications, a dispersion compensation device [6] and a light deflection device [7] are discussed, but the most fundamental application is a diffraction-type narrow-band wavelength filter. To realize the dense wavelength-division multiplexing (DWDM) network up to end users, more simplified systems and the cost reduction are required. For this purpose, a high-efficiency and compact narrow-band filter, which can be integrated with other optical components, is necessary. At present, silica-based arrayed

Fig. 1. Correspondence of real space and k space. (a) Behavior of light beam in real space. (b) Dispersion surface analysis of light propagation in k space.

waveguide grating filter is widely used, but it is essentially difficult to miniaturize. The superprism filter is expected because of this reason. However, there were no reports on the quantitative estimation of the wavelength resolution, the most important performance of such a filter. Previously, we reported the first theoretical calculation of the wavelength resolution of superprisms based on the dispersion surface analysis [8]. The result showed that a superprism can realize a sufficiently high resolution usable for the DWDM, but that the PC cannot be miniaturized to less than 1 cm^2 .

In this paper, we focus on a novel superprism, which utilizes the refraction of light at an angled output end of a PC. We name such a prism a k -vector prism. (In contrast to this, let us call the conventional superprism the S -vector prism, since it deflects the Poynting vector S in the PC.) We investigate the resolution of the k -vector prism using the dispersion surface analysis. We also perform the two-dimensional (2-D) finite-difference time-domain (FDTD) simulation of light beam propagation to verify the dispersion surface analysis.

In the next section, we will review the resolution of the S -vector prism and explain the problem of this prism. In Section III, we will introduce the k -vector prism and explain the essential difference of this prism and the S -vector prism. In Section IV, we will present the resolution of the k -vector prism, and in Section V, the light beam propagation obtained by using the 2-D FDTD method.

II. CONVENTIONAL S -VECTOR PRISM

As shown in Fig. 1(a), we consider a finite width Gaussian beam incident to a triangle lattice 2-D PC. The deflection angle of the light beam can be analyzed from an equi-frequency curve of a dispersion surface in the Brillouin zone. As shown in Fig. 1(b), the wave vector in the PC k_c is determined so that the tangential component of the incident wave vector k_{in} against the input end of the PC is conserved. The direction of the S

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Fig. 2. Schematics of dispersion surfaces of lower three bands with equi-incident-angle curves (upper) and corresponding examples of calculation (lower).

vector is determined by the gradient of the dispersion surface at cross-point A of the equi-frequency curve and the construction line. Now, we introduce the equi-incident-angle curve for the better understanding of the superprism. It is a locus of point A in the Brillouin zone against a constant incident angle and varying frequency of light. Fig. 2 shows schematics of the dispersion surface and the equi-incident-angle curve for the lowest three photonic bands and those calculated by the plane-wave expansion method. Equi-incident-angle curves shown by gray lines exhibit a radial pattern near the Γ point of the first photonic band. For higher bands, which are more important for superprisms, they become like vertical lines. If one aims at a wavelength filter for a certain incident angle of light beam, one has to utilize the dispersion characteristic along the corresponding equi-incident-angle curve.

A Gaussian beam incident to a PC can be expanded by plane waves with a Gaussian angular spectrum. The full spectral width at $1/e^2$ intensity is given as $2\theta_D = \lambda/\pi n w_0$, where λ is the wavelength in vacuum, n is the refractive index of the incident medium, and w_0 is the spot radius at $1/e^2$ intensity. For example, $2w_0 = 120 \mu\text{m}$ corresponds to $2\theta_D = 0.3^\circ$, when $\lambda = 1.55 \mu\text{m}$ and $n = 3.065$. On the other hand, light propagating in the PC is no more a Gaussian beam, since each plane-wave component is expanded into Bloch waves at the input end of the PC and affected by the complex dispersion characteristic. However, the envelope of the light intensity profile can be approximated by a Gaussian, when $2\theta_D$ is sufficiently small. Then, the beam width $2w$ in the PC far from the input end is expressed as

$$2w \cong 2 \frac{\lambda L}{\pi n w_0} p \quad (1)$$

where p is defined as $p \equiv (\partial\theta_c/\partial\theta_i)$ for the incident beam angle θ_i and the propagating beam angle in the PC θ_c . This parameter represents the beam collimating ability in the PC. In (1), L is the distance from the input end, which satisfies

