Abstract—This paper presents a basic model of an electro-mechanical energy conversion system using the resonant inductive coupling as well as the power transfer system between the coils. The proposed system utilizes two sets of two-phase air-core coils both for the stator and the mover. There exists parasitic capacitance of the coils which is connected in series. Thus the whole system has certain resonant frequencies. Two-phase voltage source connected to the primary coils is driven at the resonant frequency. Unlike conventional wireless power transfer systems using resonant inductive coupling, the proposed system can generate moving flux field. The proposed system is similar to the induction machine but it is driven at very high resonant frequency and the slip ratio is also quite high. Tendency of the generated force of the mover obtained by the FEA simulation shows that can be explained by the model derived.

I. INTRODUCTION

In recent years, wireless power transfer technology using resonant inductive coupling [1]–[14] has attracted attention. This technology, compared with the conventional wireless power transmission technology using the magnetic induction [15]–[19], the transmission distance is relatively long about 1m. The system consists of two coils at the primary and secondary side of magnetically coupled weakly, and a capacitor connected to the coils. In some cases, it connects actual capacitors in addition to the parasitic capacitance of the coils. It is found to be able to explain the frequency and efficiency of power which can be transmitted precisely by equivalent concentrated circuit. There exists the resonant frequencies by which the impedance from the primary side becomes minimal. If the Q value is high enough, large power can be transmitted at the resonant frequency.

The idea in this paper is to apply the resonant inductive coupling to the electro-mechanical energy conversion system as well as the power transfer system between the coils. Based on the best knowledge of the authors, studies utilizing magnetic resonant coupling dealing with both power transfer and electro-mechanical energy conversion in a unified manner are not found so far. In our proposed system, totally four coils are utilized. Two primary coils at the stator are allocated as shown in Fig. 1. The phase of two coils are spatially shifted 90-degree. Two secondary coils at the mover are allocated as well as the primary side. The primary coils are driven by two sinusoidal voltage sources whose phase difference is 90 degree, which generate moving flux field. The frequency of the voltage sources is near the resonant frequency of the system. When connecting the output end on the secondary side, it behaves as the induction machine. However, compared to the conventional induction machines, the slip ratio of the proposed machine is quite high which is near 100%. This is because the frequency of the voltage sources is near the resonant frequency, which is generally between several hundreds kHz and ten and several MHz, and the frequency of the mover is typically less than 10 kHz. Differences between the proposed machine and conventional induction machines are summarized as follows.

- Capacitors exist in the primary side and the secondary side, and the circuit is driven at a high frequency near the resonant frequency.
- Since the speed of the secondary side mover with respect to the driving frequency is quite small, a slip is extremely big.
- It is possible to have a large gap between the primary side and the secondary side.
- When the primary side and the secondary side are air-core coil, generated force is small.

In this paper, a basic model of the electric machine with resonant inductive coupling is proposed and generation of force is confirmed by FEA simulations.

The rest of this paper is organized as follows. Section II presents a mathematical model of the proposed electric machine where voltage equation and force equation are derived. Section III describes resonant frequency of the whole circuit considering the effect of the speed of the secondary side mover. Section IV demonstrates simulation results by using Finite
Element Analysis (FEA) and generation of force is confirmed, and Section V concludes the paper.

II. MODEL

Consider four coils located as shown in Fig. 1. Two-phase primary coils are fixed on the stator and the other two-phase secondary coils are mounted on the mover, where \( x \) is the displacement of the mover, \( l_x \) is the pitch length corresponding to \( 2\pi \) radian in electric angle of the primary two-phase coils, and \( l_y \) is the gap length between the primary coils and the secondary coils. Air-core coils are assumed in this paper. If we utilize the open-end coils both in the primary side and the secondary side, there exists parasitic capacitance of the coils which is connected in series. The equivalent circuit of this model as shown in Fig. 2. The inductive coupling between the primary coils and the secondary coils are variable according to the displacement of the mover. The inductance matrix of the coils in Fig. 2 can be described as follows.

\[
L = \begin{bmatrix}
L_1 & 0 & M_0 \cos \theta & -M_0 \sin \theta \\
0 & L_1 & M_0 \sin \theta & M_0 \cos \theta \\
-M_0 \sin \theta & M_0 \cos \theta & L_2 & 0 \\
-M_0 \sin \theta & M_0 \cos \theta & 0 & L_2
\end{bmatrix}
\]

where \( L_1 \) and \( L_2 \) are self-inductances of the primary coils and the secondary coils, respectively. \( M_0 \) is the maximum value of a mutual inductance between the primary coils and the secondary coils. \( \theta = 2\pi x/l_x \) is the electric angle of the mover. The self-inductances and the mutual inductance are given from the magnetic field distribution. Details can be found in the Appendix.

