On the Pricing Strategies with Asymmetric Information

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1. Introduction

Pricing decision is one of the most important decision area in business, which requires further works from both theoretical and empirical points of view. The comprehensive reviews of existing pricing research in both economics and marketing science have been provided by [1] and [2].

In order to theoretically abstract pricing problems from management science perspective, it is convenient to regard pricing decision faced by management as a resource allocation through price and grasp it in a hierarchical framework of principal-agent setting. Therefore a price is treated as active control variable of principal (i.e., the principal is in monoplistic situation with regard to pricing decision) and delegated agent's choice behavior is incorporated in setting price. This situation arises in various contexts of economic environments, e.g., relationships between a center and production units in a firm (transfer pricing problems), relationships between a producer and retailers, and relationships between a retailer and consumers.

Little and Shapiro [3] developed a model of pricing in marketing science based on the above notion. They present a theory of pricing goods using accepted principles of economic behavior, where maximizing behaviors on the parts of consumer and store are presumed, i.e., the customers purchase goods to maximize utility and the stores maximize profit.

Their model is quite comprehensive in dealing with pricing strategies considering principal-agent interaction but the model, however, is lacking in a viewpoint of uncertainty which is managerially important.

Price can be regarded as a control variable for which trade is performed between two parties. In addition, whenever trade takes place, both parties perceive that there exists some uncertainty with regard to the state of nature. Then it will commonly arise that one party to the trade has more relevant information about uncertainly than does the other. Recognizing this situation, we have to attempt to model the pricing strategies. In this paper we consider the model setting that takes into account of factors of asymmetric information, assuming expected return (benefit minus expense) maximizing behavior on the part of agent and expected profit maximization on the part of principal like Little and Shapiro [3] [4].

An central aim in this paper is to give an analytical approach for optimal pricing strategy which optimally exploits the agent's cooperation with superior information. Furthermore, we will try to show that under uncertainty, the
optimal pricing taking advantage of information gap is more beneficial for both principal and agent than the optimal pricing not utilizing information gap (the absence of private information) between the relationship.

Finally we discuss the case where the information is jointly observed (public information). It will be shown that there exists a pricing decision taking advantage of public information, which is more beneficial than the optimal pricing with information gap.

These results are also concerned with the role of information on modeling situation with uncertainty and hence it would give an incentive for agent or principal to install information systems to obtain relevant information.

2. Definitions and Assumptions

The following notions will be used in this paper.

$x \equiv$ quantity of the goods (resources) bought by the agent
$p \equiv$ price of the goods
$c \equiv$ marginal cost of the goods
$z \equiv$ random variable with a sample space $Z$ (the state of nature)

The agent and the principal will have the following type of preference functions:

$$F^4(x, p, z) \equiv u(x) + zx - w(px), \quad x \in R^+ \equiv \{x | x \geq 0\}$$

$$F^3(x, p) \equiv (p - c)x, \quad p > c > 0$$

We are presuming that the agent intends to maximize an expected value of his return $F^4$ and accordingly an amount $x$ can be purchased at a price $p$ for a benefit $u(x) + zx$ depending on the state of nature $z$. Furthermore, $F^4$ implies that the effect of uncertainty to his preference will be large when the quantity bought by him will increase. On the other hand, the principal's function is a usual profit maximization. Here we make the assumptions as follows.

(a) $u(x) : R^+ \rightarrow R^1$, a function of class $C^3$, $u(0) \geq 0$.

(b) $w(e) : R^+ \rightarrow R^1$, a function of class $C^3$.

(c) $u'(x) > 0$, $u''(x) < 0$ and $w'(e) \geq 0$, $w''(e) \geq 0$ for any $x, e \in R^+$, where $u'(x) = du(x)/dx$ and $u''(x) = d^2u(x)/dx^2$.

(d) $Z = [0, 2\mu]$ and $z \in Z$ is a random variable with a symmetric density function $g(z)$, and the principal and the agent agree on the structure of the probability space.