$$L > \frac{\pi n w_0^2}{\lambda} p^{-1}. \quad (2)$$

Since the beam width of (1) occupies far-field angle $2w/L$, the wavelength resolution is given by

$$\frac{\Delta\lambda}{\lambda} = \frac{(\frac{2w}{L})}{(\frac{a}{\lambda})} \frac{\partial(\frac{a}{\lambda})}{\partial\theta_c} \cong \frac{2\lambda^2}{\pi n w_0 a} \left(\frac{q}{p}\right)^{-1} \quad (3)$$

where a is the lattice constant of the PC, and q is defined as $q \equiv \partial\theta_c/\partial(a/\lambda)$. This parameter represents the angular dispersion (wavelength sensitivity) of the PC. When $\lambda = 1.55 \mu\text{m}$, $2w_0 = 120 \mu\text{m}$, and $n = 3.065$, a wavelength resolution of 0.4 nm is obtained by $q/p > 75$. In the following, we call q/p the resolution parameter.

We investigated parameters $1/p$, q , and q/p by calculating dispersion surfaces of a 2-D PC of triangular lattice elliptical airholes. We assumed the background index to be equal to the incident medium index, i.e., 3.065, the index of the airholes to be 1.0, and the size of the airholes to be laterally $0.4a$ and vertically $0.9a$. Dispersion surfaces were obtained by calculating eigen frequencies over the Brillouin zone using Ho's plane-wave expansion method [9]. To carry out differential calculations of $1/p$ and q with enough accuracy even on a sharply changing equi-frequency curve, enormous computation time is required. In this calculation, the number of plane waves was limited to 37. This number does not give a perfect convergence of the calculation, but gives outlines of results with a small error less than 1% for such lower order bands.

Fig. 3 shows calculated results for the second band. It is observed that in general, $1/p$ is low and q is high in the PC. This means that the light beam is easy diverged and its propagation is sensitive to the wavelength. These characteristics are evident on sharply changing equifrequency curves. On such curves, the resolution parameter q/p becomes rather small, since a large q is compensated by a small $1/p$. Close to these curves, however, there exist narrow regions that satisfy $q/p > 75$. Fortunately, some of them are lying along equi-incident-angle curves. It means that a high resolution is expected in a wide spectral range against a fixed incident angle. But each lateral width of these regions is very narrow. This means that the latitude of the incident angle is so narrow that the incident beam is required

Fig. 3. Shaded drawing of parameters for $\mathbf{S}$ -vector prism of a triangular lattice 2-D PC. (a) Beam collimation parameter $1/p$, (b) wavelength sensitivity parameter q , and (c) resolution parameter q/p . Equi-incident-angle curve is drawn with an interval of 5° .

to be highly collimated. For example, the width of the incident beam must be wider than $100 \mu\text{m}$ for a target wavelength resolution of 0.4 nm at $\lambda \sim 1.55 \mu\text{m}$. For such a wide beam, the PC length L reaches to over 10 cm , according to (2). Thus, we noticed that it is difficult to miniaturize the $\mathbf{S}$ -vector prism to mm^2 order.

III. PROPOSAL OF $\mathbf{k}$ -VECTOR PRISM

The $\mathbf{k}$ -vector prism is a superprism designed such that the $\mathbf{k}$ vector is deflected with small beam divergence for a small change of the wavelength. Fig. 4 shows two dispersion surfaces, which are absolutely the same, but (a) and (b) are focusing on the change of the $\mathbf{S}$ vector and the $\mathbf{k}$ vector, respectively. The $\mathbf{S}$ vector is widely changing in (a), but the change of the $\mathbf{k}$ vector is small. In this case, the refraction angle at the output end of the PC exhibits a small change for different wavelength beams. Therefore, it is necessary to completely separate the beams inside the PC for wavelength resolution. This is an essential reason that disturbs the miniaturization of the PC in the $\mathbf{S}$ -vector prism. On the other hand, the $\mathbf{k}$ vector is widely changing in (b). If we prepare an angled output end of the PC, different $\mathbf{k}$ vectors lead to different refraction angles. This allows the beam separation through the free-space propagation outside the PC, and so allows the drastic miniaturization of the PC to shorter than $100 \mu\text{m}$. By integrating a beam focusing lens at the output end or outside of the PC, the length of the free-space propagation will be further reduced, so the total filter system can be significantly miniaturized.