The voltage equation of the equivalent circuit can be described as follows.

\[
\begin{align*}
V &= RL + \frac{d}{dt}(LI) + V_C \\
I_C &= C \frac{dV_C}{dt}
\end{align*}
\]

where

\[
V = [V_{1a} \ V_{1b} \ V_{2a} \ V_{2b}]^T
\]

\[
I = [I_{1a} \ I_{1b} \ I_{2a} \ I_{2b}]^T
\]

\[
R = \text{diag}(R_1, R_1, R_2, R_2)
\]

\[
C = \text{diag}(C_1, C_1, C_2, C_2)
\]

\[
V_C = [V_{C1a} \ V_{C1b} \ V_{C2a} \ V_{C2b}]^T
\]

and \( V_{1a}, V_{2a}, V_{1b}, \) and \( V_{2b} \) are terminal voltages of the coils, \( I_{1a}, I_{2a}, I_{1b}, \) and \( I_{2b} \) are currents flowing each circuits, \( R_1 \) and \( R_2 \) are resistance of the primary coils and the secondary coils, \( C_1 \) and \( C_2 \) are parasitic capacitances of the primary coils and the secondary coils, and \( V_{C1a}, V_{C2a}, V_{C1b}, \) and \( V_{C2b} \) are voltages of those capacitors.

Generated force along with direction of the displacement \( x \) is described as follows.

\[
F = \frac{1}{2} I^T \frac{\partial L}{\partial x} I + \frac{d\theta}{dt} \frac{1}{2} I^T \frac{\partial L}{\partial \theta} I.
\]

Assume that the currents flowing through the coils are sinusoidal and given by

\[
\begin{align*}
I_{1a} &= I_1 \sin(\omega_1 t) \\
I_{1b} &= I_1 \sin(\omega_1 t - \pi/2) \\
I_{2a} &= I_2 \sin(\omega_2 t - \delta) \\
I_{2b} &= I_2 \sin(\omega_2 t - \delta - \pi/2)
\end{align*}
\]

where \( \delta \) is a phase delay between the primary coils and the secondary coils. And \( I_1 \) and \( \omega_1 \) are amplitude and angular frequency of the current of the primary coils, respectively. \( I_2 \) and \( \omega_2 \) are those of the secondary coils as well.

By substituting (10)–(13) to (1)–(8), the voltage equation is transformed as follows.

\[
\begin{align*}
V_{1a} &= R_1 I_1 \sin(\omega_1 t) + (\omega_1 L_1 - \frac{1}{\omega_1 C_1}) I_1 \cos(\omega_1 t) \\
&\quad + (\omega_2 + \omega) M_0 I_2 \cos(\omega_2 t + \theta - \delta) \\
V_{1b} &= -R_1 I_1 \cos(\omega_1 t) + (\omega_1 L_1 - \frac{1}{\omega_1 C_1}) I_1 \sin(\omega_1 t) \\
&\quad + (\omega_2 + \omega) M_0 I_2 \sin(\omega_2 t + \theta - \delta) \\
V_{2a} &= R_2 I_2 \sin(\omega_2 t - \delta) + (\omega_2 L_2 - \frac{1}{\omega_2 C_2}) I_2 \cos(\omega_2 t - \delta) \\
&\quad + (\omega_1 - \omega) M_0 I_1 \cos(\omega_1 t - \theta) \\
V_{2b} &= -R_2 I_2 \cos(\omega_2 t - \delta) + (\omega_2 L_2 - \frac{1}{\omega_2 C_2}) I_2 \sin(\omega_2 t - \delta) \\
&\quad + (\omega_1 - \omega) M_0 I_1 \sin(\omega_1 t - \theta).
\end{align*}
\]