3. Statement of the Problem

We consider the two types of optimal pricing strategies for the principal. One is a pricing which utilizes an agent having superior information (the agent
A*) and the other is a pricing which utilize an agent having no superior information (the agent A0). The agent A* can privately observe a signal z ∈ Z which is not revealed to the principal. On the other hand, the agent A0 can not observe it, i.e., there is no information gap between the agent and the principal.

The agent A* and A0's behavior (response functions), given a price p, are described as follows:

\[ \text{A*}'s \text{ problem.} \quad \text{Find } x = \hat{x}(p, z) \text{ such that it maximizes } F^A(x, p, z) \text{ for any } z. \]

\[ \text{A0}'s \text{ problem.} \quad \text{Find } x = \hat{t}(p) \text{ such that it maximizes } \int F^A(x, p, z) = F^A(x, p, \mu), \text{ where } E[. .] \text{ means the expectation.} \]

We can replace each problem by the first order conditions, respectively.

\[ \frac{\partial F^A}{\partial x} = u'(x) + z - w'(px) \cdot p = 0 \quad (1) \]

\[ \frac{\partial E[F^A]}{\partial x} = \int \{u'(x) + z - w'(px) \cdot p\} g(z) dz = u'(x) + \mu - w'(px) \cdot p = 0 \quad (2) \]

By (a), the following inequality is obtained:

\[ \frac{\partial^2 F^A}{\partial x^2} = \frac{\partial^2 E[F^A]}{\partial x^2} = u''(x) - w''(px)p^2 < 0. \quad (3) \]

Hence, there exist the solution \( \hat{t}(p, z) \geq 0 \) and \( \hat{t}(p) = \hat{t}(p, \mu) \geq 0 \) for any \( z \in Z \) and \( p \) which satisfy the equation (1) and (2) respectively, if and only if \( \frac{\partial^2 F^A}{\partial x^2} \) is nonnegative at \( x = 0 \) and negative at \( x = \infty \) for any \( z \) and \( p \), i.e., \( u'(0) + z - w'(0)p \geq 0 \) and \( u'(\infty) + z - w'(\infty)p < 0 \) for any \( z \). We will denote that region of \( p \) by \( I \). It is clear that

\[ I \neq \phi \text{ if } u'(0)/w'(0) > c \text{ and } u'(0)/w'(0) > u'(\infty)/w'(\infty) + 2\mu \]

Now let us formulate the principal's problem incorpolated with the agent A* and A0.

We define:

\[ \hat{t}^p(p) \equiv E[(p - c)\hat{t}(p, z)] \quad (4) \]

\[ \hat{t}^p(p) \equiv (p - c)\hat{t}(p) = (p - c)\hat{t}(p, \mu) \quad (5) \]

\[ P^* = \arg\max_{p \in I} \hat{t}^p(p), \quad p^0 = \arg\max_{p \in I} \hat{t}^p(p). \]

An main problem in this paper is to question whether \( P^* \) is Pareto superior to \( p^0 \), i.e., whether both principal and agent are better off.
4. The Principal’s Profit

We first introduce the following lemma of which proof is in Appendix.

**Lemma 1.** Let the assumptions \((a_1)\sim(a_4)\) and the following \((a_5)\) be satisfied.

\((a_5)\) \(u'(\cdot)\) is is strictly convex and \(w'(\cdot)\) is concave. Then for any \(p \in I\) \(\hat{r}(p, z)\) is strictly convex in \(z\).

By using the above lemma, it can be shown that \(p^*\) is more profitable than \(p^0\).