Similarly to the case of the $\mathbf{S}$ -vector prism, we defined differential parameters for the $\mathbf{k}$ -vector prism, i.e., the beam collimation parameter $p \equiv (\partial\theta_{\text{out}}/\partial\theta_{\text{in}})$, the wavelength sensitivity parameter $q \equiv \partial\theta_{\text{out}}/\partial(a/\lambda)$, and the wavelength resolution parameter q/p . Here, θ_{out} is the beam angle outside the PC. To calculate θ_{out} , we used the conservation law of the tangential $\mathbf{k}$ -vector component at the output end. These parameters for the second band of the 2-D PC of triangular lattice circular airholes were calculated in the same way as for the $\mathbf{S}$ -vector prism. The background index and the airhole index were 3.065 and 1.0 , respectively, and $2r/a = 0.624$. The number of plane waves was 37 . The angle of the output end was 30° . As shown in Fig. 5, $1/p$ is higher and q is lower than those of the $\mathbf{S}$ -vector prism. Consequently, q/p is higher than the $\mathbf{S}$ -vector prism's. This is due to the beam width expanded by the angled output end, which improves the collimation parameter outside the PC.

Fig. 4. Difference of two superprisms in real space and $\mathbf{k}$ space. (a) $\mathbf{S}$ -vector prism and (b) $\mathbf{k}$ -vector prism.

Fig. 5. Shaded drawing of parameters for $\mathbf{k}$ -vector prism in a triangular lattice 2-D PC. Others are the same as for Fig. 3.

IV. RESOLUTION OF THE $\mathbf{k}$ -VECTOR PRISM

Figs. 6 and 7 show q/p for the second band of PCs with different lattices and angles of the output end. The angle is changed in the range of 20 – 40° for the triangular lattice and 40 – 50° for the square lattice rotated by 45° . In Fig. 6(a), no high-resolution regions along an equi-incident-angle curve are seen, while in (b) and (c), incident angles of 10° and 30° give a high resolution, respectively. These regions are vertically longer than those for the $\mathbf{S}$ -vector prism, which means a wider usable spectral range. In addition, the latitude of the incident angle is much wider, which allows a high resolution even for a narrow incident beam. In Fig. 7(b) and (c), incident angles of 10° and 25° give a high resolution, respectively, and the angle latitude is further expanded. Table I summarizes the specification that satisfies $q/p = 100$. For the 45° -rotated square lattice with a 45° output end, an incident beam width of $13 \mu\text{m}$ is allowed. This enables the butt-joint of a single-mode fiber to the prism. For the same lattice with a 50° output end, $>100 \text{ nm}$ spectral range is usable at $\lambda \sim 1.55 \mu\text{m}$. To convert the resolution param-

Fig. 6. Resolution parameters for triangular lattice 2-D PCs with various angles θ_{pc} of output end. (a) $\theta_{pc} = 20^\circ$, (b) 30° , and (c) 40° .

Fig. 7. Resolution parameters for 2-D PCs of square lattice rotated by 45° with various angles θ_{pc} . (a) $\theta_{pc} = 40^\circ$, (b) 45° , and (c) 50° .

TABLE I
SPECIFICATION OF $\mathbf{k}$ -VECTOR PRISM FOR A RESOLUTION PARAMETER OF 100

Lattice/Output angle	Acceptable spot diameter	Usable wavelength range
Triangular / 30°	44 μm	1538 – 1562 nm
/ 40°	49 μm	1545 – 1554 nm
Square / 45°	13 μm	1507 – 1595 nm
/ 50°	28 μm	1498 – 1605 nm

eter of the $\mathbf{k}$ -vector prism into an absolute value of the wavelength resolution, the free-space propagation has to be involved in the discussion. Even without such discussion, we can notice a high resolution for the $\mathbf{k}$ -vector prism, when we consider a fact that the $\mathbf{S}$ -vector prism can achieve a wavelength resolution of 0.4 nm even by a resolution parameter of 75.