In addition, the generated force (9) is also transformed into the form:

\[
F = \frac{2\pi}{l_x} I_1 I_2 M_0 \sin((\omega_1 - \omega_2)t - \theta + \delta).
\]
(17), we have solutions satisfying $V_{2n} = V_{2b} = 0$ as follows.

$$\delta = - \tan^{-1} \frac{R_2}{\omega_2 L_2 - \frac{1}{\omega_2 C_2}}$$ (19)

$$I_2 = - \sqrt{\frac{\omega_2 M_0}{R_2^2 + (\omega_2 L_2 - \frac{1}{\omega_2 C_2})^2}} I_1$$ (20)

$$\omega_2 t = \omega_1 t - \theta$$

$$\omega_2 = \omega_1 - \omega.$$ (22)

The phase delay $\delta$, the amplitude $I_2$ and angular frequency $\omega_2$ of the secondary current are determined uniquely.

By substituting these solutions (19)–(22) to (14) and (15), the voltage equations in the primary circuit are simply expressed as follows.

$$V_{1a} = I_1 \sqrt{R_1^2 + (\omega_1 L_1 - \frac{1}{\omega_1 C_1})^2} \cos(\omega_1 t + \gamma) + \omega_1 M_0 I_2 \cos(\omega_1 t - \delta)$$

$$V_{1b} = I_1 \sqrt{R_1^2 + (\omega_1 L_1 - \frac{1}{\omega_1 C_1})^2} \sin(\omega_1 t + \gamma) + \omega_1 M_0 I_2 \sin(\omega_1 t - \delta)$$

$$\gamma = - \tan^{-1} \frac{R_1}{\omega_1 L_1 - \frac{1}{\omega_1 C_1}}.$$ (25)

The generated force is also simplified as follows by substituting the solutions (19)–(22) to (18).

$$F = \frac{2\pi}{L_x} \frac{(\omega_1 - \omega) M_0^2 R_2}{R_2^2 + (\omega_2 L_2 - \frac{1}{\omega_2 C_2})^2} I_1^2.$$ (26)

From this result, it turns out that magnitude of the force can be controlled by the amplitude of the primary current $I_1$ and direction of the force can be controlled by the sign of the angular frequency of the primary currents $\omega_1$.

Further calculation yields more simplified expressions as follows.

$$V_{1a} = I_1 A \cos(\omega_1 t + \beta)$$

$$V_{1b} = I_1 A \sin(\omega_1 t + \beta)$$

where

$$A = \sqrt{R_1^2 + X_1^2 + \frac{\omega_1 \omega_2 M_0^2 (\omega_1 \omega_2 M_0^2 + 2 R_1 R_2 - 2 X_1 X_2)}{R_2^2 + X_2^2}}.$$ (29)

$$\beta = - \tan^{-1} \frac{\omega_1 \omega_2 M_0^2 R_2 + R_1 (R_2^2 + X_2^2)}{\omega_1 \omega_2 M_0^2 X_2 - X_1 (R_2^2 + X_2^2)}.$$ (30)

$$X_1 = \omega_1 L_1 - \frac{1}{\omega_1 C_1}.$$ (31)

$$X_2 = \omega_2 L_2 - \frac{1}{\omega_2 C_2}.$$ (32)

Regarding the generated force, we have

$$F = \frac{2\pi}{L_x} \frac{(\omega_1 - \omega) M_0^2 R_2}{D} V_1^2$$ (33)

where

$$D = (R_1^2 + X_1^2)(R_2^2 + X_2^2) + \omega_1 \omega_2 M_0^4 (\omega_1 \omega_2 M_0^2 + 2 R_1 R_2 - 2 X_1 X_2).$$ (34)

and $V_1 = \sqrt{V_{1a}^2 + V_{1b}^2}$ is the amplitude of the voltage source in the primary circuits.

As same as typical induction machines, when extremely high frequency $\omega_1$ for the voltage source in the primary side is given, the slip frequency $\omega_2$ is also extremely high according to (22) because typical range of the operating frequency of the mover is less than 10 kHz. In a case of conventional induction machines, the secondary current hardly flows in such high frequency region because reactance of the coils in the secondary side is quite high. In contrast, the large secondary current flows in our proposed electric machine when the slip frequency $\omega_2$ is close to the resonant frequency. This point is quite different from the conventional induction machines. Therefore, the coils and the capacitors of the proposed machine should be designed so that the resonant frequency is quite higher than the operating frequency of the mover and the Q factor covers variation of the slip frequency due to the change of the operating frequency of the mover.