**Theorem 1.** Assume \((a_1)\sim(a_5)\). Then the principal is better off with \(p^*\) than with \(p^0\), i.e.,

\[ \hat{r}^p(p^*) > \hat{r}^p(p^0). \]

**Proof.** Since \(g(z)\) is symmetric, \(g(z)=g(2\mu-z)\).

Hence, we have:

\[ \hat{r}^p(p) = \int_0^\mu (p-c)\hat{r}(p, z)g(z)dz + \int_\mu^\infty (p-c)\hat{r}(p, z)g(z)dz. \]

From \(\hat{r}(p, z)+\hat{r}(p, 2\mu-z))/2 > \hat{r}(p, \mu)\) by Lemma 1, it follows that

\[ \hat{r}^p(p) > 2\int_0^\mu (p-c)\hat{r}(p, \mu)g(z)dz = (p-c)\hat{r}(p, \mu) = \hat{r}^p(p) \]

for any \(p \in I\). Thus we have the stated result.

**Remark.** There would exist \(P^*\) and \(P^0\) on \(I\), because \(\hat{r}^p=\hat{r}^p=0\) at \(p=c\) and if \(p=\{u'(0)+z\}/w'(0), \hat{r}(p, z)=0\), and hence it may be expected that \(\hat{r}^p\) attains a maximum on \(c<p<\{u'(0)+z\}/w'(0)\).

5. The Agent’s Benefit

We define as follows:

\[ \hat{f}^p(p) = E[F^A(\hat{r}(p, z), p, z)] \] (6)

\[ \hat{f}^p(p) = F^A(\hat{r}(p), p, \mu)=\hat{f}^p(\hat{r}(p), p). \] (7)

It is not clear whether or not \(A^*\) is also better off with \(P^*\) than \(A^0\) with \(P^0\) on the previous assumptions, i.e., \(\hat{f}^p(p^*) > \hat{f}^p(p^0)\). Now let us guarantee this benefit by imposing some restrictions on a class of functions.

\((a_6)\) \(w(e)=e\) and \(-u'(x)/u''(x)=ax+b\).
It is clear that \((a_0)\) implies \((a_9)\) for a \(>-1\) and hence Lemma 1 is true with \((a_9)\) for a \(>-1\). The function \(u(x)\) satisfying \((a_9)\) is usually called hyperbolic absolute risk aversion utility functions. In the case that \(w(e)=e\), we have
\[
\begin{align*}
    u'(x) &= p - z \\
    u(x) &= p - \mu 
\end{align*}
\]
for (1) and (2), and hence \(\hat{p}\) is written as
\[
\hat{p}(p, z) = \gamma(p-z), \ p \in I 
\]
(8)

We prepare the several properties before discussions. From the definition \(\gamma\), it follows that
\[
\begin{align*}
    u'(\gamma(p-z)) &= p - z. 
\end{align*}
\]
(9)

By differentiating (9) with respect to the variable \(z\), we have
\[
\begin{align*}
    -u''(\gamma(p-z))\gamma'(p-z) &= -1. 
\end{align*}
\]
(10)

\((a_9)\) implies
\[
\begin{align*}
    -u'(\gamma(p-z)) &= a\gamma(p-z)u''(\gamma(p-z)) + bu''(\gamma(p-z)). 
\end{align*}
\]
(11)

Substituting (9) and (10) into (11) gives
\[
\begin{align*}
    (p-z)\gamma'(p-z) &= a\gamma(p-z) + b 
\end{align*}
\]
(12)

Let us write down the first order condition which \(\hat{p}\) and \(\hat{p}^0\) has to satisfy.
\[
\begin{align*}
    \frac{\partial \hat{p}(p)}{\partial p} &= E[(p-c)\gamma'(p-z) + \gamma(p-z)] = 0 
\end{align*}
\]
(13)
\[
\begin{align*}
    \frac{\partial \hat{p}(p)}{\partial p} &= (p-c)\gamma'(p-\mu) + \gamma(p-\mu) = 0 
\end{align*}
\]
(14)

Now we offer the following lemma.

**Lemma 2.** Assume \((a_1)-(a_9)\) and let \(\hat{p}, \hat{p}^0\) be unimodal. If \(\alpha\) is numbers which satisfy \(a > -2 + \frac{\hat{p}^*}{\hat{p}^* - c} = -1 + \frac{c}{\hat{p}^* - c}\) or \(a > -1 + \frac{c}{\hat{p}^* - c}\), then we have \(\hat{p}^* < \hat{p}^0\).