So far, we only paid attention to the change of the $\mathbf{k}$ vector. However, even in the $\mathbf{k}$ -vector prism, the light beam propagates inside the PC according to the theory of the $\mathbf{S}$ -vector prism. Unless it is not taken into consideration, the design of the $\mathbf{k}$ -vector prism sometimes fails because the light beam may not go toward the target output end. So, such failure conditions are shadowed on the dispersion surface, as shown in Fig. 8. The high-resolution region is lying near boundaries of the shadowed region, where the propagating beam is expanded at the output end and the collimation parameter is improved. The shadowed region is more expanded for a large angle of the output end. However, this region becomes usable if the same angle output end is placed in the opposite side of the prism, as shown in Fig. 9. Under this condition, the $\mathbf{k}$ -vector prism works if it satisfies the conservation law of the tangential $\mathbf{k}$ -vector component. Thus, all the high-resolution condition in the Brillouin zone can be utilized.

V. FDTD ANALYSIS

In the analysis, we set an Yee cell size of 40 nm and a time interval of 0.08 fs. The normalized airhole diameter $2r/a$

was set to be 0.527, which is suitable for the observation of highly collimated beams. Berenger's perfectly matched layer absorbing boundary condition of 64 layers was used for the termination of the analytical space. As an incident wave, a 10- μm -wide continuous sinusoidal wave was excited for the magnetic field normal to the 2-D plane, H_z . The incident angle was 10° against normal to the PC. The normalized frequency a/λ was 0.268 or 0.274. The calculated profile of H_z is shown in Fig. 10. The incident light beam propagates in the direction of the $\mathbf{S}$ -vector in the PC, and should refract at the output end according to the $\mathbf{k}$ -vector prism theory. Since half-circles are placed at the output end [10], the collimated output beam is extracted without extra diffraction beams. For the two a/λ , the beam angle is changed by 2.1° . Actually, however, these beam angles do not agree with those expected from the simple $\mathbf{k}$ -vector conservation law. To explain this, we applied the conservation law to the $\mathbf{k}$ vector in the second Brillouin zone, as shown in Fig. 11. If the tangential component of the $\mathbf{k}$ -vector is simply conserved, the beam is extracted in direction A. But direction B, which is derived from the conservation law for the second Brillouin zone [7], rather agrees well with the FDTD calculation. The normalized frequency of the incident wave assumed in this FDTD simulation corresponds to the second band, which is lying in the second Brillouin zone of the extended zone scheme. Therefore, this result is considered to be reasonable. For such a condition, we performed the dispersion analysis again. As a result, the resolution parameter slightly decreased but was not significantly different from the result of Fig. 7.

As shown in Fig. 10, the reflection is observed at input and output ends. It can be suppressed by the modification of the interface structure [10]. But for the complete suppression of the reflection, quantitative estimation of the mode matching and the optimization of the structure are necessary. It will be the issue to be solved in the near future study.

Fig. 8. Normal output end model and distribution of usable condition for 2-D PC of square lattice rotated by 45° . (a) $\theta_{pc} = 40^\circ$, (b) 45° , and (c) 50° . The shadowed region fails the $\mathbf{k}$ -vector prism condition.

Fig. 9. Opposite output end model and distribution of usable condition. Others are the same as for Fig. 8.

Fig. 10. FDTD simulation of light propagation. (a) Calculation model, (b) profile of H_z at $a/\lambda = 0.268$, and (c) that at 0.274.

VI. CONCLUSION

We discussed the $\mathbf{k}$ -vector prism instead of the conventional $\mathbf{S}$ -vector prism for the 2-D PC with the angled output end. From the dispersion surface analysis, we estimated a resolution parameter comparable to or higher than the $\mathbf{S}$ -vector prism's, over 100 nm usable wavelength range at $\lambda \sim 1.55 \mu\text{m}$, and a large latitude of the incident angle, which enables the butt-joint of a single-mode fiber to the prism. However, we found that the optimum resolution condition and the angle of the output end are restricted by the $\mathbf{S}$ -vector in the PC. To make the whole Brillouin zone usable, the alignment of the output end must be optimized. In the FDTD simulation of light propagation, we confirmed the deflection of output light beam. This direction was explained by the dispersion surface analysis including the second Brillouin zone. At the next stage of the research, it will

Fig. 11. Analysis of light propagation by the dispersion surface including the second Brillouin zone.

be necessary to design an integrated condenser lens for a compact filter system and an interface structure for connecting an optical fiber with low reflection loss.

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