### III. Resonant Frequency

In this section, the resonant frequencies are derived considering the effect of the speed of the secondary side mover. For simplicity, we assume that $R_1 = R_2 = 0$ in this section. The condition that the impedance of the whole circuit becomes zero can be derived from the condition $A = 0$ using (29) in this case, which is as follows.

$$\omega_1 \omega_2 M_0^2 = \left(\frac{1}{\omega_1 C_1}\right) \left(\frac{1}{\omega_2 C_2}\right) = 0$$ (35)

This condition is equivalent to $D = 0$ in (34) when $R_1 = R_2 = 0$. We can see that this condition is independent from the electric angle $\theta$ of the mover. It means that the resonant frequency does not affected by the displacement of the mover. However, the resonant frequency depends on the slip frequency $\omega_0 = (\omega_1 - \omega)$ which also depends on $\omega$, the frequency of the mover.

If we assume that the inductances and the capacitance of the primary coils and the secondary coils are same, we can solve (35) with respect to $\omega_1$ under the conditions of $L_1 = L_2 = L$, $C_1 = C_2 = C$, and $\omega_2 = \omega_1 - \omega$. As the results, we obtain four solutions in complicated forms. The Taylor series expansion of the solution with respect to $\omega$ around $\omega = 0$ up to 1st order is obtained as follows.

$$\omega_{1res} = \left\{ - \frac{1}{\sqrt{(L - M_0)C}} + \omega_0, \frac{1}{\sqrt{(L - M_0)C}} - \frac{1}{\sqrt{(L + M_0)C}} + \omega_0 \right\}.$$ (36)

They represent good approximations of the resonant frequencies. From these solutions, we confirm that there are four resonant frequencies in the proposed electric machine; two are in negative frequency domain and the other two are in positive frequency domain. All resonant frequencies are shifted by a half of the frequency of the mover.
IV. FEA Simulations

In this section, we confirm the proposed model by Finite Element Analysis (FEA). Parameters used in these simulations are shown in Table I. Fig. 3–Fig. 5 show numerical results calculated by the proposed model and simulation results of frequency responses calculated by using a FEA simulation software JMAG-Designer 14.0. Especially Fig. 3 shows numerical results given by (33) and simulation results of generated force along with the moving direction of the mover in various gap length. A product of this force and the speed of the mover corresponds to the mechanical power of the machine.

We can confirm that the simulation results precisely agree with our proposed model (33). We can also observe two peak of the force at resonant frequencies. As gap length between the primary coils and the secondary coils increases, difference between two resonant frequencies becomes small. But magnitude of the force is almost constant until $l_y \leq 25$ [mm] and starts to decrease when $l_y > 25$ [mm]. This tendency will be strongly related to the scattering parameters. Fig. 4 shows simulation results of generated force perpendicular to the moving direction of the mover in various gap length. Repulsive force is generated at the lower resonant frequency...
Fig. 5. FEA simulation results of flux density under the condition of \( l_y = 25 \text{[mm]} \), \( \omega_1 = 1.45 \text{[MHz]} \).

![Image](image-url)

while attractive force is generated at the higher resonant frequency. Fig. 5 shows contour plots of the flux density under the condition of the gap length \( l_y = 25 \text{[mm]} \) and the frequency of the voltage source \( \omega_1 = 1.45 \text{[MHz]} \).