**Proof.** See Appendix.

**Corollary.** Assume \(c > 2\mu\). Since \(p \in I\) implies \(p \geq 2\mu\), the condition of lemma is replaced by \(a > -1 + \frac{c}{2\mu - c}\).

**Theorem 2.** Let the assumptions in Lemma 2 be satisfied. Then the agent \(A^*\) is no worse off with \(\hat{p}\) than the agent \(A^0\) with \(\hat{p}^0\), i.e., \(f^4(\hat{p}^*) \geq f^4(\hat{p}^0)\).

**Proof.** Since \(\gamma(p-z)\) maximize \(E[F^A]\) pointwise, we can get
\[ \hat{I}^*(p^*) = E[F^*(\gamma(p^* - z), p^*, z)] \geq F^*(\gamma(p^* - \mu), p^*, \mu) \]  

(15)

On the other hand, by (7), we have

\[ \frac{\partial \hat{I}^*(p)}{\partial p} = \frac{\partial F^*(\gamma(p - \mu), p, \mu)}{\partial p} \]

\[ = u'(\gamma(p - \mu))\gamma'(p - \mu) + \mu \gamma'(p - \mu) \]

\[ - \hat{\gamma}'(p - \mu) - \gamma(p - \mu) \]

Since \( \gamma(p - \mu) \) satisfies (2), we have

\[ \frac{\partial \hat{I}^*(p)}{\partial p} = -\gamma(p - \mu) \leq 0. \]  

(16)

Since \( F^*(\gamma(p - \mu), p, \mu) \) is decreasing with respect to \( p \) by (16) and \( p^* < p^0 \) by Lemma 2, it can be obtained that

\[ F^*(\gamma(p^* - \mu), p^*, \mu) \geq F^*(\gamma(p^0 - \mu), p^0, \mu) = \hat{I}^*(p^0). \]  

(17)

Thus (15) and (17) give the stated result.

Remark. It should be easy to see that \( \bar{p}^* = \min\{p^*, p^0\} \leq p^0 \) is also Pareto superior to \( p^0 \) without assuming \( (a) \) (i.e., \( \hat{I}^*(p^*) > \hat{I}^*(p^0) \) and \( \hat{I}^*(\bar{p}^*) \geq \hat{I}^*(p^0) \)), because \( \hat{I}^*(p) > \hat{I}^*(\bar{p}^*) \) for any \( p \), and (15) (16) and (17) are also satisfied with \( (a) \sim (a) \). This means that \( \bar{p}^* \) is a second best solution for the principal. Moreover, by Lemma 1, we can easily obtain the following inequality.

\[ \frac{\partial \hat{I}^*(p)}{\partial p} < \frac{\hat{I}^*(p)}{\partial p} \leq 0 \]

This fact means \( A^* \) is more price-sensitive than \( A^0 \), which would be intuitively clear.

This theorem can be related to value of information. Since \( p^* \) is Pareto superior to \( p^0 \), the comparative advantage of \( A^* \) over \( A^0 \) would give an amount to pay to obtain the information.

Example. We consider the following example which satisfies the assumptions in Lemma 2 (Theorem 2).

\[ F^*_A = 2\sqrt{x} + zx - px \]

\[ F^*_p = (p - 3)x \]

where \( u(x) = 2\sqrt{x}, c = 3, \) and \( z \) has a uniform distribution on the interval \([0, 2]\).

From \( u'(x) = \frac{1}{\sqrt{x}} = p - z \), we have
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\[ x = r(p - z) = (p - z)^2, \]

\[ \ddot{f}^p(p) = E[(p - 3)(p - z)^2] = \frac{2(p - 3)}{p(p - 2)}, \]

\[ \ddot{f}^p(p) = (p - 3)(p - 1)^2. \]

Thus by simple calculations, it follows that

\[ p^* = 3 + \sqrt{3}, \quad p^0 = 5 \]

\[ \ddot{f}^p(p^*) = 2 - \sqrt{3} \approx 0.27, \quad \ddot{f}^p(p^0) = 0.125 \]

\[ \ddot{f}^4(p^*) = -\log (p^* - z) \bigg|_0^2 = -\log \sqrt{3} = 0.347, \quad \ddot{f}^4(p^0) = 0.25. \]

Moreover we can check by \(-u'(x)/u''(x) = 2x,\)

\[ a = 2 > -1 + \frac{3}{p^* - 3} = -1 + \sqrt{3} \]

6. Public Information

Here we discuss the case where the information is jointly observed for both principal and agent.

We put:

\[ F^p(p) = (p - c) \hat{f}(p, z) \quad (18) \]

Now let us model the pricing decision utilizing the public information.

\[ P^{**}(z) = \arg\max_{p \in I} F^p(p) \quad \text{for any } z \quad (19) \]

\[ \ddot{f}^p(p^{**}) = E[(p^{**}(z) - c) \hat{f}(p^{**}(z), z)] \quad (20) \]

\[ \ddot{f}^4(p^{**}) = E[F^4(\hat{f}(p^{**}(z), z), p^{**}(z), z)] \quad (21) \]

The following theorem shows that there exists the incentives for both parties to share the private information.

**Theorem 3.** Assume \((a_3) \sim (a_5)\) (without \((a_4)\)). Then there exists a pricing decision \(\bar{p}(z)\) such that both parties are no worse off with \(\bar{p}(z)\) with the agent \(A^*\) than \(p^*\) with the agent \(A^*\).

**Proof.** For a fixed \(z\), we have:

\[
\frac{\partial F^4(\hat{f}(p, z), p, z)}{\partial p} = u'(\hat{f}(p, z)) \cdot \frac{\partial \hat{f}(p, z)}{\partial p} + z \cdot \frac{\partial \hat{f}(p, z)}{\partial p}
- w'(\hat{f}(p, z)) \left\{ \hat{f}(p, z) + p \cdot \frac{\partial \hat{f}(p, z)}{\partial p} \right\}
= -w'(\hat{f}(p, z)) \hat{f}(p, z) \leq 0
\]

(22)
Define $\tilde{p}(z)$ as follows:

$$\tilde{p}(z) = \begin{cases} p^*: z \in X \\ p^{**}(z): z \in Z - X \end{cases}$$

(23)

where

$$X \equiv \{ z \in Z \mid p^{**}(z) \geq p^* \}$$

By the property of $\tilde{p}(z)$,

$$\tilde{f}^p(\tilde{p}) = E[(\tilde{p}(z) - c)\hat{r}(\tilde{p}(z), z)]$$

$$= \int_X (p^* - c)\hat{r}(p^*, z)g(z)dz + \int_{Z - X} (p^{**}(z) - c)\hat{r}(p^{**}(z), z)g(z)dz.$$

Since $p^{**}(z)$ is a solution of point maximization, it can be obtained that:

$$\tilde{f}^p(\tilde{p}) \geq \int_X (p^* - c)\hat{r}(p^*, z)g(z)dz + \int_{Z - X} (p^* - c)\hat{r}(p^*, z)g(z)dz = \tilde{f}^p(p^*)$$

On the other hand, by (22) and (23),

$$F^A(\hat{r}(\tilde{p}(z), z), \tilde{p}(z), z) \geq F^A(\hat{r}(p^*, z), p^*, z) \text{ for any } z.$$

Hence we can get

$$\tilde{f}^A(\tilde{p}) \geq \tilde{f}^A(p^*).$$

Obviously, if the measure of $Z \cdot X$ is positive, both parties are better off.