![Table and Figures](table-and-images-url)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch length of coils</td>
<td>( l_z = 40 \text{[mm]} )</td>
<td></td>
</tr>
<tr>
<td>Gap length</td>
<td>( 10 \leq l_y \leq 30 \text{[mm]} )</td>
<td></td>
</tr>
<tr>
<td>Depth of stator and mover</td>
<td>( l_s = 50 \text{[mm]} )</td>
<td></td>
</tr>
<tr>
<td>Amplitude of voltage source</td>
<td>( V_1 = 5 \text{[V]} )</td>
<td></td>
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<tr>
<td>Resistance</td>
<td>( R_1 = R_2 = 5.3 \text{[m\Omega]} )</td>
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<tr>
<td>Capacitance</td>
<td>( C_1 = C_2 = 0.22 \text{[\mu F]} )</td>
<td></td>
</tr>
<tr>
<td>The number of turn of coils</td>
<td>( N_1 = N_2 = 1 )</td>
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</tbody>
</table>

V. CONCLUSION

In this paper, we proposed an electro-mechanical energy conversion system using the resonant inductive coupling and examined the basic mathematical model of the system. This model unifies the wireless power transmission system and the electro-mechanical energy conversion system using the resonant inductive coupling. The proposed system is similar to the induction machine but it is driven at very high resonant frequency and the slip ratio is also quite high (near 100%). The simulation results of the generated force by the FEA accurately agreed with the numerical results by the proposed model.

In the future, more detailed verification of the proposed model by simulations and experiments will be conducted. Improvement of the generated force is also another important subject. The proposed model will be applicable for various industrial applications such as cost-effective, light-weight large-scale generators since the machine does not require precise gap design.

VI. APPENDIX

The self-inductance and the mutual inductance are derived as follows. Consider a single straight wire, the number of turns of which is \( n \), carrying current \( I \) at the origin \( O \) shown in Fig. 7. It generates magnetic field according to the Ampere’s law. The normal component of the magnetic field is described as

\[
H_0 = \frac{x}{2\pi(x^2 + y^2)^{3/2}} nI. \tag{37}
\]

The magnetic field at the position \((x, y)\) generated by the infinite number of wires located at the one-dimensional lattice as shown in Fig. 7 is calculated as follows.

\[
H(x, y) = \sum_{k=-\infty}^{\infty} \left( H_0(x - l_x k, y) - H_0(x - l_x (2k - 1), y) \right)
= \frac{2\cosh \frac{2\pi y}{l_x}}{l_x (\cos \frac{4\pi x}{l_x} - \cosh \frac{4\pi y}{l_x})} nI. \tag{38}
\]

Therefore, the self-flux linkage is computed as

\[
\Phi_L = \mu_0 l_z n \int_{-a}^{l_z/2-a} H(x, 0) dx
= -\frac{\mu_0 l_z n^2}{\pi} \log \tan \frac{\pi a}{l_z}. \tag{39}
\]

where \( \mu_0 \) is the permeability of free space, \( a \) is the radius of area of the wires, and \( l_z \) is the depth of the wires, respectively. Hence, the self-inductance is given by

\[
L = -\frac{\mu_0 l_z n^2}{\pi} \log \tan \frac{\pi a}{l_z}. \tag{40}
\]

Fig. 6 shows a gap length dependency of generated forces in the tangent direction (a) and in the normal direction (b). We can see that the normal force decreases as the gap length increases.
The interlinkage flux on the coil whose position is \((x, l_y)\) is calculated as follows.

\[
\Phi_M = \mu_0 l_z n \int_{x}^{x+l_z/2} H(x, l_y) \, dx
\]

\[
= \mu_0 l_z n^2 I \frac{\log \left( \frac{\cosh \frac{2\pi l_x}{l_z} + \cos \frac{2\pi x}{l_x}}{\cosh \frac{2\pi l_x}{l_z} - \cos \frac{2\pi x}{l_x}} \right)}{2\pi}. \tag{41}
\]

Therefore the mutual inductance between two coils is given by

\[
M = \frac{\mu_0 l_z n^2}{2\pi} \log \frac{\cosh \frac{2\pi l_x}{l_z} + \cos \frac{2\pi x}{l_x}}{\cosh \frac{2\pi l_x}{l_z} - \cos \frac{2\pi x}{l_x}}. \tag{42}
\]

This function can be approximated by the cosine function. Its fundamental component is given by

\[
M_0 = \frac{2}{l_x} \int_0^{l_x} M \cos \left( \frac{2\pi x}{l_x} \right) \, dx
\]

\[
= \frac{2\mu_0 l_z n^2}{\pi} \exp \left( -\frac{2\pi l_y}{l_x} \right). \tag{43}
\]

REFERENCES