7. Conclusion

In this paper we have given one approach for optimal pricing decisions encountered in a trade performed between two parties (principal-agent) under uncertainty. In this situation, one party having superior information trades with the other party by purchasing the goods for a benefit at a given price.

It has been shown that the optimal pricing taking advantage of agent's superior information about uncertainty is more beneficial, that is to say, both parties are better off. Furthermore, it would give a reason for the agent to install the information system to get finer information.

Finally it has been shown that there exists a pricing decision utilizing public information which is more beneficial than the optimal pricing with information gap. This result means that there exists the incentive to share the information between two parties.

An extension of the statements obtained here to a multiagent setting will be obvious in the case that pricing separates so that each agent affects only his own choice decision. However, an extension to the case with various items of the goods would not be so easy.
Appendix

Proof of Lemma 1. We put
\[ u'(\hat{r}(p, z)) - w'(p\hat{r}(p, z))p = -z \]
\[ u'(\hat{r}(p, z)) - w'(p\hat{r}(p, z))p = -z \]

Since \( u'(. - w'(. \text{ is strictly convex by the assumptions, the following relation can be satisfied for } 0<\alpha<1. \]
\[ u'(\alpha \hat{r}(p, z_i) + (1 - \alpha) \hat{r}(p, z_i)) - w'(p(\alpha \hat{r}(p, z_i) + (1 - \alpha) \hat{r}(p, z_i)))p \]
\[ <\alpha[u'(\hat{r}(p, z_i)) - w'(p\hat{r}(p, z_i))p] + (1 - \alpha)[u'(\hat{r}(p, z_i)) - w'(p\hat{r}(p, z_i))p] \]
\[ = -(\alpha z_i + (1 - \alpha)z_i) \]
\[ = u'(\hat{r}(p, \alpha z_i + (1 - \alpha)z_i)) - w'(p\hat{r}(p, \alpha z_i + (1 - \alpha)z_i))p \]

Since \( u'(. - w'(. \text{ is monotone decreasing by } (a_3), \text{ the above relation implies} \]
\[ \alpha \hat{r}(p, z_i) + (1 - \alpha) \hat{r}(p, z_i) > \hat{r}(p, \alpha z_i + (1 - \alpha)z_i). \]

Proof of Lemma 2. We put
\[ Q = (p - c)\gamma'(p - z) + \gamma(p - z). \]

It should be easy to see \( P^*, P^0 \neq c \). Therefore it will suffice to prove the lemma that \( Q \text{ is strictly concave with respect to } z \text{ at } P^* \text{ or } P^0, \text{ because the similar technique to that in Theorem 1 will produce under the concavity} \]
\[ \frac{\partial^2(P^*)}{\partial p} < \frac{\partial^2(p^*)}{\partial p} \text{ or } \frac{\partial^2(p^0)}{\partial p} < \frac{\partial^2(p^0)}{\partial p} \]
and consequently from unimodality we can conclude \( P^* < P^0. \)

Now let us show that \( Q \text{ is concave at } P^* \). By differentiating (12) twice with respect to \( z \text{ at } P^*, \text{ we have} \]
\[ \gamma''(p^* - z)(p^* - z) = -(a + 2)\gamma''(p^* - z). \]

On the other hand, we have
\[ \frac{\partial^2 Q}{\partial z^2} = (p^* - c)\gamma''(p^* - z) + \gamma''(p^* - z). \]

and hence
\[ \frac{\partial^2 Q}{\partial z^2} = -\frac{(p^* - c)(a + 2) + p^* - z}{p^* - z} \gamma''(p^* - z) \]

Since \( \gamma''(p^* - z) > 0 \text{ by Lemma } 1, p^* - z > 0 \text{ by } p^* \in I \text{ and } z \in [0, 2\mu] \), we obtain \( \frac{\partial^2 Q}{\partial z^2} < 0 \text{ whenever } a + 2 > p^*/(p^* - c) \text{ and finally } Q \text{ become concave. It is clear that the same inequality can be obtained at } p^0. \)
References


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